THE INFLUENCE MONITORING PROBLEM

László Hajdu[1]
InnoRenew CoE
Livade 6, 6310 Izola, Slovenia
E-mail: laszlo.hajdu@innorenew.eu

Miklós Krész[2]
InnoRenew CoE
Livade 6, 6310 Izola, Slovenia
E-mail: miklos.kresz@innorenew.eu

Abstract: A new problem called influence monitoring is defined and studied for the Generalized Independent Cascade Problem. It is shown that the objective function of the problem is submodular by which the greedy algorithm can ensure an approximate ratio. The efficient implementation of the greedy method is described from algorithm design point of view.

Keywords: infection processes, influence maximization, greedy algorithm, network optimisation, network immunization

1 INTRODUCTION

The study of network infection processes plays an important role in several fields; in addition to epidemiology [9], it was successfully applied in sociology (spread of opinions) [8] or business and economics (marketing campaign [6] or risk management [3]) , to mention a few. The related research questions were formulated as a discrete optimisation problem in 2003 in the pioneering work of Kempe, Kleinberg and Tardos [10] The so-called influence maximization is defined by the following way: determine the most influential vertices in a directed edge-weighted graph (where edge weights are reflecting the infection probabilities) with respect to an infection process changing the status of the vertices according to a well-defined function of the incoming edges and incident vertices. Kempe et al proved that the problem is NP-hard, however for a wide class of infection processes the greedy algorithm provides a solution with a guaranteed approximation ratio. During the last 15 years the above work exposed an extraordinary interest in the scientific community and several generalization of the original problem were developed (for a review see e.g. [1]).

In this paper, instead of finding the most influential vertices, we will be focusing to identify those vertices through which the highest expected number of vertices are accessible in "infection chains". This "influence monitoring task" is different from the original problem, as the optimal solution is given by those vertices which are the best candidates to be "monitored" to decrease the influence of an infection process (instead of finding those vertices which are the most influential for an "outbreak"). Concerning the infection mechanism our study will concentrate on the generalization of the most widely used Independent Cascade model [6], however our approach can be generalized for all diffusion processes under the Generalized Threshold Model [10]. In this Generalized Independent Cascade Model each vertex has an initial "a priori" infection probability. We will prove that the greedy algorithm provides the same approximation factor as the influence maximization problem has and we will develop an efficient implementation of this method.

[1] Also at University of Primorska, Slovenia and University of Szeged, Hungary.
[2] Also at University of Primorska, Slovenia and University of Szeged, Hungary.

The organization of this paper is as follows. In Section 2 we will present the necessary formal background on influence maximization and formally define the influence monitoring problem. In Section 3 as a main result we will develop the greedy method with all the technical details and will describe its approximability power. Finally, in Section 4 we will give a short conclusion. Because of space constraints proofs are omitted.

2 INFLUENCE MAXIMIZATION AND MONITORING

In this paper we will consider directed weighted general graphs. Our terminology will be standard, the set of vertices and set of edges will be denoted by *V(G)* and *E(G)*, respectively. We are considering stochastic diffusion processes on networks (see e.g. [2]) in general, but because of space restrictions in this paper we will be focusing on one of the basic (and most popular) models, the Independent Cascade model. In this model the weight $w_{u,v} \in [0,1]$ for the edge *(u,v)* directed from *u* to *v* is expressing the probability by which *u* is forwarding the diffusion to *v* through this edge.

As an iterative process the *Independent Cascade (IC)* model [10] starts with a set S_0 of active vertices and in the ith iteration step the set of (previously inactive) vertices S_i will be activated by the vertices of S_{i-1}. The process will be terminated once S_i is empty. During the process a vertex *u* from S_{i-1} has a single chance to activate a neighbouring vertex *v* with probability $w_{u,v}$. If more than one vertex is trying to activate *v* in the same iteration, the attempts are realized in a random order and independently. The process is not deterministic, however the expected number $\sigma(S_0)$ of activated vertices can be used for a well-defined optimisation problem called *influence maximization* [10]: Given a positive integer k, find a set of vertex set *S* with size *k* such that $\sigma(S)$ is maximized.

