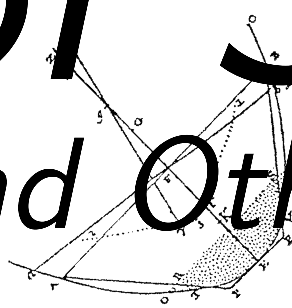
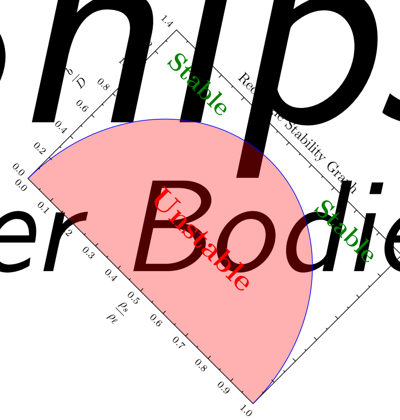
Stability of Ships and Other Bodies





Vasa on August 10, 1628

by

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Stability of Ship and Other Bodies

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"We are like dwarfs sitting on the shoulders of giants"

from The Metalogicon by John in 1159

Prologue For This Book

Version 0.5.5 Aug 22, 2021

pages 179 size 3.7M

It is very proud moment to observe all the achievements that were done in writing this book. Normally it is hard to see significance of the one book. However, it is moment one should be able to brag with amount of achievements. In fact, in a field that is so outdated and with "facts" that in entrenched and no one challenge them. For example, around what point is the ship rolling. In fact, the entrenched answer is around the metacenter which is wrong. The fact that in this book basically the author discover and demonstrated that rotation depend on cross section is because the new method that was developed.

Furthermore, the fact that no one utilized Nusselt's technique is strange. Why no one use the dimensional analysis in presenting the stability diagram? Is that presentation is so hard? As results, now the location of stable zones are clear. One indirect effect of the stability analysis provides in the calculation of the segment centroid. All the graphs showing the GZ what is referred sometime as the right hand arm are smooth. This another one of the entrenched misunderstanding of the stability field.

Still there are several topics that needed clarifications that this book did not provide. It is hope that other will start to provide significant work that merit inclusion in this book. For example, the dynamic stability analyzed in these days (2021) utilizing numerical calculations. It is not that numerical are wrong. It is what said garbage in garbage out. For example, a paper (Hu, Ramlingam, Meyerson, Eckert, and Goldstein 1992) carried the author (previous) name (Meyerson, without permission). The correct thing in the paper is the experiments that this author had done. The model is wrong because it failed to take into account right physical phenomenon. Is it the same for these stability models present in the recent books? This author is not convinced hence no section in this book can be presented on these topics. Yet all the issues related to the safety regulations should appear in future.

Version 0.5 June 4, 2021

pages 159 size 3.3M

This author introduction to ship stability was during 1974 where he was at Akko Nautical College a unique and special high school. The class was taught by a big man (size wise) as a main instructor who used to be captain and at time worked as psychologist. The first class was extremely interesting because his unique ability to handle boys[1]. However, because the class was after a war, it was carried in a condense fashion (over 8 hours a day for two weeks) to fulfill missing material. The most material taught was produced by the French Pierre Bouguer. For example, the numerical integration was done by the trapezoid method, the Meta Center etc. It is interesting that the instructor or his co–teachers did not know about the Bouguer him self.

Since that class, the author has worked on various stability[2] problems like the stability of thin film (small and large waves) in relationship to energy transfer in his Master at Tel Aviv University. Later the author work on various stability problems in compressible flow. Additionally, the author work on stability problem multi–phase flow. After the author gone through these various stability problems, coming again to ship stability this author got awaken to see how old technology has been frozen in time (over 300 years). There were two attempts that the author is aware to bring to date the ship stability science. The first one from Paul Erdös and second from Lautrup. The first is mathematician who done the mathematics correctly yet he mess up some elements from the physics point of view. The second is by a Danish physicist who had too much respect to the old tradition and yet he was able to renovate and focus on concepts into scientific like the stability diagram. This author will refer to diagram as Lautrup's diagram.

Perhaps the most amazing is the lack of dimensionless analysis of the stability of ship and other bodies. That is, what is the significance of GM=2m. Is it the same effect on a big ship as compared to a small ship? The ship size has a strong effect and dimensional analysis done only recently by several breve students from Egypt (Habib, Ali, and Nawar 2018). Nevertheless, proper analysis utilizing Nusselt's method was done in Fluid Mechanics book by this author for the first time.

This is the partial list of achievements in this book.

- The development of the angle in angle out approach for stability.
- The recognition that stability is a problem that deals solely with rotation.
- The proper usage of dimensional analysis within the stability analysis.
- Recognition of the change cross area with the height on the stability.
- Construction of stability diagram for many fundamental geometrical shapes.
- Presentation of the GM in dimensionless form for several geometrical shapes.
- Presentation of the correction gravity centroid needed to make the body stable.
- Discuss the differences between stability on solid and liquid

[1]A very talent in taking command of mischievous teen age boys. Author is ashamed of the things that his group used to do.

[2]Even actually doing every morning stability calculations for ship as part of his duty. Note really scientific but shear fact that dealing with it.

- Point to interdependence of the different rotations.

After reading this book one can answer questions that before one could not. For example, if you are on a boat which is about to be roll what should you do? Remove items and throw them overboard? or add more items like filled out buckets full of water? and why? If you were using all the all books you could not answer this question.

This book is the fourth book in the series of POTTO project books. POTTO project books are open content textbooks so everyone are welcome to joint in. The topic of stability mechanics was chosen because the discovery or the development of the Direct methods. And the fact, this author wanted to write a book that will be used by his old high school.

This book is written in the spirit of my adviser and mentor E.R.G. Eckert. Eckert, aside from his research activity, wrote the book that brought a revolution in the education of the heat transfer. Up to Eckert's book, the study of heat transfer was without any dimensional analysis. He wrote his book because he realized that the dimensional analysis utilized by him and his adviser (for his post doc), Ernst Schmidt, and their colleagues, must be taught in engineering classes. His book met with a strong criticism in which some called to "burn" his book. Today, however, there is no known place in world that does not teach according to Eckert's doctrine. It is assumed that the same kind of individual(s) who criticized Eckert's work will criticize this work. Indeed, the previous book, on compressible flow, met its opposition. For example, anonymous Wikipedia user name EMBaero claimed that the material in the book is plagiarizing, he just doesn't know from where and what. Maybe that was the reason that he felt that is okay to plagiarize the book on Wikipedia. These criticisms will not change the future or the success of the ideas in this work. As a wise person says "don't tell me that it is wrong, show me what is wrong"; this is the only reply. With all the above, it must be emphasized that this book is expected to revolutionize to some degree the field and change some of the way things are taught.

This book is written for several groups like high school who study ship stability, sea officers (even skipper), and for university students, hence the additional material is required to satisfy these conflicting interest.

The book is organized into several chapters which, as a traditional textbook, deals with a basic mathematics, than fundamental of mechanics, and fluid statics. Later several chapters will be dealing with various aspect of the floating stability.

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- **Nature of contribution:** LaTeX formatting, help on building the useful equation and important equation macros.
- **Nature of contribution:** In 2009 creating the exEq macro to have different counter for example.

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- **Date(s) of contribution(s):** April 2008
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- **I. P. Loampo** , June 2021, point to typos and, help with questioning the logic of the presentation.
- **Tousher Yang** April 2008, review of statics and thermo chapters (remain from fluid mechanics book)
- **Alessandra from Brazil** June 2021 questions several points.

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Nomenclature

α	The resulting angle for tilde, see equation (5.13), page 50
β	The large permanent angle, see equation (5.19), page 52
$\mathcal{A}$	The rotation (fix?) point at liquid surface, see equation (5.19), page 52
ℓ	Length, see equation (8.1), page 82
ρ_ℓ	The liquid density, see equation (9.3), page 105
ρ_s	The solid density, see equation (9.3), page 105
ρ	Density of the body, see equation (5.4), page 41
ρ_{fresh}	Fresh water density, see equation (3.2), page 22
ρ_{sal}	Sea water density, see equation (3.2), page 22
θ	The imposed angle of tilde, see equation (5.13), page 50
A	Cross area or area in general, see equation (5.5), page 41
A_M	Midship wetted area of the cross section, see equation (3.7), page 24
A_r, A_o	Area removed and added, see equation (5.8), page 42
A_{ls}	Liquid surface area, see equation (3.11), page 26
B	Buoyancy centerid, see equation (5.19), page 52
b	The base of the body, see equation (5.9), page 43
C_m	Maximum section Area Coefficient, see equation (3.8), page 24
C_{DW}	Dead weight coefficient, see equation (3.4), page 24
C_{ls}	Liquid surface coefficient, see equation (3.11), page 26
D	the distance to point G, see equation (9.3), page 105
d	Draft of the body normally underwater, see equation (5.9), page 43
d	the distance to point B, see equation (9.3), page 105

F The area of surface, see equation (7.1), page 71

Fr_D Froude Displacement Coefficient, see equation (3.15), page 27

g The gravity constant, see equation (8.2), page 82

GB The distance between the buoyancy centroid and the gravity centroid, see equation (9.3), page 104

GM The distance between the gravity centroid to Metacenter point, see equation (10.6), page 122

h Height in the pressure chapter, see equation (8.7), page 83

I_{xx} Moment of inertia, see equation (6.1), page 59

m Mass of the body, see equation (5.1), page 40

P The area of surface, see equation (7.1), page 71

P_{atmos} Atmospheric Pressure, see equation (7.5), page 75

R The big radius, see equation (5.9), page 42

r Coordinate in r direction, see equation (4.17), page 35

t thickness, see equation (8.7), page 83

V Volume of the body, see equation (5.1), page 40

V_0 The volume under the liquid, see equation (9.3), page 104

V_d Encompassing volume, see equation (3.1), page 22

x Coordinate in the base direction, see equation (4.0), page 30

y Coordinate in the other direction, see equation (4.0), page 30

The Book Change Log

Version 0.5.5

Aug 22, 2021 (3.9M 179 pages)

- Add the chapter on the definitions.
- Add the section on GZ diagram.
- Add the explanation to change of coordinate system for finding centroid.
- Add the section dealing centroid of circle segment utilizing the stability new method direct investigation.

Version 0.5.2

July 1, 2021 (3.5M 167 pages)

- English and layout corrections
- Add nomenclature.

Version 0.5.1

June 25, 2021 (3.5M 163 pages)

- English and layout corrections
- More examples and some corrections to the content.

Version 0.5

June 15, 2021 (3.3M 159 pages)

- Initial math/calculus gear to stability understanding,
- A small revolution in floating body stability)
- New change centroid location for added or subtracted masses
- Chapter on moment of inertia

- Chapter on Archimedes' Law
- Chapter on 2-D stability
- Chapter on GM
- Chapter on Variable Gravity Centroid

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How This Book Was Written

Initial

This book started because I found or developed a new way, after hundred years, to calculate the stability limits. While Simpson's role was penetrating to the stability field displacing the trapezoid integration there still ancient approach like lake of dimensional analysis approach that must be updated. Perhaps, this book is written by this author, who has none conforming and none traditional approach, will move the field to modern as a results of writing this book.

The book started as to attempt to teach my kids experimental physics in a hope that they will like and want to be engineers. This attempt failed to get them to be interested in engineering yet this book was born.

The famous Eckert converted the field of heat transfer to a dimensional(less) science. It hope that tables (using big ships, small ship etc.) that appear in ship stability books like (Derrett and Barrass 1999; Biran and Pulido 2013) should be located only in library learning about the history of the issue.

There are several books on stability of floating bodies but none them is open content. The approach adapted in this book is practical, and more hands–on approach. This book indent to many different and conflicting audience and that create a difficulty in writing a book. So, issue of proofs so and so are here only either to explain a point or have a solution of exams. Otherwise, this book avoids this kind of issues.

The structure developed in the author's book on fluid mechanics was adapted and used as a scaffolding for this book. This author was influenced by material that he was thought in high school but this material will be ignored in this textbook. The chapters are written in order that assume reader has little background.

The presentation of some of the chapters is different from other books because the new ideas and usability of the computers. The book does not provide the old style graphical solution methods yet provides the graphical explanation of things.

Of course, this book was written on Linux (Micro$oftLess book) using the vim editor for editing (sorry never was able to be comfortable with emacs). The graphics were done by ipe, a new graphic program that this author is learning to use (and which is user friendly with poor documentation yet there no better package.). The figures were done by GLE. The figure in cover page was created by Genick Bar-Meir, and is copyleft by him.

1 Preface

"In the beginning, the POTTO project was without form, and void; and emptiness was upon the face of the bits and files. And the Fingers of the Author moved upon the face of the keyboard. And the Author said, Let there be words, and there were words." [1].

This book, Stability of Ships and Other Bodies, describes the fundamentals when and why floating bodies are stable. In addition, it describes steps that transforms un–stable bodies to stable. This book is designed to replace all the other books and inseminate that recent developed technology and advances. The material in standard books is so entrenched, old, and outdated material that one can be only amazed. For example, concept like potential stability is not discussed or even mentioned in any of all the books that this author review.

This book written from physical point of view rather then construction point of view like all the books review by this author on stability. It is hoped that the book could be used as a reference book for people who have at least some basics knowledge of science areas such as algebra, basic physics, etc. Even without deep understating the graphs in this book can be used to find necessarily quantities needed to understand if to fix your vessel.

The structure of this book is such that many of the chapters could be usable independently. I hope this approach makes the book easier to use as a reference manual. However, this manuscript is first and foremost a textbook, and secondly a reference manual only as a lucky coincidence.

I have tried to describe why the theories are the way they are, rather than just listing "seven easy steps" for each task. This means that a lot of information is presented which is not necessary for everyone. Many in the industry, have called and emailed this

[1] To the power and glory of the mighty God. This book is only attempt to explain his power.

author with questions since this book is only source in the world of some information. These questions have lead to more information and further explanations which are not available else where.

This book is written and maintained on a volunteer basis. Like all volunteer work, there is a limit on how much effort undersign was able to put into the book and its organization. Moreover, due to the fact that English is my third language and time limitations, the explanations are not as good as if I had a few years to perfect them. Nevertheless, I believe professionals working in other engineering fields could benefit from this information.

I have left some issues which have unsatisfactory explanations in the book, marked with a Mata mark (still under work). I hope to improve or to add to these areas in the near future. Furthermore, I hope that other individuals will participate of this project and will contribute to this book (even small contributions such as providing examples or editing mistakes are needed).

I have tried to make this text of the highest quality possible and am interested in your comments and ideas on how to make it better. Incorrect language, errors, ideas for new areas to cover, rewritten sections, more fundamental material, more mathematics (or less mathematics); I am interested in it all. I am particularly interested in the best arrangement of the book. If you want to be involved in the editing, graphic design, or proofreading, please drop me a line. You may contact me via Email at "barmeir@gmail.com".

Naturally, this book contains material that never was published before (sorry cannot avoid it). Actually it is the main trust of the book showing the new way to do old things. This material never went through a close content review. While close content peer review and publication in a professional publication is excellent idea in theory. In practice, this process leaves a large room to blockage of novel ideas and plagiarism. If you would like be "peer reviews" or critic to my new ideas please send me your comment(s). Even reaction/comments from individuals like David Marshall[2].

Several people have helped me with this book, directly or indirectly. I would like to especially thank to my adviser, Dr. E. R. G. Eckert, whose work was the inspiration for this book. His approach in dimensional analysis was a key in understating concepts like what dimensional group effect the stability but also the presentation.

I encourage anyone with a penchant for writing, editing, graphic ability, LaTeX knowledge, and material knowledge and a desire to provide open content textbooks and to improve them to join me in this project. If you have Internet e-mail access, you can contact me at "barmeir@gmail.com".

[2] Dr. Marshall wrote to this author that the author should review other people work before he write any thing new (well, literature review is always good, isn't it?). Over ten individuals wrote me about this letter. I am asking from everyone to assume that his reaction was innocent one. While his comment looks like unpleasant reaction, it brought or cause the expansion of the explanation for the oblique shock. However, other email that imply that someone will take care of this author aren't appreciated.

2

Introduction to Stability

2.1 What is Stability of Floating Bodies?

The perquisite condition for stability of floating body regardless to the usage, the body (ship) has to float. This condition is controlled by Archimedes's law or principle. The stability of floating does not have stability issue unless it is in a small scale or the density isAs almost alw transitional[1]. That is, once the body is floating, unless the body density is changed. However, in under what conditions or angle a floating body will be in equilibrium is the question of ship or body stability. Floating bodies do not sink if their density is below the liquid density because lack of force or moment that cause it. On the contrary, when body is pushed down below the stability point there is a net force that push the body up and vice versa. However, the body is placed on the liquid only under certain conditions the body will remain in initial position. For example, putting extruded wood square in water will tilt to 45° (try it).

A body is placed over a solid surface will remain in the same conditions if the body is placed only, at minimum, the center gravity is over the base. The situation with solid surface is very intuitive as most readers experienced it (for example for extruded square there are 4 angles of stability). As almost always, placing bodies on liquid the stability points are complicated determined. The stability point on solid surface is different that the same body has on liquid. Even the number of stability points can be different between solid surface and liquid. For example, extruded T–shape body can 4 stability points on solid surface (orientation) while in liquid it can only one. As opposed the solid case, in liquid the ratio of the densities affects the stability. The experience has shown that the stability can be plotted as ratio of geometrical parameters as a function

[1]Lautrup suggested this possibility.

of the ratio of solid density and liquid density as shown in a typical stability diagram (see Fig. 2.1).

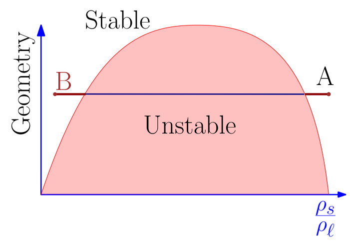


Fig. 2.1 – Typical stability diagram for solid bodies.

It is amazing that for a certain geometry the ship or floating body is stable (see point A in the Fig. 2.1) at one point and if the density ratio changes it become unstable (the line enters into the dome) and if further change in the density ratio occurs it become stable again (the line leaves the dome). This point is not obvious (at least it was not clear to this author when he took ship stability class in high school). In fact, ships have different scales for salt water and fresh water (they have different densities, and they should have different scale for different temperatures). First, this diagram will be introduced and second in the book it will be explained how to construct it.

This book is organized by building on the foundations like what is centroid and how to calculate it. Additional important fundamental concept will be introduced: the moment of inertia. The buoyancy will be introduced and analyzed. Later, these ideas will be used in the calculation of the ship stability (and floating body and in this book the word ship stability should be interpreted as floating bodies as well.) As all the books by this author, the books starts with history of ship stability.

2.2 History of Ship Stability

The history of the stability analysis is reflective of general physics and fluid mechanics science. A good summary is given by (Nowacki and Ferreiro 2003) but lacking major developments that occurred in the last 30 years. The highlights of stability analysis research show that it was important topic for a long time. Clearly having ship that does not flipped in the sea (or other water body) was important since the early time. The test was done by some individuals moving on the floating body to examine how the stable is the ship. The real understanding of the stability is tied to more advance mathematics and fluid mechanics there was no ability to examine this

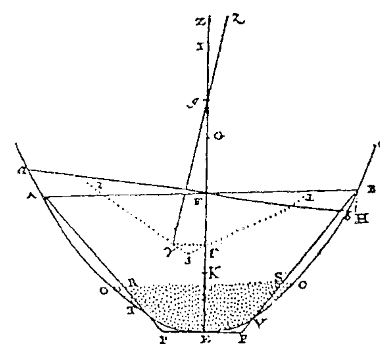


Fig. 2.2 – Bouguer Showing Metacenter.

issue. For example, Archimedes did not know about the concept of pressure hence he lack a major tool in his understanding. The early work was done by Huygens (Huygens 1967) by that time the concept of pressure and some knowledge of early calculus was available. Even the concept of "specific gravity" (specific density) was introduced by that time (density was introduced 1586 by Simon Stevin). Stevin also discovered that the forces (gravity and buoyancy) have to act in the same line as prerequisite for stability. French mathematics Paul Hoste, (1652-1700) made attempt to tackle the stability problem but fail because did know about the calculus. The famous case of the Swedish flag ship **Vesa** shown on the cover page during her maiden voyage was turned to a no–return–point and eventfully drawn. The king Gustavus Adolphus demands on ship's cannons were so high and heavy that it was ridiculously absurd showing the contempt people had to science.

Euler was requested by the Russian (at the time he was Russian Tzer kids tutor, what a lucky students) to review the work of La Croixs work (Euler 1735; Euler 1736). As usual money was the reason pushing the science forward. That was the age of discovery and ability to project power especially with a marine power was essential. During that era the ship's gunport was developed. The need to find the water line and maximum turning point before water get into the ship were important. Hence the importance of developing the science behind the stability.

Pierre Bouguer French Hydrologist (fluid mechanics) got his father royal professor post at age of 15 after his father pass away (must be very smart kid). He improved the numerical integration methods (trapezoid method)[2]. Later he derived the Metacenter concept (Bouguer 1746) see Fig. 2.2. This Metacenter method is the most used method today. Yet, when one tries to use it, it is found to be complicated and graphical representation (or numerical modeling) is commonly required. If the conversion to moment of inertia is utilized then the body must be smooth.

As results, another method namely the potential/energy principle is or could be used. In this method the energy or the potential of the system is written and utilized to find stability points. This technique was first proposed by Huygens and again because lack of calculus developed at that time he failed to work it out the technical details.

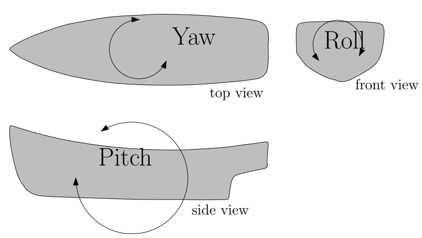


Fig. 2.3 – Typical rotation of ship/floating body.

Paul Erdös et al was the first (this author is not aware any other one who worked the to solve this problem unilizing potential energy) to have used this approach successfully (Erdös, Schibler, and Herndon 1992). Amazingly the authors were not aware the centroid calculations are well established topic and used complex integral calculations to find the centroid of trapezoid (and these calculations were done 1992!). Additionally they have made some nonessential assumptions which Mohammad Abol-

[2]This method is widely used in stability study even though there are simpler and better methods like Simpson's rule

hassani was able to fix. The calculations of centroid were not explained in the last paper (Abolhassani 2004). The potential method will be explain briefly later on. This approach utilizes mathematics without the ability to see or examine what cause what and why. In way the method abstract the physics and convert it a pure mathematical creation. The method is seeking to find the angle(s) for which the shortest vertical distance between buoyancy centroid and the gravity/mass centroid. Numerous mathematical papers (dealing with the mathematics) where published later dealing with abstract. It is the opinion of this undersign that many of these papers are without any real meaning to the stability of floating body. It is interesting to point out that because lack of physical observation ability or because the underline the equilibrium analysis it was assumed that it is a dimensionally compartmental. For floating bodies the stability is compartmental under very unique cases where the body is symmetrical and extruded. For example, using marine terminology, roll rotation creates yaw rotation because change of centroid location in x,y, and z directions.

The newest approach is Direct Examination approach and it is suggested by this undersign. The Metacenter method is probably the closest to the Direct Examination.

Example 2.1: Why G is above B **Level: Easy**

In the illustration 12.2 depicts G above B. Explain why at equilibrium stage the G and B must be in the same vertical action line.

Solution

One of the favorite question that this undersign bring to engineers. Assume that G is not the same vertical action line as the B. In that case, a moment is created and the body will rotate until G and B will on the vertical action line.

2.2.1 Recent Developments

Some developments that happened recently will be presented in this section. Among the recent developments of the ship stability is the stability diagram. The stability diagram is important tool showing the stability of a certain geometrical body. The investigation of history this diagram is not conclusive. While it is too personal for this author, as he build these diagrams and was sure that he is the first (it is a disappointment to find that someone invent them head of you). However, it seem that the credit should go toward to Lautrup (Lautrup 2011). While the coordinate system presented in Lautrup book is different (there is only one) but the concept is somewhat similar. Lautrup did not realized that this typical diagram and it can be build for different geometries and maybe he did not attempt to build for other geometries. Another advance Lautrup should be credited for is the solution of the principle axis.

All the investigation (research) on the ship stability was under the assumption of compartmental effect[3]. For example the potential energy implemented by Erdös (Erdös, Schibler, and Herndon 1992) was work under the assumption of compartmental effect. This separation is not correct and was demonstrated in (Bar-Meir 2021).

Beside the potential energy, this undersigned developed the Direct Examination method which is simpler to understand and implement (Bar-Meir 2021). Additionally, the author introduces the dimensional analysis to this area. There are quite new advances in this book and recently developed and they will be discussed here.

[3]If you aware of such work, please notify this author.

3
Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation

4
Marine Bodies Definitions

4.1 Opening Remarks on Definitions

Marine nomenclature is dominating (obviously) the ship or the floating bodies stability. While trying to find what is known and what is still black art, this author conclude that the terminology is used to complicate the topic. Furthermore, it seems that many books on ship stability written is such jargon that it is hard to read them especially if the reader is out the field. In way the chapter is a kind of dictionary or key that is provided to be able to read books in the area.

Hence, in this chapter presents a literature survey on the nomenclature. In this book a minor adaptation of some of this terminology. This old terminology is entrenched and appear as ancient relic of outdated science frozen for 300 years. Some terms were changed to reflect the advances in science or specifically the new technology developed here. Furthermore, this material is backported into other books and hence the terminology of the wider field is preferred. In a typical stability analysis (in fluid mechanics and solid mechanics) there is a typical disturbance which result in change of the field (where the disturbance grows or decayed). In ship stability, the metacenter method does not have such concept. For example, for a stability indication is whether GM positive or negative but further more the analysis assume that the metacenter is fixed regardless to the disturbance θ. Therefore, for this author, who start his career thermofluid with a ship stability class, is forced to modify many terms.

Yet, the terms like left or right of the ship are not used but port and starboard are used adapted here because they do not conflict with the standard fluid dynamics field. These terms can trace their root to Vikings (Hendrickson 2008).

The main definitions are dealing with sizes and not naming. The first definition is dealing with volume of a ship. The width of ship is denoted as b and height of ship is D and the length of the ship is s. Notice the lower case d is only part of the D and it is the distance from bottom to the liquid level[1]. In some books this quantify is denoted as T (not in this book). The displacement is volume displaced by the ship or the floating body denoted as V_0. The encompassing volume

$$V_d = d\, b\, s \tag{4.1}$$

Note that some books refers to the displacement as the displaced mass or the displaced weight (yes, these strange definitions are common).

4.1.1 Salinity and temperature effect

In the ship industry the minute change of density due to salinity and/or temperature (about 2%) appear in approximation as

$$\frac{d_{sal}}{d_{fresh}} \sim \frac{\rho_{fresh}}{\rho_{sal}} \tag{4.2}$$

The fresh water density is approximated as 1000 $[kg/m^3]$ and the density of sea water is approximated as 1025 $[kg/m^3]$. The temperature has small effect and yet also appear as seasonal. In other words, the seasonal refers to Summer, Winter, etc. Eq. (4.2) can be derived from Archimedes's formula (will be covered in Buoyancy chapter). It has pointed that temperature play role nevertheless the changes are negligible.

4.1.2 Load Lines

Load lines are marked on a vessel's side for various conditions water salinity, and temperature. Yet, the common in the industry to make them on the ship

[1]Notice that in this book the term liquid is used and not water. The reason for this is to indicate that apply to any kind of liquid and limited to fresh or sea water.

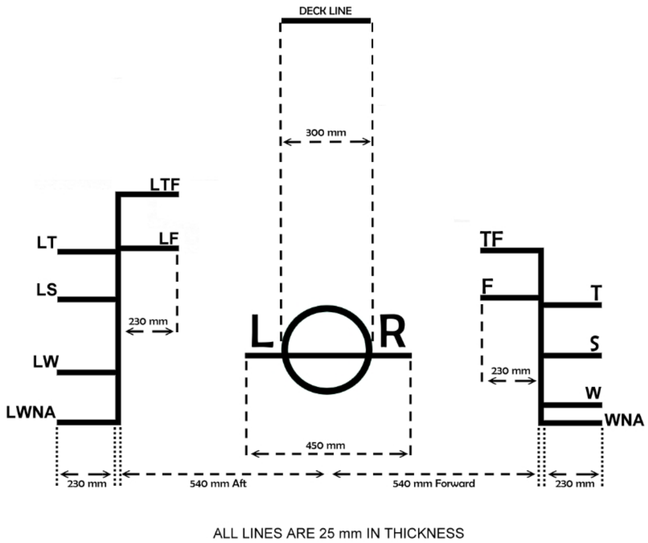


Fig. 4.1 – The first International Load Line Convention took place in 1930 after which, continue to amended to latest in 2003. Load lines based international Load Line Convention after wikimedia.org.

0.9 From the top with smaller density from the top: TF is tropical fresh water, F for fresh water, T is for tropical seawater, S is based and the summer seawater, W is the winter seawater, and WNA is winter North Atlantic. The major part of the standard is dealing the thickness of the lines etc. One can wonder how this relic still in use. The answer to that has to do with historical validation and people respect things that have been used for long.

4.1.3 Tonne Per Centimeter

This definition deals with the amount weight that increase the weight of the ship per centimeter. It is strange that this definition is not defined in a scientific way like the following

$$TPC_{proper} = \frac{dW}{dh} \tag{4.3}$$

In this case, dh refers to the infinitesimal height element and dW is the ship weight element. Regardless the definition, the assumption is that the cross area (parallel liquid surface) does not change much with the height (in other words, cross area is almost constant). In fact, this definition should be a function (example of how outdated material).

There two extreme cases of displacements one: Light Displacement two: Load Displacement is the weight of the liquid displaced by a ship when at summer time in salt water for maximum load (conflicting requirements, salt and temperature pushing to different directions). The ship's displacement lay between these two extremes. The light displacement included fuel, stores, ballast etc which referred as "deadweight". Deadweight Coefficient describes the effectiveness of the ship. It is defined as

$$C_{DW} = \frac{Light\ Displacement}{Displacement} \tag{4.4}$$

This value is a dimensionless number. Basically it describe the useless fraction of the ship weight. The typical values are

Table 4.1 – Typical values for dead weight coefficient.

Vessel Type	C_{DW}	**Vessel Type**	C_{DW}
General Cargo Ship	0.7	Carry Wheeled Cargo	0.3
Oil Tanker	0.83	Container Ship	0.6
Bulk Carrier	0.82	Passenger Liners	0.37
Gas Tanker	0.62	Cross-channel Ferry	0.2

Some of these vessels type carry heavy weight and it is very wasteful.

The block coefficient is defined as the ratio between the actual volume to encompassing rectangular (extruded rectangular).

$$C_b = \frac{V_0}{D\,b\,s} \tag{4.5}$$

C_b is used for application and regulations of vessels. The coefficient indicates how "smooth" the ship is. The ship displacing volume is denoted V_0 and is defined as

$$V_0 = C_b\,D\,b\,s \tag{4.6}$$

Another relic from the past is the Prismatic Coefficient which is the ratio of

$$C_p = \frac{V_0}{s\,A_m} \tag{4.7}$$

This coefficient represents the width of the ship if the mid–section was expended to full length of the ship. Notice that the shape encompassing the ship is not rectangular. The Maximum Sectional Area Coefficient is defined as

$$C_m = \frac{A_m}{b\,D} \tag{4.8}$$

Example 4.1: C_d calculations **Level: Easy**

A general cargo ship has a dead weight of 50k tonnes. Estimate the fully loaded displacement (W).

Solution

The value of C_d is given the table 4.1 is 0.7. Hence, the displacement is

$$Displacement = \frac{\text{Light Displacement}}{C_d} \tag{4.1.a}$$

Applying the data to the equation Eq. (4.1.a)

$$Displacement = \frac{50000}{0.7} \sim 71428.6\ [tonnes] \tag{4.1.b}$$

asically, the ship can carry about 21,428 [tonnes].

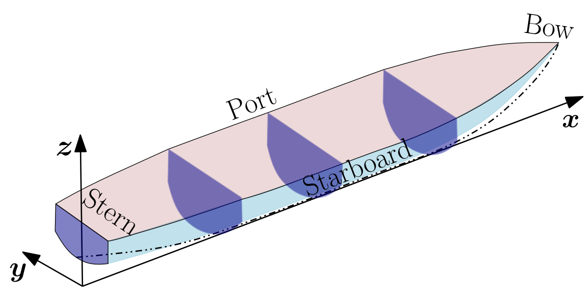


Fig. 4.2 – Ship Coordinates System suggested by DIN 81209-1.

Strangely there is a recommendation for coordinate system which hopefully will be for entire thermofluid field. However there is differences between the various standards. For example, ISO 7460 and 7463 defined different origin as compare to DIN 81209-1. In this book, this recommendation is not followed because it it does not compatible with presentation of the material. The concept of the system used by the context. It is assumed that this principle foreign to these who write the standards. For example, in two dimension it is convenient to use x, y coordinate while in three dimension is should be x, z. This is the system adapted in this book.

General coordinate system is attached to the floating body or ship. So, ship with fixed z–coordinate fixed to liquid z–coordinate is referred to as upright condition. A rotation of the z y plain are referred as heel or roll depending on the variance with time. Even keel is referred to the situation in which x–coordinate is parallel to the liquid level. Nautical terminology depreciates pitch and trim (see Fig. 12.3). If the inclination is dynamic is pitch and more or less static it is trim. The trim is defined positive when the ship bow is up and referred as trimmed by the head. The bilge region is at the bottom and there are several definitions for it.

4.1.4 Ship Length

While this books does not deal with ship design several points has to mention about ship's dimensions. The ship width is limited by the cost and the space available (canal limitations). The ship length is determined by cost and need but limited by space at designated ports. In same way the draft (depth) depends on the limited space with the exception of the large tanker which can load and unload outside the port.

The measuring how much ship is way from the "box" shape is

$$C_M = \frac{A_M}{b\,D} \tag{4.9}$$

The center cross section is referred as midship-section C_M is referred as midship coefficient. The compactness of ship is defined as

$$C_P = \frac{V_0}{s\,A_M} \sim \frac{C_b}{C_M} \tag{4.10}$$

Where A_M is the wetted area of the midship cross section. Another definition to measure compactness of the surface area at the liquid level as

$$C_{ls} = \frac{A_{ls}}{s\,b} \tag{4.11}$$

Where Liquid Surface Coefficient is the measure of compactness relative recommendation area at liquid surface and some define denotes as C_w. The liquid cross area, A_{ls}, is the area at the surface of the liquid. For completeness, addition has be mentioned as Prismatic Coefficient.

4.1.5 Ship Length

The design of the ship/boat is art more than science as indication shown that a new design is based on the old and tested design rather design totally from fresh. yet, some engineering principles can be utilized. The displacement is related to the ship length yet ships, especially smaller one, heaving a complex shape which the relationship has to account for these effects.

$$s \sim \sqrt[3]{V_0} \longrightarrow s = f(\cdots)\,\sqrt[3]{V_0} \tag{4.12}$$

The logical assumption is that the correction factor which is related to deviation factors from the extruded rectangular. These deviation factors C_b, C_M, C_P were defined earlier. The value of these is given by experience and no science but black art is used. Notice that in these calculations the actual ship width and the ship depth are unknown. The relationship The ratio of the length, s/b is determined from the requirement on the resistance to the flow (ship speed). The ratio of b/D is dictated by the stability consideration (more this issue in the book). With deviation factors given above the length of a ship can be estimated for given dead weight, DW (which related to V_0) as

$$s = \sqrt[3]{\frac{DW}{\rho\, g}}\, f(C_b, C_D) \tag{4.13}$$

There are several formulas are suggested in the literature. No information on the scientific nature of the development is equations was provide and here none can be recommended. An example of such formula is

$$s = \sqrt[3]{\frac{DW \left(\dfrac{s}{b}\right)^2 \left(\dfrac{b}{D}\right)}{\rho\, g\, C_b\, C_D}} \tag{4.14}$$

A dimensionless parameter known as Froude Displacement Coefficient to measure the stretchiness (resistance to the flow) of the vessel and is

$$Fr_D = \frac{s}{\sqrt[3]{V_0}} \tag{4.15}$$

The Froude is range from 17 for racing boat, racing yacht 7, container ship about 6.5 and Seventeenth-century first rate of 4. Beside this Froude number there additional number have the same name and defined as

$$Fr = \frac{U}{\sqrt{g\, s}} \tag{4.16}$$

Heaving is vertical movement of the ship. Sway is the linear movement in the y direction. Surge is the linear movement in the x direction. If the body is symmetrical than these movements do not affect each other. However, if the body is not a symmetrical they affect each other. Furthermore, all these moments creates all the rotational moments. The base for the interaction between these movements is clear but the calculation is beyond scope of this book. Estimate this author done on this effect show that it about percent of order of magnitude. While this number sound insignificant, in real life this can be a significant amount.

4.2 Special Dimensions of the Ships

This section is tied with historical naming and such depend on school of thought and no real uniform system is agreed. The length of the ship is defined by some as the length at the water line. The same can said on the ship width. The curvature of ship like round shape of the deck (to make sure that the water are deflected to the sea) is referred as camber. The arc created by the ship floor is referred as "bilge." The bilge exist for the several reasons, like collection the water (or liquids), strength, and etc. The other curvature surfaces are created for structure reason. However, the flare can help with the stability.

