# Khufu's Coffer 

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#### Abstract

An analysis of Khufu's coffer dimensions and location, based on Petrie's measurements. The analysis suggests that it can not be $4^{\text {th }}$ Dynasty, due to the formulas involved.


Keywords: Egyptology, Giza, geometry, archaeogeometry, $\pi$, pi, $\varphi$, golden ratio, plastic ratio $\rho$.
Best viewed and printed in colour.

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## Revision history:

1.0.0 19 September 2021 Initial version

## 1. Introduction

"The language of Giza is mathematics."

> Robert Bauval
"You will believe."
The architects of Giza
This short document is an analysis of Khufu's coffer, as measured by Petrie. [1]
I did search (Google, Google Scholar, ResearchGate, Zenodo, Academia, SCIRP) to see if these results have been published before but could not find anything.

I have seen claims along the lines of "the outer volume is exactly twice the inner volume" but the numbers do not support "exact" here. Only close. There are other relationships that are better.

This document will be expanded if anything new surfaces. It is also likely to be incorporated into the upcoming (as of September 2021) Zep Tepi Mathematics 201, the sequel to Zep Tepi Mathematics 101 [2] (henceforth ZTM101).

## A note on style

I don't like the usual phrases "The current author" or "The present author." I will refer to myself in the first person, or frequently as "we," not because I am schizophrenic but I've been using that term since childhood, and it's even more relevant now. While investigating Giza, I have had constant help from sources unknown, and they deserve due credit. Tesla experienced the same phenomenon, and could not explain the source either.
"My brain is only a receiver. In the universe there is a core from which we obtain knowledge, strength, inspiration. I have not penetrated into the secrets of this core, but I know that it exists."

Attributed to Nikola Tesla
The guides are my shepherd; I shall not wonder.
They make me ponder plans, and lead me above still waters.
They restore my hope.
They lead down the paths of mathematics to admire them.
Even though I walk through the pyramids among the shadow of death, I will have no doubts:
for they are with me;
their $\pi$ and their $\varphi$
they comfort me.
And I shall dwell
in the house of Thoth
Forever.

## 2. Notation, accuracy and methodology

### 2.1 Notation

I take the royal cubit as $\pi / 6$ metres, to 4 decimal places. (The Beautiful Cubit System, Douglas 2019 [3]). While working on ZTM101 last year it became apparent they thought 3 or 4 decimals were accurate enough, so I have used that, and things "just work."

Symbols used in this and other papers:

| Symbol | Name | Approximate / practical value | \% Accuracy to true |
| :---: | :--- | :--- | ---: |
| $\pi$ | Archimedes' constant | 3.1416 | 99.9998 |
| e | Euler's number | 2.7183 | 99.9993 |
| $\varphi$ | Golden ratio | $1.618 \quad \varphi+1=\varphi^{2}=2.618$ | 99.9979 |
| $\rho$ | Plastic number / ratio | $1.3247 \rho+1=\rho^{3}=2.3247$ | 99.9986 |
| G | Royal cubit aka cubit | $0.5236 \mathrm{~m} \quad(\pi / 6)$ |  |
| $" n$ | Petrie inches | 0.025399977 m |  |

## Table 1: Symbols, names and values

I use "cubit" for Royal cubit G .
When Petrie took his measurements, his inch was 25.399977 mm [4] rather than the current 25.4 mm . To convert his inches to $\mathbb{G}$, I use a conversion factor of 20.61419189 , from

$$
\frac{0.5236}{0.025399977}=20.61419189
$$

Petrie concluded, based on his measurements of the King's Chamber, that the G was $20.620^{\prime \prime} \pm$ $0.005^{\prime \prime}$, which is 0.5236205259 to 0.5238745256 m .

Petrie took his measurements after at least one major (in 1303) and several lesser earthquakes [5], as well as the activities of the explosive Howard Vyse, [6] so it is quite possible that the walls could have shifted slightly. Everyone assumes the chamber was built $100 \%$ perfectly, which may not be true.

Comparing the 20 G length of the king's chamber for the different values of G :

| Cubit m | Length m | Difference m |
| :---: | :---: | :---: |
| 0.5236000000 | 10.47200000 | 0.000000000 |
| 0.5236205259 (-0.005") | 10.47241052 | $0.000410520 \approx 0.41 \mathrm{~mm}$ |
| 0.5237475257 (20.62") | 10.47485051 | $0.002950514 \approx 2.95 \mathrm{~mm}$ |
| 0.5238745256 (+0.005") | 10.47749051 | $0.005490512 \approx 5.49 \mathrm{~mm}$ |

Table 2: King's chamber length for different cubit values

### 2.2 Accuracy

How accurate must things be? We have no idea what tools or technologies the builders had, what they considered "accurate" or "good enough," nor exactly how earthquakes or gunpowder and sledge-hammers have affected the relative positions over time. We can not assume that their standards were the same as ours. There is no such thing as perfect accuracy in building construction.