Influence maximization is a hard problem, even for a fixed set *S,* the computation $\sigma(S)$ *is* #P-complete [5]. Nevertheless, in their pioneering paper Kempe et al [10] showed that by random sampling and direct simulation of the diffusion process, $\sigma(S)$ can be arbitrarily approximated. Furthermore, they have proved that using the greedy hill-climbing method (in each step choosing the vertex providing the largest marginal increase in $\sigma(S)$, the optimum can be approximated within a factor of $(1 - 1/e)$ (where *e* is the base of the natural logarithm). They have actually shown that the set function $\sigma(S)$ is a monotone nonnegative submodular function. This property of $\sigma(S)$ guarantees the above approximability by a classical result of Nemhauser et al [12]. A function that maps subsets of a finite ground set to real numbers is called *submodular* if the marginal gain by adding an extra element to a set *S* cannot be higher than the marginal gain by adding the same element to a subset of *S*. For more details about submodular optimisation see [11].

The IC model was generalized in [4] with assigning infection probabilities to the vertices as well. In the *Generalized Independent Cascade* (GIC) model the weighted directed graph is extended with *a priori infection probabilities* w_v for all $v \in V(G)$ and each vertex *v* becomes active independently with probability w_v at the beginning of the process. Having a random choice of infected vertices according to the a priori probabilities, the process is executed by the IC model. Concerning the stochastic feature of the model, at the end of the process an *a posteriori probability* w_v^* is obtained for each vertex *v*. As a generalization the sum of the values w_v^* provides the generalized function $\sigma(w)$. We will suppose that a graph *G* is given with infection probabilities on the edges as well as on the vertices for the rest of the paper.

With respect to the above stochastic process it can be a natural question to describe the “local influence” of vertices. Concerning the interpretation of the IC model an “instance infection” is a branching process (see e.g. [7] Chapter 21) an unweighted directed forest as an (unweighted) subgraph of G. Starting from the initially infected vertices a particular realization of infection in the cascade corresponds to a forest where each iteration will determine the next level of this forest. Therefore, concerning the whole stochastic process of G, it is a distribution of unweighted forests sampled from G. Each forest will be referred as a *forest instance infection* or simply *forest instance*.

Now considering a forest instance F and a vertex set $S \subseteq V(F)$, the *local influence value* $\mu_F(S)$ is defined as the number of descendants of S in F, i.e. the number of vertices accessible by a directed path from a vertex of S. Note that the elements of S are also counted in this manner (with paths of length zero). Finally, the *local influence index* $\mu_G(S)$ for the weighted graph G and $S \subseteq V(G)$ is the expected value of $\mu_F(S)$ for a random forest instance F in G. In this paper our goal is to optimize the local influence index:
In the *influence monitoring problem* for a given weighted graph G and positive integer k, determine the vertex set S with size k for which $\mu_G(S)$ provides the maximum value.

3 THE GREEDY ALGORITHM

We will be developing a greedy heuristic with a guaranteed approximation ratio. For that we are extending the concept of Kempe et al [10] for graph sampling with "live" and "blocked" edges (see also "complete simulation" in [4]). Concerning the attempt of infecting vertex v by vertex u with a probability $w_{u,v}$, we can consider it as flipping a coin of bias $w_{u,v}$ and generating unweighted edge to be "live" with probability $w_{u,v}$. Since each attempt is independent, we can "flip" for each edge independently and generating an unweighed graph instance. Concerning the result of GIC it has no relevance when the independent coin flips are realized, we can make it at the beginning of the process. In such a graph instance an infection from a vertex u to a vertex v is activated if and only if there exists a directed path from u to v. However, in contrast to influence maximization, in our influence monitoring problem the probability of activating by a particular edge in a graph instance is important (concerning multiple attempts to the same vertex); we will discuss it later.

Extending the sampling methodology of Kempe et al [10] by a *graph instance infection or graph instance* in short we will mean an unweighted graph sampling each edge *(u,v)* according to $w_{u,v}$ and determining initial activated vertices in a random manner according to a priori infection probabilities. Similarly to influence maximization and the original sampling of Kempe et al [10], graph G is equivalent to the a distribution of graph instances G' and $\mu_G(S)$ can be obtained as a linear combination of different $\mu_{G'}(S)$ values according to this distribution. The advantage of this approach is twofold. On one hand it provides a framework for approximating $\mu_G(S)$ by simulation (see complete simulation in [4]). On the other hand as linear combination of submodular functions is also submodular, it will be enough for an arbitrary graph instance G' that $\mu_{G'}(S)$ is submodular. As a consequence, by the results of [12] and considering the monotone and nonnegative properties of $\mu_G(S)$ we will get the guaranteed approximation for the greedy method.