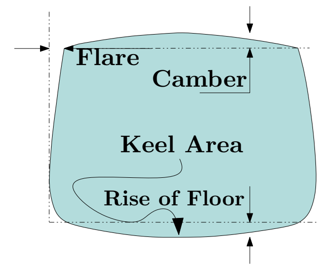


Fig. 4.3 – Mid ship cross section to illustrate various definitions.

4.3 Summary

This chapter deals with topics related to ship structure and design. For example, different ships were introduced and various international standards discussed. These definitions are indirectly related to ship stability. This discussion was add to the book because there was a demand for it. Moreover, the main usage of the stability is for the marine industry. Some of the material related to dimensional analysis but it is covered because the dimensionless aspects are not the main issue.

5

The Mathematical Background for Stability

In this chapter is review for most readers and new for others and intend to cover minimum topics related to stability of floating body. These topics are present so that one with some background in trigonometry could deal with the mathematics that encompass in this book. Hence without additional reading, this book could be used by most readers.

5.1 Differentiation

What is differentiation and why it is important? In trigonometric there is a function call tangent for short $\tan$, that reader should know, the ratio the sides of triangle. A parabolic function shown Fig. 5.1 has a point x, x^2 denoted by the red point. To find the tangent to the function a triangle is build starting at red point. The second point is at $(x + \Delta x, x^2)$ and the third is on the graph vertically from second point denoted by green point. Clearly the line connecting the second and third point is above the graph (the purple point). To minimize the distance between the purple line (see the figure)

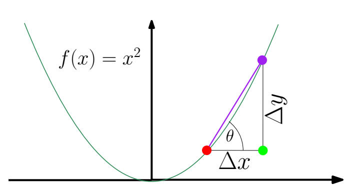


Fig. 5.1 – Parabola to explain derivative. Not to scale and point the triangle starts is at (x, x^2) (red point)

and the graph, the triangle has to shrink. The tangent of the angle of the triangle at it is

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

The value of Δy an be ascertained the difference between the function value at x and $x + \Delta x$ as

$$\Delta y = f(x + \Delta x) - f(x) = (x + \Delta x)^2 - x^2$$

The first term on right hand side can be expand as

$$\Delta y = \cancel{x^2} + 2\,x\,\Delta x + (\Delta x)^2 - \cancel{x^2} = 2\,x\,\Delta x + (\Delta x)^2$$

The tangent in this case is

$$\tan\theta = \frac{2\,x\,\Delta x + (\Delta x)^2}{\Delta x} = 2\,x + \Delta x$$

When Δx become smaller and smaller (about zero) the value of the tangent is $2x$. This example of differentiation explains the concept and it has been extend. In this book the concept was explained and other derivative of other functions should obtained from table or other sources. There is no need to emphasis here. The reason that differentiation is extremely important because almost all the calculations in stability requires understanding this concept. Ultimately, most of the calculations ended to simple arithmetic. The derivative are denote as dy/dx and they are actually $\tan\theta$.

$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{5.1}$$

The tangent can be used to evaluate the at some distance from point x as

$$f(x + \Delta x) \approx f(x) + \Delta x \overbrace{\frac{dy}{dx}}^{\tan\theta} \tag{5.2}$$

It must be note that a short notation of derivative is $f'(x)$ or even shorter hand is simply f'.

5.2 Integration

The integration is calculation of the area under the curve. The area is made of many trapezoids as shown in Fig. 5.2. Without any proof but simply using the intuition integration is opposite of differentiation. The explain to that is based on the following. Note the explanation is not rigorous but just the underline. The integration is denoted by the symbol $\int$. For example, the function (F) can be defined as

$$F(x) = \int_0^x f(t)dt$$

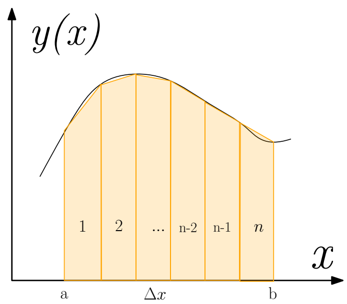


Fig. 5.2 – The trapezoid method to explain the numerical integration.

In this case, the notation of dt and t is just a dummy variable and the integral from point 0 to point x. Integration is done by adding small rectangular shape thus,

$$F(x+\Delta x) = \int_0^x f(t)dt + \overbrace{\int_x^{x+\Delta x} f(t)dt}^{\sim \Delta x\, f(x)} \tag{5.3}$$

For small distance of Δx the value of the function is constant. Hence, the second integral can be written as over the brace was the width and height given.

$$F(x) + \Delta x\, F'(x) \approx \int_0^x f(t)\, dt + \int_x^{x+\Delta x} f(t)\, dt \tag{5.4}$$

The first term on the left is equal to first term on the right. Thus, the second the term on left has to equal to second of the left

$$\cancel{\Delta x} F'(x) \approx \cancel{\Delta x} f(x) \tag{5.5}$$

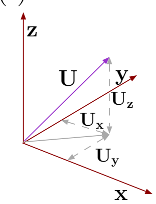


Fig. 5.3 – Vector in Cartesian coordinates system.

Thus $F'(x) \approx f(x)$ and this consider fundamental equation or law of calculus. To find the integral of function is done by "guessing." For example, it was found before that the derivative of x^2 is $2x$. Hence, the integral of $2x$ is x^2.

This a brief introduction to calculus and no mean it meat to be complete and comprehensive. Yet, the reader should be able to understand some of the mathematical explanation.

5.3 Vectors and Coordinates System

The are many coordinate systems which are used to describe location or direction. In this book two systems will be discussed, Cartesian, Cylindrical coordinate. The Cartesian system describes the location with three linear coordinate normally call, x, y, z. The

Cylindrical coordinate measures with a distance from the center on the plane and the angle and z.

Vector is a quantity with direction as oppose to scalar. The length of the vector in Cartesian coordinates (the coordinates system is relevant) is

$$\|\boldsymbol{U}\| = \sqrt{{U_x}^2 + {U_y}^2 + {U_z}^2} \tag{5.6}$$

Vector can be normalized and in Cartesian coordinates depicted in Figure 5.3 where U_x is the vector component in the x direction, U_y is the vector component in the y direction, and U_z is the vector component in the z direction. Thus, the unit vector is

$$\widehat{\boldsymbol{U}} = \frac{\boldsymbol{U}}{\|\mathrm{U}\|} = \frac{U_x}{\|\boldsymbol{U}\|}\hat{\boldsymbol{i}} + \frac{U_y}{\|\boldsymbol{U}\|}\hat{\boldsymbol{j}} + \frac{U_z}{\|\boldsymbol{U}\|}\hat{\boldsymbol{k}} \tag{5.7}$$

and general orthogonal coordinates

$$\widehat{\boldsymbol{U}} = \frac{\boldsymbol{U}}{\|\mathrm{U}\|} = \frac{U_1}{\|\boldsymbol{U}\|}\mathrm{h}_1 + \frac{U_2}{\|\boldsymbol{U}\|}\mathrm{h}_2 + \frac{U_3}{\|\boldsymbol{U}\|}\mathrm{h}_3 \tag{5.8}$$

Vectors have some what similar rules to scalars which will be discussed in the next section.

5.3.1 Vector Algebra

Vectors obey several standard mathematical operations which are applicable to scalars. The following are vectors, $\boldsymbol{U}$, $\boldsymbol{V}$, and $\boldsymbol{W}$ and for in this discussion a and b are scalars. Then the following can be said

1. $(\boldsymbol{U} + \boldsymbol{V}) + \boldsymbol{W} = (\boldsymbol{U} + \boldsymbol{V} + \boldsymbol{W}) = \boldsymbol{U} + (\boldsymbol{V} + \boldsymbol{W})$
2. $\boldsymbol{U} + \boldsymbol{V} = \boldsymbol{V} + \boldsymbol{U}$
3. Zero vector is such that $\boldsymbol{U} + \boldsymbol{0} = \boldsymbol{U}$
4. Additive inverse $\boldsymbol{U} - \boldsymbol{U} = 0$
5. $a\,(\boldsymbol{U} + \boldsymbol{V}) = a\boldsymbol{U} + a\boldsymbol{V}$
6. $a\,(b\boldsymbol{U}) = a\,b\boldsymbol{U}$

The multiplications and the divisions have somewhat different meaning in a scalar operations. There are two kinds of multiplications for vectors. The first multiplication is the "dot" product which is defined by equation (5.9). The results of this

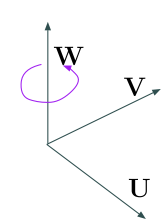


Fig. 5.4 – The right hand rule, multiplication of $\boldsymbol{U} \times \boldsymbol{V}$ results in $\boldsymbol{W}$.

multiplication is scalar but has no negative value as in regular scalar multiplication.

$$\boldsymbol{U}\cdot\boldsymbol{V} = \overbrace{|\boldsymbol{U}|\cdot|\boldsymbol{V}|}^{\text{regular scalar multiplication}} \cos\overbrace{\left(\angle(\boldsymbol{U},\boldsymbol{V})\right)}^{\text{angle between vectors}} \tag{5.9}$$

The second multiplication is the "cross" product which in vector as opposed to a scalar as in the "dot" product. The "cross" product is defined in an orthogonal coordinate ($\widehat{h}_1$, $\widehat{h}_2$, and $\widehat{h}_3$) as

$$\boldsymbol{U}\times\boldsymbol{V} = |\boldsymbol{U}|\cdot|\boldsymbol{V}|\sin\overbrace{\left(\angle(\boldsymbol{U},\boldsymbol{V})\right)}^{angle}\widehat{\boldsymbol{n}} \tag{5.10}$$

where θ is the angle between $\boldsymbol{U}$ and $\boldsymbol{V}$, and $\widehat{\boldsymbol{n}}$ is a unit vector perpendicular to both $\boldsymbol{U}$ and $\boldsymbol{V}$ which obeys the right hand rule. The right hand rule is referred to the direction of resulting vector. Note that $\boldsymbol{U}$ and $\boldsymbol{V}$ are not necessarily orthogonal. Additionally note that order of multiplication is significant. This multiplication has a negative value which means that it is a change of the direction.

One of the consequence of this definitions in Cartesian coordinates is

$$\widehat{\boldsymbol{i}}^2 = \widehat{\boldsymbol{j}}^2 = \widehat{\boldsymbol{k}}^2 = 0 \tag{5.11}$$

In general for orthogonal coordinates this condition is written as

$$\widehat{\boldsymbol{h_1}}\times\widehat{\boldsymbol{h_1}} = \widehat{\boldsymbol{h_1}}^2 = \widehat{\boldsymbol{h_2}}^2 = \widehat{\boldsymbol{h_3}}^2 = 0 \tag{5.12}$$

where $\boldsymbol{h_i}$ is the unit vector in the orthogonal system.

In right hand orthogonal coordinate system

$$\begin{array}{ll} \widehat{\boldsymbol{h_1}}\times\widehat{\boldsymbol{h_2}} = \widehat{\boldsymbol{h_3}} & \widehat{\boldsymbol{h_2}}\times\widehat{\boldsymbol{h_1}} = -\widehat{\boldsymbol{h_3}} \\ \widehat{\boldsymbol{h_2}}\times\widehat{\boldsymbol{h_3}} = \widehat{\boldsymbol{h_1}} & \widehat{\boldsymbol{h_3}}\times\widehat{\boldsymbol{h_2}} = -\widehat{\boldsymbol{h_1}} \\ \widehat{\boldsymbol{h_3}}\times\widehat{\boldsymbol{h_1}} = \widehat{\boldsymbol{h_2}} & \widehat{\boldsymbol{h_1}}\times\widehat{\boldsymbol{h_3}} = -\widehat{\boldsymbol{h_2}} \end{array} \tag{5.13}$$

The "cross" product can be written as

$$\boldsymbol{U}\times\boldsymbol{V} = (U_2\,V_3 - U_3\,V_2)\,\widehat{\boldsymbol{h_1}} + (U_3\,V_1 - U_1\,V_3)\,\widehat{\boldsymbol{h_2}} + (U_1\,V_2 - U_2\,V_1)\,\widehat{\boldsymbol{h_3}} \tag{5.14}$$

Equation (5.14) in matrix form as

$$\boldsymbol{U}\times\boldsymbol{V} = \begin{pmatrix} \widehat{\boldsymbol{h_1}} & \widehat{\boldsymbol{h_2}} & \widehat{\boldsymbol{h_3}} \\ U_2 & U_2 & U_3 \\ V_2 & V_2 & V_3 \end{pmatrix} \tag{5.15}$$

The most complex of all these algebraic operations is the division. The multiplication in vector world have two definition one which results in a scalar and one which results in a vector. Multiplication combinations shows that there are at least four possibilities of combining the angle with scalar and vector. The reason that these current combinations, that is scalar associated with $\cos\theta$ vectors is associated with $\sin\theta$, is that these combinations have physical meaning. The previous experience is that help to define multiplication help to definition the division. The number of the possible combinations of the division is very large. For example, the result of the division can be a scalar combined or associated with the angle (with $\cos$ or $\sin$), or vector with the angle, etc. However, these above four combinations are not the only possibilities (not including the left hand system). It turn out that these combinations have very little[1] physical meaning. Additional possibility is that every combination of one vector element is divided by the other vector element. Since every vector element has three possible elements the total combination is $9 = 3 \times 3$. There at least are two possibilities how to treat these elements. It turned out that combination of three vectors has a physical meaning. The three vectors have a need for additional notation such of vector of vector which is referred to as a tensor. The following combination is commonly suggested

$$\frac{\boldsymbol{U}}{\boldsymbol{V}} = \begin{pmatrix} \dfrac{U_1}{V_1} & \dfrac{U_2}{V_1} & \dfrac{U_3}{V_1} \\ \dfrac{U_1}{V_2} & \dfrac{U_2}{V_2} & \dfrac{U_3}{V_2} \\ \dfrac{U_1}{V_3} & \dfrac{U_2}{V_3} & \dfrac{U_3}{V_3} \end{pmatrix} \tag{5.16}$$

One such example of this division is the pressure which the explanation is commonality avoided or eliminated from the fluid mechanics books including the direct approach in this book.

This tenser or the matrix can undergo regular linear algebra operations such as finding the eigenvalue values and the eigen "vectors." Also note the multiplying matrices and inverse matrix are also available operation to these tensors.

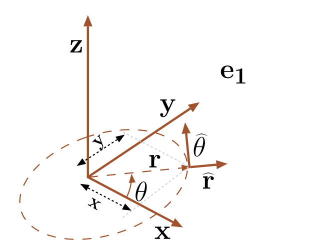


Fig. 5.5 – Cylindrical Coordinate System.

5.3.2 Cylindrical Coordinates

The cylindrical coordinates are commonality used in situations where there is line of symmetry or kind of symmetry. This kind situations occur in pipe flow even if the pipe

[1]This author did find any physical meaning these combinations but there could be and those the word "little" is used.

is not exactly symmetrical. These coordinates reduced the work, in most cases, because problem is reduced a two dimensions. Historically, these coordinate were introduced for geometrical problems about 2000 years ago[2]. The cylindrical coordinates are shown in Figure 5.5. In the figure shows that the coordinates are r, θ, and z. Note that unite coordinates are denoted as $\widehat{r}$, $\widehat{\theta}$, and $\widehat{z}$. The meaning of $\overrightarrow{r}$ and $\widehat{r}$ are different. The first one represents the vector that is the direction of $\widehat{r}$ while the second is the unit vector in the direction of the coordinate r. These three different rs are some what similar to any of the Cartesian coordinate. The second coordinate θ has unite coordinate $\widehat{\theta}$. The new concept here is the length factor. The coordinate θ is angle. In this book the dimensional chapter shows that in physics that derivatives have to have same units in order to compare them or use them. Conversation of the angel to units of length is done by length factor which is, in this case, r. The conversion between the Cartesian coordinate and the Cylindrical is

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan\frac{y}{x} \qquad z = z \tag{5.17}$$

The reverse transformation is

$$x = r\,\cos\theta \qquad y = r\,\sin\theta \qquad z = z \tag{5.18}$$

The line element and volume element are

$$ds = \sqrt{dr^2 + \left(r\,d\theta\right)^2 + dz^2} \qquad dr\,r\,d\theta\,dz \tag{5.19}$$

5.4 Numerical Analysis

The integration and the derivation sometimes cannot or hard to perform. In fact, the first integration method was introduced by the father of modern, Bouguer, Pierre namely the trapezoid method. First, the differentiation will be introduced[3]. The concept numerical differentiation is direct as

$$\frac{\Delta y}{\Delta x} \sim \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{\Delta x} \tag{5.20}$$

This method is called a forward differentiation or one-sided differentiation. The similar methods is backward differentiation is

$$\frac{\Delta y}{\Delta x} \sim \frac{y_1 - y_0}{x_1 - x_0} = \frac{y_1 - y_0}{\Delta x} \tag{5.21}$$

[2]Coolidge, Julian (1952). "The Origin of Polar Coordinates". American Mathematical Monthly 59: 7885. http://www-history.mcs.st-and.ac.uk/Extras/Coolidge_Polars.html. Note the advantage of cylindrical (polar) coordinates in description of geometry or location relative to a center point.

[3]It hard to discuss intelligently the issue of differentiation without understanding Taylor series. Taylor series will be presented hopefully in the next version. It is assumed that that they are known to the reader.

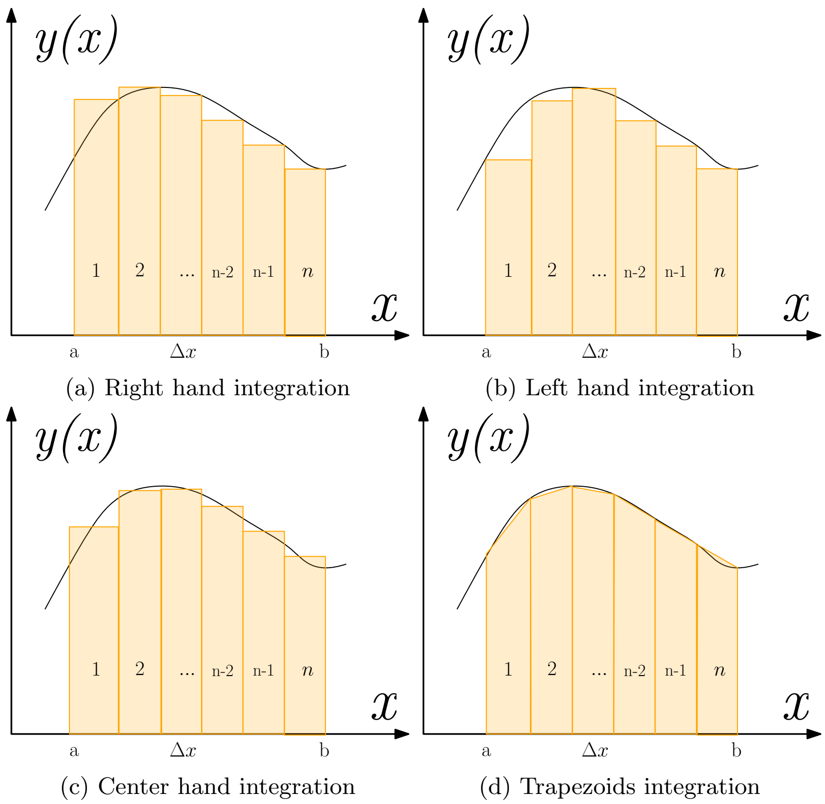


Fig. 5.6 – Four different integration methods showing the difference graphically between the actual function to the integrated area. It can be seem that Bouguer's method (trapezoid) is much better technique than these methods.

If the distance between the point is uniform then improvements can be made. Another approach is the take the two sides which sometimes referred to as centered differentiation

$$\frac{\Delta y}{\Delta x} \sim \frac{y_2 - y_0}{x_2 - x_0} = \frac{y_2 - y_0}{2\,\Delta x} \tag{5.22}$$

This equation is more precise under certain or even (most) certain circumstance.

5.5 Integration

Integration is calculating the area under the "graph." The simplest method is left or right rectangular which is basically the value of the function (data point) times $Deltax$. Better technique was invented by Bouguer, Pierre. In this methods it is assumed that the area can be calculated by a trapezoid. For practical purpose if distance between the data points is equal, the calculation is

$$A = \left(\frac{y_0 + y_n}{2} + \sum_{i=1}^{n-1} y_i \right) \Delta x \tag{5.23}$$

5.6 Simpson's Rule

A better method is what is called Simpson Rule and it should be called Kepler rule. The idea behind that methods is that the function can be represented by parabola so it close the actual function. In this case the area is even equal parts (intervals) because the nature of the method.

Simpson's Rule is just a tool and the derivation can be found on numerous places. However, actual procedure is presented here. For example if the interval is split into two sub intervals than the equation is

$$\text{Area} = (2\,\Delta x) \left(\frac{y_o + 4\,y_1 + y_2}{6} \right)$$

The minimal larger influence on the results. Simpson's rule gives you the following estimate for the area under the curve:

$$\text{Area} = (2\,\Delta x) \left(\frac{1}{6} \right) [(y_o + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \cdots + (\cdots y_n)] \tag{5.24}$$

We can combine terms here by exploiting the following pattern in the coefficients:

1	4	1				
+		1	4	1		
+				1	4	1
1	4	2	4	2	4	1

Simpson's rule get the final form

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3} \left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n \right) \tag{5.25}$$

Example 5.1: Simpson Rule **Level: Easy**

A function $y\,(x)$ is given by the table of values. Approximate the area under the function between point $x = 0$ and and the point $x = 4$ using Simpsons Rule with 4 subintervals.

x	0	1	2	3	4
y	2	7	12	10	5

Solution

For $n = 4$ subintervals, Simpsons rule is reduced into

End of Ex. 5.1

$$A = \frac{\Delta x}{3}\left[y(0) + 4\,y(1) + 2\,y\left(x_2\right) + 4\,y\left(x_3\right) + y\left(x_4\right)\right]$$

Every subintervals is

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

Using the values from the table and calculate the approximate value of the area

$$A \approx \frac{1}{3}\left[2 + 4\cdot 7 + 2\cdot 12 + 4\cdot 10 + 5\right] = \frac{1}{3}\left[2 + 28 + 24 + 40 + 5\right] = \frac{1}{3}\cdot 99 = 33 \qquad (5.1.a)$$

5.7 Summary

Mathematics is the language of science. It is hard to see how one can understand concept as stability without knowledge of calculus. While the mathematics in stability does not expand to two dimensions or even three dimensions. In the chapter dealing moment of inertia and centroid of area or volume or mass requires knowledge integration. This author experience was such that once intro was given and taken seriously the material could be understood. Mathematics is only the tool and not core what this book is focus on.

6
Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation

7 Mass Centroid

The mass (or gravity) centroid is divided into two sections, first, the mass centroid and two, area centroid (two–dimensional body with equal distribution mass). Additionally, the change of center of mass due to addition or subtraction of mass plus discrete areas are presented. The reason for the last addition, is that explain core explanation of the stability analysis.

7.1 Actual Mass Centroid

In many engineering problems, the knowledge of mass centroid is required to make the calculations especially in stability. This concept is derived from the fact that a body has a centroid (mass/gravity) which interacts with other bodies and that this force acts on the center (equivalent force)[1]. It turns out that this concept is very useful in calculating rotations, moment of inertia, etc. The mass centroid doesn't depend on the coordinate system and on the way it is calculated. The physical meaning of the mass centroid is that if a straight line force acts on the body in away through the center of gravity, the body will not rotate. In other words, if a body will be held by one point it will be enough to hold the body in the direction of the mass centroid. Note, if the body isn't be held through the mass centroid, then a moment, in additional to the force, is required (to prevent the body for rotating). It is convenient to use the Cartesian system to explain this concept. Suppose that the body has a distribution of the mass (density, ρ) as a function of the location. The density "normally" defined as mass per volume. Here,

[1](Lautrup 2011) has suggested when a body is long this statement needs corrections in variable gravity field. To encourage participation of students, it is suggested here small projects. If you a student or someone how interested in the field, and like your name mentioned, contact this author for a project discussion.

the line density is referred to density mass per unit length in the x direction.

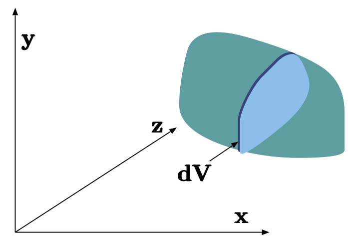


Fig. 7.1 – Description of how the center of mass is calculated.

In x coordinate, the center will be defined as

$$,\overline{x} = \frac{1}{m}\int_V x \overbrace{\rho(x)\, dV}^{dm} \tag{7.1}$$

Here, the dV element has finite dimensions in y–z plane and infinitesimal dimension in x direction see Figure 7.1. Also, the mass, m is the total mass of the object. It can be noticed that center of mass in the x–direction isn't affected by the distribution in the y nor by z directions. In same fashion the center of mass can be defined in the other directions as following

x_i of Center Mass

$$\bar{x}_i = \frac{1}{m}\int_V x_i\, \rho(x_i) dV \tag{7.2}$$

where x_i is the direction of either, x, y or z. The density, $\rho(x_i)$ is the line density as function of x_i. Thus, even for solid and uniform density the line density is a function of the geometry. When finite masses are combine the total mass Eq. (7.2) converted into

$$\bar{x} = \frac{\sum x_i\, m_i}{\sum m_i} \tag{7.3}$$

where i denotes every mass in the system.

Example 7.1: Calculation of ship centroid **Level: Easy**

An example of ship' weights is given in the following table.

End of Ex. 7.1

Table 7.1 – Ship Data for typical calcualtion of gravity ceneroid. Notice that it done in this example for z direction only

Your Ship			
Item	$z_i\,[m]$	$W_i(Weight)\,[ton]$	$z_i\,W_i[ton \times m]$
Main engines	7	2,000	14,000
Anchors and cable	12	150	1,800
Lifeboats	10	20	200
Fuel	0.75	120	80
Stores	150	15	2,250
Hull structure	9	18,000	162,000
Cargo	8	20,000	160,000
Smuggled material	15	10	150
Total	n/a	40,315	340,490

Solution

The solution is obtained by utilizing Eq. (7.1) and it is

$$\overline{z} = \frac{\sum z_i\,W_i}{\sum W_i} = \frac{340,490}{40,315} \sim 8.446[m] \tag{7.1.a}$$

7.2 Approximate Centroid of Area

The previous section dealt with the body was a three dimensional characteristics. There are cases where the body can be approximated as a two-dimensional shape because the body is with a thin with about uniform density. Consider a uniform thin body with a constant thickness shown in Figure 7.2 which has density, ρ. Thus, equation (7.1) can be transferred into

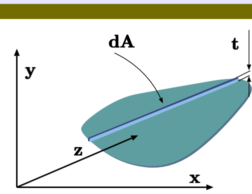


Fig. 7.2 – Thin body centroid of mass/area schematic.

$$\bar{x} = \frac{1}{\underbrace{t\,A}_{V}\,\rho} \int_V x\,\overbrace{\rho\,t\,dA}^{dm} \tag{7.4}$$

The density, ρ and the thickness, t, are constant and, therefore, can be canceled. Thus equation (7.4) can be transferred into

Approximate x_i of Center Mass

$$\bar{x_i} = \frac{1}{A}\int_A x_i dA \tag{7.5}$$

when the integral now over only the area as oppose over the volume. Eq. (7.5) can also be written for discrete areas as

$$\bar{x_i} = \frac{\sum x_i\, A_i}{\sum A_i} \tag{7.6}$$

It must be noted that area A_i can be positive or negative. The meaning of negative area in this context is subtraction of area.

7.3 Change of Centroid Location Due to Area Change

Solid body with one area added and one area removed. The old centroid marked "o" the new centroid marked "n" and area removed "r" and area added "a." This section deals with a change centroid location when an area is add or subtracted from a given area with a know centroid (or unknown). This topic is important when a centroid of area was found or previously calculated and hence only change needed. Furthermore, while the location can be recalculated for some problems, the change or its direction has more importance as it will be discussed in greater detail in, chapter on stability of floating bodies on page 113. The centroid of body in Fig. 7.3 is denoted at point "o" (old). The centroid of the added and removed areas are at points "a" (added) and "r" (removed), respectively. The point "n" (new) is the centroid after modification. A special case when the added area is equal to the subtracted area and its application will be discussed in an example below. It has to be noted that added and subtracted areas do not have to be continuous. Utilizing Eq. (7.6) for the identical areas reads for this case as

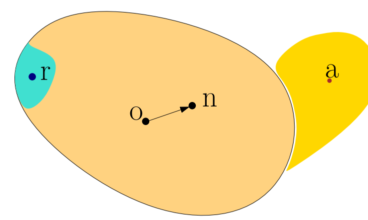


Fig. 7.3 – Solid body with removed and added area.

$$x_n = \frac{x_o\, A_o + x_r\, A_r - x_a\, A_a}{A_o + A_r - A_a} \tag{7.7}$$

In a special case where subtracted area is equal to added area ($A_r = A_a$) Eq. (7.7) is reduced to

$$x_n = x_o + x_r\,\frac{A_r}{A_o} - x_a\,\frac{A_a}{A_o} \tag{7.8}$$

Change of Centroid

$$\bar{x} - x_o = \frac{A_r}{A_o}\left(x_r - x_a\right) \tag{7.9}$$

Finding the centroid location should be done in the most convenient coordinate system since the location is coordinate independent. There should be a sign convention to determine the centroid direction movement so that the direction should be immediately expressed in the result. However, faults were found in several options that were considered[2].

[2]If you have a good method/technique please consider discussing it with this author.

Example 7.2: Center of Circle inside Circle **Level: Easy**

A circle with a radius, r has a cut out from a larger circle with radius, R where $R > r$ (see Fig. 7.4). The distance between the center of the larger circle and the small circle is x. Calculate the centroid of the circle that a smaller circle was cut out of it. Assume that x is small enough so that the small circle is whole.

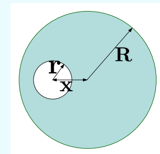


Fig. 7.4 – Subtraction of circle from a large circle for calculating the new center.

Solution

The change in the centroid is only the direction of x. It should be noted that for $x = 0$, the centroid is at $x = 0$ and $y = 0$ that is the centroid is at the center of the larger circle. For larger distance up to the $x = R - r$ the centroid can be calculated utilizing Eq. (7.7) reads

$$\Delta x = \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2} (x - 0) = x \left(\frac{r}{R}\right)^2 \tag{7.2.a}$$

Notice that x is the distance between the two centers while Δx is the change in the centroid location. Additionally, if the removed circle is not on the x coordinate then these calculations can be reused. For instance, if the cut is at angle, θ, the change will be along straight line from the center of the large circle at the distance that was obtained in Eq. (7.2.a). The conversion to a regular coordinate system could be done by utilizing simple trigonometric functions.

Example 7.3: Centroid of Trapezoid **Level: Complex**

In some situations a rectangle shape is transferred to a trapezoid shape while the area size remain constant. For example, a body floating (in liquid) has a rectangle shape. Rotation (hypothetical or real) of the floating body will result in a change to a trapezoid. This situation is used to check whether a body is stable. Assume that the rectangle body is transferred into a trapezoid with the exact same area see Fig. 7.5. Calculate the change of the centroid in the y and x directions, the total change, and the angle of change α. Note that α is measured from point (b/2,d).

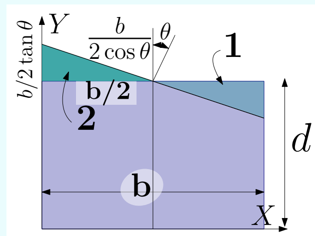


Fig. 7.5 – Trapezoid created from a tilted rectangle shape.

Solution

The rectangular centroid is located at $b/2, d/2$ (see table 3). The two triangles must be identical to keep the condition of constant area. It also implies that two triangles must start from the same point at the "center" (point $x = b/2$ and $y = d$ to be denoted Point A). Eq. (7.9) dictates that the change is due to the addition and subtraction of the triangles. The change can be expressed as

$$\Delta\left(A_i\, y_i\right) = \overbrace{\left(\not{d} + \frac{b\,\tan\theta}{6}\right)}^{y_2}\overbrace{\frac{b^2\,\tan\theta}{8}}^{A} - \overbrace{\left(\not{d} - \frac{b\,\tan\theta}{6}\right)}^{y_1}\overbrace{\frac{a^2\,\tan\theta}{8}}^{A} \tag{7.3.a}$$

continue Ex. 7.3

It can be noticed that the two triangles contribute identical value in the direction of raising the center. After simplifications and canceling Eq. (7.3.a) reads

$$\Delta\left(A_i\, y_i\right) = \cancel{2}^{1}\left(\frac{b\,\tan\theta}{\cancel{6}_{3}}\right)\times \frac{b^2\,\tan\theta}{8} = \frac{b^3\tan^2\theta}{24} \tag{7.3.b}$$

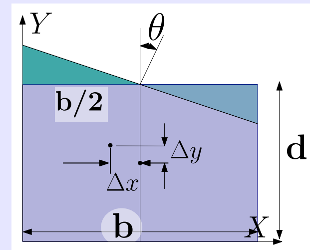


Fig. 7.6 – Mass Center Moving to New Location center. Notice that the actual size is exaggerated.

It can be observed that the net change in the y direction is

$$\Delta y = \frac{\dfrac{b^3\tan^2\theta}{24}}{\underbrace{b\,d}_{total\ area}} = b\left(\frac{b}{d}\right)\frac{\tan^2\theta}{24} \tag{7.3.c}$$

The change depends on the rectangle width and the ratio of the height to the width. The change in the x direction is

$$\Delta\left(A_i\, x_i\right) = \overbrace{\frac{5\,b}{6}}^{x_1=b-b/6}\overbrace{\frac{b^2\,\tan\theta}{8}}^{A_1} - \overbrace{\frac{b}{6}}^{x_2=1/3\times a/2}\overbrace{\frac{b^2\,\tan\theta}{8}}^{A_2}\Bigg|_{A1=A2=A} = \frac{b^3\tan\theta}{12} \tag{7.3.d}$$

This change is to the left of the old centroid. Or the net change is

$$\Delta x = \frac{b^3\tan\theta}{12}\overbrace{\frac{1}{b\,d}}^{1/A} = b\,\frac{b}{d}\,\frac{\tan\theta}{12} \tag{7.3.e}$$

Note that for small angle $\theta \sim \tan\theta$ therefore $\theta >> \theta^2$ and hence, $\Delta x >> \Delta y$. The total change Δr can expressed as a function of

continue Ex. 7.3

Δy and Δx to read

$$\Delta r = \sqrt{\Delta y^2 + \Delta x^2}$$

or in a dimensionless form as

$$\frac{\Delta r}{b} = \sqrt{\frac{\Delta y^2}{b^2} + \frac{\Delta x^2}{b^2}} \tag{7.3.f}$$

Substituting Eq. (7.3.e) and Eq. (7.3.c) into Eq. (7.3.f) reads

$$\frac{\Delta r}{b} = \sqrt{\left(\frac{b}{d}\right)^2 \frac{\tan^2\theta}{12^2} + \left(\frac{b}{d}\right)^2 \frac{\tan^4\theta}{24^2}} \tag{7.3.g}$$

rearranging the Eq. (7.3.g) provides

$$\frac{\Delta r}{b} = \frac{b}{d}\frac{\tan\theta}{12}\sqrt{1 + \frac{\tan^2\theta}{4}} \tag{7.3.h}$$

The angle is measured from a specific point. In this case the point is the connection between the two triangles. The reason for this selection is that this point will be used in stability issue. It can be noticed that angle, α, of the new center can be obtained from the straight triangle with one side of $(d/2 - \Delta y)$ and Δx.

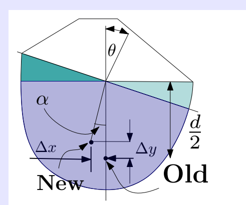


Fig. 7.7 – Movement of area center to new location due to the change of shape.

$$\alpha = \tan^{-1}\frac{\Delta x}{d/2 - \Delta y} \tag{7.3.i}$$

Substituting the the values for Δx and Δy yields

$$\alpha = \tan^{-1}\frac{b\dfrac{b}{d}\dfrac{\tan\theta}{12}}{\dfrac{d}{2} - b\,\dfrac{b}{d}\dfrac{\tan^2\theta}{24}}$$

End of Ex. 7.3

with additional simplifications

$$\alpha = \tan^{-1}\left(\frac{\frac{b^2}{d}\frac{\tan\theta}{12}}{\frac{b^2}{d}\left(\frac{d^2}{2\,b^2}-\frac{\tan^2\theta}{24}\right)}\right) \tag{7.3.j}$$

It is interesting to point out that the α is related to the ratio of d/b and to the rotating angle, θ. For small angle, $\tan\theta \cong \theta$ and $d/b \geq 1/8$ (or even smaller) Eq. (7.3.j) becomes

$$\alpha = \tan^{-1}\left(\frac{2\,\tan\theta}{12\left(\frac{d}{b}\right)^2 - \tan^2\theta^{\sim 0}}\right) \cong \tan^{-1}\left(\frac{\tan\theta}{6}\left(\frac{b}{d}\right)^2\right) \tag{7.3.k}$$

Thus under the condition of $(b/d)^2 = 6$, the change in α is equal θ. This is a limiting case which determines in certain situations whether a floating body is stable and it will be discussed in Chapter 12 on page 113.

Example 7.4: Centroid of Triangle

Level: Complex

Triangle turns to a trapezoid as shown in Fig. 7.8. Calculate the change in the centroid (in the directions of x, y, r, α) due to change of angle θ. Hint due to the symmetry, it does note matter rectangular turns to left or the right.