Note that "close" in context of this discussion refers to practical measurements on a building project using unknown instruments, not something on the scale of modern micro-electronics.

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## 3. Dimensions

Petrie gives the coffer dimensions as:

|  |  <br> Outer length | 89.62 |
| :--- | :--- | :--- |
| Outrie inches " |  |  |
| Outer height | 38.50 |  |
| Inner length | 41.31 |  |
| Inner width | 78.09 |  |
| Inner depth | 26.81 |  |
| In.42 |  |  |

## Table 3: Petrie's measurements for the coffer

Let us first deal with the existing claim regarding the ratio of outer volume to inner volume.

$$
\frac{\text { outer volume }}{\text { inner volume }}=\frac{89.62 \times 38.50 \times 41.31}{78.09 \times 26.81 \times 34.42}=\frac{142534.7847}{72061.46762}=1.977961169
$$

which is close to 2.000 but could be better. The accuracy to 2.000 is only $98.898 \%$.
With that out of the way, we can begin.

1. The width of 38.50 " converts to 1.867645368 G. I suggest that the design intent was 1.87 ©. The difference is 1.23 mm .

What is special about 1.87? Firstly, it is the length of a digit in cm . [3] $28 \times 1.87 \mathrm{~cm}=52.36 \mathrm{~cm}$. Secondly, and more importantly, it is the rounded value of $\pi \varphi / \mathrm{e}$.

$$
\begin{aligned}
& \frac{\pi \varphi}{e}=\frac{3.14159265 \ldots \times 1.6180339887 \ldots}{2.718281828459 \ldots}=1.870006134 \ldots \\
& \text { or } \\
& \qquad \frac{3.1416 \times 1.618}{2.7183}=1.869958724 \ldots
\end{aligned}
$$

So we can say that the width is $(\pi \varphi / e)$. .
Trying to convert the width to palms and fingers does not work:

$$
\begin{gathered}
38.5 "=1.867645368 G=0.9778991145 \mathrm{~m} \\
\frac{97.78991145 \mathrm{~cm}}{1.87}=52.29407029 \text { digits }
\end{gathered}
$$

which leaves you with 0.294 digits ( 5.423 mm ) after assigning the 52 digits to cubits, palms, etc. So we clearly need to work in decimal cubits.
2. The length of $89.62^{\prime \prime}$ converts to $4.347490334 €$. I would take the intent to be $1.87 \times \rho^{3}$.

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$1.87 \times 2.3247=4.347189 \quad$ (accuracy to above is $99.993 \%$ )
Which can be written as

$$
\frac{\pi \varphi \rho^{3}}{e}
$$

A formula beautiful enough to rival Euler's identity. This is the key formula for the coffer, used for a length here, and later for area and volume.
3. The area of the base is then

$$
\text { Area }=\text { width } \times \text { length }=1.87 \times 1.87 \rho^{3}=1.87^{2} \rho^{3}=8.12924343
$$

or

$$
\left(\frac{\pi \varphi}{e}\right)^{2} \rho^{3}
$$

Convert that to $\mathrm{m}^{2}$

$$
8.12924343 \times 0.5236^{2}=2.2286888666
$$

which is close to

$$
\frac{\pi}{\sqrt{2}}=\frac{3.1416}{1.4142}=2.221467968 \quad(99.676 \% \text { accurate. })
$$

4. Petrie gives the height as $41.31^{\prime \prime}$, which converts to 2.003959224 , so the target height was clearly 2 6.
5. The outer volume is then

Volume $=$ width $\times$ length $\times$ height $=1.87 \times 1.87 \rho^{3} \times 2=1.87^{2} \rho^{3} \times 2=16.25848686 \mathscr{\epsilon}^{3}$
or
$\left(\frac{\pi \varphi}{e}\right)^{2} 2 \rho^{3}$, or numerically twice the area of the base. The accuracy to $10 \varphi$ is $99.517 \%$.
That converts to $2.333882771 \mathrm{~m}^{3}$, or effectively $21 / 3 \mathrm{~m}^{3}$.
6. The area of the inside length is

Area $=$ length $\times$ depth $=78.06 \times 34.42=2686.8252$, which converts to $6.322761263 \mathbb{q}^{2}$, which converts to $1.733429007 \mathrm{~m}^{2}$.