Concerning the efficient implementation of the greedy method as well as proving submodularity of $\mu_{G'}(S)$ we will need to solve the following problem.

Problem 1. *Given a graph instance G' of G and $S \subseteq V(G)$. Determine a function $\mu^*_{G'}(S, v)$ which satisfies the following conditions for each vertex $v \in V(G)$:*

- $\mu_{G'}(S) = \sum_{v \in S} \mu^*_{G'}(S, v)$
- $\mu_{G'}(S \cup \{v\}) = \mu_{G'}(S) + \mu^*_{G'}(S, v)$ for each $v \in V(G) - S$

In order to solve the above problem, starting from the initially infected vertices of *G'* build up the directed breadth first search forest of *G'*. We will keep those edges only which are connecting two neighbouring levels of the forest. This reduced graph will be denoted by *BF[G']* and the set of edges of *BF[G']* is denoted by *BF[E']*. This graph is representing the collection of potential infection instances by IC in *G'* in such a way that the initially infected vertices are on $level_1$. Vertices in $level_2$ will be infected from $level_1$ and generally the vertices in $level_{i+1}$ are infected from $level_i$.

Suppose now that we have *r* levels in *BF[G']*. Clearly, for any vertex *v* in $level_r$, $\mu^*_{G'}(S,v) = 1$. Notice that starting from $level_r$ we can recursively backward calculate $\mu^*_{G'}(S,v)$. Indeed, if the indegree of *v* is *d*, then the infection can be realized by any incoming edge to *v* with probability *1/d*. By this observation it is easy to organize the calculation, which is described described in details in Algorithm 1.

Algorithm 1

Input: Breadth-first reduction *BF[G']* of graph instance *G'* with *r* levels and dedicated vertex set $S \subseteq V(G')$
Output: For each vertex $v \in V(G')$ the value of a function $\mu^*_{G'}(S,v)$ satisfying conditions described in Problem 1
  For each vertex *v* at $level_r$ let $\mu^*_{G'}(S,v) = 1$
 for *i=r* **downto** *2* **do**
  for each vertex *v* at $level_i$
    for each incoming edge *(u,v)*
      let $label_{u,v}=1/d$ (where *d* is the indegree of *v* in *BF[G']*
  for each vertex *u* at $level_{i-1}$
    $\mu^*_{G'}(S,u) = 1 + \sum_{u' \notin S \& (u,u)' \in BF[E']} label_{u,u'} \cdot \mu^*_{G'}(S,u')$
 end for

It can be proved by induction that the function $\mu^*_{G'}(S,v)$ produced by Algorithm 1 has the required properties as summarized below.

Proposition 1. *For each* $v \in V(G')$, *the function* $\mu^*_{G'}(S,v)$ *in Algorithm 1 satisfies the conditions of Problem 1 and the method runs in linear time of E(G').*

Analyzing Algorithm 1, based on the conditions defined in Problem 1 we can obtain the submodular property.

Proposition 2. *Function* $\mu_{G'}(S)$ *is submodular.*

As discussed earlier, the submodularity of $\mu_G(S)$ follows from Proposition 2 and it is also clear that $\mu_G(S)$ is monotone and non-negative.

Corollary 1. *Function* $\mu_G(S)$ *is monotone, non-negative and submodular.*

By Corollary 1 we can design the greedy method with guaranteed approximation described in Algorithm 2. Note that instead of the mean value we are using the sum for $\mu_G(S,v)$ in order to simplify calculation.