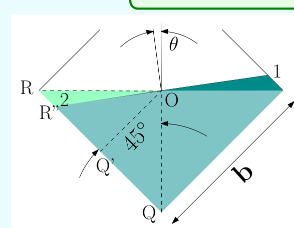


Fig. 7.8 – Triangle turns to trapezoid.

Solution

The added and subtracted areas are shown in Fig. 7.8.

$$\Delta(A_i\,x_i) = A\,(x_1 - x_2) \tag{7.4.a}$$

continue Ex. 7.4

The triangle △ROQ is an isosceles triangle with 45°. The angle ∢RR"O is $(180° - 45° - \theta)$. Law of sines can be written for △RR"O as

$$\frac{\overline{RR''}}{\sin\theta} = \frac{b/\sqrt{2}}{\sin(135° - \theta)} \tag{7.4.b}$$

$\sin(135°-\theta)$ according the trigonometry identies equal to $(\sin\theta + \cos\theta)/\sqrt{2}$ and for small angle of θ Eq. (7.4.b) becomes

$$\overline{RR''} = \frac{b/\cancel{\sqrt{2}}\,\sin\theta}{1/\cancel{\sqrt{2}}\,(\sin\theta + \cos\theta)} = \left.\frac{b\,\sin\theta}{\cancelto{1}{\cos\theta} + \cancelto{\sim 0}{\sin\theta}}\right|_{\theta\sim 0} = b\,\sin\theta \tag{7.4.c}$$

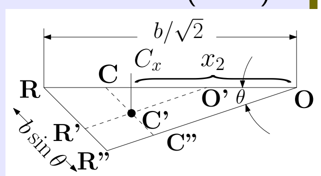


Fig. 7.9 – The centroid of the small triangle. Point C' is the centroid location. Line $\overline{R'O'}$ is parallel to base Line $\overline{R''O}$.

Note that for small θ the standard approximation for $\sin\theta$ and $\cos\theta$ were used. The height of $\triangle ROR''$ is b/2 (it is the same height as for $\triangle ROQ$ line $\overline{Q'O}$ see Fig. 7.8). Hence the area of $\triangle ROR'$ is

$$A_{\triangle ROR'} = \frac{\overline{RR'} \times \frac{b}{2}}{2} \sim \frac{b^2\,\sin\theta}{4} \tag{7.4.d}$$

x_2 can be ascertained from table 3 and other data (geometry) from Fig. 7.9. The dashed lines in the Fig. 7.9 are parallel to their respective base. Further, these lines are at one third of the distance. The distance $\overline{OR} = b/\sqrt{2}$ (It can be verified by looking at the isosceles triangle △RQ'O that has $\overline{O\,R}$ against $\pi/2$ angle.). The distance $\overline{O\,C} = 2/3 \times \overline{O\,R} = \sqrt{2}\,b/3$. The distance $\overline{C\,C''} = 2/3 \times \overline{R\,R'}$. Hence, $\overline{C\,C''} = 2\,b\,\sin\theta/3$. The line $\overline{R'\,O'}$ split the line $\overline{C\,C''}$ exactly in the middle (why? because $\overline{C''\,C'}$ is two third of $\overline{R\,R''}$). Thus,

$$\overline{C\,C'} = \frac{b\,\sin\theta}{3} \tag{7.4.e}$$

continue Ex. 7.4

The triangle △ C'C_x,C is an isosceles triangle with $\pi/2$ angle. Hence, the line $\overline{C'C_x} = \overline{CC_x}$ is $b\,\sin\theta/(3\sqrt{2})$. x_2 can be found by

$$x_2 = \overline{O\,C'} - \overline{C'\,C_x} = \frac{\sqrt{2}\,b}{3} - \frac{b\,\sin\theta}{3\sqrt{2}} \tag{7.4.f}$$

after simplification, Eq. (7.4.f)

$$x_2 = \frac{\sqrt{2}\,b}{3}\left(1 - \cancelto{0}{\frac{\sin\theta}{2}}\right) \cong \frac{2\,b}{3\sqrt{2}} \tag{7.4.g}$$

The distance in the y_2 direction is $\Delta y = \frac{a\,\sin\theta}{3\sqrt{2}}$. With the values of x_2 and y_2 the values of x_1 and y_1 are also known. Utilizing Eq. (7.9) it can be written that

$$\Delta\left(A_i\,x_i\right) = \cancelto{1}{2}\underbrace{\left(\frac{b^2\sin\theta}{\cancelto{1}{4}}\right)}_{A}\underbrace{\left(\frac{\cancelto{1}{2}\,b}{3\sqrt{2}}\right)}_{x} = \frac{b^3\sin\theta}{3\sqrt{2}} \tag{7.4.h}$$

The change in the x direction is

$$\Delta x = \underbrace{\frac{1}{b\times b/2}}_{A}\underbrace{\frac{b^3\sin\theta}{3\sqrt{2}}}_{\Delta(A_i\,x_i)} = \frac{\sqrt{2}\,b\,\sin\theta}{3} \tag{7.4.i}$$

The other direction can be obtained in similar fashioned

$$\Delta\left(A_i\,y_i\right) = \cancelto{1}{2}\underbrace{\left(\frac{b^2\sin\theta}{\cancelto{1}{4}}\right)}_{A}\underbrace{\frac{b\,\sin\theta}{3\sqrt{2}}}_{y} = \frac{b^3\,sin^2\theta}{3\sqrt{2}} \tag{7.4.j}$$

and the net change in Δy is

$$\Delta y = \underbrace{\frac{2}{b^2}}_{1/A}\frac{b^3\,sin^2\theta}{3\sqrt{2}} = \frac{\sqrt{2}\,b\,\sin^2\theta}{3} \tag{7.4.k}$$

End of Ex. 7.4

As it was shown earlier, the change in angle, α is from a straight triangle with one side is Δx and the other side $\dfrac{b}{3\sqrt{2}} - \Delta y$ which is

$$\frac{b}{3\sqrt{2}} - \Delta y = \frac{b}{3\sqrt{2}}\left(1 - 2\,\sin^2\theta\right)$$

The angle of the change is

$$\alpha = \tan^{-1}\left(\frac{\dfrac{\cancel{b}}{\cancel{3}\sqrt{2}}\left(1 - 2\,\sin^2\theta\right)}{\dfrac{\sqrt{2}\,\cancel{b}\,\sin\theta}{\cancel{3}}}\right) = \tan^{-1}\left(\frac{\left(1 - 2\,\sin^2\theta\right)}{2\sin\theta}\right) \tag{7.4.l}$$

For small angle θ Eq. (7.4.l) becomes

$$\alpha \cong \tan^{-1}\left(\frac{\theta}{2}\right) \tag{7.4.m}$$

The total change is the same as in Eq. (7.3.f)

$$\frac{\Delta r}{b} = \sqrt{\left(\frac{\sqrt{2}\,\sin\theta}{3}\right)^2 + \left(\frac{\overbrace{2\,\sin^2\theta}^{\sim 0}}{3}\right)^2} \tag{7.4.n}$$

and for small angle $\theta << 1$

$$\frac{\Delta r}{b} = \frac{\sqrt{2}\,\sin\theta}{3} \cong \frac{\sqrt{2}\,\theta}{3} \tag{7.4.o}$$

7.4 Change of Mass Centroid Due to Addition or Subtraction of Mass in 3D

This topic is extension of the previous topic of two dimensions change of centroid. All bodies are three dimensions thus when no symmetry or extrudation[3] exist the full analysis has to be done. Furthermore, it is interesting to point to the phenomenon none symmetrical body the change and be in a third dimension. This topic to be discussed in stability Chapter.

[3] The word "extrudation" means same meaning it has in blender (software) for example, a flat area that is expanded vertically.

A centroid of a slob is located in point "o" and additional mass depicted as "a" and the removed/subtracted mass "r" and again the new location of centroid is denoted as "n."

$$x_n = \frac{m_o\,x_o + m_a\,x_a - m_r\,x_r}{m_o - m_a + m_r} \tag{7.10}$$

As before the special case of equal subtracted and added material Eq. (7.10) converted into

$$x_n = \frac{m_o\,x_o + m\;(x_a - x_r)}{m_o} \tag{7.11}$$

when the density is uniform, Eq. (7.11) can be written

$$x_n = \frac{V_o\,x_o + V\;(x_a - x_r)}{V_o} \longrightarrow x_n - x_o = \frac{V}{V_o}(x_a - x_r) \tag{7.12}$$

Example 7.5: Center of Circle inside Circle **Level: Complex**

Calculate the change in the centroid of cylinder with length d,that one wedge is added and one is removed. In Fig. 7.10 describing only one wedge in additional to second side (which is not drawn in the figure.). Calculate the tiding angle, α from the center of the cylinder.

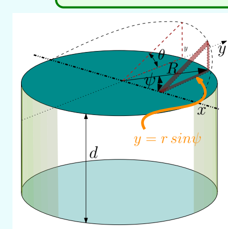


Fig. 7.10 – Mass centroid of cylinder with added wedge and subtracted wedge.

Solution

First the centroid of the wedge has to be calculated and Eq. (7.2) can be used as

$$x_i = \frac{\int x_i\,dV}{\int dV} \tag{7.5.a}$$

The ratio shown in Eq. (7.12) indicates that it is sufficient to ascertain over only half of wedge. The centroid of mass of cylinder wedge is not given in any table in this book and is not readily available other places. Hence, this work will be presented (a bit more than what should be presented in this book.). The volume

continue Ex. 7.5

of the wedge is

$$V = 2 \int_0^r A\, dx \tag{7.5.b}$$

Notice that instead to carry the integration from r to $-r$ it is carried from 0 to r and multiplied by two. The circle equation is $x^2 + y^2 = r^2$. The infinitesimal volume made from the triangle (the brown color) with a base of $\sqrt{r^2 - x^2}$ and the height of $\sqrt{r^2 - x^2}\,\tan\theta$ and infinitesimal thickness, dx, to be integrated as

$$V = \cancel{2} \int_0^r \frac{\overbrace{(r^2 - x^2)\tan\theta}^{A}}{\cancel{2}}\, dx = \tan\theta \int_0^r \left(r^2 - x^2\right)\, dx \tag{7.5.c}$$

A new variable is defined as $\xi = x/r$ when $x = r \longrightarrow \rho = 1$, and equation Eq. (7.5.c) can be transformed

$$V = r^3\,\tan\theta \int_0^r \left(1 - \frac{x^2}{r^2}\right) \frac{dx}{r} = \\ r^3\,\tan\theta \int_0^1 \left(1 - \xi^2\right)\, d\xi = \frac{2\,r^3\,\tan\theta}{3} \tag{7.5.d}$$

The brown triangle with infinitesimal thickness has mass centroid at 2/3 of the length of the triangle (from the center or in other words cylinder center). The mass centroid can be calculated utilizing the similar approach but simpler in cylindrical coordinates with the boundaries of zero to $\pi/2$. The integration of infinitesimal volume (similar to previous calculations where the base is $r\cos\psi$) can be carried as

$$\int x_i\, dV = 2 \int_0^{\pi/2} \overbrace{\left(\frac{\cancel{2}}{3}\, r\cos\psi\right)}^{y_i} \frac{\overbrace{(r\,\cos\psi)^2\tan\theta}^{A}}{\cancel{2}} \overbrace{\cos\psi\, r\, d\psi}^{''dx''} \tag{7.5.e}$$

The choice of the coordinate system base on convenience. Eq. (7.5.e) can be written as

$$\int x_i\, dV = \frac{2\,r^4\,\tan\theta}{3} \int_0^{\pi/2} (\cos\psi)^4\, d\psi \tag{7.5.f}$$

End of Ex. 7.5

The integral is readily available and can be done by integration by part as

$$\int x_i\, dV = \frac{2\, r^4\, \tan\theta}{3} \times \left(\frac{\sin\psi\, \cos^3\psi}{4} + \frac{3\, \cos\psi \sin\psi}{8} + \frac{3\,\psi}{8} \right) \Bigg|_0^{\pi/2} = \frac{r^4\, \pi\, \tan\theta}{8} \tag{7.5.g}$$

The first and the second terms vanished at the boundaries. Substituting the volumes of both integrals into Eq. (7.5.a) provides

$$x_i = \frac{\dfrac{r^4\, \pi\, \tan\theta}{16}}{\dfrac{2\, r^3\, \tan\theta}{3}} = \frac{3\,\pi\, r}{8} \tag{7.5.h}$$

It is interesting to point out that x_i is not a function of the angle θ. The total change based on Eq. (7.12) is

$$\Delta x = \frac{2\, V\, x_i}{V_0} = \frac{\dfrac{2\, r^4\, \pi\, \tan\theta}{8}}{\pi\, r^2\, d} = \frac{r^2\, \tan\theta}{4\, d} \tag{7.5.i}$$

The angle of change, α under assumption of small Δy can be obtained as

$$\alpha \cong tan^{-1} \frac{\Delta x}{\xi} = \tan^{-1} \left(\frac{\dfrac{r^2\, \tan\theta}{4\, d}}{\xi} \right) = \tan^{-1} \left(\frac{r^2\, \tan\theta}{4\, d\, \xi} \right) \tag{7.5.j}$$

Example 7.6: Wedge centroid **Level: Intermediate**

In Ex. 7.5 Δy was assumed to be zero. Is this assumption is correct or/and under what conditions it is correct. Hint: first calculate Δy and then use the results to the estimated results.

Solution

under construction

7.4.1 A Small Change in Angle of Rotation

This section is dealing with a special topic of change of area due to rotation when the area (volume) is constant topic very is important to stability analysis. The change of the area in Ex. 7.5 dealt with a specific geometry. This procedure can be generalized or even simplified the procedure. The process of calculating the change of the centroid can be converted for small angle.

$$x_i = \frac{\int x\, dV}{\int dV} = \frac{\int x \overbrace{x\,\tan\theta}^{h}\, dA}{V} = \frac{\tan\theta \int x^2\, dA}{V} \tag{7.13}$$

The term in the nominator is called the Moment of Inertia and will be discussed in the following section. The Moment of Inertia symbolized by I_{xx} and Eq. (7.13) by

$$x_i = \frac{\tan\theta\, I_{xx}}{V} \tag{7.14}$$

Notice that I_{xx} is a function of the cross section only and is half of the cross section. Hence for the total moment of inertia double the half (see next section for explanation). The volume of the small wedge is calculated below. The total change is defined in Eq. (7.12)

Δx due to change in θ

$$\Delta x = \frac{\cancel{V}}{V_0} \overbrace{\frac{\tan\theta\, I_{xx}}{\cancel{V}}}^{2\,x_1} = \tan\theta\, \frac{I_{xx}}{V_0} \tag{7.15}$$

It is remarkable that the change location of centroid can be determined from knowing/calculating the moment of inertia of the cross section and by the displaced volume.

The change in centroid in the y direction and done in a similar fashion when noticing that the y is actually $y/2$ (when first y refers to the y of the distance and the second refers to its value.). For small angle it can be written that

$$y_i = \frac{\int y\, dV}{V} = \frac{\int y/2\, \overbrace{y\, dA}^{dV}}{V} = \frac{\int (\tan\theta\, x)^2\, dA}{2\,V} = \frac{\tan^2\theta \int x^2\, dA}{2\,V} \tag{7.16}$$

Again as in the x direction Eq. (7.17) moment of inertia should be recognized and it can be written as

$$y_i = \frac{\tan^2\theta\, I_{xx}}{2\,V} \tag{7.17}$$

Hence,

Δy due to change in θ

$$\Delta y = \frac{\cancel{V}}{V_0}\overbrace{\frac{\tan^2\theta\, I_{xx}}{2\cancel{V}}}^{2\,x_1} = \tan^2\theta\,\frac{I_{xx}}{2\,V_0} \tag{7.18}$$

The difference between in the changes is very significant. The amount change in y smaller by a factor. For practical purposes, the change in y can be ignored.

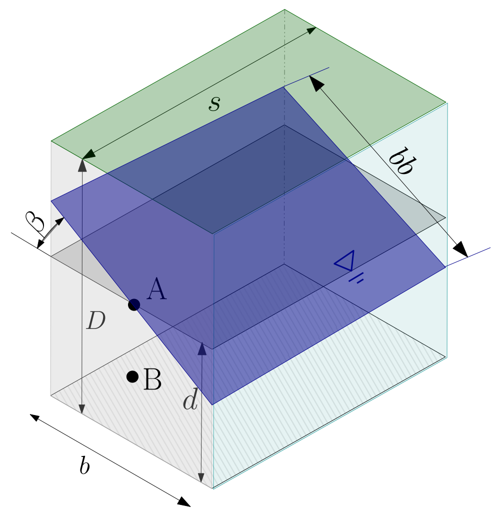


Fig. 7.11 – Rectangular Floating in Various Angles. Notice that blue surface represents all the positions that the body can have up to point that is "fail" on the side.

Example 7.7: Centroid of cylinder **Level: Complex**

Repeat Ex. 7.3 and **??** with using Eq. (7.15). Use as a base the calculate all change for all the angle $0 < \theta < 60°$. Comment on the geometrical limitations. In these two cases the volume is fix and the moment of inertia is varies. The moment of inertia is discussed in the Chapter 15. For the purpose of this exercise assume that moment inertia for rectangular is $I = ab^3/12$ see the book Appendix table 3. Where a is the length around the rotation occurs and b is width of the rectangular.

Solution

continue Ex. 7.7

As long as the geometry remains the same this analysis can be assumed to be correct. Also note to that the β and θ are diffident angle: β is the angle that is as location for which is checked to see if it is stable, while θ is the small angle forced on the body to see the body will remain at β. Fig. 7.11 depicts the liquid surface plane (the blue color). The question refers two edge cases, one when $\beta = 0$ and two when $\beta = 45°$. Thus, the solution here will deals with all the angles. So all the properties have to express as a function of angle β. The volume is constant regardless to β. The moments inertia is a strong of β. The distance bb shown in the figure is

$$bb = \frac{b}{\cos\beta} \tag{7.7.a}$$

Hence, the moment of inertia is

$$I_{xx} = \frac{s\left(\dfrac{b}{\cos\theta}\right)^3}{12} = \frac{s\,b^3}{12\,\cos^3\beta} \tag{7.7.b}$$

The volume of the body is

$$V = d\,b\,s \tag{7.7.c}$$

The change in buoyancy centroid Eq. (7.15) is

$$\Delta x = \tan\theta \frac{\dfrac{\cancel{s}\,\cancel{b^3}^{\,b^2}}{12\,\cos^3\theta}}{d\,\cancel{b}\,\cancel{s}} \longrightarrow \frac{\Delta x}{b} = \frac{b}{d}\frac{\tan\theta}{12\,\cos^3\beta} \tag{7.7.d}$$

The two cases in the question are $\beta = 0$ and $\beta = 45°$ can be substituted into Eq. (7.7.d) to get the answers which are plotted on Fig. 7.12. The graph depict Eq. (7.7.d) for the dimensionless parameters, angle of inclination, β, and ratio of the moment of inertia to the volume (below). The results show that as the inclination angle, β, has a dramatic effect. For large angle (30°–40°) the change in centroid getting larger and larger. Additionally, larger value of b/d (width and depth) ratio the effect become more significant. In the case of $\beta = 0$ and $\beta = 45°$ the equations

End of Ex. 7.7

respectively are: 0 case;

$$\frac{\Delta x}{b} = \frac{b}{d}\frac{\tan\theta}{12} \tag{7.7.e}$$

The distance AB is $d/2$ thus tangent α is

$$\tan\alpha = \frac{\Delta x}{AB} = \frac{\dfrac{b^2}{d}\dfrac{\tan\theta}{12}}{d/2} \longrightarrow$$

$$\alpha = \tan^{-1}\left\{\frac{1}{6}\left(\frac{b}{d}\right)^2 \tan\theta\right\} \tag{7.7.f}$$

45°;

$$\frac{\Delta x}{b} = \frac{b}{d}\frac{\tan\theta}{12\,\dfrac{1}{2\sqrt{2}}} \longrightarrow \frac{\Delta x}{b} = \frac{\sqrt{2}}{3}\frac{b\tan\theta}{d} \tag{7.7.g}$$

In this case AB =d/$(3\sqrt{2})$. For the second case, the case angle α is

$$\alpha = \tan^{-1}\left\{2\left(\frac{b}{d}\right)^2 \tan\theta\right\} \tag{7.7.h}$$

Before the general expression could be written as formula for the centroid has to found.

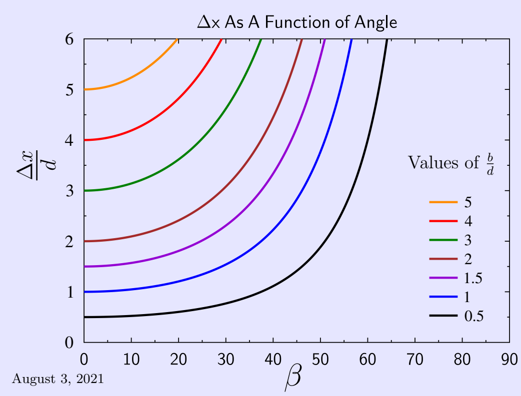


Fig. 7.12 – Change of Buoyancy in The x Direction.

It can be observed from the analysis the change of Δx varies and this will be used in the stability analysis. There several conclusions that can be drawn so far. As long geometry remain the same, larger change on centroid appear for larger angle. When happen when the change of number of angle appear. In other words, if the surface reaches to upper or the lower corner then the analysis has to be modified.

Example 7.8: Ellipse centroid

Level: Intermediate

Repeat Ex. 7.7 for cylinder standing up. Cross section area is ellipse with minor axis of D while the major radius changes and is function of the geometrical identity as $r_m = r/cos\beta$. Using this information construct a map similar to the above example.

Solution

The moment of inertia of the ellipse (see figure table 3) is

$$I_{xx} = \frac{\pi\, r \left(\dfrac{r}{\cos\beta}\right)^3}{4} \longrightarrow I_{xx} = \frac{\pi\, r^4}{4\,\cos^3\beta} \tag{7.8.a}$$

The volume, again, is constant and equable to

$$V = \pi\, r^2\, d \tag{7.8.b}$$

The change in centroid is

$$\Delta x = \tan\theta \frac{\dfrac{\cancel{\pi} \cancelto{r^2}{r^4}}{4\,\cos^3\beta}}{\cancel{\pi}\cancelto{1}{r^2}\, d} \longrightarrow \frac{\Delta x}{r} = \frac{\tan\theta\, r}{4\,\cos^3\beta\, d} \tag{7.8.c}$$

Remark

The difference between this Eq. (7.8.c) and Eq. (7.7.d) is the coefficient.

Generally the change in centroid for extruded shape will be in a general form of

$$\frac{\Delta x}{b} = CC\, \frac{geometrical\ parameter}{geometrical\ parameter}\, \frac{\tan\theta}{\cos^3\beta} \tag{7.19}$$

Where CC is a coefficient that depend on the geometry. This form change when the shape of the body is radially diffident. Large ships can be considered as a large box with a round bottom. Thus the analysis provide here in relevant to these bodies close to "boxes." However, for small boat have more complicate shape and it can be numerically analyzed. It also can be noticed that bodies that change shape with z coordinate like cone the centroid change as well as the center of the surface area (at liquid surface).

7.5 Transformation of Coordinates

Sometimes the information on the centroid is provided in different coordinate system. In this case a demonstration on how to transfer the coordinate is present. A trapezoid centroid are given normally in coordinate system that attached to base as shown below.

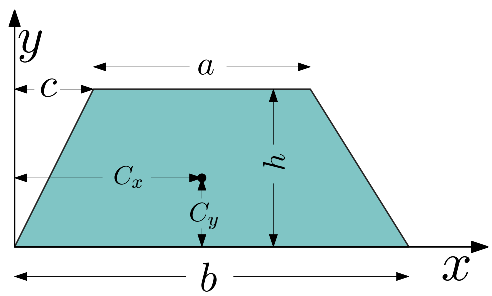


Fig. 7.13 – Standard trapezoid with given data and coordinate system.

For this trapezoid in this specific coordinate system the center is given utilizing the given names in the following:

$$c_x = \frac{a^2 + b^2 + 2\,a\,c + b\,c + a\,b}{3\,(a+b)} \tag{7.20a}$$

$$c_y = \frac{h\,(2\,a + b)}{3\,(a+b)} \tag{7.20b}$$

To ascertain the location according to the new specific dimensions. It can be noticed that the following "translation" should be made when the two Parallel lines. Utilizing the equations Eq. (7.20) to the dimensions shown Fig. 7.13 to apply to Fig. 7.14. It reduces the labor in handling with the use of the following definitions are used $\Psi = b \tan\beta/2$, $h \longrightarrow b$ $a \longrightarrow d - \Psi$ $b \longrightarrow d + \Psi$ $c \longrightarrow 2\,\Psi$.

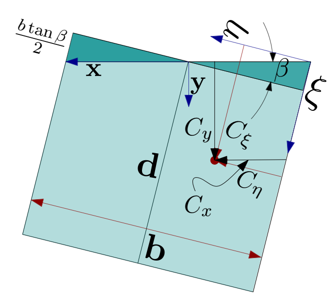


Fig. 7.14 – Trapezoid in a new coordinate system to be transformation from old system.

Several identies can be observed;

Table 7.2 – Trapezoid properties and equations

Trapezoid properties and equations	
Transformation	Identities Units
$\Psi = b/2\,\tan\beta$	$a + b = 2\,d$
$h \longrightarrow b$	$a^2 + b^2 = 2\,d^2 + 2\,\Psi^2$
$a \longrightarrow (d - \Psi)$	$a\,b = d^2 - \Psi^2$
$b \longrightarrow (d + \Psi)$	$2\,a\,c = 4\,d\,\Psi - 4\,Psi^2$
$c \longrightarrow 2\,\Psi$	$\mathrm{b\,c} = 2\,d\,\Psi + 2\,\Psi^2$
	$2\,a\,c + b\,c = 6\,d\,\Psi - 2\,\Psi^2$

Eq. (7.20) or more specifically Eq. (7.20a) can be evaluated

$$C_\xi = \frac{d}{2} + \Psi - \frac{\Psi^2}{6\,d} \tag{7.21}$$

or in dimensionless form as

$$\frac{C_\xi}{d} = \frac{1}{2} + \frac{\Psi}{d} - \frac{1}{6}\left(\frac{\Psi}{d}\right)^2 \tag{7.22a}$$

or in terms of b and $\tan\beta$ as

$$\frac{C_\xi}{d} = \frac{1}{2} + \frac{1}{2}\frac{b}{d}\tan\beta - \frac{1}{24}\left(\frac{b}{d}\right)^2\tan^2\beta \tag{7.22b}$$

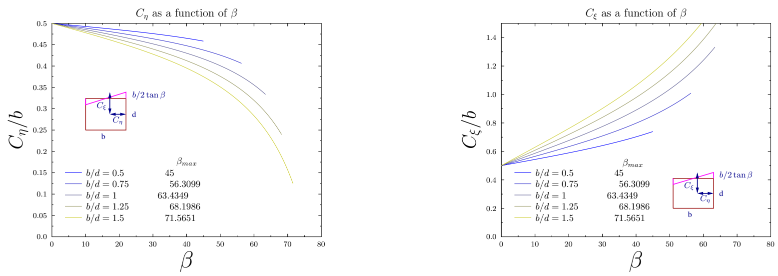
For the η direction

$$C_\eta = \frac{b\,(2\,(d-\Psi) + (d+\Psi))}{6\,d} = \frac{b\,(3\,d - \Psi)}{6\,d} = \frac{b}{2} - \frac{b\,\Psi}{6\,d} \tag{7.22c}$$

and dimensionless form as

$$\frac{C_\eta}{d} = \frac{1}{2}\,\frac{b}{d} - \frac{b\,\Psi}{6\,d^2} = \frac{1}{2}\,\frac{b}{d} - \frac{1}{12}\left(\frac{b}{d}\right)^2\tan\beta \tag{7.22d}$$

The ratio of C_ξ is related to C_x by β through the equation above and and the same can said for the other pair (C_y, C_η). Assume that β is rotating at a constant angular speed then the centroid is rotating but in different angular velocity. This fact means that the centroid will not be under the point G or even A. The results are depicted in Fig. 7.16.



(a) The distance of C_η as a function of β.

(b) The distance of C_ξ as a function of β.

Fig. 7.16 – Distance of the center as a function of β. The distance is measured from the top corner and it is also moving with β as for C_ξ (it does not effect C_η).

The corrected distance is depicted in Fig. 7.17.

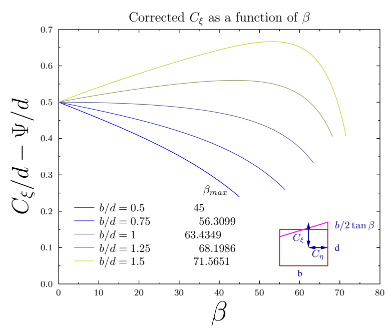


Fig. 7.17 – Corrected C_ξ as a function of β.

It can be noted that for small value b/d the distance increases which is very important factor for instability.

7.6 Appendix

As a side kick, the integral that was canceled before Eq. (7.15) can be calculated as following

$$\int dV = \int x \tan\theta dA = \tan\theta \int x \, dA = \tan\theta \bar{x} \, A \tag{7.23}$$

The value of $\tan\theta$ is constant in the integration and the value $\bar{x}$ is the average height of wedge. The value of $\bar{x}$ is a function of θ but not its location and the A cross area is not function of θ. Sometime of values $\bar{x}$ are tabulated and hence the integration can be readily available.

8

Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation to different coordinate systems (linear, rotational). Furthermore, the product of inertia will be introduced.

8.1 Moment of Inertia for Mass

The moment of inertia turns out to be an essential part for the calculations of rotating bodies and stability. Furthermore, it turns out that the moment of inertia has much wider applicability and usefulness than one assumes initially. Moment of inertia of mass is defined as

Moment of Inertia

$$I_{rrm} = \int_V \rho r^2 dV \tag{8.1}$$

If the density is constant then equation (8.1) can be transformed into

$$I_{rrm} = \rho \int_V r^2 \, dV \tag{8.2}$$

The moment of inertia is independent of the coordinate system used for the calculation, but dependent on the location of axis of rotation relative to the body. Some people define the radius of gyration as an equivalent concepts for the center of mass concept and which means if all the mass were to locate in the one point/distance and to obtain the same of moment of inertia.

$$r_k = \sqrt{\frac{I_m}{m}} \tag{8.3}$$

The body has a different moment of inertia for every coordinate/axis and they are

$$\begin{aligned} I_{xx} &= \int_V {r_x}^2 dm = \int_V (y^2 + z^2)\, dm \\ I_{yy} &= \int_V {r_y}^2 dm = \int_V (x^2 + z^2)\, dm \\ I_{zz} &= \int_V {r_z}^2 dm = \int_V (x^2 + y^2)\, dm \end{aligned} \tag{8.4}$$

$$\begin{aligned} I_{xx} &= \int_V {r_x}^2 dm = \int_V (y^2 + z^2)\, dm \\ I_{yy} &= \int_V {r_y}^2 dm = \int_V (x^2 + z^2)\, dm \\ I_{zz} &= \int_V {r_z}^2 dm = \int_V (x^2 + y^2)\, dm \end{aligned} \tag{8.5}$$

8.2 Moment of Inertia for Area

8.2.1 General Discussion

For body with thickness, t and uniform density the following can be written

$$I_{xxm} = \int_m r^2 dm = \rho\, t \overbrace{\int_A r^2 dA}^{\text{moment of inertia for area}} \tag{8.6}$$

The moment of inertia about axis is x can be defined as

Moment of Inertia

$$I_{xx} = \int_A r^2 dA = \frac{I_{xxm}}{\rho\, t} \tag{8.7}$$

where r is distance of dA from the axis x and t is the thickness. Any point distance can be calculated from axis x as

$$x = \sqrt{y^2 + z^2} \tag{8.8}$$

Thus, Eq. (8.7) can be written as

$$I_{xx} = \int_A \left(y^2 + z^2\right) dA \tag{8.9}$$

In the same fashion for other two coordinates as

$$I_{yy} = \int_A \left(x^2 + z^2\right) dA \tag{8.10}$$

$$I_{zz} = \int_A \left(x^2 + y^2\right) dA \tag{8.11}$$

8.3 The Parallel Axis Theorem

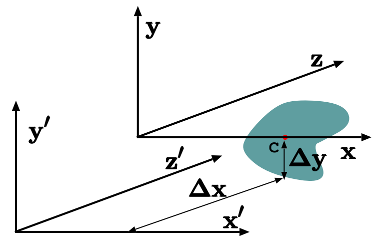


Fig. 8.1 – The schematic that explains the summation of moment of inertia.

This section deals with transformation of the moment of from one system to a parallel system. The moment is depended on the coordinate system. The dependency is because moment of inertia changes when the body rotates in different coordinates. The moment of inertial can be calculated for any axis. The knowledge about one axis can help calculating the moment of inertia for a parallel axis. I_{xx} is denoted as the moment of inertia about axis xx which is at the center of mass/area. The moment of inertia for axis $x^{'}$ is

$$I_{x^{'}x^{'}} = \int_A {r^{'}}^2 dA = \int_A \left({y^{'}}^2 + {z^{'}}^2\right) dA = \int_A \left[(y+\Delta y)^2 + (z+\Delta z)^2\right] dA \quad (8.12)$$

Eq. (8.12) can be expended as

$$I_{x^{'}x^{'}} = \overbrace{\int_A \left(y^2+z^2\right) dA}^{I_{xx}} + \overbrace{2\int_A \left(y\,\Delta y + z\,\Delta z\right) dA}^{=0} + \int_A \left((\Delta y)^2 + (\Delta z)^2\right) dA \quad (8.13)$$

The first term in Eq. (8.13) on the right hand side is the moment of inertia about axis x and the second them is zero. The second therm is zero because it integral of center about center thus is zero. The third term is a new term and can be written as

$$\int_A \overbrace{\left((\Delta y)^2 + (\Delta z)^2\right)}^{constant} dA = \overbrace{\left((\Delta y)^2 + (\Delta z)\right)}^{r^2} \overbrace{\int_A^2 dA}^{A} = r^2\, A \quad (8.14)$$

Hence, the relationship between the moment of inertia at xx and parallel axis $x^{'}x^{'}$ is

Parallel Axis Equation

$$I_{x^{'}x^{'}} = I_{xx} + r^2\, A \quad (8.15)$$

The moment of inertia of several areas is the sum of moment inertia of each area see Figure 8.2 and therefore,

$$I_{xx} = \sum_{i=1}^{n} I_{xx\,i} \tag{8.16}$$

If the same areas are similar thus

$$I_{xx} = \sum_{i=1}^{n} I_{xx\,i} = n\, I_{xx\,i} \tag{8.17}$$

Equation (8.17) is very useful in the calculation of the moment of inertia utilizing the moment of inertia of known bodies. For example, the moment of inertia of half a circle is half of whole circle for axis at the center of circle. The moment of inertia can then move the center of area.

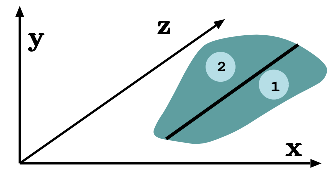


Fig. 8.2 – The schematic to explain the summation of moment of inertia.

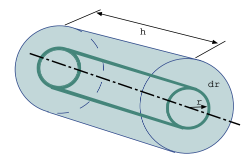


Fig. 8.3 – Cylinder with an element for calculation moment of inertia.

8.4 Examples of Moment of Inertia

Example 8.1: Moment of Cylinder **Level: Easy**

Calculate the moment of inertia for the mass of the cylinder about center axis which height of h and radius, r_0, as shown in Figure 8.3. The material is with an uniform density and homogeneous.

Solution

The element can be calculated using cylindrical coordinate. Here the convenient element is a shell of thickness dr which shown in Figure 8.3 as

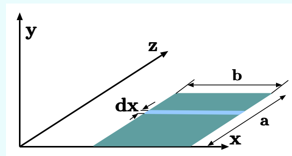
$$I_{rr} = \rho \int_V r^2 dm = \rho \int_0^{r_0} r^2 \overbrace{h\, 2\, \pi\, r\, dr}^{dV} =$$

$$\rho\, h\, 2\, \pi \frac{{r_0}^4}{4} = \frac{1}{2} \rho h \pi {r_0}^4 = \frac{1}{2} m\, {r_0}^2 \tag{8.1.a}$$

The radius of gyration is

$$r_k = \sqrt{\frac{\frac{1}{2}\, m {r_0}^2}{m}} = \frac{r_0}{\sqrt{2}} \tag{8.1.b}$$

Example 8.2: Rectangular moment of Inertia **Level: Easy**



Calculate the moment of inertia of the rectangular shape shown in Figure 8.4 around x coordinate.

Fig. 8.4 – Description of rectangular in x–y plane for calculation of moment of inertia.

Solution

The moment of inertia is calculated utilizing equation (8.9) as following

$$I_{xx} = \int_A \left(\overbrace{y^2}^{0} + z^2 \right) dA = \int_0^a z^2 \overbrace{b dz}^{dA} = \frac{a^3 \, b}{3}$$

This value will be used in later examples.

Example 8.3: Rectangular moment of Inertia **Level: Intermediate**

To study the assumption of zero thickness, consider a simple shape to see the effects of this assumption. Calculate the moment of inertia about the center of mass of a square shape with a thickness, t compare the results to a square shape with zero thickness.

