

Quantum Strong Measurements

Francesco R. Ruggeri Hanwell, N.B. Sept. 28, 2021

In classical physics there exists deterministic motion i.e. x and t are coupled as $x(t)$ which allows for quantities such as velocity and acceleration to also be coupled. Measurements do not affect this coupled motion. In quantum mechanics, $V(x)$ we argue is really an average of probabilistic impulse hits. Thus quantum mechanics is already a statistical theory i.e. one has resonance/equilibrium states yet one does want measurements to create new resonances. They can, however, destroy resonances. As a result, we argue there are two sets of probabilities, measurement related ones $P(x)$ and $P(p)$ and resonance creating probabilities $P(p/x)$. The resonance probabilities are based on a kind of invariance/nonlocal periodic probability scheme which must create local quantities such as $V(x)$ or $KE(x)$ (kinetic energy) through interference/superposition. The measurement scheme cannot discern the various $P(p/x)$ because otherwise it would create a new resonance, so it only sees the superposed result. This holds for both bound and free type interactions which we argue are both statistical.

Classical and Quantum Schemes

In classical physics, measurements of a deterministic system $x(t)$ do not affect the system. For example, photons bouncing off of classical objects during measurements do not affect interactions.

Quantum mechanics starts from a different view point, we argue, namely that of an average local quantity $V(x)$ the potential which is responsible for local interactions in the classical world. In quantum mechanics a set of nonlocal impulse hit probabilities combine to form $V(x)$ i.e. $V(x) = \sum_p V_k \exp(ikx)$ where k is the momentum change due to the impulse and $\exp(ikx)$ is a complex probability. One may note that $\exp(ikx)$ is periodic and nonlocal which is necessary for it to recreate a local potential. Thus interactions in a quantum system are probabilistic and governed by the $\exp(ikx)$ periodicity which has physical implications (i.e. leads to wavy type patterns). Quantum particles also have an $\exp(ipx)$ periodically invariant probability and give rise to a complex probability distribution $\sum_p a(p) \exp(ipx)$ which is called the wavefunction. We note that $\exp(ipx)$ is a dynamical probability because it is $\exp(ipx)$ for a forward moving particle and $\exp(-ipx)$ for a backwards moving one. For real probabilities there would be no such distinction. This distinction allows for probability or superposition of probability to indicate an overall direction of motion.

Quantum mechanics is constrained to produce $KE(x)$ (kinetic energy) and $V(x)$, classical quantities, on average. These are local quantities which are created through the superposition/interference of periodic (positive/negative probability) invariant (nonlocal) probabilities $\exp(ipx)$. Thus there seem to be two worlds. One is based on probability, although a strange one with complex (dynamic) periodic invariant/nonlocal probabilities which create crests and troughs and the second is based on a local probability based on interference and producing real values.

Quantum Measurements

When making quantum measurements, one deals a probabilistic system based on various types of resonances. The quantum instrument does not act like a component of $V(x)$ i.e. $V_k \exp(ikx)$. It cannot detect the dynamical complex probability, but rather sees only the magnitude e.g. $W^*(x)W(x)$ where $W(x) = \sum_p a(p) \exp(ipx)$.

The quantum measurement may break a resonance, but it is not part of the $V_k \exp(ikx)$ impulse scenario of creating resonances. Resonance breaking may occur in various different ways. One way involves a wavefunction which is the sum of various resonances as it is a statistical quantity. A measurement may establish that the system is only one of these resonances i.e. the quantum system collapses into a particular resonance. Another similar scenario is to have a quantum measurement break a resonance into a constituent “resonance” piece. For example, if one has $W(x) = \sum_p a(p) \exp(ipx)$ representing a bound state resonance with energy E_n , a measurement of momentum may break this resonance and knock out the single particle which is found to be in a momentum state of p . The particle is now in a momentum resonance in the sense that it is described by $\exp(ipx)$, but the resonance with $V(x)$ has been broken. Similarly one may have a spatial density $W^*(x)W(x)$ with a wavy pattern and have a quantum measurement measure the density by breaking systems in an ensemble at various points x and sensing the density.

One Directional Statistical Equilibrium/Resonance

A quantum bound state may be viewed as an equilibrium/resonance, but it is peculiar in that the periodic, nonlocal probabilities $\exp(ipx)$ extend to plus/minus infinite even though the particle usually has most of its probability within the classical turning points. Probability extending past the classical turning point is thought of tunnelling. It seems one must be careful with the idea of $\exp(ipx)$ extending to infinite as one does not consider a particle as having a probability to be at infinite. Rather $\exp(ipx)$ seems to be a math object, with superposition of such objects predicting what quantum measurements detect. For example, for a particle in an infinite potential well, $W(x)=0$ outside the well. Thus quantum measurements detect nothing there, yet $W(x)$ is composed of $\sum_p a(p) \exp(ipx)$ even outside the well. $a^*(p)a(p)$ does not give the probability of finding the particle in a state of momentum p outside the well because $W^*(x)W(x)=0$ outside the well. Thus superpositions/interference states are very important as they are linked to physical measurements. The form $\exp(ipx)$ extending to infinite, however, is useful in calculations.