Algorithm 2

Input: Weighted graph G and positive integer k
Output: Vertex set S with cardinality k by greedy with respect to $\mu_G(S)$
Let $S = \emptyset$
For $i=1$ **to** k **do**
    Generate graph instances
    For each graph instance G' and for each vertex $v \in V(G)$, calculate $\mu^*_{G'}(S, v)$ according to *Algorithm 1*
    Let $\mu^*_G(S, v)$ be the sum of the $\mu^*_{G'}(S, v)$ values of all graph instances for each vertex $v \in V(G)$
    Let $S = S \cup \{v\}$ with v having maximum in $\mu^*_G(S, v)$
end for

Now we can summarize our finding by a direct consequences of the previous results.

Theorem 1. *Algorithm 1 provides a solution approximating the influence monitoring problem within a factor of* $(1 - 1/e - \varepsilon)$*, where where e is the base of the natural logarithm and ε is an arbitrary small positive number depending on the simulation.*

Note that the scale of approximation can depend on the simulation (the number and distribution of the graph instances), however, similarly to the influence maximization problem [10], this gap can be arbitrarily small.

4 CONCLUSION

In this paper we have introduced the influence monitoring problem for the Generalized Independent Cascade Model. We have proved that the objective function for this problem is nonnegative, monotone and submodular, thus using the classical result of Nemhauser et al, it is shown that the greedy method provides a solution within an approximation factor of $(1 - 1/e - \varepsilon)$, where where e is the base of the natural logarithm and ε is an arbitrary small positive number depending on the simulation. As a further research we are extending the results for a wider class of problems and will show the efficiency of the methodology with a comprehensive testing on artificial and real networks.

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Helena Sofia Rodrigues, Business School, Instituto Politécnico de Viana do Castelo; Center for Research and Development in Mathematics and Applications (CIDMA), Universidade de Aveiro
Črtomir Rozman, University of Maribor, Faculty of Agriculture and Life Sciences
Ângela Silva, Business School, Instituto Politécnico de Viana do Castelo; COMEGI, Universidade Lusíada, Portugal
Tadeusz Trzaskalik, University of Economics, Department of Operational Research Katowice, Poland
Tomasz Wachowicz, University of Economics in Katowice, Katowice, Poland
Lidija Zadnik Stirn, University of Ljubljana, Biotechnical Faculty, Ljubljana, Slovenia
Anja Žnidaršič, University of Maribor, Faculty of Organizational Sciences, Kranj, Slovenia
Janez Žerovnik, University of Ljubljana, Faculty of Mechanical Engineering, Ljubljana, Slovenia

Preface

This volume, Proceedings of the 16th International Symposium on Operational Research, called SOR'21, contains papers presented at SOR'21 (https://sor.fov.um.si/), organised by Slovenian Society INFORMATIKA (SDI), Section for Operational Research (SOR), University of Maribor, Faculty of Organisational Sciences, Kranj, Slovenia (FOV), and University of Ljubljana, Faculty of Mechanical Engineering, Ljubljana, Slovenia (UL FS). The SOR'21 symposium, held 22-24 September 2021, was originally planned to take place in Bled, Slovenia, but was moved online due to the situation of COVID-19 in Slovenia and beyond. The volume contains blind peer-reviewed papers or abstracts of papers presented at the symposium.

The opening address at SOR'21 was given by Prof. Dr. Lidija Zadnik Stirn, President of SOR, Mr. Niko Schlamberger, President of SDI, representatives of FOV and UL FS, Prof. Dr. Mario Jadrić, President of Croatian Operational Research Society (CRORS), Dr Sarah Fores, manager of The Association of European Operational Research Societies (EURO), and presidents/representatives of some others Operational Research Societies from abroad.

SOR'21 is the scientific event in the field of Operational Research, another in the traditional series of biennial international OR conferences organised in Slovenia by SDI-SOR. It is the continuation of fifteen previous symposia. The main objective of SOR'21 is to promote knowledge, interest and education in the field of OR in Slovenia, Europe and worldwide in order to build the intellectual and social capital essential for maintaining the identity of OR, especially at a time when interdisciplinary cooperation is proclaimed as particularly important for solving problems in today's challenging times. By joining IFORS and EURO, the SDI-SOR has also agreed to collaborate with different disciplines, i.e., to balance the depth of theoretical knowledge in OR and the understanding of theory, methods, and problems in other fields within and outside OR. We believe that SOR'21 creates the advantage of these goals, contributes to the quality and reputation of OR by presenting and sharing new developments, opinions and experiences in the theory and practise of OR.