Solution

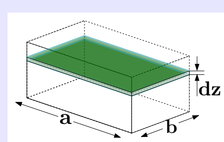


Fig. 8.5 – A square element for the calculations of inertia of two-dimensional to three–dimensional deviations.

The moment of inertia of transverse slice about y' (see Fig. 8.5) is

$$dI_{xxm} = \rho \overbrace{dy}^{t} \overbrace{\frac{b \, a^3}{12}}^{I_{xx}} \qquad (8.3.a)$$

End of Ex. 8.3

The transformation into from local axis x to center axis, x' can be done as following

$$dI_{x'x'm} = \rho dy \left(\overbrace{\frac{b\,a^3}{12}}^{I_{xx}} + \overbrace{\underbrace{z^2}_{r^2} \underbrace{b\,a}_{A}}^{r^2\,A} \right) \tag{8.3.b}$$

The total moment of inertia can be obtained by integration of equation (8.3.b) to write as

$$I_{xxm} = \rho \int_{-t/2}^{t/2} \left(\frac{b\,a^3}{12} + z^2\,b\,a \right) dz = \rho\,t\,\frac{a\,b\,t^2 + a^3\,b}{12} \tag{8.3.c}$$

Comparison with the thin body results in

$$\frac{I_{xx}\,\rho\,t}{I_{xxm}} = \frac{b\,a^3}{t^2\,b\,a + b\,a^3} = \frac{1}{1 + \dfrac{t^2}{a^2}} \tag{8.3.d}$$

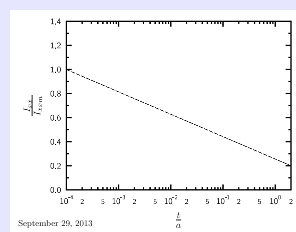


Fig. 8.6 – The ratio of the moment of inertia of two-dimensional to three–dimensional.

It can be noticed right away that equation (8.3.d) indicates that ratio approaches one when thickness ratio is approaches zero, $I_{xxm}(t \rightarrow 0) \rightarrow 1$. Additionally it can be noticed that the ratio a^2/t^2 is the only contributor to the error[1]. The results are present in Fig. 8.6. I can be noticed that the error becomes significant very fast even for small values of t/a while the width of the box, b has no effect on the error.

[1] This ratio is a dimensionless number that commonly has no special name. This author suggests to call this ratio as the B number.

Example 8.4: Moment of Inertia in z direction **Level: Intermediate**

Calculate the rectangular moment of Inertia for the rotation trough center in zz axis (axis of rotation is out of the page). Hint, construct a small element and build longer build out of the small one. Using this method calculate the entire rectangular. Calculate the rectangular moment of Inertia for the rotation trough

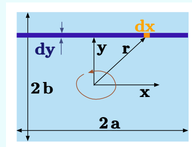


Fig. 8.7 – Rectangular Moment of inertia in the z direction.

Solution

The moment of inertia for a long element with a distance y shown in Figure 8.7 is

$$d\,I_{zz}\Big|_{dy} = \int_{-a}^{a} \overbrace{\left(y^2 + x^2\right)}^{r^2} dy\,dx = \frac{2\,\left(3\,a\,y^2 + a^3\right)}{3}\,dy \tag{8.4.a}$$

The second integration (no need to use (8.15), why?) is

$$I_{zz} = \int_{-b}^{b} \frac{2\,\left(3\,a\,y^2 + a^3\right)}{3}\,dy \tag{8.4.b}$$

Results in

$$I_{zz} = \frac{a\,\left(2\,a\,b^3 + 2\,a^3\,b\right)}{3} = \overbrace{A}^{4\,a\,b}\left(\frac{(2a)^2 + (2b)^2}{12}\right) \tag{8.4.c}$$

Or

Example 8.5: Parabola Moment of Inertia **Level: Intermediate**

Calculate the center of area and moment of inertia for the parabola, $y = \alpha x^2$, depicted in Figure 8.8. Hint, calculate the area first. Use this area to calculate moment of inertia. There are several ways to approach the calculation (different infinitesimal area).

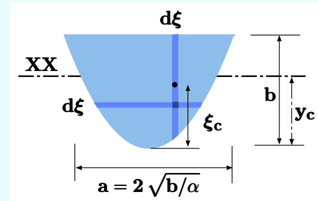


Fig. 8.8 – Parabola for calculations of moment of inertia and the area centroid.

Solution

For $y = b$ the value of $x = \sqrt{b/\alpha}$. First the area inside the parabola calculated as

$$A = 2 \int_0^{\sqrt{b/\alpha}} \overbrace{(b - \alpha\xi^2) d\xi}^{dA/2} = \frac{2(3\,\alpha - 1)}{3} \left(\frac{b}{\alpha}\right)^{\frac{3}{2}}$$

The center of area can be calculated utilizing equation (7.5). The center of every element is at, $\left(\alpha\,\xi^2 + \dfrac{b - \alpha\xi^2}{2}\right)$ the element area is used before and therefore

$$x_c = \frac{1}{A} \int_0^{\sqrt{b/\alpha}} \overbrace{\left(\alpha\xi^2 + \frac{(b - \alpha\xi^2)}{2}\right)}^{x_c} \overbrace{(b - \alpha\xi^2) d\xi}^{dA}$$

$$= \frac{3\,\alpha\,b}{15\,\alpha - 5} \qquad (8.5.a)$$

The moment of inertia of the area about the center can be found using in equation (8.5.a) can be done in two steps first calculate the moment of inertia in this coordinate system and then move the coordinate system to center. Utilizing equation (8.9) and doing the integration from 0 to maximum y provides

$$I_{x'x'} = 4 \int_0^b \xi^2 \overbrace{\sqrt{\frac{\xi}{\alpha}}\, d\xi}^{dA} = \frac{2\,b^{7/2}}{7\,\sqrt{\alpha}}$$

End of Ex. 8.5

Utilizing equation (8.15)

$$I_{xx} = I_{x'x'} - A\,\Delta x^2 = \overbrace{\frac{4\,b^{7/2}}{7\,\sqrt{\alpha}}}^{I_{x'x'}} - \overbrace{\frac{3\,\alpha-1}{3}\,\left(\frac{b}{\alpha}\right)^{\frac{3}{2}}}^{A}\overbrace{\left(\frac{3\,\alpha\,b}{15\,\alpha-5}\right)^2}^{(\Delta x = x_c)^2}$$

or after working the details results in

$$I_{xx} = \frac{\sqrt{b}\,\left(20\,b^3 - 14\,b^2\right)}{35\,\sqrt{\alpha}}$$

Example 8.6: Triangle Moment of Inertia

Level: Complex

Calculate the moment of inertia of strait angle triangle about its y axis as shown in the Figure on the right. Assume that base is a and the height is h. What is the moment when a symmetrical triangle is attached on left? What is the moment when a symmetrical triangle is attached on bottom? What is the moment inertia when $a \longrightarrow 0$? What is the moment inertia when $h \longrightarrow 0$?

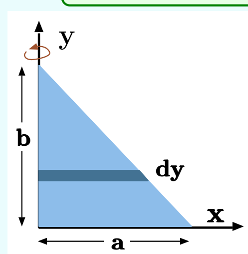


Fig. 8.9 – Triangle for example 8.6.

Solution

The right wedge line equation can be calculated as

$$\frac{y}{h} = \left(1 - \frac{x}{a}\right)$$

or

$$\frac{x}{a} = \left(1 - \frac{y}{h}\right)$$

Now using the moment of inertia of rectangle on the side (y) coordinate (see example 8.2)

$$\int_0^h \frac{a\,\left(1-\dfrac{y}{h}\right)^3 dy}{3} = \frac{a^3\,h}{4}$$

End of Ex. 8.6

For two triangles attached to each other the moment of inertia will be sum as $\dfrac{a^3\,h}{2}$.
The rest is under construction.

8.5 Product of Inertia

In addition to the moment of inertia, the product of inertia is commonly used. Here only the product of the area is defined and discussed. The product of inertia defined as

Moment of Inertia

$$I_{x_i\,x_j} = \int_A x_i\,x_j dA \tag{8.18}$$

For example, the product of inertia for x and y axis is

$$I_{xy} = \int_A x\,y dA \tag{8.19}$$

Product of inertia can be positive or negative value as oppose the moment of inertia. The calculation of the product of inertia isn't different much for the calculation of the moment of inertia. The units of the product of inertia are the same as for moment of inertia.

8.5.0.1 Transfer of Axis Theorem

Same as for moment of inertia there is also similar theorem.

$$I_{x'\,y'} = \int_A x^{'}\,y^{'} dA = \int_A (x+\Delta x)\,(y+\Delta y) dA \tag{8.20}$$

expanding equation (8.20) results in

$$I_{x'\,y'} = \overbrace{\int_A x\,y dA}^{I_{xy}} + \overbrace{\int_A x\,\Delta y dA}^{\Delta y \overbrace{\int_A x\,dA}^{0}} + \overbrace{\int_A \Delta x\,y dA}^{\Delta x \overbrace{\int_A y\,dA}^{0}} + \overbrace{\int_A \Delta x\,\Delta y dA}^{\Delta x\,\Delta y\,A} \tag{8.21}$$

The final form is

$$I_{x'\,y'} = I_{xy} + \Delta x\,\Delta y\,A \tag{8.22}$$

There are several relationships should be mentioned

$$I_{xy} = I_{yx} \tag{8.23}$$

Symmetrical area has zero product of inertia because integration of odd function (asymmetrical function) left part cancel the right part.

Example 8.7: Triangle Moment of Inertia **Level: Intermediate**

Calculate the product of inertia of straight wedge triangle. Assume that body is two dimensional.

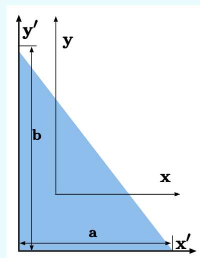


Fig. 8.10 – Product of inertia for triangle.

Solution

The equation of the line is

$$y = \frac{a}{b}x + a$$

The product of inertia at the center is zero. The total product of inertia is

$$I_{x'y'} = 0 + \overbrace{\frac{a}{3}}^{\Delta x}\overbrace{\frac{b}{3}}^{\Delta y}\overbrace{\left(\frac{a\,b}{2}\right)}^{A} = \frac{a^2\,b^2}{18}$$

8.6 Principal Axes of Inertia

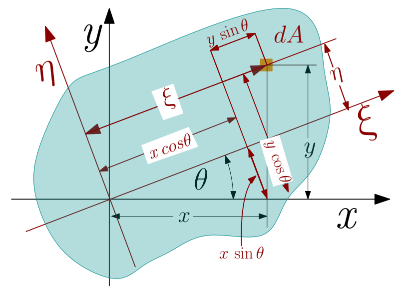


Fig. 8.11 – Rotated coordinate system to explain rotation of axes.

The element area shown in Fig. 8.11 can be measured in two coordinate systems x–y and ξ–η. The transformation of from one system to another can be expressed by the geometrical distance shown in the figure.

$$\xi = x\,\cos\theta + y\,\sin\theta \tag{8.24}$$

and

$$\xi = y\,\cos\theta - x\,\sin\theta \tag{8.25}$$

Using these identities and writing by collecting all the right terms yields the following relationship

$$I_{\xi\xi} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2}\cos 2\theta - I_{xy}\sin 2\theta \tag{8.26a}$$

$$I_{\eta\eta} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2}\cos 2\theta + I_{xy}\sin 2\theta \tag{8.26b}$$

$$I_{\xi\eta} = \frac{I_{xx} - I_{yy}}{2}\sin 2\theta + I_{xy}\,\cos 2\theta \tag{8.26c}$$

This Eq. (8.26) can be manipulated to show

$$I_{\xi\xi} + I_{\eta\eta} = I_{xx} + I_{yy} \tag{8.27}$$

By differentiating the Eq. (8.26a) with respect to θ and setting the result to zero to obtain

$$\frac{dI_{\xi\xi}}{d\theta} = -2\left(\frac{I_{xx} - I_{yy}}{2}\right)\sin 2\theta - 2\,I_{xy}\cos 2\theta \tag{8.28}$$

Thus, the maximum or minimum is at

$$\tan 2\theta_{principal} = \frac{2\,I_{xy}}{I_{yy} - I_{xx}} \tag{8.29}$$

Using these results it can be shown that the maximum/minimum moment of inertia is

$$I_{\substack{max\\min}} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + {I_{xy}}^2} \tag{8.30}$$

The topic of Mohrs Circle is not discussed her because it is out the possible scop of this book. Only for completion presentation of 3D tensor of the moment of inertia is presented.

The inertia matrix or inertia tensor is

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{8.31}$$

In linear algebra it was shown that for some angle equation (8.31) can be transform into

$$\begin{bmatrix} I_{x'x'} & 0 & 0 \\ 0 & I_{y'y'} & 0 \\ 0 & 0 & I_{z'z'} \end{bmatrix} \tag{8.32}$$

System which creates equation (8.32) referred as principle system.

9 Pressure

9.1 Introduction

The concept of pressure is very important in stability. As opposed to regular treatment in Fluid Mechanics books or class, it will treated as a scalar and care will be given to minimize the actual complex mathematical details. It is assumed that some of the readers of this book lack the mathematical background, hence the complexity will be abstracted.

In simple terms, pressure is the ratio of force and the area such as

$$P = \frac{F}{A} \tag{9.1}$$

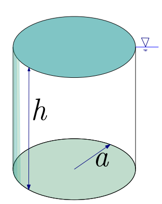


Fig. 9.1 – Cylinder to explain the pressure concept.

The force, F is a vector as well the area, A. So, division of vector by vector creates a complex creation with nine (9) terms. However, here only one term is used since the pressure is equal in all three directions and there no other terms (at no flow situations). Here, the pressure is assumed to be acting force magnitude divided by the area in the same direction. When the "bureaucratic" formality is cleared, a simple pressure explanation can be presented. A cylinder of liquid shown in the Fig. 9.1 the top is at liquid surface and is weight

$$W = \overbrace{\underbrace{\pi\, a^2}_{A}\; h}^{V}\; \rho\, g \tag{9.2}$$

The weight is the force that acting on the surface at the bottom of the cylinder. The pressure at the surface according to the definition 9.1 is

$$\frac{\cancel{\pi a^2} h \rho g}{\cancel{\pi a^2}} = \rho g h \tag{9.3}$$

The pressure in stationary liquid is simply depends on the density and the height (and the gravity). The pressure is not depends on the area.

This concept that pressure depends solely on the depth of liquid is extended to the fact that there is no need for it be in a continuous line. That is the pressure constant pressure lines that goes where the liquid is connected. For example, in Fig. 9.2 depicts constant pressure line across solid walls.

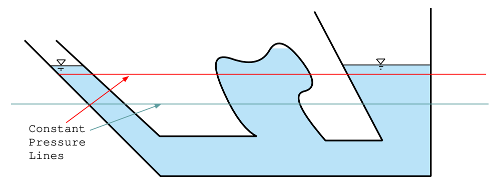


Fig. 9.2 – Constant pressure lines across solid boundaries. It can be noticed that free surface are equal while the close could be higher.

In fact the this idea used in may places as way to measure two locations to be in the same height.

9.2 Fluid Forces on Surfaces

The forces that fluids (at static conditions) extracts on surfaces are very important for engineering purposes. This section deals with these calculations which are divided into two categories, straight surfaces and curved surfaces. These issues especially applied to floating bodies and ships.

9.2.1 Fluid Forces on Straight Surfaces

A motivation is needed before going through the routine of derivations. Initially, a simple case will be examined. Later, how the calculations can be simplified will be shown.

Example 9.1: Pressure on Door **Level: Intermediate**

A straight vertical ship door is exposed to salt water at density of ρ_{ws}. The width of door is 1[m] and its length is 2[m] and door hinges are located at the surface of the water. The calculate the total force acted on the door and the moment the water acts on the door. If water was acting both sides of the door the net force and momentum is zero. Why?

continue Ex. 9.1

Solution

The pressure that acting on door varied from zero at the top (since it is at the water surface) and maximum pressure at the bottom. To avoid using the mathematics the two values can be taken and averaged and multiply to get the force. The pressure at the bottom is $\rho\, g\, h$ and the height is given the h and thus average pressure

$$P_{avg} = \frac{\rho\, g\, h}{2} \tag{9.1.a}$$

The total force will be

$$F = \frac{\overbrace{\rho\, g\, h}^{P}}{2}\, A = 1000 \times 9.8 \times 2/2 \times \overbrace{2 \times 1}^{A} \sim 19,600[N] \tag{9.1.b}$$

The force is incredible almost 20 tons. According to same logic the moment (assuming that this concept is introduced somewhere else before) will be at the center and value will be 20 $ton \times m$. At this part a bit must be inserted and check against these assumptions. It is regular procedure to take an very small element in which the pressure is uniform and add these elements. The element chosen is an element dh and the width of the door. The pressure in the element $(a\, dh)$ is uniform since the element depth is uniform. The element force is $dF = \rho\, g\, h\, a\, dh$ and total force is

$$F = \int_0^h \rho\, g\, h\, a\, dh = \rho\, g\, a\, \frac{h^2}{2} \tag{9.1.c}$$

which identical to what was computed using the average pressure. Note the integral of h is $h^2/2$. To find the point of equivalent force (that force substitute pressure) obtained by

$$h_{effective} = \frac{M}{F} = \frac{\int_0^h h\, \overbrace{\cancel{\rho}\, \cancel{g}\, h\, \cancel{a}\, dh}^{dF}}{0.5\, \cancel{\rho}\, \cancel{g}\, \cancel{a}\, h^2} = \frac{\overbrace{\int_0^h h^2\, dh}^{h^3/3}}{0.5\, h} = \frac{2\, h}{3} \tag{9.1.d}$$

End of Ex. 9.1

The result is the center of triangle which is the shape of the pressure. So while the pressure assumption was correct the center of pressure is at center of the triangle.

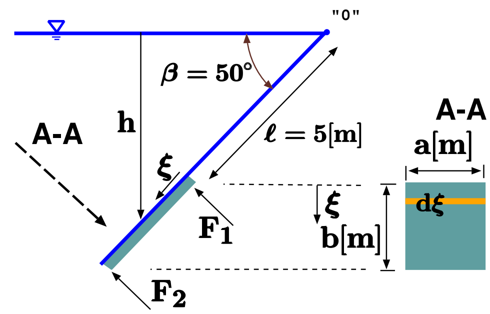


Fig. 9.3 – Rectangular area under pressure.

Example 9.2: Pressure on Rectangular

Level: Intermediate

Consider a rectangular shape gate as shown in Fig. 9.3. Calculate the minimum forces, F_1 and F_2 to maintain the gate in position. Assuming that the atmospheric pressure can be ignored.

Solution

The forces can be calculated by looking at the moment around point "O." The element of moment is $a\,d\xi$ for the width of the gate and is

$$dM = \overbrace{P\;\underbrace{a\,d\xi}_{dA}}^{dF}(\ell + \xi) \tag{9.2.a}$$

The pressure, P can be expressed as a function ξ as the following

$$P = g\,\rho\,(\ell + \xi) sin\beta$$

The liquid total moment on the gate is

$$M = \int_0^b g\,\rho\,(\ell + \xi)\sin\beta\,a\,d\xi(\ell + \xi)$$

The integral can be simplified as

$$M = g\,a\,\rho\,\sin\beta\int_0^b(\ell + \xi)^2 d\xi \tag{9.4}$$

End of Ex. 9.2

The solution of the above integral is

$$M = g\,\rho\,a\,\sin\beta\left(\frac{3\,b\,l^2 + 3\,b^2\,l + b^3}{3}\right)$$

This value provides the moment that F_1 and F_2 should extract. Additional equation is needed. It is the total force, which is

$$F_{total} = \int_0^b g\,\rho\,(\ell + \xi)\sin\beta\,a\,d\xi$$

The total force integration provides

$$F_{total} = g\,\rho\,a\,\sin\beta\int_0^b (\ell + \xi)d\xi = g\,\rho\,a\,\sin\beta\left(\frac{2\,b\,\ell + b^2}{2}\right)$$

The forces on the gate have to provide

$$F_1 + F_2 = g\,\rho\,a\,\sin\beta\left(\frac{2\,b\,\ell + b^2}{2}\right)$$

Additionally, the moment of forces around point "O" is

$$F_1\,\ell + F_2(\ell + b) = g\,\rho\,a\,\sin\beta\left(\frac{3\,b\,l^2 + 3\,b^2\,l + b^3}{3}\right)$$

The solution of these equations is

$$F_1 = \frac{(3\,\ell + b)\;a\,b\,g\,\rho\,\sin\beta}{6}$$

$$F_2 = \frac{(3\,\ell + 2\,b)\;a\,b\,g\,\rho\,\sin\beta}{6}$$

The above calculations are time consuming and engineers always try to make life simpler. Looking at the above calculations, it can be observed that there is a moment of area in Eq. (9.4) and also a center of area. These concepts have been introduced in Chapter 15. Several represented areas for which moment of inertia

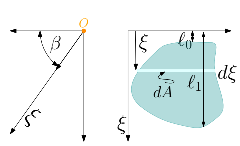


Fig. 9.4 – Schematic of submerged area to explain the center forces and moments.

and center of area have been tabulated in
Chapter 15. These tabulated values can be used to solve this kind of problems.

9.2.1.1 Symmetrical Shapes

Consider the two–dimensional symmetrical area that are under pressure as shown in Figure 9.4. The symmetry is around any axes parallel to axis x. The total force and moment that the liquid extracting on the area need to be calculated. First, the force is

$$F = \int_A P dA = \int (P_{atmos} + \rho\, g\, h) dA = A\, P_{atmos} + \rho\, g \int_{\ell_0}^{\ell_1} \overbrace{(\xi + \ell_0) \sin\beta}^{h(\xi)} dA \quad (9.5)$$

In this case, the atmospheric pressure can include any additional liquid layer above layer "touching" area. The "atmospheric" pressure can be set to zero.

The boundaries of the integral of equation (9.5) refer to starting point and ending points not to the start area and end area. The integral in equation (9.5) can be further developed as

$$F_{total} = A\, P_{atmos} + \rho\, g\, \sin\beta \left(\ell_0\, A + \overbrace{\int_{\ell_0}^{\ell_1} \xi dA}^{x_c\, A} \right) \quad (9.6)$$

In a final form as

Total Force in Inclined Surface

$$F_{total} = A \left[P_{atmos} + \rho\, g\, \sin\beta\, (\ell_0 + x_c) \right] \quad (9.7)$$

The moment of the liquid on the area around point "O" is

$$M_y = \int_{\xi_0}^{\xi_1} P(\xi) \xi dA \quad (9.8)$$

$$M_y = \int_{\xi_0}^{\xi_1} (P_{atmos} + g\, \rho\, \overbrace{h(\xi)}^{\xi\, \sin\beta}) \xi dA \quad (9.9)$$

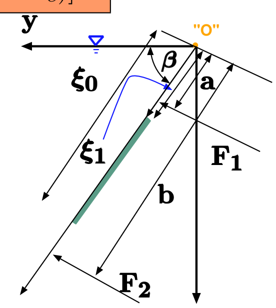


Fig. 9.5 – The general forces acting on submerged area.

Or separating the parts as

$$M_y = P_{atmos} \overbrace{\int_{\xi_0}^{\xi_1} \xi dA}^{x_c\, A} + g\, \rho\, \sin\beta \overbrace{\int_{\xi_0}^{\xi_1} \xi^2 dA}^{I_{x'x'}} \quad (9.10)$$

The moment of inertia, $I_{x'x'}$, is about the axis through point "O" into the page. Equation (9.10) can be written in more compact form as

Total Moment in Inclined Surface

$$M_y = P_{atmos}\, x_c\, A + g\, \rho\, \sin\beta I_{x'x'} \tag{9.11}$$

Example 9.2 can be generalized to solve any two forces needed to balance the area/-gate. Consider the general symmetrical body shown in Fig. 9.5 which has two forces that balance the body. Equations (9.7) and (9.11) can be combined the moment and force acting on the general area. If the "atmospheric pressure" can be zero or include additional layer of liquid. The forces balance reads

$$F_1 + F_2 = A\, \left[P_{atmos} + \rho\, g\, \sin\beta\, \left(\ell_0 + x_c\right)\right] \tag{9.12}$$

and moments balance reads

$$F_1\, a + F_2\, b = P_{atmos}\, x_c\, A + g\, \rho\, \sin\beta I_{x'x'} \tag{9.13}$$

The solution of these equations is

$$F_1 = \frac{\left[\left(\rho\, \sin\beta - \frac{P_{atmos}}{g\, b}\right)\, x_c + \ell_0\, \rho\, \sin\beta + \frac{P_{atmos}}{g}\right]\, b\, A - I_{x'x'}\, \rho\, \sin\beta}{g\, (b-a)} \tag{9.14}$$

and

$$F_2 = \frac{I_{x'x'}\, \rho\, \sin\beta - \left[\left(\rho\, \sin\beta - \frac{P_{atmos}}{g\, a}\right)\, x_c + \ell_0\, \rho\, \sin\beta + \frac{P_{atmos}}{g}\right]\, a\, A}{g\, (b-a)} \tag{9.15}$$

In the solution, the forces can be negative or positive, and the distance a or b can be positive or negative. Additionally, the atmospheric pressure can contain either an additional liquid layer above the "touching" area or even atmospheric pressure simply can be set up to zero. In symmetrical area only two forces are required since the moment is one dimensional. However, in non–symmetrical area there are two different moments and therefor three forces are required. Thus, additional equation is required. This equation is for the additional moment around the x axis (see for explanation in Figure 9.6). The moment around the y axis is given by equation (9.11) and the total force is given by (9.7). The moment around the x axis (which was arbitrary chosen) should be

$$M_x = \int_A y\, P dA \tag{9.16}$$

Substituting the components for the pressure transforms equation (9.16) into

$$M_x = \int_A y\, \left(P_{atmos} + \rho\, g\, \xi\, \sin\beta\right) dA \tag{9.17}$$

The integral in equation (9.16) can be written as

$$M_x = P_{atmos} \overbrace{\int_A y\, dA}^{A\, y_c} + \rho\, g\, \sin\beta \overbrace{\int_A \xi\, y\, dA}^{I_{x'y'}} \tag{9.18}$$

The compact form can be written as

Moment in Inclined Surface

$$M_x = P_{atmos}\, A\, y_c + \rho\, g\, \sin\beta\, I_{x'y'} \tag{9.19}$$

The product of inertia was presented in Chapter 15. These equations (9.7), (9.11) and (9.19) provide the base for solving any problem for straight area under pressure with uniform density. There are many combinations of problems (e.g. two forces and moment) but no general solution is provided. Example to illustrate the use of these equations is provided.

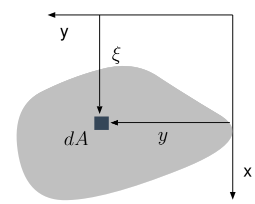


Fig. 9.6 – The general forces acting on non symmetrical straight area.

Example 9.3: Pressure on Non Symeterical **Level: Intermediate**

Calculate the forces which required to balance the triangular shape shown in the Figure 9.7.

Solution

The three equations that needs to be solved are

$$F_1 + F_2 + F_3 = F_{total} \tag{9.3.a}$$

The moment around x axis is

$$F_1\, b = M_y \tag{9.3.b}$$

The moment around y axis is

$$F_1\, \ell_1 + F_2\, (a + \ell_0) + F_3\, \ell_0 = M_x \tag{9.3.c}$$

The right hand side of these equations are given before in equations (9.7), (9.11) and (9.19).
The moment of inertia of the triangle around x is made of two triangles (as shown in the Figure (9.7) for triangle 1 and 2).

continue Ex. 9.3

Triangle 1 can be calculated as the moment of inertia around its center which is $\ell_0 + 2 * (\ell_1 - \ell_0)/3$. The height of triangle 1 is $(\ell_1 - \ell_0)$ and its width b and thus, moment of inertia about its center is $I_{xx} = b(\ell_1 - \ell_0)^3/36$. The moment of inertia for triangle 1 about y is

$$I_{xx1} = \frac{b(\ell_1-\ell_0)^3}{36} + \overbrace{\frac{b(\ell_1-\ell_0)}{3}}^{A_1} \overbrace{\left(\ell_0 + \frac{2(\ell_1-\ell_0)}{3}\right)^2}^{\Delta {x_1}^2}$$

The height of the triangle 2 is $a - (\ell_1 - \ell_0)$ and its width b and thus, the moment of inertia about its center is

$$I_{xx2} = \frac{b[a-(\ell_1-\ell_0)]^3}{36} + \overbrace{\frac{b[a-(\ell_1-\ell_0)]}{3}}^{A_2} \overbrace{\left(\ell_1 + \frac{[a-(\ell_1-\ell_0)]}{3}\right)^2}^{\Delta {x_2}^2}$$

and the total moment of inertia

$$I_{xx} = I_{xx1} + I_{xx2}$$

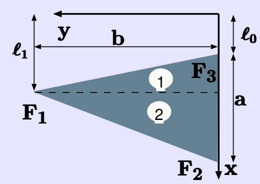


Fig. 9.7 – The general forces acting on a non symmetrical straight area.

The product of inertia of the triangle can be obtain by integration. It can be noticed that upper line of the triangle is $y = \frac{(\ell_1-\ell_0)x}{b} + \ell_0$. The lower line of the triangle is $y = \frac{(\ell_1-\ell_0-a)x}{b} + \ell_0 + a$.

$$I_{xy} = \int_0^b \left[\int_{\frac{(\ell_1-\ell_0)x}{b}+\ell_0}^{\frac{(\ell_1-\ell_0-a)x}{b}+\ell_0+a} x\, y\, dx \right] dy = \frac{2\, a\, b^2\, \ell_1 + 2\, a\, b^2\, \ell_0 + a^2\, b^2}{24}$$

The solution of this set equations is

$$F_1 = \overbrace{\left[\frac{a\, b}{3}\right]}^{A} \frac{\left(g\, \left(6\, \ell_1 + 3\, a\right) + 6\, g\, \ell_0\right)\, \rho\, \sin\beta + 8\, P_{atmos}}{24},$$

End of Ex. 9.3

$$\frac{F_2}{\left[\frac{a\,b}{3}\right]} = -\frac{\left((3\,\ell_1 - 14\,a) - \ell_0\left(\frac{12\,\ell_1}{a} - 27\right) + \frac{12\,{\ell_0}^2}{a}\right) g\,\rho\,\sin\beta}{72} - \frac{\left(\left(\frac{24\,\ell_1}{a} - 24\right) + \frac{48\,\ell_0}{a}\right) P_{atmos}}{72},$$

$$\frac{F_3}{\left[\frac{a\,b}{3}\right]} = \frac{\left(\left(a - \frac{15\,\ell_1}{a}\right) + \ell_0\left(27 - \frac{12\,\ell_1}{a}\right) + \frac{12\,{\ell_0}^2}{a}\right) g\,\rho\,\sin\beta}{72} + \frac{\left(\left(\frac{24\,\ell_1}{a} + 24\right) + \frac{48\,\ell_0}{a}\right) P_{atmos}}{72}$$

9.2.2 Pressure Center

In the literature, pressure centers are commonly defined. These definitions are mathematical in nature and has physical meaning of equivalent force that will act through this center. The definition is derived or obtained from equation (9.11) and equation (9.19). The pressure center is the distance that will create the moment with the hydrostatic force on point "O." Thus, the pressure center in the x direction is

$$x_p = \frac{1}{F}\int_A x\,P\,dA \tag{9.20}$$

In the same way, the pressure center in the y direction is defined as

$$y_p = \frac{1}{F}\int_A y\,P\,dA \tag{9.21}$$

To show relationship between the pressure center and the other properties, it can be found by setting the atmospheric pressure and ℓ_0 to zero as following

$$x_p = \frac{g\,\rho \sin\beta\, I_{x'x'}}{A\,\rho\,g\,\sin\beta\,x_c} \tag{9.22}$$

Expanding $I_{x'x'}$ according to equation (8.12) results in

$$x_p = \frac{I_{xx}}{x_c\,A} + x_c \tag{9.23}$$

and in the same fashion in y direction

$$y_p = \frac{I_{xy}}{y_c\,A} + y_c \tag{9.24}$$

It has to emphasis that these definitions are useful only for case where the atmospheric pressure can be neglected or canceled and where ℓ_0 is zero. Thus, these limitations diminish the usefulness of pressure center definitions. In fact, the reader can find that direct calculations can sometimes simplify the problem.

9.3 Pressure on Ship

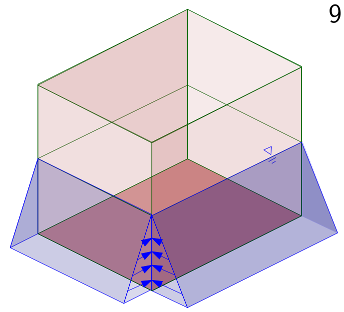


Fig. 9.8 – Pressure on a floating block.

While the fundamentals of the pressure where discussed before it worth while to review with a typical body like a block. The pressure around the block has a shap of triangle. It can be noticed that pressure on the front is identical to the pressure at the back. Hence it cancel each other. The same can be said for the right side and the left side. Similar argument is made for diagonal surfaces. For diagonal surface the net force in any direction is just the "shadow" in that direction times the pressure. The pressure is the same in the same depth and hence it cancel. The only side that pressure is not cancel it the pressure on the bottom. The pressure is total (force $= P\,A$) in the weight of the ship or the body. When the body is total immersed in the liquid, the force at the bottom is reduced by the force at the top. The net force in the vertical direction, at the stationary conditions, is the weight of the body.

9.4 Summary

The main points that this chapter was indent to illustrate are the following. Pressure lines are constant depth form the surface. The pressure acting surfaces from both sides has zero force and zero moment because there is a counter force and momentum. On straight surfaces the forces are at 2/3 of the distance.

10 Buoyancy

10.1 Introduction

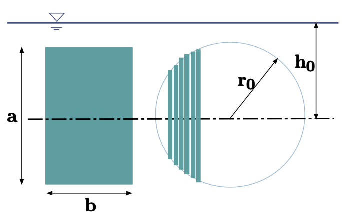


Fig. 10.1 – Schematic of Immersed Cylinder.

One of the oldest known scientific research on fluid mechanics relates to buoyancy due to question of money was carried by Archimedes. Archimedes principle is related to question of density and volume. While Archimedes did not know much about integrals, he was able to capture the essence. Here, because this material is presented in a different era, more advance mathematics will be used. While the question of the stability was not scientifically examined in the past, the floating vessels structure (more than 300 years ago) show some understanding[1].

The total net forces the liquid and gravity exact on a body are considered as a buoyancy issue while the moment these force considered as a stability issue. The buoyancy issue was solved by Archimedes and for all practical purpose is really solved

[1]This topic was the author's high school class name (ship stability). It was taught by people like these, 300 years ago and more, ship builders who knew how to calculate $\bar{GM}$ but weren't aware of scientific principles behind it. If the reader wonders why such a class is taught in a high school, perhaps the name can explain it: Sea Officers High School (or Acco (sometimes spell Akko) Nautical College)

issue. Furthermore, as a derivative issue, the stability in the perpendicular direction to the liquid surface is a solved problem which did not cause any real question (like oscillating of body is solved problem). While there are recent papers which deal the issue but they do solve any issue in this respect. However, the rotation stability is issue that continue to be evolved even after this work. There three approaches that deal with issue which are in historical order are Metacenter, Potential, and Direct Examination[2]. These stability issues will discussed in the next several chapters.

To understand this issue, consider a cubical and a cylindrical body that is immersed in liquid and center in a depth of, h_0 as shown in Fig. 10.1. The force which hold the cylinder at the place made from the integration of the pressure around the surface of the bodies. The forces on square geometry body are made only of vertical forces because the two sides cancel each other. However, on the vertical direction, the pressure on the two surfaces are different. On the upper surface the pressure is $\rho\, g\, (h_0 - a/2)$. On the lower surface the pressure is $\rho\, g\, (h_0 + a/2)$.

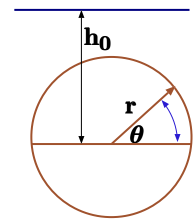


Fig. 10.2 – The floating forces on Immersed Cylinder.

The force due to the liquid pressure per unit depth (into the page) is

$$F = \rho\, g\, \left((h_0 - a/2) - (h_0 + a/2) \right) \ell\, b = -\rho\, g\, a\, b\, \ell = -\rho\, g\, V \tag{10.1}$$

In this case the ℓ represents a depth (into the page). Rearranging equation (10.1) to be

$$\frac{F}{V} = \rho\, g \longrightarrow F = V\, \rho\, g \tag{10.2}$$

The force acting on the immersed body is equal to the weight of the displaced liquid. This analysis can be generalized by noticing two things. All the horizontal forces are canceled. Any body that has a projected area that has two sides, those will cancel each other in the perpendicular to surface direction. Another way to look at this point is by approximation. For any two rectangle bodies, the horizontal forces are canceling each other. Thus, even these bodies are in contact with each other, the imaginary pressure make it so that they cancel each other.

On the other hand, any shape is made of many small rectangles. The force on every rectangular shape is made of its weight of the volume. Thus, the total force is made of the sum of all the small rectangles which is the weight of the sum of all volume.

In illustration of this concept, consider the cylindrical shape in Figure 10.1. The force per area (see Figure 10.2) is

$$dF = \overbrace{\rho\, g\, (h_0 - r\, \sin\theta)}^{P}\, \overbrace{\sin\theta\, r\, d\theta}^{dA_{vertical}} \tag{10.3}$$

[2]The first method was developed 300 years ago, the potential was developed about 30 years ago and Direct Examination is present here for the first time.