We consider an example. Imagine one has two electron sources at two different places in three-space and that two electrons with the same momentum p move in straight lines reaching a point (x_1, y_1, z_1) at the same time. The form $\exp(ipx)$ seems to be based on momentum magnitude as one may always consider a coordinate system for each electron such that it moves along the z axis with $p_z=p$ the magnitude. This gives the sense of “physical” cycling and phase shifts if one particle travels a longer distance than the other. There are, however, many different ways to create the same phase shift (i.e. various potentials which do not change

momentum such as the magnetic vector potential). If no potentials are present, then it is sometimes said that the different lengths traveled lead to the phase difference.

We argue, however, that this statement is based on a classical type view of wave motion. A bound state is composed of $\exp(ipx)$, $\exp(-ipx)$ type terms which extend to infinite so the creation of electron (i.e. a source) in a statistical picture should be a boundary condition, we argue. If one has $W(x)=0$ behind the source and $\exp(ipx)$ in front of the source, one may find the Fourier transform of a step function times $\exp(ipx)$. In such a case, $W(x)$ is actually composed of all $\exp(ikx)$ weights everywhere which combine to give $W(x)=0$ behind the source and $\exp(ipx)$ in front. This is the statistical picture. If there are two electrons which are identical at (x_1, y_1, z_1) one needs to create a superposition i.e.

$$\text{Sum over } k \quad a_k \exp(i k L_1) + \text{Sum over } j \quad a_j \exp(i j L_2) \quad ((1))$$

Here L_1 and L_2 are the lengths from the two electron sources to x_1, y_1, z_1 . From a classical wave point of view, one has different phase shifts i.e

$$\exp(ip L_1) + \exp(ip L_2) \quad ((2))$$

This is sometimes interpreted as indicating that there is a physical cycling (e.g. zitterbewegung), but one cannot ensure that each electron starts with the same phase at its creation point. Thus the inner cycling picture seems to be problematic. Rather we consider the creation point as a boundary condition. The interference/superposition $((1))$ (or $((2))$) is used because the quantum measurement cannot distinguish between $\exp(ip L_1)$ and $\exp(ip L_2)$. (Otherwise one would use an AND product approach not OR sum.) Furthermore if the quantum measurement could discern the state $\exp(ip L_1)$, $\exp(ip L_2)$ it could form a quantum resonance with these. In such a case it would be like a potential and not a measurement. In other words, for a bound state $V(x)$ the potential is not like an external quantum measurement.

Finally one takes the modulus squared of $((1))$ or $((2))$ to find the intensity at x_1, y_1, z_1 . Even though this is a one directional flow quantum problem it is still treated as a statistical “steady state” type of equilibrium problem. Time-independent scattering problems are treated in a similar statistical steady state manner.

Dynamical Quantum Probability $P(p/x)$

The quantum probability $\exp(ipx)$ associated with probabilistic impulse hits from a potential is peculiar in that it extends to $\pm$ infinite which makes it seem like a mathematical object. For example, $W^*(x)W(x)=0$ for $x < a$ means that the superposition, which is what the quantum measurement detects, cannot find a particle for $x < a$ even though $\exp(ipx)$ in $W(x)=\text{Sum } a(p)\exp(ipx)$ is well defined for $x < a$. Interestingly, there are two different probabilities for a forward and backward moving particle $\exp(ipx)$ and $\exp(-ipx)$. These ensure that for a bound state with $a(p)=a(-p)$ one has a real wavefunction indicating no overall net flow to the right or left although there are pockets with motion to the right or left at different x points.

Conclusion

In conclusion, we argue that quantum mechanics is a statistical system with two sets of probabilities $P(x)$ or $P(p)$ and $P(p/x)$ which is complex. This makes it very different from classical statistical mechanics. $P(p/x)$ is periodic and invariant in space i.e. nonlocal to allow it to create local spatial functions such as $KE(x)$ or $V(x)$ the potential i.e. $V(x) = \sum_k V_k \exp(ikx)$. $V_k \exp(ikx)$ represents an impulse hit of k which interacts with $\exp(ipx)$ of a particle to establish a momentum distribution $W(x) = \sum_p a(p) \exp(ipx)$. $\exp(ipx)$ is two dimensional to handle flow to the right and left and extends to $\pm$ infinite. Its positive/negative values allow for creating local values.

If a particle cannot be found at say $x < a$ then $W(x) = 0$ for $x < a$, but $\exp(ipx)$ terms still exist for $x < a$, only the superposition disappears. We thus argue that it is the superposition which a quantum measurement detects. It cannot interact with $\exp(ipx)$ to create a resonance, but it may destroy resonances i.e. wavefunction collapse. Quantum measurements lead to local discernible results e.g. $\text{density}(x)$ which still reflect the wavy behaviour of $\exp(ipx)$ albeit in a superposition scenario. For a one dimensional type of problem, e.g. two electrons created at different sources and meeting at x_1, y_1, z_1 , we argue that statistical steady state type arguments still hold (as they do in scattering) and that the source point may be thought of as a boundary condition with $W(x) = 0$ behind the source and $W(x) = \exp(ipx)$ in front. Alternatively, one may think of the various $\exp(ikx)$ terms creating an $\exp(ipx)$ in front of the source and think in terms of phase shifts, but these do not need to be linked to physical cycling because one does not know the initial phase of a created electron. Quantum measurements again cannot discern $\exp(ip L_1)$ and $\exp(ip L_2)$ from the two electrons and so an OR probability situation is used and the modulus squared taken.