SOR'21 was highlighted by five distinguished keynote speakers. The first part of Proceedings SOR'21 contains invited abstracts, presented by five outstanding scientists: Assist. Prof. Nikolina Ban, University of Innsbruck (UIBK), Department of Atmospheric and Cryospheric Sciences, Innsbruck, Austria, Assist. Prof. Vedran Kojić, University of Zagreb, Faculty of Economics & Business, Zagreb, Croatia, Prof. Panos Patrinos, KU Leuven, Department of Electrical Engineering (ESAT), STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics, Leuven, Belgium, Prof. Suresh P. Sethi, Eugene McDermott Chair Professor of Operations Management, Director, Center of Intelligent Supply Networks, Naveen Jindal School of Management, The University of Texas at Dallas, Dallas, USA, and Prof. Jerneja Žganec Gros, Alpineon Ltd, Ljubljana, Slovenia.

The Proceedings includes 118 papers or abstracts by 240 authors. Most of the authors of the contributed papers came from Slovenia (82), then Croatia (52), Hungary (23), Portugal (23), Serbia (17), Poland (9), Czech Republic (8), Slovak Republic (7), Spain (6), Netherlands (4), Bosnia and Herzegovina (2), Austria (1), Belgium (1), France (1), Germany (1), Romania (1), Ukraine (1), United Kingdom (1), and United States of Amerika (1). The papers published in the Proceedings are divided into Plenary Lectures (5 abstracts), eleven special sessions: Application of Operational Research in Smart Cities (6 papers), Computational Mathematical Optimization (7 papers and 6 abstracts), Data Science – Methodologies and Case Studies (10 papers), Graph Theory and Algorithms (2 papers), *High-Performance Computing and Big Data (3 papers), Industry & Society 5.0: Optimization in Industrial and Human Environments (6 papers), International Projects in Operations Research (2 papers), Lessons Learned from the COVID-19 Pandemic: Applications of Statistical and OR Methods (8 papers), Logistics and Sustainability (9 papers), Operational Research in Ageing Studies and Social Innovations (5 papers), Operations Research in Agricultural Economics and Farm Management (5 papers), and eight sessions: Econometric Models and Statistics (6 papers), Environment and Social Issues (5 papers), Finance and Investments (6 papers), Location and Transport, Graphs and their Applications (5 papers), Mathematical Programming and Optimization (5 papers and 1 abstract), Multi-Criteria Decision-Making (10 papers), Theory of Games (3 papers), and Problems Approaching OR (3 papers).*

Proceedings of the previous fifteen International Symposia on Operational Research organised by the Slovenian Section on Operational Research, listed at https://www.drustvo-informatika.si/sekcije/sor/sor-publikacijepublications/, are indexed in the following secondary and tertiary publications: Current Mathematical Publications, Mathematical Review, Zentralblatt fuer Mathematik/ Mathematics Abstracts, MATH on STN International and CompactMath, INSPEC. It is expected that Proceedings SOR'21 will be covered by the same bibliographic databases.

The success of the scientific events at SOR'21 and of the present conference proceedings should be seen because of joint efforts. On behalf of the organisers, we would like to express our sincere gratitude to all those who assisted us in the preparation of the event. Without the dedicated and advice of the active members of the Slovenian Operations Research Section, we would not have been able to attract so many top-class speakers from all over the world. Many thanks to them. In addition, we would like to express our deepest gratitude to the prominent keynote speakers, the members of the Programme and Organising Committees, the reviewers who improved the quality of SOR'21 with their useful suggestions, the section chairs and all the numerous people - far too many to list individually here - who helped in organizing of the 16th International Symposium on Operational Research SOR'21 and compiling this proceedings. Finally, we thank the authors for their efforts in preparing and presenting the papers that made the 16th Symposium on Operational Research SOR'21 a success.

We would like to give special thanks to the Partnership for Advanced Computing in Europe (PRACE) for their financial support.

Ljubljana and Kranj, September 22, 2021

Samo Drobne
Lidija Zadnik Stirn
Mirjana Kljajić Borštnar
Janez Povh
Janez Žerovnik
(Editors)

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