The total force will be the integral of the equation (10.3)

$$F = \int_0^{2\pi} \rho\, g\, (h_0 - r\, \sin\theta)\, r\, d\theta\, \sin\theta \tag{10.4}$$

Rearranging equation (10.3) transforms it to

$$F = r\, g\, \rho \int_0^{2\pi} (h_0 - r\, \sin\theta) \sin\theta\, d\theta \tag{10.5}$$

The solution of equation (10.5) is

$$F = -\pi\, r^2\, \rho\, g \tag{10.6}$$

The negative sign indicate that the force acting upwards. While the horizontal force is

$$F_v = \int_0^{2\,\pi} (h_0 - r\, \sin\theta) \cos\theta\, d\theta = 0 \tag{10.7}$$

10.2 Examples

Example 10.1: Floating Log

Level: Intermediate

To what depth will a long log with radius, r, a length, ℓ and density, ρ_s in liquid with density, ρ_ℓ. Assume that $\rho_\ell > \rho_s$. You can provide the angle or the depth as the solution.

Solution

This example actually deals with two dimensional problem. Archimedes formula reads

$$V\, \rho_s = V_0\, \rho_\ell \tag{10.1.a}$$

The equations for both volumes linearly depended on the length s as

$$A \cancel{s}\, \rho_s = A_0 \cancel{s}\, \rho_\ell \tag{10.1.b}$$

where A_0 is the area of the segment. The area of segment as a function of θ is

$$A = \frac{r^2\, (\theta - \sin\theta)}{2} \tag{10.1.c}$$

End of Ex. 10.1

Substituting Eq. (10.1.c) into Eq. (10.1.b) results in

$$\pi \cancel{r^2} \rho_s = \left(\frac{\cancel{r^2} \left(\theta - \sin\theta \right)}{2} \right) \rho_\ell \tag{10.1.d}$$

which can be somewhat simplified as

$$\frac{2\,\pi\,\rho_s}{\rho_\ell} = \left(\theta - \sin\theta \right) \tag{10.1.e}$$

From practical point of view, this example is solved. Nevertheless, for the fun of the math the $\sin\theta$ can be expanded in Taylor series as

$$\frac{2\,\pi\,\rho_s}{\rho_\ell} = \frac{\theta^3}{3!} - \frac{\theta^5}{5!} + \frac{\theta^7}{7!} + \ldots \tag{10.1.f}$$

And for small θ (or small ratio of the density) Eq. (10.1.f) is reduced into

$$\theta \sim \sqrt[3]{\frac{12\,\pi\,\rho_s}{\rho_\ell}} \quad \theta \lessapprox 0.2 \tag{10.1.g}$$

Which means that $\frac{\rho_s}{\rho_\ell} < 0.0002$ for this approximation to be valid.

Typical examples to explain the buoyancy are of the vessel with thin walls put upside down into liquid. The second example of the speed of the floating bodies. Since there are no better examples, these examples are a must.

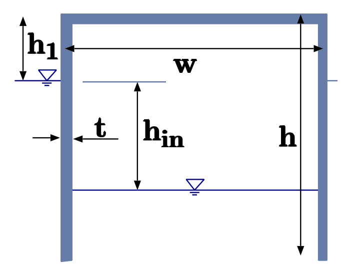


Fig. 10.3 – Schematic of a thin wall floating body.

Example 10.1:
A cylindrical body, shown in Figure 10.3 ,is floating in liquid with density, ρ_l. The body was inserted into liquid in a such a way that the air had remained in it. Express the maximum wall thickness, t, as a function of the density of the wall, ρ_s liquid density, ρ_l and the surroundings air temperature, T_1 for the body to float. In the case where thickness is half the maximum, calculate the pressure inside the container. The container diameter is w. Assume that the wall thickness is small compared with the other dimensions ($t << w$ and $t << h$).

Solution

The air mass in the container is

$$m_{air} = \overbrace{\pi\, w^2\, h}^{V} \overbrace{\frac{P_{atmos}}{R\,T}}^{\rho_{air}}$$

The mass of the container is

$$m_{container} = \left(\overbrace{\pi\, w^2 + 2\,\pi\, w\, h}^{A} \right) t\, \rho_s$$

The liquid amount enters into the cavity is such that the air pressure in the cavity equals to the pressure at the interface (in the cavity). Note that for the maximum thickness, the height, h_1 has to be zero. Thus, the pressure at the interface can be written as

$$P_{in} = \rho_l\, g\, h_{in}$$

On the other hand, the pressure at the interface from the air point of view (ideal gas model) should be

$$P_{in} = \frac{m_{air}\, R\, T_1}{\underbrace{h_{in}\, \pi\, w^2}_{V}}$$

Since the air mass didn't change and it is known, it can be inserted into the above equation.

$$\rho_l\, g\, h_{in} + P_{atmos} = P_{in} = \frac{\left(\pi\, w^2\, h\right) \overbrace{\frac{P_{atmos}}{R\,T_1}}^{\rho} R\, T_1}{h_{in}\, \pi\, w^2}$$

The last equation can be simplified into

$$\rho_l\, g\, h_{in} + P_{atmos} = \frac{h\, P_{atmos}}{h_{in}}$$

And the solution for h_{in} is

$$h_{in} = -\frac{P_{atmos} + \sqrt{4\, g\, h\, P_{atmos}\, \rho_l + {P_{atmos}}^2}}{2\, g\, \rho_l}$$

and

$$h_{in} = \frac{\sqrt{4\, g\, h\, P_{atmos}\, \rho_l + {P_{atmos}}^2} - P_{atmos}}{2\, g\, \rho_l}$$

The solution must be positive, so that the last solution is the only physical solution.

End Solution

Advance material can be skipped

Example 10.2:
Calculate the minimum density an infinitely long equilateral triangle Triangle shape (three equal sides) has to be so that the sharp end is in the water. Assume that the body is stable (it is not for most).

Solution

The solution demonstrates that when $h \longrightarrow 0$ then $h_{in} \longrightarrow 0$. When the gravity approaches zero (macro gravity) then

$$h_{in} = \frac{P_{atmos}}{\rho_l\, g} + h - \frac{h^2\, \rho_l\, g}{P_{atmos}} + \frac{2\, h^3\, {\rho_l}^2\, g^2}{{P_{atmos}}^2} - \frac{5\, h^4\, {\rho_l}^3\, g^3}{{P_{atmos}}^3} + \cdots$$

This "strange" result shows that bodies don't float in the normal sense. When the floating is under vacuum condition, the following height can be expanded into

$$h_{in} = \sqrt{\frac{h\, P_{atmos}}{g\, \rho_l}} + \frac{P_{atmos}}{2\, g\, \rho_l} + \cdots$$

which shows that the large quantity of liquid enters into the container as it is expected.

Archimedes theorem states that the force balance is at displaced weight liquid (of the same volume) should be the same as the container, the air. Thus,

$$\overbrace{\pi\, w^2\, (h - h_{in})\, g}^{\text{net displayed water}} = \overbrace{\left(\pi\, w^2 + 2\, \pi\, w\, h\right)\, t\, \rho_s\, g}^{\text{container}} + \overbrace{\pi\, w^2\, h\, \left(\frac{P_{atmos}}{R\, T_1}\right)\, g}^{\text{air}}$$

If air mass is neglected the maximum thickness is

$$t_{max} = \frac{2\, g\, h\, w\, \rho_l + P_{atmos}\, w - w\, \sqrt{4\, gh\, P_{atmos}\, \rho_l + {P_{atmos}}^2}}{\left(2\, g\, w + 4\, g\, h\right)\, \rho_l\, \rho_s}$$

The condition to have physical value for the maximum thickness is

$$2\, g\, h\, \rho_l + P_{atmos} \geq \sqrt{4\, gh\, P_{atmos}\, \rho_l + {P_{atmos}}^2}$$

The full solution is

$$t_{max} = -\frac{\left(w\, R\, \sqrt{4\, gh\, P_{atmos}\, \rho_l + {P_{atmos}}^2} - 2\, g\, h\, w\, R\, \rho_l - P_{atmos}\, w\, R\right)\, T_1 + 2\, g\, h\, P_{atmos}\, w\, \rho_l}{\left(2\, g\, w + 4\, g\, h\right)\, R\, \rho_l\, \rho_s\, T_1}$$

In this analysis the air temperature in the container immediately after insertion in the liquid has different value from the final temperature. It is reasonable as the first approximation to assume that the process is adiabatic and isentropic. Thus, the temperature in the cavity immediately after the insertion is

$$\frac{T_i}{T_f} = \left(\frac{P_i}{P_f}\right)$$

The final temperature and pressure were calculated previously. The equation of state is

$$P_i = \frac{m_{air}\, R\, T_i}{V_i}$$

The new unknown must provide additional equation which is

$$V_i = \pi\, w^2\, h_i$$

Thickness Below The Maximum

For the half thickness $t = \frac{t_{max}}{2}$ the general solution for any given thickness below maximum is presented. The thickness is known, but the liquid displacement is still unknown. The pressure at the interface (after long time) is

$$\rho_l\, g\, h_{in} + P_{atmos} = \frac{\pi\, w^2\, h \frac{P_{atmos}}{R\, T_1} R\, T_1}{\left(h_{in} + h_1\right)\, \pi\, w^2}$$

which can be simplified to

$$\rho_l\, g\, h_{in} + P_{atmos} = \frac{h\, P_{atmos}}{h_{in} + h_1}$$

The second equation is Archimedes' equation, which is

$$\pi\, w^2 \left(h - h_{in} - h_1\right) = \left(\pi\, w^2 + 2\, \pi\, w\, h\right) t\, \rho_s\, g\big) + \pi\, w^2\, h \left(\frac{P_{atmos}}{R\, T_1}\right) g$$

End Solution

End Advance material

Example 10.3:
A body is pushed into the liquid to a distance, h_0 and left at rest. Calculate acceleration and time for a body to reach the surface. The body's density is $\alpha\, \rho_l$, where α is ratio between the body density to the liquid density and ($0 < \alpha < 1$). Is the body volume important?

SOLUTION

The net force is

$$F = \overbrace{V\, g\, \rho_l}^{\text{liquid weight}} - \overbrace{V\, g\, \alpha\, \rho_l}^{\text{body weight}} = V\, g\, \rho_l \left(1 - \alpha\right)$$

But on the other side the internal force is

$$F = m\, a = \overbrace{V\, \alpha \rho_l}^{m}\, a$$

Thus, the acceleration is

$$a = g\left(\frac{1-\alpha}{\alpha}\right)$$

If the object is left at rest (no movement) thus time will be ($h = 1/2\,a\,t^2$)

$$t = \sqrt{\frac{2\,h\alpha}{g(1-\alpha)}}$$

If the object is very light ($\alpha \longrightarrow 0$) then

$$t_{min} = \sqrt{\frac{2\,h\,\alpha}{g}} + \frac{\sqrt{2\,g\,h}\;\alpha^{\frac{3}{2}}}{2\,g} + \frac{3\,\sqrt{2\,g\,h}\,\alpha^{\frac{5}{2}}}{8\,g} + \frac{5\,\sqrt{2\,g\,h}\,\alpha^{\frac{7}{2}}}{16\,g} + \cdots$$

From the above equation, it can be observed that only the density ratio is important. This idea can lead to experiment in "large gravity" because the acceleration can be magnified and it is much more than the reverse of free falling.

End Solution

Example 10.4:
In some situations, it is desired to find equivalent of force of a certain shape to be replaced by another force of a "standard" shape. Consider the force that acts on a half sphere. Find equivalent cylinder that has the same diameter that has the same force.

SOLUTION

The force act on the half sphere can be found by integrating the forces around the sphere. The element force is

$$dF = (\rho_L - \rho_S)\,g\,\overbrace{r\;\cos\phi\;\cos\theta}^{h}\overbrace{\cos\theta\;\cos\phi\;\overbrace{r^2\,d\theta\,d\phi}^{dA}}^{dA_x}$$

The total force is then

$$F_x = \int_0^{\pi}\int_0^{\pi}(\rho_L - \rho_S)\,g\cos^2\phi\cos^2\theta\,r^3\,d\theta\,d\phi$$

The result of the integration the force on sphere is

$$F_s = \frac{\pi^2\,(\rho_L - \rho_S)\,r^3}{4}$$

The force on equivalent cylinder is

$$F_c = \pi\,r^2\,(\rho_L - \rho_S)\,h$$

These forces have to be equivalent and thus

$$\frac{\pi \not{r} \, \cancel{(\rho_L - \rho_S)} \, r^{\not{3}}}{4} = \not{\pi} \not{r}^{\not{2}} \, \cancel{(\rho_L - \rho_S)} \, h$$

Thus, the height is

$$\frac{h}{r} = \frac{\pi}{4}$$

End Solution

Example 10.5:

In the introduction to this section, it was assumed that above liquid is a gas with inconsequential density. Suppose that the above layer is another liquid which has a bit lighter density. Body with density between the two liquids, $\rho_l < \rho_s < rho_h$ is floating between the two liquids. Develop the relationship between the densities of liquids and solid and the location of the solid cubical. There are situations where density is a function of the depth. What will be the location of solid body if the liquid density varied parabolically.

SOLUTION

In the discussion to this section, it was shown that the net force is the body volume times the density of the liquid. In the same vein, the body can be separated into two: one in first liquid and one in the second liquid. In this case there are two different liquid densities. The net force down is the weight of the body $\rho_c \, h \, A$. Where h is the height of the body and A is its cross section. This force is balance according to above explanation by the two liquid as

$$\rho_c \cancel{h A} = \cancel{A h} \left(\alpha \, \rho_l + (1-\alpha)\rho_h \right)$$

Where α is the fraction that is in low liquid. After rearrangement it became

$$\alpha = \frac{\rho_c - \rho_h}{\rho_l - \rho_h}$$

the second part deals with the case where the density varied parabolically. The density as a function of x coordinate along h starting at point ρ_h is

$$\rho(x) = \rho_h - \left(\frac{x}{h}\right)^2 (\rho_h - \rho_l)$$

Thus the equilibration will be achieved, A is canceled on both sides, when

$$\rho_c \, h = \int_{x_1}^{x_1+h} \left[\rho_h - \left(\frac{x}{h}\right)^2 (\rho_h - \rho_l) \right] dx$$

After the integration the equation transferred into

$$\rho_c \, h = \frac{(3\,\rho_l - 3\,\rho_h) \; x1^2 + (3\,h\,\rho_l - 3\,h\,\rho_h) \; x1 + h^2\,\rho_l + 2\,h^2\,\rho_h}{3\,h}$$

And the location where the lower point of the body (the physical), x_1, will be at

$$X_1 = \frac{\sqrt{3}\,\sqrt{3\,h^2\,{\rho_l}^2 + \left(4\,\rho_c - 6\,h^2\,\rho_h\right)\,\rho_l + 3\,h^2\,{\rho_h}^2 - 12\,\rho_c\,\rho_h} + 3\,h\,\rho_l - 3\,h\,\rho_h}{6\,\rho_h - 2\,\rho_l}$$

For linear relationship the following results can be obtained.

$$x_1 = \frac{h\,\rho_l + h\,\rho_h - 6\,\rho_c}{2\,\rho_l - 2\,\rho_h}$$

In many cases in reality the variations occur in small zone compare to the size of the body. Thus, the calculations can be carried out under the assumption of sharp change. However, if the body is smaller compare to the zone of variation, they have to accounted for.

End Solution

Example 10.6:

A hollow sphere is made of steel ($\rho_s/\rho_w \cong 7.8$) with a t wall thickness. What is the thickness if the sphere is neutrally buoyant? Assume that the radius of the sphere is R. For the thickness below this critical value, develop an equation for the depth of the sphere.

Solution

The weight of displaced water has to be equal to the weight of the sphere

$$\rho_s\,\not{g}\,\frac{4\,\pi\,R^3}{3} = \rho_w\,\not{g}\left(\frac{4\,\pi\,R^3}{3} - \frac{4\,\pi\,\left(R-t\right)^3}{3}\right) \tag{10.VI.a}$$

after simplification equation (10.VI.a) becomes

$$\frac{\rho_s\,R^3}{\rho_w} = 3\,t\,R^2 - 3\,t^2\,R + t^3 \tag{10.VI.b}$$

Equation (10.VI.b) is third order polynomial equation which it's solution (see the appendix) is

$$\begin{aligned} t_1 &= \left(-\frac{\sqrt{3}\,i}{2} - \frac{1}{2}\right)\left(\frac{\rho_s}{\rho_w}R^3 - R^3\right)^{\frac{1}{3}} + R \\ t_2 &= \left(\frac{\sqrt{3}\,i}{2} - \frac{1}{2}\right)\left(\frac{\rho_s}{\rho_w}R^3 - R^3\right)^{\frac{1}{3}} + R \\ t_3 &= R\left(\sqrt[3]{\frac{\rho_s}{\rho_w} - 1} + 1\right) \end{aligned} \tag{10.VI.c}$$

The first two solutions are imaginary thus not valid for the physical world. The last solution is the solution that was needed. The depth that sphere will be located depends on the ratio of t/R which similar analysis to the above. For a given ratio of t/R, the weight displaced by the sphere has to be same as the sphere weight. The volume of a sphere cap (segment) is given by

$$V_{cap} = \frac{\pi\, h^2\, (3R - h)}{3} \tag{10.VI.d}$$

Spherical volume Where h is the sphere height above the water. The volume in the water is

$$V_{water} = \frac{4\,\pi\, R^3}{3} - \frac{\pi\, h^2\, (3R - h)}{3} = \frac{4\,\pi\, \left(R^3 - 3\,R\,h^2 + h^3\right)}{3} \tag{10.VI.e}$$

When V_{water} denotes the volume of the sphere in the water. Thus the Archimedes law is

$$\frac{\rho_w\, 4\,\pi\, \left(R^3 - 3\,R\,h^2 + h^3\right)}{3} = \frac{\rho_s\, 4\,\pi\, \left(3\,t\,R^2 - 3\,t^2\,R + t^3\right)}{3} \tag{10.VI.f}$$

or

$$\left(R^3 - 3\,R\,h^2 + h^3\right) = \frac{\rho_w}{\rho_s}\left(3\,t\,R^2 - 3\,t^2\,R + t^3\right) \tag{10.VI.g}$$

The solution of (10.VI.g) is

$$h = \left(\frac{\sqrt{-f R\,\left(4\,R^3 - f R\right)}}{2} - \frac{f R - 2\,R^3}{2}\right)^{\frac{1}{3}} + \frac{R^2}{\left(\frac{\sqrt{-f R\,\left(4\,R^3 - f R\right)}}{2} - \frac{f R - 2\,R^3}{2}\right)^{\frac{1}{3}}} \tag{10.VI.h}$$

Where $-fR = R^3 - \frac{\rho_w}{\rho_s}\left(3\,t\,R^2 - 3\,t^2\,R + t^3\right)$ There are two more solutions which contains the imaginary component. These solutions are rejected.

End Solution

Example 10.7:

One of the common questions in buoyancy is the weight with variable cross section and fix load. For example, a wood wedge of wood with a fix weight/load. The general question is at what the depth of the object (i.e. wedge) will be located. For simplicity, assume that the body is of a solid material.

Solution

It is assumed that the volume can be written as a function of the depth. As it was shown in the previous example, the relationship between the depth and the displaced liquid volume of the sphere. Here it is assumed that this relationship can be written as

$$V_w = f(d, \text{other geometrical parameters}) \tag{10.VII.a}$$

The Archimedes balance on the body is

$$\rho_\ell V_a = \rho_w V_w \tag{10.VII.b}$$

$$d = f^{-1} \frac{\rho_\ell V_a}{\rho_w} \tag{10.VII.c}$$

End Solution

Example 10.8:
In example 10.7 a general solution was provided. Find the reverse function, f^{-1} for cone with 30° when the tip is in the bottom.

Solution

First the function has to built for d (depth).

$$V_w = \frac{\pi\, d \left(\dfrac{d}{\sqrt{3}}\right)^2}{3} = \frac{\pi\, d^3}{9} \tag{10.VIII.a}$$

Thus, the depth is

$$d = \sqrt[3]{\frac{9\,\pi\,\rho_w}{\rho_\ell\, V_a}} \tag{10.VIII.b}$$

End Solution

Example 10.9:
Extruded isosceles triangle is submerged in liquid and the net force of this triangle is F. Outside the liquid the weight of the triangle is W. Calculate the ratio of solid density and liquid density. What is the volume of the wood? Assume the liquid density is known.

Solution

$$W = \rho_s\, V\, g \tag{10.IX.a}$$

where ρ_s density of the wood, V is the volume of the wood. In the above equation there are two unknowns and another equation is needed. In the liquid net force on submerged triangle is

$$F = W - \rho_\ell\, V\, g = \rho_s\, V\, g - \rho_\ell\, V\, g = V\, g\, (\rho_s - \rho_\ell) \tag{10.IX.b}$$

from Eq. (10.IX.a) the volume can be expressed and substitute in Eq. (10.IX.b)

$$F = \frac{W}{\rho_s\, \not{g}} \not{g}\, (\rho_s - \rho_\ell) \tag{10.IX.c}$$

and ultimately the solid density is

$$\frac{\rho_s}{\rho_\ell} = \frac{W}{W - F} \tag{10.IX.d}$$

This found density ratio can be used to calculate the volume

$$V = \frac{W}{\rho_s\, g} \tag{10.IX.e}$$

End Solution

10.3 Applications of Buoyancy

Example 10.10:

Instrument used to measure the density liquid is called a hydrometer. One possible design is a big heavy spherical ball attached to thin glass with a pipe of diameter, D. The spherical ball is designed to weighed down so the rig will float straight up. The hydrometer is placed in two liquids with different densities. Assume that $\rho_1 > \rho_2$. What is the difference in height above surface for these two situations. Assume the air has no weight.

SOLUTION

Again this question deals with Archimedes' law. In fact this device has a long history and it was "invented" numerous times. This question deals with the calibration of this device. The volume of the spherical attachment (acting like a ballast) denoted as V_0. The pipe has lower density than the liquid under the examination. The total mass of the hydrometer is fix and denoted as m_0 and total weight is $m_0\, g$. According to Archimedes law it should be placed by the buoyancy force.

$$m_0\, g = \rho_1 \left(V_0 + l_1 \overbrace{\pi\, D^2/4}^{A} \right) \tag{10.X.a}$$

Where l_1 is the length of the pipe in the case 1.

In the case 2 the same equation can be written

$$m_0\, g = \rho_2 \,\left(V_0 + l_2\, \pi\, D^2/4 \right) \tag{10.X.b}$$

The length can be expressed from Eq. (10.X.a) as

$$l_1 = \frac{\dfrac{m_0\, g}{\rho_1} - V_0}{\pi\, D^2/4} \tag{10.X.c}$$

and the difference in the height is

$$\Delta h = \frac{\dfrac{m_0\, g}{\rho_2} - V_0}{\pi\, D^2/4} - \frac{\dfrac{m_0\, g}{\rho_1} - V_0}{\pi\, D^2/4} \tag{10.X.d}$$

End Solution

Example 10.11:
Use the solution of Example 10.10 to convert it to a dimensional form.

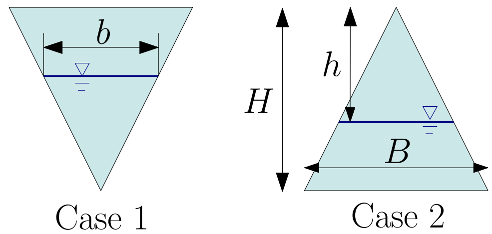


Fig. 10.4 – caption.

Example 10.12:
Extruded wooden isosceles triangle can float in liquid in many positions. For this excises consider two positions triangle floats on the head or the bottom. There is difference how much height triangle will be above the liquid. Draw the ratio of these two heights as a function of the density of of the wood to the liquid. That is the density of the wood obey $0 < \frac{\rho_s}{\rho_\ell} < 1$. *Show that the length of the triangle is irrelavente for this calculations. This question is a preparation of the stability as this height above the liquid.*

Solution

While the question deals with geometry and it effects on physics. Archimedes' law dictates that the area in both cases must be same. The relationship between the densities ratio and the area has to be established. In Case 1 the Archimedes' law is

$$\rho_s \frac{B\,H}{\cancel{2}} \cancel{L\,g} = \rho_\ell \frac{h_1\,b_1}{\cancel{2}} \cancel{L\,g} \tag{10.XII.a}$$

where index 1 and 2 refer to the case1 and case 2 respectively. The geometry identity relates the different sides of the triangle for both cases as

$$\frac{H}{B} = \frac{h}{b} \longrightarrow b = \frac{B\,h}{H} \tag{10.XII.b}$$

Eq. (10.XII.a) utilizing Eq. (10.XII.b) as

$$\rho_s\,B\,H = \rho_\ell\,{h_1}^2 \frac{B}{H} \longrightarrow \frac{\rho_s}{\rho_\ell} = \left(\frac{h_1}{H}\right)^2 \tag{10.XII.c}$$

For the case 2 the Archimedes's law is (now without the length and the gravity)

$$\rho_s \frac{B\,H}{\cancel{2}} = \rho_\ell (H - h_2) \frac{b_2 + B}{\cancel{2}} \tag{10.XII.d}$$

Substituting Eq. (10.XII.b) into Eq. (10.XII.d) yields

$$\rho_s\, B\, H = \rho_\ell (H - h_2) \left(\frac{B\, h_2}{H} + B \right) \tag{10.XII.e}$$

Eq. (10.XII.f) can be simplified as

$$\frac{\rho_s}{\rho_\ell} = \left(1 - \frac{h_2}{H}\right) \left(\frac{h_2}{H} + 1\right) \longrightarrow 1 - \left(\frac{h_2}{H}\right)^2 \tag{10.XII.f}$$

It common to denote two new parameters as $\bar{\rho} = \rho_s/\rho_\ell$ and $\bar{h} = h/H$. Hence Eq. (10.XII.f) can be written as

$$\bar{\rho} = 1 - \bar{h}^2 \tag{10.XII.g}$$

or case 1 as

$$\bar{\rho} = \bar{h}^2 \tag{10.XII.h}$$

Eqs. (10.XII.h) and (10.XII.g) are drawn Fig. 10.5.

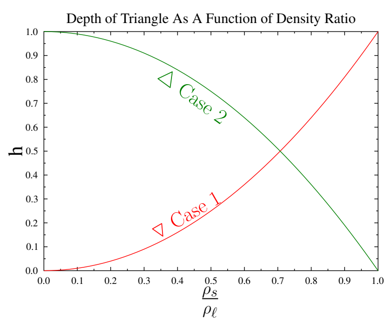


Fig. 10.5 – The size of the tip height above the liquid. The red line represents the case

The density ratio is approaching Zero the most the triangle is out the liquid. Notice that for case 1 (upside and color red) the zero means that h most the triangle is out of the liquid. While in case 2 the situation is the opposite. Notice that $h \longrightarrow 1$ most of the triangle is out of the liquid. For both cases, the function is a parabola.

End Solution

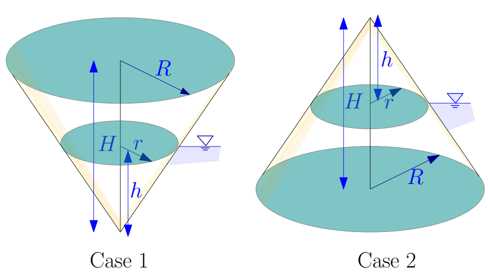


Fig. 10.6 – Floating cone height in liquid as a function of density ratio.

Example 10.13:
As in Example 10.12 repeat the calculation for a symmetrical cone shown in Fig. 10.6.

SOLUTION

The point of this example is not merely to repeat Example 10.12 but rather to show the idea of symmetry of calculations. That is, the solution in the more convenient (at least for this author) will be used to obtain the solution in the second case.

The cone has several properties that are well documented which include

Table 10.1 – Basic Cone properties

Cone Properties		
Name	Equation(s)	Units
V	$\frac{1}{3}\pi R^2 H$	none
Centroid	$H/4$	none

Archimedes' laws written in this case as

$$\rho_s \frac{\cancel{\pi} R^2 H}{\cancel{3}} = \rho_\ell \frac{\cancel{\pi} r^2 h}{\cancel{3}} \longrightarrow \rho_s R^2 H = \rho_\ell r^2 h \tag{10.XIII.a}$$

Geometrical relationship for the cone are

$$\frac{r}{R} = \frac{h}{H} \longrightarrow r = \frac{R h}{H} \tag{10.XIII.b}$$

Combine Eqs. (10.XIII.a) and (10.XIII.b) yields

$$\rho_s R^2 H = \rho_\ell \left(\frac{R h}{H}\right)^2 h \longrightarrow \frac{\rho_s}{\rho_\ell} = \left(\frac{h}{H}\right)^3 \tag{10.XIII.c}$$

Now Eq. (10.XIII.c) and from symmetry shown in Example 10.12 that case 2 the relationship is

$$\rho_s R^2 H = \rho_\ell \frac{\rho_s}{\rho_\ell} = 1 - \left(\frac{h}{H}\right)^3 \tag{10.XIII.d}$$

Additional argument or rational is that ratio should be same because the volume will be from both sides.

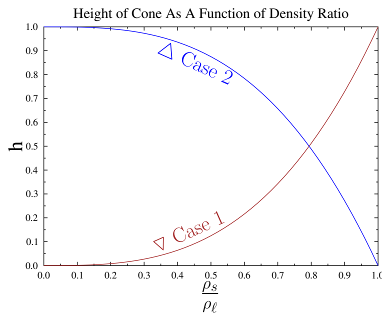


Fig. 10.7 – Cone height for various densities ratio of solid to liquid. Because the most the volume is the the base the claiming is slow and become rapid at ratio of 0.6

End Solution

10.4 Practical questions

This section indent to deal with practical problems that may or may not creates physical questions.

Example 10.14:

The ship Ever Given blocked the Suez Canal in on 23 March 2021 because navigation that might or might relate to stability. With this introduction this ship length dimensions became famous. Assume that Ever Given is a rectangular (it is not far away from the shape of the ship). For the following shape calculate the buoyancy centroid. When the ship is empty draft is $14.5[m]$ *and when the ship is the fully loaded the draft is* $16[mm]$ *The ship's width is* $59[m]$ *and ship length is about* $399[m]$*. Assuming that the ship is made from uniform material what the distance* GB*?*

Solution

The width and the length of the ship do effect the buoyancy centroid. The centroid is half of the depth. In this case, the value is $7.25[m]$ when empty and $8[m]$ when the ship is full.

End Solution

11 Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation

12

Direct Examination

12.1 Introduction

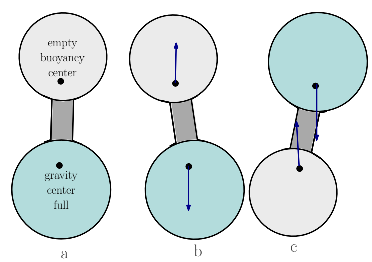


Fig. 12.1 – Schematic of floating bodies.

Simplistically, the stability (of floating body) is divided into three categories. When moments/forces are such that they returned the immersed body to its original position state is referred to as the stable body and vice versa. The third state is when the couple forces do have zero moment, it is referred to as the neutral stable. An example of such situation is a rounded body, like a marble, on flat surface[1]

Floating **uniform** density bodies are, as it can be observed, are inherently "unstable" because the gravity centroid is always above buoyancy centroid (case c in Fig. 12.1). Only at extreme cases where liquid density is almost equal to the density of solid body it will be neutral stability. Bodies with none uniform densities can be both situations, in stable and unstable. The bodies with none uniform density can be arranged so that the mass centroid in lower position and the buoyancy centroid. This fact can be illustrated by Fig. 12.4 where the mass/gravity centroid can be at lower point than the buoyant center. The discussion here will be focused on uniformed bodies as they provide more complicated situations. The none uniformed bodies are like uniform bodies but with a movable gravity centroid. To understand the unstable zone consider Fig. 12.1 which shows a body made of a hollow balloon and a heavy sphere connected by a thin and

[1]It happen that the same for ball (spherical) and for the same reasons.

light rod in three different configurations. The left one (a) shows the sphere just under the balloon in middle (b) there is a slight deviation from the previous case. Case 3 depicts (right side) almost opposite to case (a). This arrangement has mass centroid close to the middle of the sphere. The buoyant centroid is below the middle of the balloon. If this arrangement is inserted into liquid and will be floating, the balloon will be on the top and sphere on the bottom Fig. 12.1a. Tilting the body with a small angle from its resting position creates a shift in the forces direction to return original state (examine Fig. 12.1a). These forces create a moment which wants to return the body to the resting (original) position. When the body is at the position shown in Fig. 12.1c, the body is unstable and any tilt from the original position creates moment that will further continue to move the body from its original position. This analysis doesn't violate the second law of thermodynamics because it takes energy to move the body to the unstable situation.

When a solid object is placed on a straight horizontal surface as long as the gravity centroid is placed above the contact surface between the body and the surface, it is stable (see Fig. 12.2). The body can be stable in other situations but they equivalent to the above statement. Yet, when a body (solid) is placed on top of liquid, the conditions of stability are more complicated. It is not longer sufficient for the gravity centroid to be about the contact area (the same area as in the solid). Furthermore, in one liquid the body can be stable while in another liquid the body is unstable. Even the number of stable points and their locations are different between the liquid and the solid.

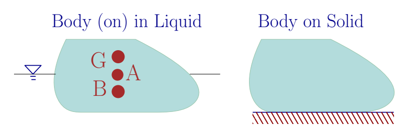


Fig. 12.2 – Center of mass arbitrary floating body on solid surface and in liquid.

12.2 Centroid of Floating Body or Buoyancy Centroid

To carry this analysis a new concept has to introduce, the centroid of displaced liquid or Buoyancy Centroid which is denoted "B." The pressure center discussed in Section 9.2.2 in this section expanded to deals with the equivalents force that acting on the floating bodies. To illustrate this point consider an arbitrary shape floats on liquid shown in Fig. 12.2. It was shown, in this book, that the force acting on floating body must be only in the vertical direction. Furthermore, the liquid pressure must be balanced the displaced liquid. The equivalent force of the pressure acting on the body in equilibrium can be obtained from calculating the (mass) centroid of the displaced liquid. Note that the above statement is correct for arbitrary density (for example, if the density, $\rho = f(h)$). If the body is not in equilibrium with the floating force, it does not act at the center of mass. The location and direction of the force is some distance from the mass centroid yet in the vertical direction under stationary conditions.

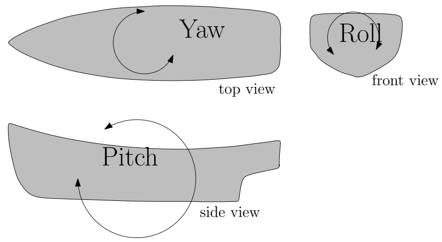


Fig. 12.3 – Typical rotation of ship/floating body.

12.3 Introduction to Direct Examination Method

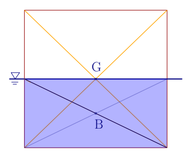


Fig. 12.4 – Schematic of Cubic showing the body centroid (G) and buoyancy centroid (G).

A cubic (for example made of pine) is inserted into liquid. In this specific case, half the block floats above liquid line. The cubic mass (weight) is in the middle of the cubic (assuming uniform density). However the buoyancy center is the middle of the volume under the water (see Fig. 12.4). This situation is similar to Fig. 12.1c. However, any experiment of this cubic shows that the cubic is stable only under special conditions. Small amount of tilting of the cubic results in immediate returning away from the original position. For example, under the conditions where the solid density is half of the liquid, the distance between GB (also AB) is exactly quarter of the side ($b/4$) as it can be observed from the drawing. The location of gravity centroid is fixed at $b/4$ and centroid of the immersed part is $b/4$ and hence $GB = b/2 - b/4 = b/4$. The buoyancy force is the weight of the cubic. When the buoyancy centroid is exactly under the mass/gravity centroid (of the cubic), it can be in equilibrium. What happen when the buoyancy force and gravity force are slightly deviate from the equilibrium? This question is the core of stability analysis.

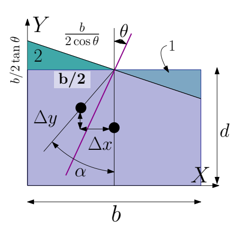


Fig. 12.5 – The Change Of Angle Due Tilting.

The stability can be answered by looking in what direction the moment created. If the moment tries to return it to "original" and tries to keep the two forces in the same line then the situation is stable. This topic is explained in several stages. In Ex. 7.3 examined the change angle α, to change of imposed angle θ. Fig. 12.5 describes the new location of the inclination of the body by purple line. When the centroid point appears right to the purple line the body is unstable and conversely the body is stable (to be on the left hand side of the purple line $\alpha \geq \theta$).

In Ex. 7.7 it was shown that

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{6}\left(\frac{b}{d}\right)^2\right) \cong \tan^{-1}\left(\frac{\theta}{6}\left(\frac{b}{d}\right)^2\right) \tag{12.1}$$

In this case $b/d = 2$ and hence Eq. (12.1) reads $4/3 > 1$ and thus $\alpha > \theta$ because θ is a monotonic linear increase function. Thus, this situation is unstable and the cubic will tilted away.