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This is effectively $\sqrt{3} \mathrm{~m}^{2} .1 .733429007^{2}$ is 3.004776121 , so $99.9176 \%$ accurate.
7. The inside width is $26.81^{\prime \prime}$, which converts to 1.30056032 G.

$$
\frac{\sqrt{\pi^{2}+\varphi^{2}}}{e} \approx 1.3
$$

8. The interior volume is

Volume $=$ length $\times$ width $\times$ depth $=78.06 \times 26.81 \times 34.42=72033.78361$, which converts to 8.22313241 G $^{3}$
8.22 is $\pi \varphi^{2}$, rounded. The values 822 and 411 are used in the Giza site plan.[2]
9. The sum of the thickness of the four sides is
$5.67+5.87+5.89+5.82=23.25$ " , which converts to 1.127863761 G .
$\sqrt[4]{1.618}=1.127832563$, accuracy is $99.997 \%$.
10. We do not have the coffer lid, but the design implies that it existed. Given the current dimensions, I would guess that the lid increased the outside height from $2 \mathbb{E}$ to 2.3247 f , i.e. $\rho^{3}$. 0.3247 f is 17 cm , which is reasonable, given the coffer height.
11. The total outer volume would then be

$$
\text { Outer volume }=\text { length } \times \text { width } \times \text { height }=1.87 \times 1.87 \times \rho^{3} \times \rho^{3}=\left(1.87 \rho^{3}\right)^{2}
$$

or

$$
\left(\frac{\pi \varphi \rho^{3}}{e}\right)^{2}
$$

In other words, the volume is numerically the length squared.
12. The value also rounds to e $\mathrm{m}^{3}$ (rounded).

$$
\begin{aligned}
\left(1.87 \rho^{3}\right)^{2} & =18.8980522 \epsilon^{3} \quad(99.797 \% \text { accurate. }) \\
e m^{3}=2.7183 \div 0.5236^{3} & =18.9364459 \epsilon^{3}
\end{aligned}
$$

13. Similarly, the area of the short side with lid is $1.87 \rho^{3}$, numerically the same as the length.

$$
\left(\frac{\pi \varphi \rho^{3}}{e}\right)
$$

14. So short side $-->$ length $-->$ volume all revolve around $\pi \varphi \rho^{3} / e$.
15. We can write the volume in terms of the irrationals:

$$
\begin{aligned}
& \text { Volume }=\text { length } \times \text { breadth } x \text { height } \\
&= \frac{\pi \varphi}{e} \rho^{3} \times \frac{\pi \varphi}{e} \times \rho^{3} \\
&=\left(\frac{\pi \varphi \rho^{3}}{e}\right)^{2} \epsilon^{3}
\end{aligned}
$$

It's rather curious the volume is also a square: length squared is volume, and area of short end with lid squared is volume.

I have a nagging suspicion that I'm still missing something here, perhaps it will surface later.

## 4. Location

We don't know the original placement of the coffer within the king's chamber.
However, while it had clearly been moved when Petrie measured it, the numbers may imply "yes, but not much."

An estimate of the mass, using a typical / average granite density of $2750 \mathrm{~kg} / \mathrm{m}^{3}$

$$
\begin{aligned}
& \text { outer volume - inner volume }=(89.62 \times 38.5 \times 41.31)-(78.06 \times 26.81 \times 34.42)=70501.00109{ }^{" 3} \\
& =8.048155 \mathrm{G}^{3} \\
& =1.1553 \mathrm{~m}^{3} \\
& =3177 \mathrm{~kg}
\end{aligned}
$$

So, not too easy to move around, even without the lid.
Petrie gives the distances from the corners (NE, NW and SW, top and base) to the north and west walls as

|  | $N E$ to $N$ wall | $N W$ to $N$ wall | $N W$ to $W$ wall | $S W$ to $S$ wall |
| :--- | :--- | :--- | :--- | :--- |
| Top | 47.70 | 48.90 | 53.34 | 56.50 |
| Base | 48.35 | 50.06 | 56.54 |  |

Table 4: Petrie's distances to the nearest walls
Dealing with the north side first, the average of the four values is $48.7525^{\prime \prime}$, which converts to 2.364996904 . I would argue, given the use of $\rho^{3}$ in the length, that the original distance, if it was set "precisely" at all, was 2.3247 E , or $\rho^{3} \mathrm{c}$. The difference is 2.1 cm .

The average distance to the west wall is $54.925^{\prime \prime}$, which converts to 2.664426541 f .

Similarly, I would argue for an original value of 2.618 G , or $\varphi^{2} \mathrm{G}$. The difference is 2.43 cm .
The current position gives a north value accuracy of $98.296 \%$, and the west value accuracy of 98.2575\%.

## 5. Conclusion

When measured using cubits or metres, Khufu's coffer reveals startling use of famous mathematical constants, which, as far as we know, the $4^{\text {th }}$ dynasty did not know. The dimensions are also ingenious in their interplay.

We must thus either accept that these dimensions are pure chance, or they are not random, and the coffer is not $4^{\text {th }}$ dynasty. We know that the dimensions of the coffer are out of sync with normal ratios for the typical dynastic sarcophagus.

## 6. Acknowledgements

Thanks as always to my "guides," whoever or whatever they are, for their constant prompting and odd ideas, which frequently lead to shocking discoveries and amazed laughter. I wish I knew them.

Thanks to the team behind the Libertinus fonts [7].

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