The prerequisite condition for stability is that the gravity centroid and the buoyancy centroid must lay on the same vertical line. When this condition is not met the situation is unstable. For cylindrical shape, (z–coordinate which is perpendicular to liquid surface)[2] this condition is fulfilled at all angles. Hence the cylindrical shape fulfills the prerequisite stability requirement. However, for all other shapes this condition is fulfilled only under specific angles. For example, in a rectangular (square) extruded shape this requirement appear in two angles: one vertically up and one at $45°$. Eq. (7.22) describes the change of buoyancy center as a function of the rotation[3]. The change in buoyancy centroid does not follow the change in the gravity rotation for the rectangular. In other words the two centerids are not on the same vertical line. Thus, for practical purposes there are only two angles that this condition occur and thus can be stable. The first location was role out and now the second angle is under the investigation. [4]

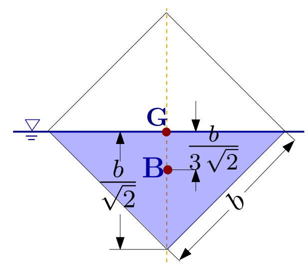


Fig. 12.6 – Cubic on the side (45°) stability analysis.

When the cubic is floating at $45°$ degree the mass centroid remains in the same location. Yet, the immersed volume (area) is now an isosceles triangle with two 45 degree angles. Note the gravity centroid and buoyancy centroid are on the same vertical line. To keep the same volume (area) the base is at the corner of the cubic shown in Fig. 12.6. Under the same procedure shown in Fig. 7.8 the relationship between the α is given as by Eq. (7.7.h). For the sake of explanation, the equation is copied to here

$$\alpha = \tan^{-1}\left\{2\left(\frac{b}{d}\right)^2 \tan\theta\right\} \tag{12..a}$$

Here, evidently, $1 \leq 2(b/d)^2$ and $b/d = 2$ and thus it allays larger than one. That means that the moment will return to the body back and therefore the body is stable in this situation. It has to emphasis that it referred to specific density (in other words that body density is half of the liquid).

[2]Note what is the definition of surface direction.

[3]in that case it was denoted β to differentiate from the small rotation θ.

[4]From thermodynamical point of view there must be at least one point in which body must be stable. Other wise the second law will be violated and the body because perpetual motion machine. There is not official proof for such point that this author is aware of. Nevertheless, this proof should be build geometrical consideration in which the are based on the potential energy analysis.

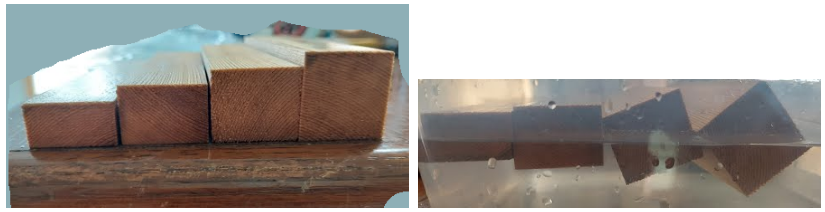
From geometrical consideration (see Table 3 page 175) center is $1/3$ of the height. The height is $b/\sqrt{2}$ and hence the AB (more importantly GB) is $b/3\sqrt{2}$. It can be noticed that in this case the value GB is smaller than the GB distance in the upright situation, that is $b/4 > b/3\sqrt{2}$. The value of GB (or AB) in the upright is about $0.01429774\,b$ larger than the tilted case but any other configuration. Yet for both cases the forces are identical (why? because body has the same mass yet the moment is smaller due to a small leverage.). This point is actually the base for the energy method.

In this case, all the situations are "unstable" (the term unstable is used because G is above B and therefor forces are pointing to each other) yet the case with the 45 degree is the least "unstable" (shown in Fig. 12.1c) because when turned the moment turns body to the original state. Hence, the (45 degree) location is the most stable. Also note body has the smallest moment (the force is the same). This topic is related to curve of dynamical stability and Moseley's formula (for stability not rays). Yet, this topic will not be covered in this book.

In other geometries and/or other densities of liquid and floating body, this kind of analysis has to done to determine the least "unstable" situation. This analysis can be done in a conventional way which will presented first and in new innovative approach. The conventional method introduces a new geometrical location which used to describe the stability while this location is physical it requires calculations and it is not "visible." While the conventional approach is used by many, now this undersign recommends to utilize the new direct examination method. Additional advance of the new method is simple explain and demonstration of the concepts. For instance, the diagrams build with the help of this methods shows the transition from the stability to unstable because change of density ratio. This method also shows the important parameters and their combination. Thus, the GM should not appear but the the ratio of GM/D should be used.

The potential method is simpler and practical but requires some theoretical understanding and abstraction of the physics.

12.4 The Direct Examination



(a) Ceder block on solid surface (b) Ceder block in liquid

Fig. 12.8 – Four blocks with different ratios of D/b = 1:1, 1:0.666, 1:1.2, 1:2 depicted in the figure. It can be observed that 1:2 and 1:5 are stable while the boundary ratio of 1:1.2 is slightly moving from the flat situation. The material used in this experiment is old grow ceder on water. The density ratio is about 0.55. This result is with agreement with the model.

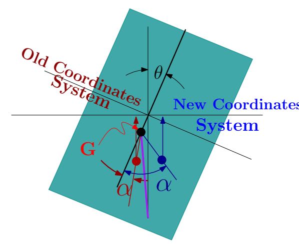


Fig. 12.7 – Arbitrary body rotates in θ and the buoyancy centroid rotates in α. The brown α shows the case of stable scenario. The purple depicts the large α not stable case.

The introduction provided an example for 2–D examination of a square body under two situations. First, upright square was examined and found to unstable (the specific case). Second, $45°$ case which was found to be stable. In this section the general principles of the direct examination are described. As it was shown, the critical point for the stability requirement is $\alpha = \theta$ which determines where are boundaries. Hence, the position under investigation is a given small tilting angle, θ, results in angle α representing the change in buoyancy centroid. If $\alpha > \theta$ then the body under the investigation is stable and vice versa. The examples provided earlier are in three dimensions that could be represented in 2-D i.e. a long square. In reality, the body is not completely symmetrical. Thus, even a body is given a small rotation in one direction the buoyancy centroid can move in 3-D. However, in most cases, the change in the other directions is "minor" and can be ignored. The quantitative test is the ratio α/θ[5]. The value of this ratio indicts how much stable the body at a specific position. Most of the calculations could be done numerically.

The core of this idea mentioned in the introduction and it will be expanded here. There are two possibilities one with $\alpha < \theta$ shown in brown in Fig. 12.7 and two with $\alpha > \theta$ shown in purple in Fig. 12.7.The angle is measured from the gravity centroid with one wing at the old coordinate and second wing is line from the gravity centroid to the new buoyancy center. For large α the buoyancy center rotates back to restore the body toward the original state. In other words, it creates moment that return original angle.

[5]it must be noted that this ratio really does not require finding either θ or α.

(a) Foam on solid surface all stable

(b) Smaller foam blocks in liquid

(c) Larger foam blocks in liquid

Fig. 12.9 – Six blocks with different ratios of D/b = 1:1, 1:0.89, 1:0.789, 1:0.61, 1:0.537, and 0.39. It can be observed that all with the exception of 1:0.39 are stable while the boundary ratio of 1:0.39 tend to be unstable. The material used in this experiment is polystyrene on water. The density ratio is about 0.05. This results is with agreement with the model.

Example 12.1:
What are the minimum conditions for 3D effects?

Solution

The cause of 3D effect is the asymmetry in two directions "opposite to the motion at question." That is a ship that perfectly symmetrical along the length of the ship but "front" (bow) and "back" (stern) are asymmetrical (for various reasons) the centroid of the ship move along back and forth (between the bow and the stern) as result ship has yaw rotation. (that is for example, roll creates yaw).

End Solution

It was shown that Eq. (7.15) the relationship is

$$\alpha = \tan^{1} \frac{\overbrace{\tan\theta}^{\tan\theta\sim\theta} \dfrac{I_{xx}}{V_0}}{GB'} \cong \frac{\theta \dfrac{I_{xx}}{V_0}}{GB} \tag{12.2}$$

Where point A is the location of the intersection of the vertical line that goes through the buoyancy centroid and intersect with liquid surface/plane.

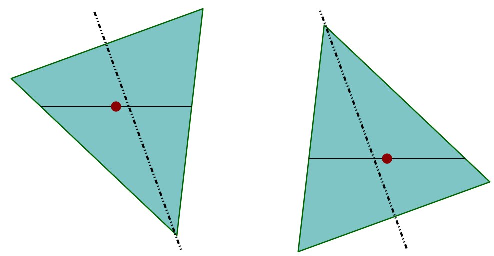


Fig. 12.10 – The change of centroid in a tilde body. The tilde increase the angle (right triangle), α and hence make the body more stable.

Equation Eq. (12.2) is written for a very small angle θ when the change of y is very small. And for practical application the stability condition is

Stability Governing

$$GB \leq \frac{I_{xx}}{V_0} \tag{12.3}$$

This equation is correct only under the condition that point A is fixed. Point A is fixed only when floating body when the area does not change with the height. For example, if body is extruded rectangular or cylinder then A is fixed. However, floating bodies like extruded triangle or a cone do not have a fixed centroid (at the cross area at the liquid level). The analysis of such situation will be in the next version of the book. Nevertheless, a simple motivation is provided. Grossly, there are two zones separated by a border that the previous discussion dealt with. In other words, the border is the bodies that centroid are fixed (body with constant area as a function of the height). Bodies that the cross section is increasing with the height like cone that its pinnacle is in the liquid. In that case, centroid of the cross section area is moving with the angle. To illustrate this point, consider the extruded triangle depicted in Fig. 12.10. On the left the centroid moves to the left and on the right the centroid moves to right. The movement of the centroid to the right (right body in Fig. 12.10) increases the angle, α, which make the body more stable and vise versa. Note, for case where the cross section area decreases with the height, a regular analysis is sufficient because the shifting of point A increase the stability.

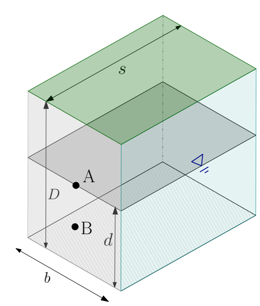


Fig. 12.11 – Rectangular body floating in a liquid for stability analysis.

Example 12.2:
What are the conditions that extruded rectangular shape will be floating stable in a liquid (see Fig. 12.11). Assume that the dimensions of the rectangular are $s >> b$ long and the crosse section is b the width and D the height are same magnitude. The example is repeat one of the previous example.

Solution

The governing equation Eq. (12.3) determines the stability conditions. In this case, GB is given by $D/2 - d/2$ the moment of inertia given in the book $b^3\,s/12$. The volume is $V_0 = d\,b\,s$.

$$\frac{D}{2} - \frac{d}{2} \le \frac{\dfrac{\cancelto{b^2}{b^3}\,\cancel{s}}{12}}{d\,\cancel{b}\,\cancel{s}} \tag{12.II.a}$$

rearrange Eq. (12.II.a) reads

$$6\,(D-d) \le \frac{b^2}{d} \tag{12.II.b}$$

The relation between the different heights (Archimedes' law) is

$$\rho_\ell\, d = \rho_s\, D \tag{12.II.c}$$

Substituting Eq. (12.II.c) into Eq. (12.II.a) reads

$$6\,\overbrace{\frac{\rho_s\,D}{\rho_\ell}}^{d}\left(D - \overbrace{\frac{\rho_s\,D}{\rho_\ell}}^{d}\right) \le b^2 \longrightarrow 6\,\frac{\rho_s\,D}{\rho_\ell}\left(D - \frac{\rho_s\,D}{\rho_\ell}\right) \ge b^2 \tag{12.II.d}$$

Eq. (12.II.e) can be rearrange to be written as

$$\frac{b}{D} \ge \sqrt{6\frac{\rho_s}{\rho_\ell}\left(1 - \frac{\rho_s}{\rho_\ell}\right)} \tag{12.II.e}$$

The results of Eq. (12.II.e) are depicted in Fig. 12.12. It can be noticed that (as expected) for large values of b/D the body is stable. However, when the density ratio is very small ($\frac{\rho_s}{\rho_\ell} \longrightarrow 0$) or very large ($\frac{\rho_s}{\rho_\ell} \longrightarrow 1$) (solid density is close to liquid density) even for small value the ratio of geometries the body is stable (not intuitive). In the mid range of densities requires a larger ratio of b/D. Note that edge close range $\rho_s/\rho_\ell \longrightarrow 0$ or $\rho_s/\rho_\ell \longrightarrow 1$ this analysis is not applied.

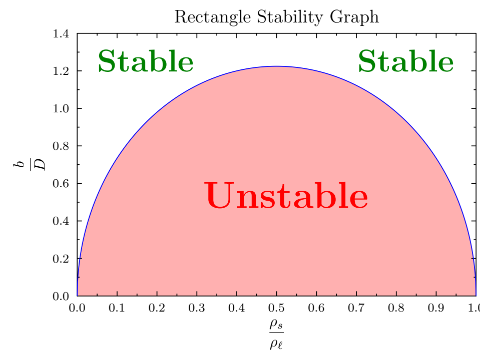


Fig. 12.12 – Extruded rectangular body stability analysis.

End Solution

12.4.0.1 1, 2, and 3 corners in the liquid

This topic should be part of dimensional analysis. In view to make part of the coherent flow of material it is present here. Nevertheless, it is add to here as a temporary place holder.

This discussion deals with uniform density. When extruded rectangular floating body in liquid there is two regimes. These two regimes are separated by half point ($\rho_s = 0.5\rho_\ell$). At this limiting case when a square turning to $45°$ there are three corners (or one if half corner is considered to be out) in the liquid. Otherwise, there are two corners in the liquid at all time. When ($\rho_s > 0.5\rho_\ell$) then there are situations where two corners or three corners inside the liquid. There are no situation with only one corner. Conversely, in the case ($\rho_s < 0.5\rho_\ell$) there are only one corner or two corners in the liquid.

Example 12.3:

A long extruded isosceles triangle is placed base down in a liquid (shown in Fig. 12.13). Analyze the stability for this case. Assume that the base and the height of the triangle are provided.

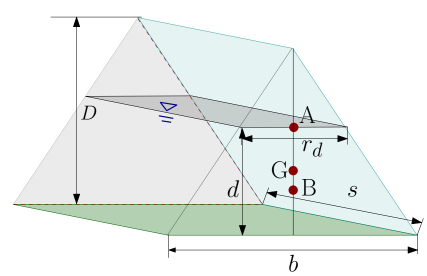


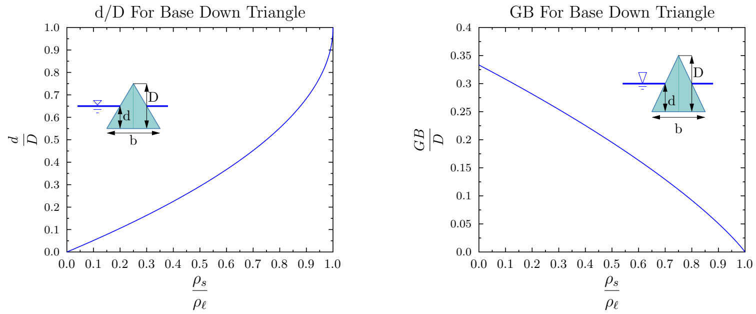
Fig. 12.13 – Floating base down triangle in liquid. The Points A and B are representation to actual location which is at the center.

SOLUTION

The mass centroid of the triangle is 1/3 of the height for upside down triangle. The governing equation for straight cross section area requires that

$$GB \leq \frac{I_{xx}}{V_0} \tag{12.III.a}$$

It was shown in Example 10.12 the relation between the density ratio and the height is



(a) d/D as function for base down triangle.

(b) GB as a function of $\frac{d}{D}$. The figure demonstrates that relationship can be approximated by a straight line. Notice that the functions do not go to one by geometrical considerations.

Fig. 12.14 – intermediate stages

given by Eq. (10.XII.f) and provides

$$\frac{\rho_s}{\rho_\ell} = 1 - \left(\frac{D-d}{D}\right)^2 \longrightarrow \frac{d}{D} = 1 - \sqrt{1 - \frac{\rho_s}{\rho_\ell}} = f\left(\frac{\rho_s}{\rho_\ell}\right) \tag{12.III.b}$$

In this case, the cross section decreases with the height of the triangle and hence if the triangle is stable without considering the movement of point A it will be more stable. The point B is centroid of trapezoid and it is given by Eq. (7.20b) (the notations have to be fixed). Point B after notations converted is

$$B = \frac{d}{3}\left(\frac{2\overbrace{\left(\frac{(D-d)\,\cancel{b}}{D}\right)}^{r_d} + \cancel{b}}{\frac{(D-d)\,\cancel{b}}{D} + \cancel{b}}\right) \longrightarrow \frac{B}{D} = \frac{d}{3\,D}\left(\frac{2\left(\frac{(D-d)}{D}\right) + 1}{\frac{D-d}{D} + 1}\right) \tag{12.III.c}$$

Observing the Fig. 12.14b that function can be approximated as a linearly function. As approximation Eq. (12.III.c) can be written as a straight line The distance GB is

$$\frac{GB}{D} = \frac{G}{D} - \frac{B}{D} \longrightarrow \frac{GB}{D} = \frac{1}{3} - \frac{d}{3\,D}\left(\frac{2\left(\frac{D-d}{D}\right) + 1}{\frac{D-d}{D} + 1}\right) \tag{12.III.d}$$

The moment of inertia is of rectangular is ${r_d}^3\, s/12$ and the immersed volume is

$$V = s\left(\frac{b\,D}{2} - \frac{(D-d)\,r_d}{2}\right) \longrightarrow V = \frac{b\,D\,s}{2}\left(1 - \left(\frac{D-d}{D}\right)^2\right) \tag{12.III.e}$$

The moment of inertia is

$$I_{xx} = \frac{\left(\overbrace{\frac{(D-d)\,b}{D}}^{r_d}\right)^3 s}{12} \longrightarrow I_{xx} = \frac{s}{12}\left(\frac{(D-d)\,b}{D}\right)^3 \tag{12.III.f}$$

Substituting the various values into the governing equation

$$\frac{D}{3}\left(1 - \frac{d}{,D}\left(\frac{2\left(\frac{D-d}{D}\right)+1}{\frac{D-d}{D}+1}\right)\right) \le \frac{\frac{\not s}{12}\left(\frac{(D-d)\,b}{D}\right)^3}{\frac{b\,D\,\not s}{2}\underbrace{\left(1-\left(\frac{D-d}{D}\right)^2\right)}_{\rho_s/\rho_\ell}} \tag{12.III.g}$$

Or after reaarrangment it can be written as

$$2\left(1 - \frac{d}{D}\left(\frac{2\left(\frac{D-d}{D}\right)+1}{\frac{D-d}{D}+1}\right)\right)\frac{\left(\frac{\rho_s}{\rho_\ell}\right)}{\left(\frac{(D-d)}{D}\right)^3} \le \frac{b^3}{b\,D^2} = \left(\frac{b}{D}\right)^2 \tag{12.III.h}$$

The results of Eq. (12.III.h) are drawn in the following exhibit.

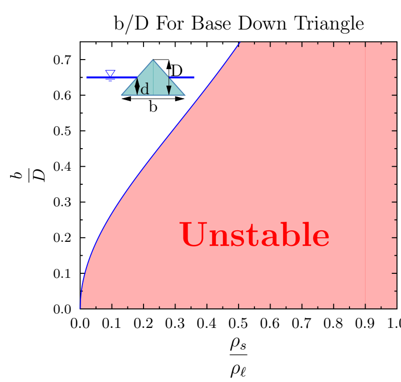


Fig. 12.15 – Base down triangle stability diagram.

The Fig. 12.15 exhibits the stability diagram of the base down triangle. As oppose to the extruded rectangular which has a dome, the base down triangle has no ability to be stable at high density ratio. The reason for it is that moment of inertia approaches to zero while the volume is increasing. On the other side, when the peak was in the liquid then at high density ratio the body is more stable. The position that body will be have that two conditions are satisfied. That it was studied yet and the significance was not explored.

End Solution

Example 12.4:

A cylinder is floating on a liquid when z coordinate is upright. Under what conditions the cylinder is stable. Is 3–D effects appears in the stability analysis of the cylinder under the condition in this question.

Solution

There is no 3-D effects because the cylinder is symmetrical in both directions around the x axis and the y axis. The condition for stability is

$$GB \leq \frac{I_{xx}}{V_0} \tag{12.IV.a}$$

The moment of inertia of circle is given in table 3 $I_{xx} = \pi\, r^4/4$. The volume of the submerged part is $\pi\, r^2\, d$. The location of point A $A = D/2$ and the location of $B = d/2$. The last part is to related between submerged volume to total volume as

$$d\, \rho_\ell = D\, \rho_s \tag{12.IV.b}$$

Armed with all the components Eq. (12.IV.a) can be written as

$$\frac{D}{2} - \frac{d}{2} \leq \frac{\dfrac{\cancel{\pi}\, \cancel{r^4}^{\,r^2}}{4}}{\cancel{\pi}\, \cancel{r^2}\, d}$$

which can be rearranged as

$$\frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \leq \frac{r^2}{4\, d} = \frac{r^2}{4\, \dfrac{D\, \rho_s}{\rho_\ell}}$$

and finally get the form as

$$\frac{r}{D} \geq \sqrt{\frac{2\, \rho_s}{\rho_\ell}\left(1 - \frac{\rho_s}{\rho_\ell}\right)} \tag{12.IV.c}$$

It can be observed that the smallest possible value of the Eq. (12.IV.c) when the ratio ($\rho_s/\rho_\ell = 0.5$) and in that case, $r > \sqrt{2}D$. The results are presented in Fig. 12.16 The

(a) Foam on solid surface all stable

(b) Smaller foam blocks in liquid

(c) Larger foam blocks in liquid

strange fact is the stability line appears symmetrical as the rectangular shape in regard to densities ratio.

End Solution

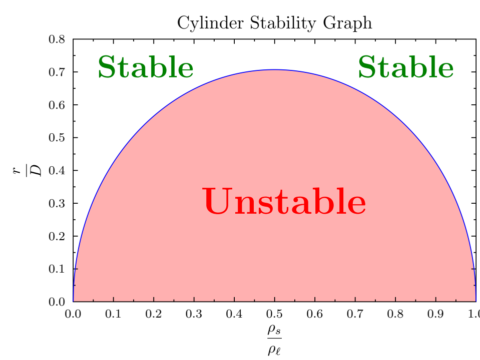


Fig. 12.16 – Cylinder in upright position stability line.

Example 12.5:
Repeat example 12.3 when the base is in the liquid.

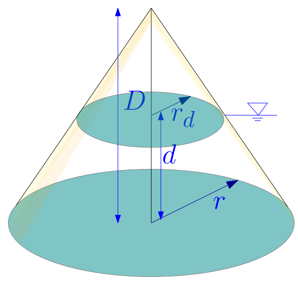


Fig. 12.18 – Cone stability calculations showing various parameters.

Example 12.6:
Calculable the requirements for symmetrical cone which base on the bottom to be stable floating on liquid.

Solution

The cone has several properties that are well documented which include volume: $V = \pi\, r^2\, D/3$ the centroid: $D/4$ The location of A is independent of the liquid and is at the centroid which is at $D/4$. The location of the buoyancy center (centroid of truncated cone) conical frustum is at

$$B = \frac{D\left(1 + \dfrac{2\, r_d}{r} + 3\left(\dfrac{r_d}{r}\right)^2\right)}{4\left(1 + \dfrac{r_d}{r} + \left(\dfrac{r_d}{r}\right)^2\right)} = \bar{C_1}\left(\frac{r_d}{r}\right) \tag{12.VI.a}$$

The centroid is similar to complete cone with a correction factor. The ratio of r_d/r can be in The ratio of $r_d/r = 1 - d/D$ results in

$$B = \frac{D\left(6 - \dfrac{8\, d}{D} + 3\left(\dfrac{d}{D}\right)^2\right)}{4\left(3 - 3\dfrac{d}{D} + \left(\dfrac{d}{D}\right)^2\right)} = C_1\left(\frac{d}{D}\right) \tag{12.VI.b}$$

The volume is the same

$$V_0 = \frac{\pi\, D\, r^2}{3}\left(1 + \frac{r_d}{r} + \left(\frac{r_d}{r}\right)^2\right) = \bar{C_2}\left(\frac{r_d}{r}\right) \tag{12.VI.c}$$

or substituting $r_d/r = 1 - d/D$ results in

$$V_0 = \frac{\pi\, D\, r^2}{3}\left(3 - 3\,\frac{d}{D} + \left(\frac{d}{D}\right)^2\right) = C_2\left(\frac{d}{D}\right) \tag{12.VI.d}$$

The correction factors, C_1 and C_2, approach zero when $r_d \longrightarrow 0$. The cone weight and the displaced liquid are identical hence

$$\rho_s \, V_{cone} = \rho_\ell V_0 \tag{12.VI.e}$$

or explicitly

$$\rho_s \frac{\pi\, D \cancel{r^2}}{\cancel{3}} = \rho_\ell \frac{\pi\, D \cancel{r^2}}{\cancel{3}} \left(3 - 3\,\frac{d}{D} + \left(\frac{d}{D}\right)^2 \right) \tag{12.VI.f}$$

which can be manipulated to be

$$\frac{d}{D} = \frac{3 + \sqrt{9 + \dfrac{4\,\rho_s}{\rho_\ell}}}{2} \tag{12.VI.g}$$

when $\rho_s \longrightarrow 0$ then $d = 0$ which was expected. The solution with the negative sign was rejected for physical reasons.

The condition for stability expressed in Eq. (12.2) can simplified as

$$GB = \frac{I_{xx}}{V_0} \tag{12.VI.h}$$

These two last equations related the densities to the displacement.

$$\frac{D}{4} - C_1 \left(\frac{d}{D}\right) = \frac{\dfrac{\pi\, {r_d}^4}{4}}{C_2 \left(\dfrac{d}{D}\right)} \tag{12.VI.i}$$

The solution of equations (12.VI.i) provide the relationship between r and D and densities ratio so limiting case of stability is established. For this chapter this point is a stopping point. The relationships found here can be drawn.

End Solution

The examination up to this point was whether the body is stable or not. The question in which angle the body is stable was not discussed. The trial and error and methods can be used however, another methods can be used. For example, the extruded rectangular has moment of inertia which is increases with the inclination angle, β $(1/\cos^3\theta)$. In some situation as depicted in the figures the inclination change to some degree or point and no further. To sharpener the question, again more tilding the rectangular results in larger moment of inertia while the volume remain the same. Thus, according to this argument all the body should continue to rotate maximum moment of inertia. In fact, the experiments that this author has done show that body is stable around that several points and they not continues (with the exception of cylinder "type" on the side). The reason for singularly of these points is that another condition has to be fulfilled. The buoyancy centerid must be on the same line vertical line with the buoyancy centerid. So the question should be locate all the angle in which the buoyancy and gravity centerids are on the same line. Then check if that point is stable. For instance, in the case of rectangular and case of $\rho_s/\rho_e ll$ is the same line are on angles $0°$ and $45°$.

(a) Ceder block in cotton seed oil

(b) Douglass fir cylinder in cotton seed oil

(c) Foam block in cotton seed oil

Fig. 12.19 – The some block that were used in previous experiments with water depicted in the current figure. For the ceder blocks it shows that they are more stable. While for the foam block becomes less stable. I can be observed that 1:2 and 1:5 are stable while the boundary ratio of 1:1.2 is slightly moving from the flat situation. The material used in this experiment is old grow ceder on water. The density ratio is about 0.55. This result is with agreement with the model.

Example 12.7:
Express the moment of inertia for rectangular body as a function of the inclination. Start with complete vertical position. Assume that the trapezoid shape remains through process. Note that the moment of inertia of rectangular is $bh^3/12$.

SOLUTION

The surface has depth of s into the page and the width of the surface is $b/\cos\theta$. The location of A is independent of inclination. The buoyancy centroid at B can

$$\frac{\Delta y}{b} = \left(\frac{b}{d}\right)^2 \frac{\tan^2\theta}{12} \tag{12.VII.a}$$

$$\frac{\Delta y}{b} = \left(\frac{b}{d}\right) \frac{\tan\theta}{6} \tag{12.VII.b}$$

$$\alpha = \tan^{-1}\left(\frac{2\,\tan\theta}{6\left(\frac{d}{b}\right) - \tan^2\theta}\right) \tag{12.VII.c}$$

The location of buoyancy centroid is at

$$GB' = \sqrt{\left(\frac{d}{2} - \Delta y\right)^2 + (\Delta x)^2} \tag{12.VII.d}$$

The critical length GB" is obtained by

$$GB'' = GB'\,cos\,|\alpha - \theta| \tag{12.VII.e}$$

The moment of inertia can be expressed as

$$I_{xx} = \frac{s\,b^3}{12\cos^3\theta} \tag{12.VII.f}$$

End Solution

Example 12.8:
One way to make a ship to be more hydrodynamical is by making the body as narrow as possible. Suppose that two opposite sides triangle (prism) is attached to each other to create a long "ship" see Fig. 12.20. Supposed that $\mathrm{b/h} \longrightarrow\sim 0$ the body will be unstable. On the other side if the $\mathrm{b/h} \longrightarrow\sim \infty$ the body is very stable. What is the minimum ratio of $\mathrm{b/h}$ that keep the body stable at half of the volume in liquid (water). Assume that density ratio is ρ_s/ρ_ℓ. Is there limits on the density ratio.

Solution

The answer the question requires solve the governing equation. In this case the moment of inertia of half triangle is $hb^3/24$ for rotating triangle around the base. There are four half triangles and thus the total moment of inertia is $hb^3/6$. The immersed volume of the body is

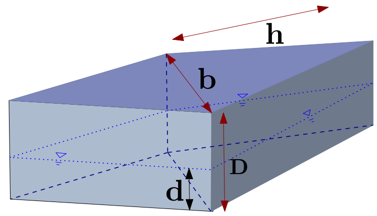


Fig. 12.20 – Stability of two triangles put tougher.

$$V_0 = 2\left(\frac{b\,h\,d}{2}\right) = b\,h\,d$$

The relationship of the density ratio to heights (Archimedes Law) is

$$d = \frac{D\,\rho_s}{\rho_\ell} \tag{12.VIII.a}$$

The governing equation is

$$\frac{D}{2} - \frac{d}{2} = \frac{\frac{\cancelto{b^2}{b^3}\,\cancel{h}}{6}}{\cancel{b}\,\cancel{h}\,d} \longrightarrow \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) = \frac{1}{6}\,\frac{b^2}{d}$$

or in a dimensional form as

$$\left(1 - \frac{\rho_s}{\rho_\ell}\right)\frac{\rho_s}{\rho_\ell} = \frac{1}{3}\left(\frac{b}{D}\right)^2 \tag{12.VIII.b}$$

According to Eq. (12.VIII.b) the stability is affected by the density ratio and by the ratio of b/D. Furthermore, according to this model the stability is not function of length, h. However, according to this model it is build on the idea that that there in marine terminology there is no pitch (rotating in the other direction). If h is the same order as b then this model is fail to take into account the stability of the other direction. Hence, with these hand waving argument h must at least two time of b.

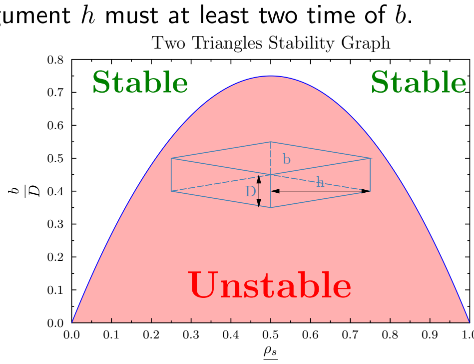


Fig. 12.21 – Two attached triangles stability diagram. Shown that minimum b is at least in the same size of D.

End Solution

Example 12.9:

This example was expansion the question 3.4 by Lautrup (Lautrup 2011). Extruded triangle with a shape pointed point down is placed in a liquid. When alpha angle is very sharp the triangle is not very stable. On the other hand, when the angle is very wide the triangle is stable. Assuming that the area change does not affect the analysis. what is the minimum angle for with the body is stable for arbitrary density ratio ($0 \leq \rho_r \leq 1$). This example only provide the trends and it sufficient of the make of the point of the range where to expect the stable zone.

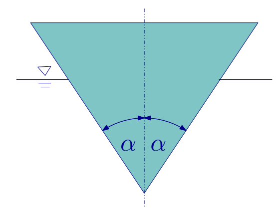


Fig. 12.22 – Triangle With The Peak Down stability analysis.

Solution

The Symmetry dictated that it is sufficient to balance half the triangle. The density ratio related to the draft as

$$\rho_s \frac{D\,D\tan\alpha}{2} = \rho_\ell \frac{d\,d\tan\alpha}{2} \longrightarrow \frac{\rho_s}{\rho_\ell} = \left(\frac{d}{D}\right)^2 \tag{12.IX.a}$$

The moment of inertia of the of the cross section area is

$$I_{xx} = \frac{s\,(2\,d\,\tan\alpha)^3}{12} \tag{12.IX.b}$$

The volume is

$$V_0 = \frac{\cancel{2}\,s\,d^2\tan\alpha}{\cancel{2}} \longrightarrow V_0 = s\,d^2\tan\alpha \tag{12.IX.c}$$

The governing equation is

$$G - B = \frac{I_{xx}}{V_0} = \frac{\dfrac{2\,\cancel{s}\,(d\,\tan\alpha)^3}{3}}{\cancel{s}\,d^2\tan\alpha} = \frac{2\,d\,\tan^2\alpha}{3} \tag{12.IX.d}$$

The distance $G - B$ the difference between the two triangles. Hence

$$\frac{2\,D}{3} - \frac{2\,d}{3} = \frac{2\,d\,\tan^2\alpha}{3} \longrightarrow \left(1 - \frac{d}{D}\right) = \frac{d}{D}\,\tan^2\alpha \tag{12.IX.e}$$

The value of limit α is

$$\alpha = \tan^{-1}\sqrt{\left(1 - \frac{d}{D}\right)\frac{D}{d}} \tag{12.IX.f}$$

This results is exhibited in the following graph. It is caution that Eq. (12.IX.f) represents the trends only and additional correction has to be taken into account.

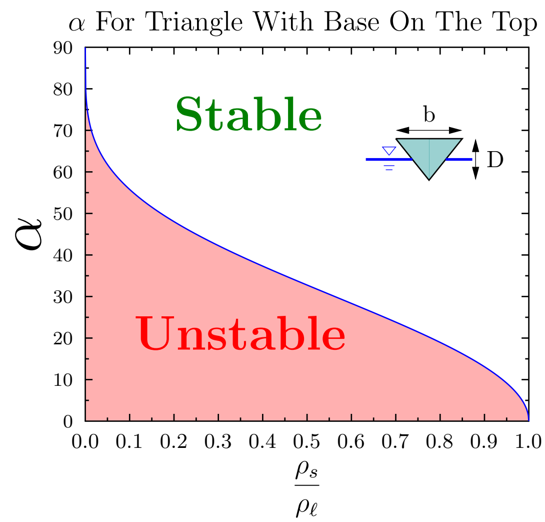


Fig. 12.23 – α as function of density ratio.

It can be observed from Fig. 12.23 that for small density ratios a large angle is required. This results is obvious and it is based on intuitive. However, it expect that required angle will have "jump" at the initial and the final zone. For large range the change is approaching a linear. Again, it must be noted that this results is not the final because the change of the rotation line and large tilde angle is applied then the triangle might not be stale.

End Solution

12.4.1 Usage of Stability to find Centroid

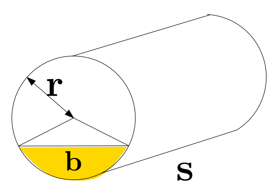


Fig. 12.24 – Circular shape for finding centroid.

The stability arguments can be used to find the centroid of fraction of circle. A cylinder is naturally stable and no matter what the value of the density ratio (location of liquid level). Hence, this calculation is based on this concept/idea, the extruded circle must obey the stability equation equation as

$$\frac{I_{xx}}{V_0} = G - B \tag{12.4}$$

Since G is fix for all circles while B is unknown. The left side of the equation can be reduced. For example, for $\rho_s/\rho_\ell = 0.5$ it becomes

$$\frac{I_{xx}}{V_0} = \frac{\dfrac{\cancel{8}(2r)^3}{12}}{\dfrac{\cancel{8}\pi\, r^2}{2}} = \frac{4\, r}{3\, \pi} \tag{12.5}$$

Thus, the centroid is found and the depth (s) is irrelevant but to the results but is useful tool in calculation the centroid of any fraction of the circle. In general, the moment of inertia of and area of the golden area can be evaluated and the direct expression is

$$B = G - \frac{I_{xx}}{V_0} = \frac{r}{2} - \frac{b^3/12}{A} \tag{12.6}$$

The area is readily available and hence center of buoyancy can be calculated.

The question that must follow this discussion is whether the circular contour is some kind of boundary between stable and unstable contours? The investigation so far shows and indicants that it is the case but there should additional stipulation which was not formulated yet. For example, a removal of small section of the circle make the body unstable and the "circle" will turn away.

12.5 Potential Energy Approach

This method was suggested by Erdös at el and was slightly improved by Abolhassani. This method based on the idea that a derivative of potential energy can provide a location or locations where a system has a minimum (or maximum) and thus it is potential of stability point[6]. The energy used in this scenario is the gravitational energy that is expressed as

$$U_{sys} = (M + m)\, g\, h_{M+m} = g\, (m\, h_m + M\, h_M) \tag{12.7}$$

Where subscript sys referred to the entire system. The m is referred to floating body and M is referred to the displaced liquid in other words to the mass if the liquid was filling the submerged volume. The logic to the last definition is that it represents the potential of the buoyancy force acting in the center immersed part. The change in the potential is due to the change in the angle

$$\frac{dU_{sys}}{d\theta} = 0 \tag{12.8}$$

The condition that angle, θ is by checking the second derivative if it positive or negative. In away doing example it will repetitive of the moment method converting it to potential and going over the mathematics. This book is more focus on the physics and therefore it not presented.

[6]This topic should be discussed elementary physics class and not fluid mechanics textbook. However, if there will be a significant request it will be briefly discussed.

13 Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation

14 Metacenter

14.1 Introduction

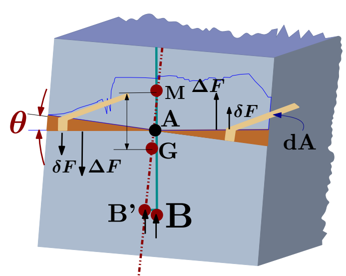


Fig. 14.1 – Stability analysis of floating body.

The last chapter two methods were discussed: the Direct Examination and the abstract Potential Energy. The cousin of the direct examination is the Metacenter method. In Metacenter method, the ship or the floating body is given a small rotation or tilde (as in Direct Examination) and instead of resulting angle, a distance from the gravity centroid to new imaginary location (Metacenter) is measured. If the distance G to M is point up it considered to possible and vice versa. If the distance is positive then the body is stable and if it is negative the body is unstable. The advantages and short falls will be discussed at the end of the Chapter.

Assuming the general body is floating and it is at a certain configuration. The body is tilted by a given small angle, θ, and the resulting distance is measured. As opposed the perceived public opinion, the tilting point is around point **A** and not around imaginary point **M**[1]. If the body will be rotating around point **M** it will not keep the displaced volume constant. The immersed part of the body centroid translates to a new location, **B'** as shown in Fig. 14.1. The mass (gravity) centroid is still at the same old location since the body itself does not change (on moving gravity center later). This deviation of the buoyant centroid from the old buoyant centroid, **B**, is similarly as it was done in the Direct Examination. The right brown area (volume) in Fig. 14.1 is displaced by the same area (really the volume) on left since the weight of the body didn't change[2]so the total immersed area (volume) is constant.

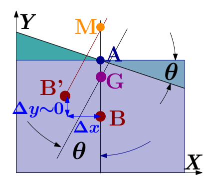


Fig. 14.2 – Description to demonstrate what is Metacenter.

In the previous chapter it was shown that Δx or BB' is

$$BB' = \frac{I_{xx}\,\tan\theta}{V_0} \tag{14.1}$$

the point where the gravity force direction is intersecting with the center line of the cross section is referred as Metacenter point, **M**. The location of the Metacenter point can be obtained from the geometry as Metacenter point! definition

$$\overline{BM} = \frac{\overline{BB'}}{\sin\theta} \tag{14.2}$$

And combining equations (14.1) with (14.2) yields

$$\overline{BM} = \frac{I_{xx}\,\tan\theta}{V_0\,\sin\theta} \tag{14.3}$$

For small angle ($\theta \sim 0$)

$$\lim_{\theta\to 0}\frac{\sin\theta}{\tan\theta} \sim 1 \tag{14.4}$$

It is remarkable that the results is independent of the angle. Looking at Fig. 14.2, the geometrical quantities[3] can be related as

$$\overline{GM} = \overbrace{\frac{I_{xx}}{V_0}}^{\overline{BM}} - \overline{BG} \tag{14.5}$$

[1]For fixed A point.

[2]It is correct to state: area only when the body is extruded. However, when the body is not extruded, the analysis is still correct because the volume and not the area should be used.

[3]Alternative explanation will be provided at the end of the chapter.

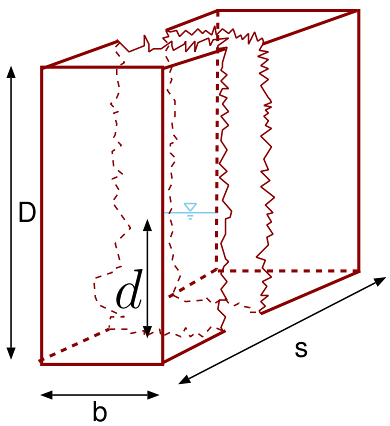


Fig. 14.3 – Cubic body dimensions for stability analysis.

Eq. (14.5) can also written for solid body as

$$\overline{GM} = \overbrace{\frac{\rho_\ell \, I_{xx}}{\rho_s \, V_{body}}}^{\overline{BM}} - \overline{BG} \tag{14.6}$$

To understand these principles consider the following examples.

14.2 Application of GM

All the terms in Eq. (14.5) normally provided and it is simply plugging them into the Eq. (14.5) and obtaining the results. Illustrate these points an extensive example is provided.

Example 14.1:

In Fig. 14.3 depicts the extruded rectangular with various dimensions. Assume that the body is solid with density below the liquid density, calculate the $\overline{GM}$ for various dimensions.

<u>SOLUTION</u>

The governing equation is

$$GM = \frac{I_{xx}}{V_0} - \overline{BG} \tag{14.I.a}$$

As before the densities is used to related

$$V_0 \, \rho_\ell = V_{body} \, \rho_s \longrightarrow d \, \rho_\ell = D \, \rho_s \tag{14.I.b}$$

Point G is located at $D/2$ and point B is located at $d/2$. Moment of inertia is $I = b^3\, s/12$ and the volume is $V_0 = d\, s\, b$ Armed with these data Eq. (14.I.a) becomes

$$GM = \frac{\frac{b^3 \cancel{s}}{12}}{d \cancel{s} b} - \left(\frac{D}{2} - \frac{d}{2}\right) \tag{14.I.c}$$

or in dimensionless form as

$$\frac{GM}{D} = \frac{1}{12}\left(\frac{b}{D}\right)^2 \left(\frac{\rho_\ell}{\rho_s}\right) - \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{14.I.d}$$

Plotting the results of various density and b/D provides the following figure

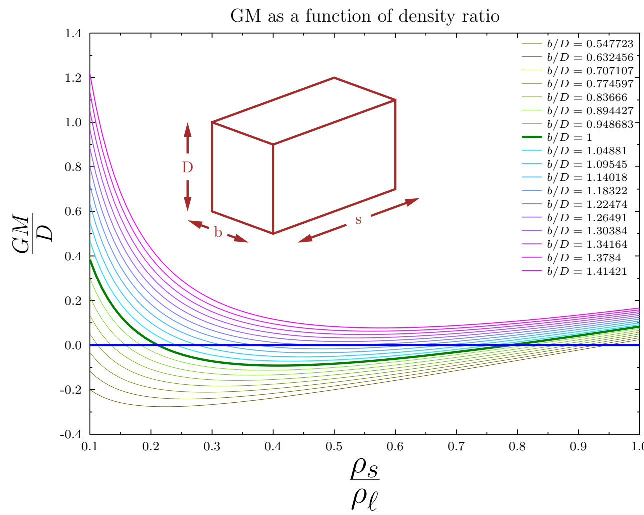


Fig. 14.4 – GM of Rectangular shape with various dimensions.

The rectangular has larger GM when floating on very heavy liquid. It is more stable if it is lighter. The blue line differentiate between positive and to negative $\overline{GM}$ values The Fig. 14.4 exhibits the $\overline{GM}$ as function of the density ratio for various ratio of b/D. The figure demonstrates that there is a minimum with every graph that is around the $\rho_s = 0.5\rho_\ell$. For some ratios of b/D the figure demonstrates that $\overline{GM}$ is negative. As solid density approaches to liquid density, the body becomes more stable and even with positive $\overline{GM}$ for some b/D ratios. At mid range density range the body is less stable.

End Solution

Example 14.2:
Assume that you are on a floating body (boat or ship) and it is about turn to it side. what should you in order to save the floating body? Throw items over board or bring more things to ship like your raft that is normally tied to your boat?

Solution

If the ship or the boat is light that throwing items will make more stable. On the other the boat is almost full and you should add more items and make it as heavy as you can (even pump water into the ship). It is common to have a maximum load marking on the ship or boat. Normally this point should be design in about 30% of the ship displacement. Thus, if the convention is applied that it better to throw as much as possible. The reason that maximum mark exist is or should be for stability reasons. Load about that point will the ship unstable (below safety factor).

As anecdote of this author, on his ship mechanic duty exam (on a missile boat) a common question was what to do when ship shows signs of turning. The proper answer was to pump and throw overboard everything as possible out. The question was originated by someone experienced it first hand without any the theoretical understanding.

End Solution

Example 14.3:
A cylinder with a radius, r and a length D is floating on a liquid. Calculate the $\overline{GM}$ for various densities ratios and ratios of r/D.

Solution

This example basically repeat example 14.1 for cylinder. The immersed volume is $\pi\, d\, r^2$ The moment of inertia of circular shape is $\pi\, r^4/4$.

$$GM = \frac{\cancel{\pi} r^4/4}{\cancel{\pi} d\, r^2} - \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{14.III.a}$$

or in a clear form

$$GM = \frac{r^2}{4\,d} - \frac{D}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{14.III.b}$$

Or in a dimensionless form as

$$\frac{GM}{D} = \frac{1}{4}\left(\frac{r}{D}\right)^2 \frac{\rho_\ell}{\rho_s} - \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) \tag{14.III.c}$$

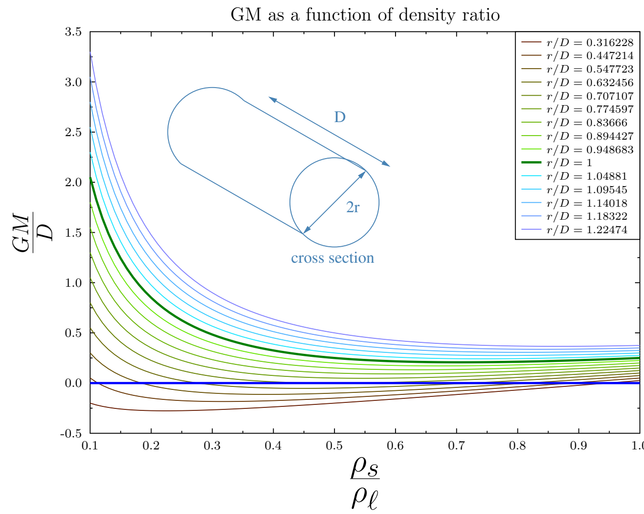


Fig. 14.5 – GM as a function of density ratio for various r/D.

End Solution

14.2.1 Unstable Bodies, Negative $\overline{GM}$

What happen when the $\overline{GM}$ negative values or in other words the height (draft) increases beyond the critical height ratio? The body will flip into the side and turn to the next stable point (angle). This is not a hypothetical question, but rather practical. This stable point (that is body at new angle) will be point where body will be rotating around. At that point, the possible rotation swing to the deck is smaller and thus the ship is less stable.

This situation happens when a ship is overloaded with containers above the maximum height. In commercial ships, the fuel is stored at the bottom of the ship and thus the mass centroid (point G) is changing during the voyage. So, the ship that was stable (positive $\overline{GM}$) leaving the initial port might became unstable (negative $\overline{GM}$) before reaching the destination port. It is the responsibility of the officer (captain) to be aware of this pitfall.

14.3 The $\overline{GZ}$ diagram

A typical approach in stability instead of looking at the value of $\overline{GM}$ a different imaginary distance is checked. This distance is what called the right hand arm. This refers to distance between the gravity and buoyancy forces. In Fig. 14.6 depicts the distance $\overline{GZ}$. This distance is perpendicular to the line $B'M$. The line GB_e is parallel to

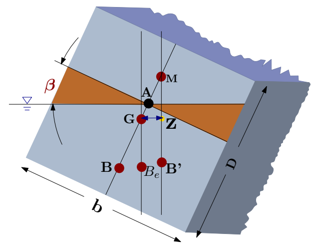


Fig. 14.6 – The right arm distance is display as $\overline{GZ}$ as a function of β

$B'M$. B_e is the buoyancy point that obtained when the equilibrium is achieved. In other words, this point is the point the regular B' that travels through until the stability point. It can be noticed that his change occurs when the body is rotated and it no question stability. In this analysis, the question is the location of buoyancy centroid. This examination is based on the size of the right turning arm. When the arm length is zero the floating body is stable or that is point like watershed line after which the body will turn to next stable point.

In this analysis, the buoyancy centroid will be determined. The simple way to explain this point is by example. First body that will be dealt is the extruded rectangular like the one show in Fig. 14.6. Utilizing Eq. (7.20) provides the buoyancy centroid. For simplicity, first the vertical distance between the buoyancy centroid to the liquid of the water is calculated. Later, the horizontal distance between the gravity centroid and the buoyancy centroid.

The vertical distance can asses by finding first distance $B'L$ which can ascertained by

$$B'L = C_\xi - LN = C_\xi - C_\eta \tan\beta \tag{14.7}$$

The vertical distance is then

$$\overline{VD} = B'L \cos\beta = (C_\xi - C_\eta \tan\beta) \cos\beta \tag{14.8}$$

Dividing equation by draft, D yields

$$\frac{\overline{VD}}{D} = \left(\frac{C_\xi}{D} - \frac{C_\eta}{D} \tan\beta\right) \cos\beta \tag{14.9}$$

Where C_ξ/D can be obtained from Eq. (7.21) plus utilizing Archimedes's formula as

$$\frac{C_\xi}{D} = \frac{\rho_r}{2} + \frac{b}{D}\frac{\tan\beta}{2} - \frac{\tan^2\beta}{24}\left(\frac{b}{D}\right)^2 \frac{1}{\rho_r} \tag{14.10}$$

Where ρ_r is the density ratio of the solid body over the liquid. Utilizing Eq. (7.22d)

$$\frac{C_\eta}{D} = \frac{b}{2\,D} - \frac{1}{12}\left(\frac{b}{D}\right)^2 \frac{\tan\beta}{\rho_r} \tag{14.11}$$

Before the results are presented the maximum angle β has to addressed. It can notice that the displaced volume is fix and independent of the angle of inclination. The maximum angle that body can turn before it get over the edge is

$$\tan\beta_{max} = \begin{cases} \frac{2\,d}{b} = \dfrac{2\,\rho_r}{\dfrac{b}{D}}, & \text{for } \rho_r \leq 0.5. \\ \frac{2\,(D-d)}{b} = \dfrac{2\,(1-\rho_r)}{\dfrac{b}{D}}, & \text{otherwise.} \end{cases} \tag{14.12}$$

The results are exhibited in Fig. 14.7. The maximum angle has a maximum at $\rho_r = 0.5$ with is results that the other corner is going to immersed in the liquid for a large value of density ratio. While, this point could be seem as trivial it is very important even with the calculations. At this point, the rate of change for moment of inertia obtains it minimum. This is a suspicious point in which it could be a stable point (e.g point where the body rotating around).

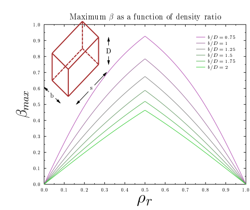
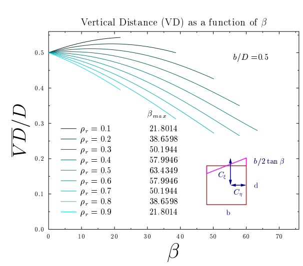
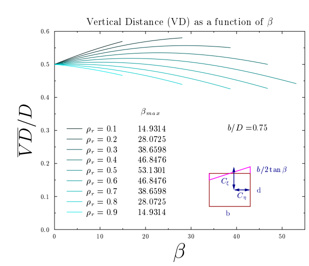


Fig. 14.7 – The maximum beta angle that before the corner is immersed in the liquid

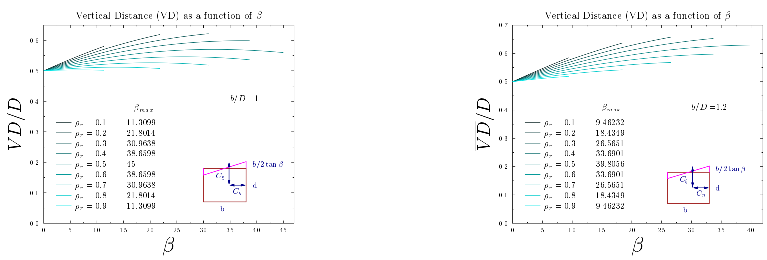
At this point the discussion can turn back to distances vertical and horizontal. The horizontal distance GZ has to go through several stages. First the vertical distance from the buoyancy (referred to as VD).



(a) Vertical distance for $b/D = 0.5$



(b) Vertical distance for $b/D = 0.75$



(c) Vertical distance for $b/D = 1.0$ (d) Vertical distance for $b/D = 1.2$

Fig. 14.9 – Vertical distance for rectangular extruded shape showing for 4 different b/D ratio as function of the turning angle β.

Fig. 14.9 shows the results of the vertical distance as a function for various density ratio and various b/D ratio. For small values value of b/D the vertical distance become smaller. For example, for $b/D = 0.5$ almost all the values are trending down. While the values treading up for large values see Fig. 14.8d. This trend is consistent with the Potential Energy is based on. According to this method, the shorter distance of $\overline{BM}$ the more stable the situation is. Here, the stable situations will have a trend going up because they are stable and vice versa. The intermediate stage has a mix situation where part trending up and part trending down which indication that the body is stable only under certain conditions which is consistent with previous observations.

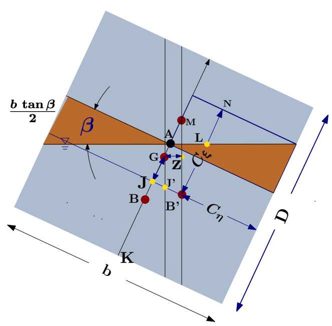


Fig. 14.10 – Schematic for naming for the $\overline{GZ}$ for vertical and horizontal distances. Notice, some of the nomenclatures are adapted from page 65.

The distance $\overline{GZ}$ is calculated utilizing the nomenclatures that used in Fig. 14.10. The distance $\overline{GJ}$ is obtained by

$$\overline{GJ} = d - \overline{JK} - \overline{AG} \tag{14.13}$$

Where these distances are evaluated as $\overline{JK} = d + \Psi - C_\xi$ and $\overline{AG} = d - D/2$. Utilizing these definitions yields

$$GJ = \frac{D}{2} + C_\xi - (\Psi + d) \tag{14.14}$$

The dimensionless value of C_ξ is given by Eq. (14.10). Utilizing the GJ to obtain the next value in the chain of values as

$$\overline{JJ'} = \overline{GJ} \tan\beta \tag{14.15}$$

with this value

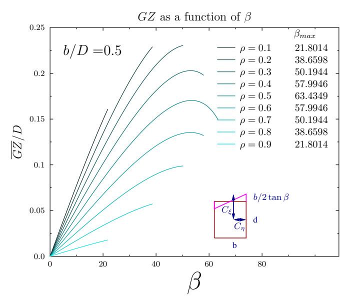
$$\overline{B'J'} = \overline{JB'} - \overline{JJ'} \tag{14.16}$$

where the value of $\overline{JB'}$ can be observed from the diagram as

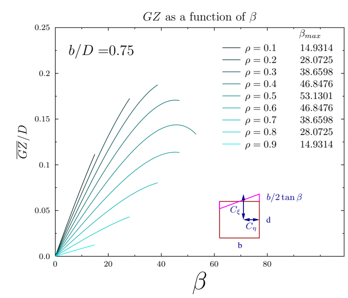
$$\overline{JB'} = \frac{b}{2} - C_\eta \tag{14.17}$$

and finally

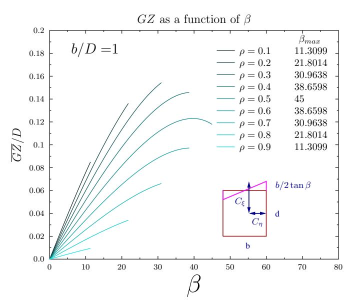
$$B'J'_\perp = GZ = B'J' \cos\beta \tag{14.18}$$



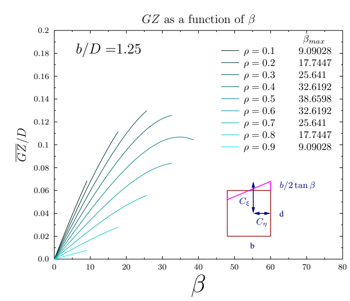
(a) GZ as a function β for b/D=0.5



(b) GZ as a function β for b/D=0.75



(c) GZ as a function β for b/D=1.0



(d) GZ as a function β for b/D=1.25

Fig. 14.12 – GZ as a function of the β for various b/D. It can be noticed that the angle is smaller for larger density ratio.

Several trends can be observed in Fig. 14.12. First it must be noted that of the abscissa scale is different for these graphs. As the ratio of b/D incenses the distance GZ decreases. The maximum is achieved earlier for larger b/D values. After the body reaches the maximum angle, new equation describes by the new equations and it is not shown in the figure. The change will manifest itself in the direction but still will be continues that is no jump in the angle.

14.4 Stability of Body with Shifting Mass Centroid

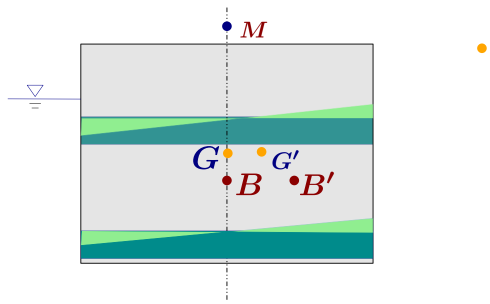


Fig. 14.13 – The effects of liquid movement on the $\overline{GM}$.

Ships and other floating bodies carry liquid or have a load which changes the mass location during tilting of the floating body. For example, a ship that carries wheat grains where the cargo is not properly secured to the ship. The movement of the load (grains, furniture, and/or liquid) does not occur in the same speed as the body itself or the displaced outside liquid. Sometimes, the slow reaction of the load, for stability analysis, is enough to be ignored. Exact analysis requires taking into account these shifting mass speeds. However, here, the extreme case where the load reacts in the same speed as the tilting of the ship/floating body is examined. For practical purposes, it is used as a limit for the stability analysis. There are situations where the real case approaches to this extreme. These situations involve liquid with a low viscosity (like water, alcohol) and ship with low natural frequency (later on the frequency of the ships). Moreover, in this analysis, the dynamics are ignored and only the statics is examined (see Figure 14.13).

A body is loaded with liquid "L" and is floating in a liquid "F" as shown in Figure 14.13. When the body is given a tilting position the body displaces the liquid on the outside. At the same time, the liquid inside is changing its mass centroid. The moment created by the inside displaced liquid is

$$M_{in} = g\, \rho_{l\,L} \theta I_{xx\,L} \tag{14.19}$$

Note that $I_{xx\,L}$ isn't the identical to the moment of inertia of the outside liquid interface.

The change in the mass centroid of the liquid "L" then is

$$\overline{G_1G_1'} = \underbrace{\frac{\not{g}\,\not{\rho_{lL}}\theta I_{xx\,L}}{\not{g}\,V_L\,\not{\rho_{lL}}}}_{\text{Inside liquid weight}} = \frac{I_{xx}\,\theta_L}{V_L} \tag{14.20}$$

Equation (14.20) shows that $\overline{GG'}$ is only a function of the geometry. This quantity, $\overline{G_1G_1'}$, is similar for all liquid tanks on the floating body.

The total change of the vessel is then calculated similarly to center area calculations.

$$\not{g}\,m_{total}\,\overline{GG'} = \overset{0}{\cancel{g\,m_{body}}} + \not{g}\,m_f\overline{G_1G_1'} \tag{14.21}$$

For more than one tank, it can be written as

$$\overline{GG'} = \frac{g}{W_{total}}\sum_{i=1}^{n}\overline{G_iG_i}\rho_{l_i}V_i = \frac{g}{W_{total}}\sum_{i=1}^{n}\frac{I_{xxbi}}{V_{bi}} \tag{14.22}$$

A new point can be defined as G_c. This point is the intersection of the center line with the vertical line from G'.

$$\overline{G\,G_c} = \frac{\overline{GG'}}{\sin\theta} \tag{14.23}$$

The distance that was used before $\overline{GM}$ is replaced by the criterion for stability by $\overline{G_c\,M}$ and is expressed as

$$\overline{G_c\,M} = \frac{g\,\rho_A\,I_{xx\,A}}{\rho_s V_{body}} - \overline{BG} - \frac{1}{m_{total}}\frac{I_{xxb}}{V_b} \tag{14.24}$$

If there are more than one tank partially filled with liquid, the general formula is

$$\overline{G_c\,M} = \frac{g\,\rho_A\,I_{xx\,A}}{\rho_s V_{body}} - \overline{BG} - \frac{1}{m_{total}}\sum_{i=1}^{n}\frac{I_{xxbi}}{V_{bi}} \tag{14.25}$$

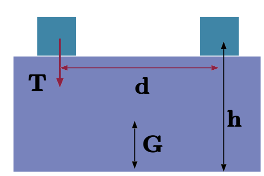


Fig. 14.14 – Measurement of GM of floating body.

One way to reduce the effect of the moving mass center due to liquid is done by substituting a single tank with several tanks. The moment of inertial of the combine two tanks is smaller than the moment of inertial of a single tank. Increasing the number of tanks reduces the moment of inertia. The engineer could design the tanks in such a way that the moment of inertia is operationally changed. This control of the stability, $\overline{GM}$, can be achieved by having some tanks spanning across the entire body with tanks spanning on parts of the body. Movement of the liquid (mostly the fuel and water) provides way to control the stability, GM, of the ship.

14.4.1 Neutral frequency of Floating Bodies

The examination of the GZ can be treated as linear for small range. Hence, this fact can be basically making the ship or the floating body act as it is a pendulum in some senses. Yet at the present case the mass or the moment of inertia is not easy to calculate. In movement of pendulum in air the added mass and the add moment of inertia are hard to calculated and is not negligible. The governing equation for the pendulum is

$$\ell\ddot{\theta} - g\,\theta = 0 \tag{14.26}$$

Where here ℓ is length of the rode (or the line/wire) connecting the mass with the rotation point. Thus, the frequency of pendulum is $\frac{1}{2\,\pi}\sqrt{\frac{g}{\ell}}$ which measured in Hz. The period of the cycle is $2\,\pi\,\sqrt{\ell/g}$. Similar situation exists in the case of floating bodies. The basic differential equation is used to balance and is

$$\overbrace{I\ddot{\theta}}^{rotation} - \overbrace{V\,\rho_s\,\overline{GM}\,\theta}^{rotating\;moment} = 0 \tag{14.27}$$

In the same fashion the frequency of the floating body is

$$\frac{1}{2\,\pi}\sqrt{\frac{V\,\rho_s\,\overline{GM}}{I_{body}}} \tag{14.28}$$

and the period time is

$$2\,\pi\sqrt{\frac{I_{body}}{V\,\rho_s\,\overline{GM}}} \tag{14.29}$$

In general, the larger $\overline{GM}$ the more stable the floating body is. Increase in $\overline{GM}$ increases the frequency of the floating body. If the floating body is used to transport humans and/or other creatures or sensitive cargo it requires to reduce the $\overline{GM}$ so that the traveling will be smoother.

14.4.2 Limitations

This topic is dealing with several aspects of fluid mechanics like wave propagation on shallow liquid and the add mass. Admittedly these topics are very advance and require considerable amount of material to explain them. Simplified explanation is provided. The take away of this section to illustrate that the analysis is is provides the limit and the actual movement are some what different. When the ship or the floating body undergoes rolling,the shape of the body is fixed (to very large degree) while the shape or the configuration of the liquid in the container or the tank is not. The actual time to get the shape that should be assumed to be in a steady state. Steady state refers to a state which obtained when the particles stay at that location for ever (again approximate definition). How long it takes for the liquid to reach a steady state is very hard to calculate. However, it (to ascertain) is a much easier to do. First, it has to define the characteristic time (the typical time to for liquid to move from side to side).

The movement of the liquid from side to side is done by a wave (this claim is without a proof and more about it will appear in the book of "Basic of Fluid Mechanics" by this author). The speed of the wave over liquid in a shallow liquid is approximated by gravity and the liquid depth.

$$U \sim \sqrt{h\,g} \tag{14.30}$$

This equation provides a reasonable estimate of the velocity. The time required is then

$$t_c = \frac{b/2}{\sqrt{h\,g}} \tag{14.31}$$

When writing Eq. (14.31), it was assumed that the average distance is about the half the width ($b/2$) (liquid is moving side to side). Several additional assumptions have to be made to find the characteristic time liquid movement. What is the

When the body or the ship is rolling the

14.5 Metacentric Height, $\overline{GM}$, Measurement

The metacentric height can be measured by finding the change in the angle when a weight is moved on the floating body.

Moving the weight, T a distance, d then the moment created is

$$M_{weight} = T\,d \tag{14.32}$$

This moment is balanced by

$$M_{righting} = W_{total}\overline{GM}_{new}\,\theta \tag{14.33}$$

Where, W_{total}, is the total weight of the floating body including measuring weight. The angle, θ, is measured as the difference in the orientation of the floating body. The metacentric height is

$$\overline{GM}_{new} = \frac{T\,d}{W_{total}\,\theta} \tag{14.34}$$

If the change in the $\overline{GM}$ can be neglected, equation (14.34) provides the solution. The calculation of $\overline{GM}$ can be improved by taking into account the effect of the measuring weight. The change in height of G is

$$\not{g}\,m_{total}\,G_{new} = \not{g}\,m_{ship}\,G_{actual} + \not{g}\,T\,h \tag{14.35}$$

Combining equation (14.35) with equation (14.34) results in

$$\overline{GM}_actual = \overline{GM}_{new}\,\frac{m_{total}}{m_{ship}} - h\,\frac{T}{m_{ship}} \tag{14.36}$$

The weight of the ship is obtained from looking at the ship depth.

14.6 Stability of Submerged Bodies

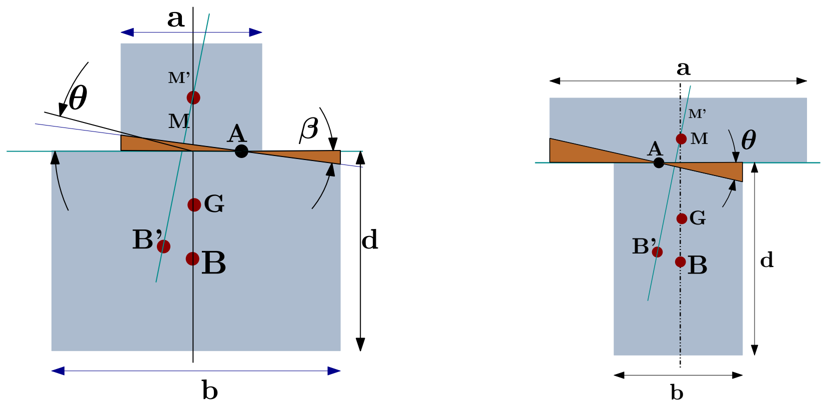
The analysis of submerged bodied is different from the stability when the body lays between two fluid layers with different density. When the body is submerged in a single fluid layer, then none of the changes of buoyant centroid occurs. Thus, the mass centroid must be below than buoyant centroid in order to have stable condition. Stable conditions

However, all fluids have density varied in some degree. In cases where the density changes significantly, it must be taken into account. For an example of such a case is an object floating in a solar pond where the upper layer is made of water with lower salinity than the bottom layer(change up to 20% of the density). When the floating object is immersed into two layers, the stability analysis must take into account the changes of the displaced liquids of the two liquid layers. The calculations for such cases are a bit more complicated but based on the similar principles. Generally, this density change helps to increase the stability of the floating bodies. This analysis is out of the scope of this book (for now).

14.7 Stability of None Systematical or "Strange" Bodies

This topic was address somewhat in the previous chapter and will be expanded in the following versions of this book. Previously, the discussion dealt with smooth transition in which the cross section area is a smooth function of the height. In section the discussion deals with abrupt change in the cross section. The change is more profound. Consider two extreme cases, one where narrow immersed body and two the wide part immersed. The rotation point A is not rotating and move in two different directions depending on cross section area (as function of the height) direction. Clearly without calculating any data, one can observed that the body with the wide immersed part rotating point A moves to the right while the other case the rotating point A moves to left. In, the change if direct of the rotating point change the way calculations are carried out. Never the lest it is clear wide immersed body is more stable.

While most floating bodies are symmetrical or semi–symmetrical, there are situations where the body has a "strange" and/or un–symmetrical body. The calculations will appear in the following versions. These are on the same considerations that that were discussed in the previous chapter.



(a) Floating Body with abrupt transition to small on the top (b) Floating Body with abrupt transition to large on the top

Fig. 14.15 – Calculations of $\overline{GM}$ for abrupt shape body.

15 Moment of Inertia

As the Chapter 7 on the centroid was divided, the chapter on moment of inertia is divided into moment of inertia of mass and area. Additionally this chapter another issue that is transformation

16 Variable Density Floating Bodies

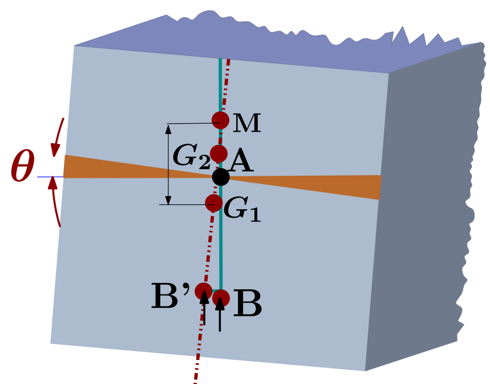


Fig. 16.1 – Body with movable gravity centroid.

Ship and other floating bodies do not have a constant density. In other words, the gravity centroid moves depend on the loading to a different location in the ship or body. That point is different from the centroid of the entire body. However, the buoyancy centroid is in the same location. located before as it depend on the displaced volume. If the gravity centroid is below the buoyancy centroid then the body is stable and no further calculations are needed. However, for most cases the issue to find how much to change the location of the gravity centroid to the vessel is stable. This chapter is dealing with the requirement on location of the gravity centroid for given geometry.

Previously the gravity location was given for the question was given a rotation of the body by θ what is the resulting α. Now the question is more complicated. Given the geometry, what are the requirements on the location of the gravity centroid to make the body stable. A new triangle can be build that is now made from tree points B, B' and G.

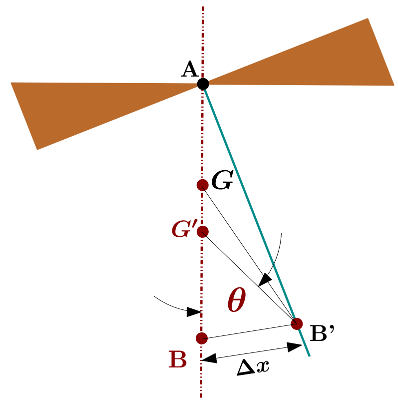


Fig. 16.2 – Diagram to explain the calculation for safe G.

The geometry (law of sines) it can be written and utilizing Eq. (7.15)

$$\frac{\Delta x}{\sin\theta} = \frac{BG'}{\sin 90°} \longrightarrow \frac{\cancel{\tan\theta}\dfrac{I_{xx}}{V_0}}{\cancel{\sin\theta}} = BG' \tag{16.1}$$

Or in a clear form

$$BG' = \frac{I_{xx}}{V_0} \tag{16.2}$$

The location of gravity in solid body (or floating body) is at G and normally is at the centroid of the entire body. The distance required to lower the gravity to make the body stable is GG'. Hence

$$GG' = BG - BG' \longrightarrow GG' = BG - \frac{I_{xx}}{V_0} \tag{16.3}$$

After the required distance is calculated, a weight needed to be attached, to bottom, or other place. The weight size is based on the calculations explained in **??**.

Example 16.1:

In example 12.2 dealt with rectangular shape which has zone with were unstable. Here in this example, a possible correction will be calculated (GG').

SOLUTION

The Fig. 12.12 shows the shape where the rectangular is unstable and for that range the required change is calculated. Eq. (12.II.e) is copied here for the boundary.

$$\frac{b}{D} = \sqrt{6\,\frac{\rho_s}{\rho_\ell}\left(1 - \frac{\rho_s}{\rho_\ell}\right)} \tag{16.I.a}$$

and

$$\frac{d}{D} = \frac{\rho_s}{\rho_\ell} \tag{16.I.b}$$

In this case $BG = D/2 - d/2$ and thus the GG' is

$$GG' = \left(\frac{D}{2} - \frac{d\,D}{2\,D}\right) - \frac{1}{12}\,\frac{b^2\,D}{D^2}\,\frac{\rho_\ell}{\rho_s} \tag{16.I.c}$$

and can be combined

$$\frac{GG'}{D} = \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) - \frac{1}{12}\left(\frac{b}{D}\right)^2\frac{\rho_\ell}{\rho_s} \tag{16.I.d}$$

It can be observed that the largest change should be made when the density is when the liquid density is relatively very law. Most large ships are with density ratio on the upper limit has the change is relatively small.

End Solution

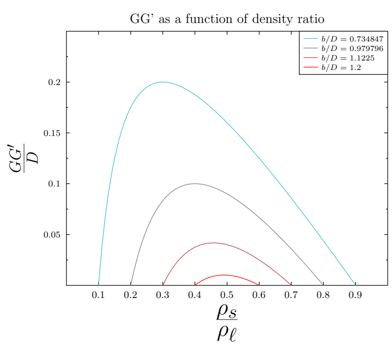


Fig. 16.3 – Requirements on gravity centroid for rectangular shape body.

The next is similar for the cylinder that was dealt before.

Example 16.2:

Earlier in the book in Fig. 12.21 the stability of cylinder was discussed. Here, the correction that need to be applied so that cylinder will be stable (GG').

<u>Solution</u>

The Fig. 12.12 shows the shape where the rectangular is unstable and for that range the required change is calculated. stability zone.

$$\frac{r}{D} = \sqrt{2\,\frac{\rho_s}{\rho_\ell}\left(1 - \frac{\rho_s}{\rho_\ell}\right)} \tag{16.II.a}$$

and

$$\frac{d}{D} = \frac{\rho_s}{\rho_\ell} \tag{16.II.b}$$

In this case $BG = D/2 - d/2$ and thus the GG' is

$$GG' = \left(\frac{D}{2} - \frac{d\,D}{2\,D}\right) - \frac{1}{4}\,\frac{r^2\,D}{D^2}\,\frac{\rho_\ell}{\rho_s} \tag{16.II.c}$$

and can be combined

$$\frac{GG'}{D} = \frac{1}{2}\left(1 - \frac{\rho_s}{\rho_\ell}\right) - \frac{1}{4}\left(\frac{r}{D}\right)^2\frac{\rho_\ell}{\rho_s} \tag{16.II.d}$$

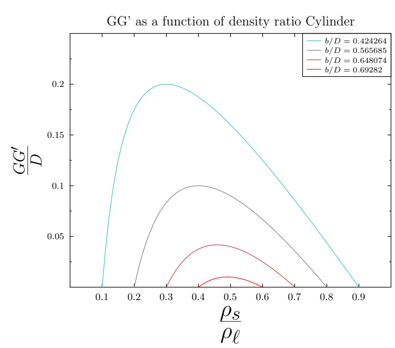


Fig. 16.4 – Requirements on gravity centroid for cylinder shape body.

The Fig. 16.4 is seem to the replica (hopefully) not a mistake.

End Solution

Example 16.3:

The famous ship Ever Giving for this excise can be consider as extruded square box with the following dimension L $= 400[m]$ b $= 60[m]$ and the D $= 60[m]$. The maximum draft (the depth) the ship can be in the liquid is $16[m]$. What is the minimum requirement on gravity centroid.

SOLUTION

The ratio of $b/D = 1$ in this case. From the value of d and D the ratio of density can be estimated as $16/60 = 0.267$ and observing from Fig. 16.3 the value of $GG'/D = 0.075$ thus, the value of GG' $= 60[m] \times 0.075 \sim 4.5[m]$ For uniform body the gravity centroid has to be at half of D thus it is has to be at $30[m]$. Thus, subtracting the correction it became that gravity centroid has to be at least below the $25[m]$ from the bottom of the ship.

End Solution

17

Dimensional Analysis

This chapter is dedicated to my adviser, Dr. E.R.G. Eckert.

Genick Bar-Meir

17.1 Introductory Remarks

The reasons that this chapter is included in this book to eliminate the tables for various situation but to have dimensionless presentation so tables should be included in the equations them self. For example, in Metacenter method the parameter GM is not given in dimensionless form. What is the meaning of GM 1[m] while it say the ship is stable and does not indicate if it is very stable or not. One has to get additional information. The indication that this question of stability is has relatively small of parameters which indicate that it has to none–dimensionalized.
What is dimensionless analysis? Dimensional analysis refers to techniques dealing with units or conversion to a unitless system. The definition of dimensional analysis is not consistent in the literature which span over various fields (mostly in the thermo–fluid field) (Buckingham 1914; Buckingham 1915b; Buckingham 1915a). Possible topics that dimensional analysis deals with are consistency of the units, change order of magnitude, applying from the old and known to unknown (see the Book of Ecclesiastes), and creation of group parameters without any dimensions. In this chapter, the focus is on the creation the creation of dimensionless groups. These techniques gave birth to dimensional parameters which have a great scientific importance (Görtler 1975; Lang-

haar 1980). Since the 1940s[1], the dimensional analysis is taught and written in all fluid, heat and mass transfer textbooks. The approach or the technique used in some of these books is referred to as Buckingham–π–theory. The π–theory was coined by Buckingham. However, there is another technique which is referred to in the literature as the Nusselt's method. Both these methods attempt to reduce the number of parameters which affect the problem and reduce the labor in solving the problem. The key in these techniques lays in the fact of consistency of the dimensions of any possible governing equation(s) and the fact that some dimensions are reoccurring. The Buckingham–π goes further and no equations are solved and even no knowledge about these equations is required. In Buckingham's technique only the dimensions or the properties of the problem at hand are analyzed. This author is aware of only a single class of cases were Buckingham's methods is useful and or can solve the problem namely the pendulum class problem (and similar).

The dimensional analysis was independently developed by Nusselt and improved by his students/co–workers (Schmidt, Eckert) in which the governing equations are used as well. Thus, more information is put into the problem and thus a better understanding on the dimensionless parameters is extracted. The advantage or disadvantage of these similar methods depend on the point of view. The Buckingham–π technique is simpler while Nusselt's technique produces a better result. Sometime, the simplicity of Buckingham's technique yields insufficient knowledge or simply becomes useless. When no governing equations are found, Buckingham's method has usefulness. It can be argued that these situations really do not exist in the Thermo–Fluid field. Nusselt's technique is more cumbersome but more precise and provide more useful information. Both techniques are discussed in this Chapter. The advantage of the Nusselt's technique are: a) compact presentation, b)knowledge what parameters affect the problem, c) easier to extent the solution to more general situations. In very complex problems both techniques suffer from in inability to provide a significant information on the effective parameters such multi–phase flow etc.

It has to be recognized that the dimensional analysis provides answer to what group of parameters affecting the problem and not the answer to the problem. In fact, there are fields in thermo–fluid where dimensional analysis, is recognized as useless. For example, the area of multiphase flows there is no solution based on dimensionless parameters (with the exception of the rough solution of Martinelli). In the Buckingham's approach it merely suggests the number of dimensional parameters based on a guess of all parameters affecting the problem. Nusselt's technique provides the form of these dimensionless

[1]The history of dimensional analysis is complex. Several scientists used this concept before Buckingham and Nusselt (see below history section). Their work culminated at the point of publishing the paper Buckingham's paper and independently constructed by Nusselt. It is interesting to point out that there are several dimensionless numbers that bear Nusselt and his students name, Nusselt number, Schmidt number, Eckert number. There is no known dimensionless number which bears Buckingham name. Buckingham's technique is discussed and studied in Fluid Mechanics while almost completely ignored by Heat and Mass Transfer researchers and their classes. Furthermore, in many advance fluid mechanics classes Nusselt's technique is used and Buckingham's technique is abandoned. Perhaps this fact can be attributed to tremendous influence Nusselt and his students had on the heat transfer field. Even, this author can be accused for being bias as the Eckert's last student. However, this author observed that Nusselt's technique is much more effective as it will demonstrated later.

parameters, and the relative relationship of these parameters.

17.1.1 Brief History

The idea of experimentation with a different, rather than the actual, dimension was suggested by several individuals independently. Some attribute it to Newton (1686) who coined the phrase of "great Principle of Similitude." Later, Maxwell a Scottish Physicist played a major role in establishing the basic units of mass, length, and time as building blocks of all other units. Another example, John Smeaton (8 June 1724–28 October 1792) was an English civil and mechanical engineer who study relation between propeller/wind mill and similar devices to the pressure and velocity of the driving forces. Jean B. J. Fourier (1768-1830) first attempted to formulate the dimensional analysis theory. This idea was extend by William Froude (1810-1871) by relating the modeling of open channel flow and actual body but more importantly the relationship between drag of models to actual ships. While the majority of the contributions were done by thermo–fluid guys the concept of the equivalent or similar propagated to other fields. Aiméem Vaschy, a German Mathematical Physicist (1857–1899), suggested using similarity in electrical engineering and suggested the Norton circuit equivalence theorems. Rayleigh probably was the first one who used dimensional analysis (1872) to obtain the relationships between the physical quantities (see the question why the sky is blue story).
Osborne Reynolds (1842–1912) was the first to derive and use dimensionless parameters to analyze experimental data. Riabouchunsky[2] proposed of relating temperature by molecules velocity and thus creating dimensionless group with the byproduct of compact solution (solution presented in a compact and simple form).
Buckingham culminated the dimensional analysis and similitude and presented it in a more systematic form. In the about the same time (1915, Wilhelm Nusselt (November 25, 1882 – September 1, 1957), a German engineer, developed the dimensional analysis (proposed the principal parameters) of heat transfer without knowledge about previous work of Buckingham.

17.1.2 Theory Behind Dimensional Analysis

In chemistry it was recognized that there are fundamental elements that all the material is made from (the atoms). That is, all the molecules are made from a combination of different atoms. Similarly to this concept, it was recognized that in many physical systems there are basic fundamental units which can describe all the other dimensions or units in the system. For example, isothermal single component systems (which does not undergo phase change, temperature change and observed no magnetic or electrical effect) can be described by just basic four physical units. The units or dimensions are, time, length, mass, quantity of substance (mole). For example, the dimension or the units of force can be constructed utilizing Newton's second law i.e. mass times acceleration $\longrightarrow m\,a = M\,L/t^2$. Increase of degree of freedom, allowing this system

[2] Riabouchunsky, Nature Vol 99 p. 591, 1915

to be non–isothermal will increase only by one additional dimension of temperature, θ. These five fundamental units are commonly the building blocks for most of the discussion in fluid mechanics (see Table of basic units 17.1).

Table 17.1 – Basic Units of Two Common Systems

Standard System			Old System		
Name	Letter	Units	**Name**	Letter	Units
Mass	M	$[kg]$	**Force**	F	$[N]$
Length	L	$[m]$	**Length**	L	$[m]$
Time	t	$[sec]$	**Time**	t	$[sec]$
Temperature	θ	$[°\text{C}]$	**Temperature**	T	$[°\text{C}]$
Additional Basic Units for Magnetohydrodynamics					
Electric Current	A	$[A]mpere$	**Electric Current**	A	$[A]mpere$
Luminous Intensity	cd	$[cd]$ candle	**Luminous Intensity**	cd	$[cd]$ candle
Chemical Reactions					
Quantity of substance	$\mathfrak{M}$	mol	**Quantity of substance**	$\mathfrak{M}$	mol

The choice of these basic units is not unique and several books and researchers suggest a different choice of fundamental units. One common selection is substituting the mass with the force in the previous selection (F, t, L, mol, Temperature). This author is not aware of any discussion on the benefits of one method over the other method. Yet, there are situations in which first method is better than the second one while in other situations, it can be the reverse. Other selections are possible but not common and, at the moment, will not be discussed here.

Example 17.1:
What are the units of force when the basic units are: mass, length, time, temperature (M, L, t, θ)? What are the units of mass when the basic units are: force, length, time, temperature (F, L, t, T)? Notice the different notation for the temperature in the two systems of basic units. This notation has no significance but for historical reasons remained in use.

Solution

These two systems are related as the questions are the reversed of each other. The connection between the mass and force can be obtained from the simplified Newton's second law $F = m\,a$ where F is the force, m is the mass, and a is the acceleration. Thus, the units of force are

$$F = \frac{M\,L}{t^2} \tag{17.I.a}$$

For the second method the unit of mass are obtain from Equation (17.I.a) as

$$M = \frac{F\,t^2}{L} \tag{17.I.b}$$

End Solution

The number of fundamental or basic dimensions determines the number of the combinations which affect the physical[3] situations. The dimensions or units which affect the problem at hand can be reduced because these dimensions are repeating or reoccurring. The Buckingham method is based on the fact that all equations must be consistent with their units. That is the left hand side and the right hand side have to have the same units. Because they have the same units the equations can be divided to create unitless equations. This idea alludes to the fact that these unitless parameters can be found without any knowledge of the governing equations. Thus, the arrangement of the effecting parameters in unitless groups yields the affecting parameters. These unitless parameters are the dimensional parameters. The following trivial example demonstrates the consistency of units

Example 17.2:
Newton's equation has two terms that related to force $F = m\,a + \dot{m}\,U$. Where F is force, m is the mass, a is the acceleration and dot above $\dot{m}$ indicating the mass derivative with respect to time. In particular case, this equation get a form of

$$F = m\,a + 7 \tag{17.II.a}$$

where 7 represent the second term. What are the requirement on equation (17.II.a)?

SOLUTION

Clearly, the units of $[F]$, $m\,a$ and 7 have to be same. The units of force are $[N]$ which is defined by first term of the right hand side. The same units force has to be applied to 7 thus it must be in $[N]$.

End Solution

Suppose that there is a relationship between a quantity a under the question and several others parameters which either determined from experiments or theoretical consideration which is of the form

$$D = f(a_1, a_2, \cdots, a_i, \cdots, a_n) \tag{17.1}$$

[3]The dimensional analysis also applied in economics and other areas and the statement should reflect this fact. However, this book is focused on engineering topics and other fields are not discussed.

where D is dependent parameters and $a_1, a_2, \cdots, a_i, \cdots, a_n$ are have independent dimensions. From these independent parameters $a_1, a_2, \cdots, a_i$ have independent dimensions (have basic dimensions). This mean that all the dimensions of the parameters $a_{i+1}, \cdots, a_n$ can be written as combination of the the independent parameters $a_1, a_2, \cdots, a_i$. In that case it is possible to write that every parameter in the later set can written as dimensionless

$$\frac{a_{i+1}}{{a_1}^{p_1}, {a_2}^{p_2}, \cdots, {a_i}^{p_i}} = \text{dimensionless} \tag{17.2}$$

The "non–basic" parameter would be dimensionless when divided by appropriately and selectively chosen set of constants $p_1, p_2, \cdots, p_i$.

Example 17.3:

In a experiment, the clamping force is measured. It was found that the clamping force depends on the length of experimental setup, velocity of the upper part, mass of the part, height of the experimental setup, and leverage the force is applied. Chose the basic units and dependent parameters. Show that one of the dependent parameters can be normalized.

Solution

The example suggest that the following relationship can be written.

$$F = f(L, U, H, \tau, m) \tag{17.III.a}$$

The basic units in this case are in this case or length, mass, and time. No other basic unit is need to represent the problem. Either L, H, or τ can represent the length. The mass will be represented by mass while the velocity has to be represented by the velocity (or some combination of the velocity). Hence a one possible choice for the basic dimension is L, m, and U. Any of the other Lengths can be represented by simple division by the L. For example

$$\text{Normalize parameter} = \frac{H}{L} \tag{17.III.b}$$

Or the force also can be normalized as

$$\text{Another Normalize parameter} = \frac{F}{m\, U^2\, L^{-1}} \tag{17.III.c}$$

The acceleration can be any part of acceleration component such as centrifugal acceleration. Hence, the force is mass times the acceleration.

End Solution

The relationship (17.1) can be written in the light of the above explanation as

$$\frac{D}{{a_1}^{p_1}, {a_2}^{p_2}, \cdots, {a_i}^{p_1}} = F\left(\frac{a_{i+1}}{{a_1}^{p_{i+1,1}},\, {a_2}^{p_{i+1,2}}, \cdots,\, {a_i}^{p_{i+1,i}}}, \cdots, \frac{a_n}{{a_n}^{p_{n,1}},\, {a_n}^{p_{n,2}}, \cdots,\, {a_n}^{p_{n,i}}}\right) \tag{17.3}$$

where the indexes of the power p on the right hand side are single digit and the double digits on the on the right hand side. While this "proof" shows the basic of the Buckingham's method it actually provides merely the minimum number of the dimension parameters. In fact, this method entrenched into the field while in most cases provides incomplete results. The fundamental reason for the erroneous results is because the fundamental assumption of equation (17.1). This method provides a crude tool of understanding.

17.1.3 Dimensional Parameters Application for Experimental Study

The solutions for any situations which are controlled by the same governing equations with same boundary conditions regardless of the origin the equation. The solutions are similar or identical regardless to the origin of the field no matter if the field is physical, or economical, or biological. The Buckingham's technique implicitly suggested that since the governing equations (in fluid mechanics) are essentially are the same, just knowing the parameters is enough the identify the problem. This idea alludes to connections between similar parameters to similar solution. The non–dimensionalization i.e. operation of reducing the number affecting parameters, has a useful by–product, the analogy in other words, the solution by experiments or other cases. The analogy or similitude refers to understanding one phenomenon from the study of another phenomenon. This technique is employed in many fluid mechanics situations. For example, study of compressible flow (a flow where the density change plays a significant part) can be achieved by study of surface of open channel flow. The compressible flow is also similar to traffic on the highway. Thus for similar governing equations if the solution exists for one case it is a solution to both cases.

The analogy can be used to conduct experiment in a cheaper way and/or a safer way. Experiments in different scale than actual dimensions can be conducted for cases where the actual dimensions are difficult to handle. For example, study of large air planes can done on small models. On the other situations, larger models are used to study small or fast situations. This author believes that at the present the Buckingham method has extremely limited use for the real world and yet this method is presented in the classes on fluid mechanics. Thus, many examples on the use of this method will be presented in this book. On the other hand, Nusselt's method has a larger practical use in the real world and therefore will be presented for those who need dimensional analysis for the real world. Dimensional analysis is useful also for those who are dealing with the numerical research/calculation. This method

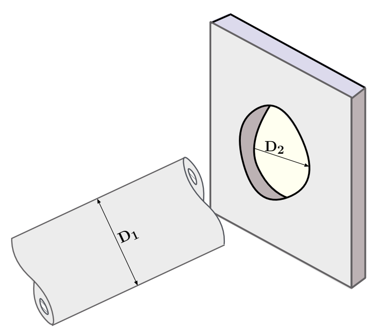


Fig. 17.1 – Fitting rod into a hole.

supplement knowledge when some parameters should be taken into account and why.

Fitting a rod into a circular hole (see Figure 17.1) is an example how dimensional analysis can be used. To solve this problem, it is required to know two parameters; 1) the rode diameter and 2) the diameter of the hole. Actually, it is required to have only one parameter, the ratio of the rode diameter to the hole diameter. The ratio is a dimensionless number and with this number one can tell that for a ratio larger than one, the rode will not enter the hole; and a ratio smaller than one, the rod is too small. Only when the ratio is equal to one, the rode is said to be fit. This presentation allows one to draw or present the situation by using only one coordinate, the radius ratio. Furthermore, if one wants to deal with tolerances, the dimensional analysis can easily be extended to say that when the ratio is equal from 0.99 to 1.0 the rode is fitting, and etc. If one were to use the two diameters description, further significant information will be needed. In the preceding simplistic example, the advantages are minimal. In many real problems this approach can remove clattered views and put the problem into focus. Throughout this book the reader will notice that the systems/equations in many cases are converted to a dimensionless form to augment understanding.

17.1.4 The Pendulum Class Problem

The only known problem that dimensional analysis can solve (to some degree) is the pendulum class problem. In this section several examples of the pendulum type problem are presented. The first example is the classic Pendulum problem.

Example 17.4:

Derive the relationship for the gravity [g], frequency [ω] and length of pendulum [ℓ]. Assume that no other parameter including the mass affects the problem. That is, the relationship can be expressed as

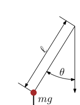


Fig. 17.2 – Figure for example (17.4).

$$\omega = f(\ell, g) \tag{17.IV.a}$$

Notice in this problem, the real knowledge is provided, however in the real world, this knowledge is not necessarily given or known. Here it is provided because the real solution is already known from standard physics classes.[4]

Solution

The solution technique is based on the assumption that the indexical form is the appropriate form to solve the problem. The Indexical form

$$\omega = C_1 \times \ell^a g^b \tag{17.IV.b}$$

The solution functional complexity is limited to the basic combination which has to be in some form of multiplication of ℓ and g in some power. In other words, the multiplication

[4]The reader can check if the mass is assumed to affect the problem then, the result is different.

of $\ell\, g$ have to be in the same units of the frequency units. Furthermore, assuming, for example, that a trigonometric function relates ℓ and g and frequency. For example, if a $\sin$ function is used, then the functionality looks like $\omega = \sin(\ell\, g)$. From the units point of view, the result of operation not match i.e. $(sec \neq \sin(sec))$. For that reason the form in equation (17.IV.b) is selected. To satisfy equation (17.IV.b) the units of every term are examined and summarized the following table.

Table 17.2 – Units of the Pendulum Parameters

Parameter	Units	Parameter	Units	Parameter	Units
ω	t^{-1}	ℓ	L^1	g	$L^1 t^{-2}$

Thus substituting of the Table 17.2 in equation (17.IV.b) results in

$$t^{-1} = C_1 \left(L^1\right)^a \, \left(L^1\, t^{-2}\right)^b \Longrightarrow L^{a+b} t^{-2\,b} \tag{17.IV.c}$$

after further rearrangement by multiply the left hand side by L^0 results in

$$L^0 t^{-1} = C\, L^{a+b} t^{-2\,b} \tag{17.IV.d}$$

In order to satisfy equation (17.IV.d), the following must exist

$$0 = a + b \quad and \quad -1 = \frac{-2}{b} \tag{17.IV.e}$$

The solution of the equations (17.IV.e) is $a = -1/2$ and $b = -1/2$.

End Solution

What was found in this example is the form of the solution's equation and frequency. Yet, the functionality e.g. the value of the constant was not found. The constant can be obtained from experiment for plotting ω as the abscissa and $\sqrt{\ell/g}$ as ordinate.

According to some books and researchers, this part is the importance of the dimensional analysis. It can be noticed that the initial guess merely and actually determine the results. If, however, the mass is added to considerations, a different result will be obtained. If the guess is relevant and correct then the functional relationship can be obtained by experiments.

17.2 Buckingham–π–Theorem

All the physical phenomena that is under the investigation have n physical effecting parameters such that

$$F_1(q_1, q_2, q_3, \cdots, q_n) = 0 \tag{17.4}$$

where q_i is the "i" parameter effecting the problem. For example, study of the pressure difference created due to a flow in a pipe is a function of several parameters such

$$\Delta P = f(L,\, D,\, \mu,\, \rho,\, U) \tag{17.5}$$

In this example, the chosen parameters are not necessarily the most important parameters. For example, the viscosity, μ can be replaced by dynamic viscosity, ν. The choice is made normally as the result of experience and it can be observed that ν is a function of μ and ρ. Finding the important parameters is based on "good fortune" or perhaps intuition. In that case, a new function can be defined as

$$F(\Delta P, L, D, \mu, \rho, U) = 0 \tag{17.6}$$

Again as stated before, the study of every individual parameter will create incredible amount of data. However, Buckingham's[5] methods suggested to reduce the number of parameters. If independent parameters of same physical situation is m thus in general it can be written as

$$F_2(\pi_1, \pi_2, \pi_3, \cdots, \pi_m) = 0 \tag{17.7}$$

If there are n variables in a problem and these variables contain m primary dimensions (for example M, L, T), then the equation relating all the variables will have (n-m) dimensionless groups.

There are 2 conditions on the dimensionless parameters:

1. Each of the fundamental dimensions must appear in at least one of the m variables
2. It must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

In the case of the pressure difference in the pipe (Equation (17.6)) there are 6 variables or $n = 6$. The number of the fundamental dimensions is 3 that is $m = 3$ ([M], [L], [t]) The choice of fundamental or basic units is arbitrary in that any construction of these units is possible. For example, another combination of the basic units is time, force, mass is a proper choice. According to Buckingham's theorem the number of dimensionless groups is $n - m = 6 - 3 = 3$. It can be written that one dimensionless parameters is a function of two other parameter such as

$$\pi_1 = f\left(\pi_2, \pi_3\right) \tag{17.8}$$

If indeed such a relationship exists, then, the number of parameters that control the problem is reduced and the number of experiments that need to be carried is considerably smaller. Note, the π–theorem does not specify how the parameters should be selected nor what combination is preferred.

[5]E. Buckingham, "Model Experiments and the Forms of Empirical Equations," Transactions of the American Society of Mechanical Engineers, Vol. 37, 1915.

17.2.1 Construction of the Dimensionless Parameters

In the construction of these parameters it must be realized that every dimensionless parameters has to be independent. The meaning of independent is that one dimensionless parameter is not a multiply or a division of another dimensional parameter. In the above example there are three dimensionless parameters which required of at least one of the physical parameter per each dimensionless parameter. Additionally, to make these dimensionless parameters independent they cannot be multiply or division of each other.

For the pipe problem above, ℓ and D have the same dimension and therefore both cannot be chosen as they have the same dimension. One possible combination is of D, U and ρ are chosen as the recurring set. The dimensions of these physical variables are: $D = [L^1]$, velocity of $U = [L\, t^{-1}]$ and density as $\rho = [M\, L^{-3}]$. Thus, the first term D can provide the length, $[L]$, the second term, U, can provide the time $[t]$, and the third term, ρ can provide the mass $[M]$. The fundamental units, L, t, and M are length, time and mass respectively. The fundamental units can be written in terms of the physical units. The first term L is the described by D with the units of $[L]$. The time, $[t]$, can be expressed by D/U. The mass, $[M]$, can be expressed by $\rho\, D^3$. Now the dimensionless groups can be constructed by looking at the remaining physical parameters, ΔP, D and μ. The pressure difference, ΔP, has dimensions of $[M\, L^{-1}\, t^{-2}]$ Therefore, $\Delta P\, M^{-1}\, L\, t^2$ is a dimensionless quantity and these values were calculated just above this line. Thus, the first dimensionless group is

$$\pi_1 = \overbrace{\Delta P}^{[M\,L^{-1}\,t^{-2}]} \overbrace{\frac{1}{\rho\, D^3}}^{[M^{-1}]} \overbrace{D}^{[L]} \overbrace{\frac{D^2}{U^2}}^{[t^2]} = \overbrace{\frac{\Delta P}{\rho\, U^2}}^{unitless} \tag{17.9}$$

The second dimensionless group (using D) is

$$\pi_2 = \overbrace{D}^{[L]} \overbrace{\ell^{-1}}^{[L^{-1}]} = \frac{D}{L} \tag{17.10}$$

The third dimensionless group (using μ dimension of $[M\, L^1\, t^{-1}]$) and therefore dimensionless is

$$\pi_3 = \mu \overbrace{\frac{1}{D^3\, \rho}}^{[M^{-1}]} \overbrace{D}^{[L]} \overbrace{\frac{D}{U}}^{[t]} = \frac{\mu}{D\, U\, \rho} \tag{17.11}$$

This analysis is not unique and there can be several other possibilities for selecting dimensionless parameters which are "legitimately" correct for this approach.

There are roughly three categories of methods for obtaining the dimensionless parameters. The first one solving it in one shot. This method is simple and useful for a small number of parameters. Yet this method becomes complicated for large number of parameters. The second method, some referred to as the building blocks method, is described above. The third method is by using dimensional matrix which is used mostly by mathematicians and is less useful for engineering purposes.

The second and third methods require to identification of the building blocks. These building blocks are used to construct the dimensionless parameters. There are several requirements on these building blocks which were discussed on page 170. The main point that the building block unit has to contain at least the basic or fundamental unit. This requirement is logical since it is a building block. The last method is mostly used by mathematicians which leads and connects to linear algebra. The fact that this method used is the hall mark that the material was written by mathematician. Here, this material will be introduced for completeness sake with examples and several terms associated with this technique.

17.2.2 Similarity and Similitude

One of dimensional analysis is the key point is the concept that the solution can be obtained by conducting experiments on similar but not identical systems. The analysis here suggests and demonstrates[6] that the solution is based on several dimensionless numbers. Hence, constructing experiments of the situation where the same dimensionless parameters obtains could, in theory, yield a solution to problem at hand. Thus, knowing what are dimensionless parameters should provide the knowledge of constructing the experiments.

In this section deals with these similarities which in the literature some refer as analogy or similitude. It is hard to obtain complete similarity. Hence, there is discussion how similar the model is to the prototype. It is common to differentiate between three kinds of similarities: geometric, kinetics, and dynamic. This characterization started because historical reasons and it, some times, has merit especially when applying Buckingham's method. In Nusselt's method this differentiation is less important.

Geometric Similarity

One of the logical part of dimensional analysis is how the experiences should be similar to actual body they are supposed to represent. This logical conclusion is an add–on and this author is not aware of any proof to this requirement based on Buckingham's methods. Ironically, this conclusion is based on Nusselt's method which calls for the same dimensionless boundary conditions. Again, Nusselt's method, sometimes or even often, requires similarity because the requirements to the boundary conditions. Here[7] this postulated idea is adapted.

Under this idea the prototype area has to be square of the actual model or

$$\frac{A_p}{A_m} = \left(\frac{\ell_{1prototype}}{\ell_{1model}}\right)^2 = \left(\frac{\ell_{2p}}{\ell_{2m}}\right)^2 \tag{17.12}$$

[6]This statement is too strong. It has to be recognized that the results are as good as the guessing which in most cases is poor.

[7]Because this book intend to help students to pass their exams, this book present what most instructors required. It well established that this over–strict requirement and under Nusselt's method it can be overcome.

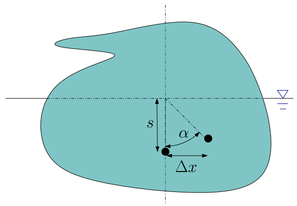


Fig. 17.3 – Floating body showing alpha and other dimensions.

where ℓ_1 and ℓ_2 are the typical dimensions in two different directions and subscript p refers to the prototype and m to the model. Under the same argument the volumes change with the cubes of lengths.

In some situations, the model faces inability to match two or more dimensionless parameters. In that case, the solution is to sacrifice the geometric similarity to minimize the undesirable effects. For example, river modeling requires to distort vertical scales to eliminate the influence of surface tension or bed roughness or sedimentation.

Kinematic Similarity

The perfect kinetics similarity is obtained when there are geometrical similarity and the motions of the fluid above the objects are the same. If this similarity is not possible, then the desire to achieve a motion "picture" which is characterized by ratios of corresponding velocities and accelerations is the same throughout the actual flow field. It is common in the literature, to discuss the situations where the model and prototype are similar but the velocities are different by a different scaling factor.

The geometrical similarity aside the shapes and counters of the object it also can requires surface roughness and erosion of surfaces of mobile surfaces or sedimentation of particles surface tensions. These impose demands require a minimum on the friction velocity. In some cases the minimum velocity can be $U_{min} = \sqrt{\tau_w/\rho}$. For example, there is no way achieve low Reynolds number with thin film flow.

Dynamics Similarity

The dynamic similarity has many confusing and conflicting definitions in the literature. Here this term refers to similarity of the forces. It follows, based on Newton's second law, that this requires similarity in the accelerations and masses between the model and prototype.

Example 17.5:
*Repeat **??** using Nusselt's technique assuming the body are with uniform density.*

SOLUTION

The governing equation for the stability of floating body is

$$\theta = \alpha \tag{17.13}$$

where θ is the arbitrary turning angle and α is the angle resulting from change of the centroid of submerge volume due to the change in θ. The relationship between the various geometrical parameters is determined connection according to Eq. (17.13). That relationship requires look at the component of triangle at Fig. 17.3. The base of the triangle is determined by

$$\Delta x = x_n - x_0 = \frac{V}{V_0} \left(x_a - x_r \right) \tag{17.V.a}$$

and x_a and x_r is related to ratio

$$x_a = \frac{\int x \, dV}{\int dV} = \frac{\int x (x \tan\theta) d(x) \, dx}{V} = \frac{\tan\theta \int x^2 \overbrace{d(x)\, dx}^{dA}}{V} \tag{17.V.b}$$

or using the definition of moment of inertia Eq. (17.V.b) can be transferred into

$$x_a = \frac{\tan\theta \, I_{xx}}{V} \tag{17.V.c}$$

After the opposite side calculation, adjacent side (GB center of (B) buoyancy (G) center of gravity) thus (G-B). The weight can be estimated as $m = \rho_\ell \, A \, B = \rho_s \, A \, G$. When A is typical cross section, thus

$$GB = G \left(1 - \frac{\rho_s}{\rho_\ell} \right) \tag{17.V.d}$$

Combining equations (17.V.c) and (17.V.d) results in

$$\alpha = \frac{\dfrac{\tan\theta \, I_{xx}}{V}}{G \left(1 - \frac{\rho_s}{\rho_\ell} \right)} \tag{17.V.e}$$

Observation of Eq. (17.V.e) so dimensional group $\frac{\rho_s}{\rho_\ell}$, and $\frac{I_{xx}}{G\,V}$ While the analysis was clumsy and rough it provides dimensionless parameters while Buckingham's method fails dramatically.

End Solution

17.3 Summary

The two dimensional analysis methods or approaches were presented in this chapter. Buckingham's π technique is a quick "fix approach" which allow rough estimates and relationship between model and prototype. Nusselt's approach provides an heavy duty approach to examine what dimensionless parameters effect the problem. It can be shown that these two techniques in some situations provide almost similar solution. In other cases, these technique proves different and even conflicting results. The dimensional analysis technique provides a way to simplify models (solving the governing equation by experimental means) and to predict effecting parameters.

Table 3 – Moments of Inertia for various plane surfaces about their center of gravity (full shapes)

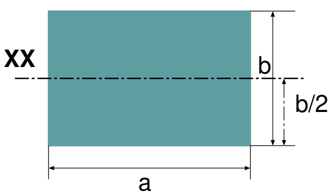
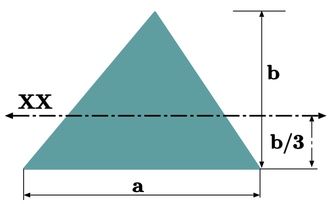
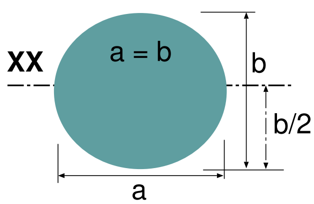
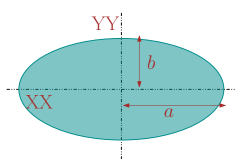
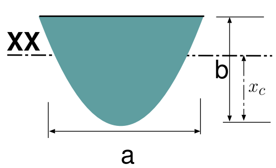
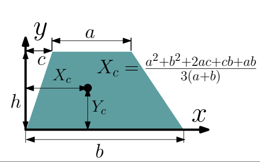
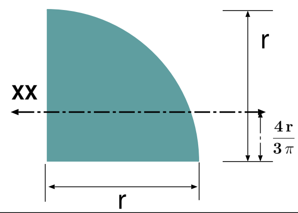
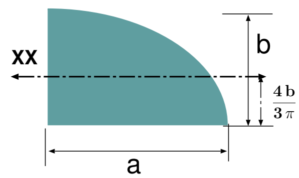
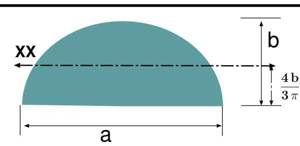
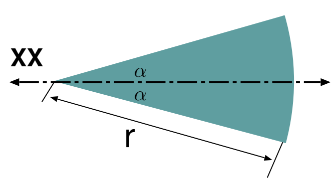
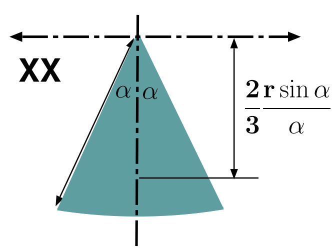
Shape Name	Picture description	x_c, y_c	A	I_{xx}
rectangle		$\frac{b}{2}$; $\frac{a}{2}$	$a\,b$	$\frac{ab^3}{12}$
Triangle		$\frac{a}{3}$	$\frac{a\,b}{3}$	$\frac{ab^3}{36}$
Circle		$\frac{b}{2}$	$\frac{\pi\,b^2}{4}$	$\frac{\pi b^4}{64}$
Ellipse		$\frac{a}{2}\ \frac{b}{2}$	$\frac{\pi\,ab}{4}$	$\frac{ab^3}{64}$
$y = \alpha x^2$ Parabola		$\frac{3\,\alpha\,b}{15\,\alpha-5}$	$\frac{6\alpha-2}{3} \times \left(\frac{b}{\alpha}\right)^{\frac{3}{2}}$	$\frac{\sqrt{b}\left(20\,b^3-14\,b^2\right)}{35\,\sqrt{\alpha}}$
Trapezoid		$Y_c = \frac{h(2a+b)}{3(a+b)}$	$\frac{h(a+b)}{2}$	$\frac{h^3(3a+b)}{12}$

Table 4 – Moment of inertia for various plane surfaces about their center of gravity

Shape Name	Picture description	x_c, y_c	A	I_{xx}
Quadrant of Circle		$\frac{4\,r}{3\,\pi}$	$\frac{\pi\,r^2}{4}$	$r^4\left(\frac{\pi}{16}-\frac{4}{9\pi}\right)$
Ellipsoidal Quadrant		$\frac{4\,b}{3\,\pi}$	$\frac{\pi\,a\,b}{4}$	$a\,b^3\left(\frac{\pi}{16}-\frac{4}{9\pi}\right)$
Half of Elliptic		$\frac{4\,b}{3\,\pi}$	$\frac{\pi\,a\,b}{4}$	$a\,b^3\left(\frac{\pi}{16}-\frac{4}{9\pi}\right)$
Circular Sector		0	$2\alpha\,r^2$	$\frac{r^4}{4}\left(\alpha-\frac{1}{2}\sin 2\alpha\right)$
Circular Sector		$\frac{2}{3}\frac{r\,\sin\alpha}{\alpha}$	$2\alpha\,r^2$	$I_{x'x'} = \frac{r^4}{4}\left(\alpha+\frac{1}{2}\sin 2\alpha\right)$

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