

# Optimally weighted ensemble for sub-seasonal forecasting

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## Introduction

- Seasonal forecasts are issued as the arithmetic mean (equal weights: EW) of an ensemble of simulations initialized by assimilating observations.
- During the operational production of the forecast (considering the buffer-time needed to ensure timely delivery), new unused observations can be available (typically a week).

### Objective

Defining an **optimal weighted average (OW)** that can use independent data to enhance the skill of the forecast on a subseasonal time scale with respect to **equal weighted averages (EW)**.

### Setup of the experiment

#### Forecasts $\mathbf{x}$ :

The Norwegian Climate Prediction Model (**NorCPM**) is providing skillful seasonal forecasts (Wang et al. 2019). A **60 member ensemble reanalysis reanalysis** has been recently produced by assimilating monthly **sea-surface temperature (SST) and T-S profile** data with the ensemble Kalman filter. Retrospective hindcasts of 60 members - initialised from the reanalysis - have been produced for the **period 1985 to 2010 with 4 start dates per year**.

#### Observation $\mathbf{y}$ :

A weekly NOAA sea-surface, optimally interpolated (Reynolds et al. 2002) after the analysis month is used to determine the weights.

### Estimation of the Optimal Weights (OW)

For a forecast  $\mathbf{x}^n$  of index  $n$  ( $1 \leq n \leq N$ ,  $N$  is the size of the ensemble) and a grid point  $i$ , the weight is determined by:

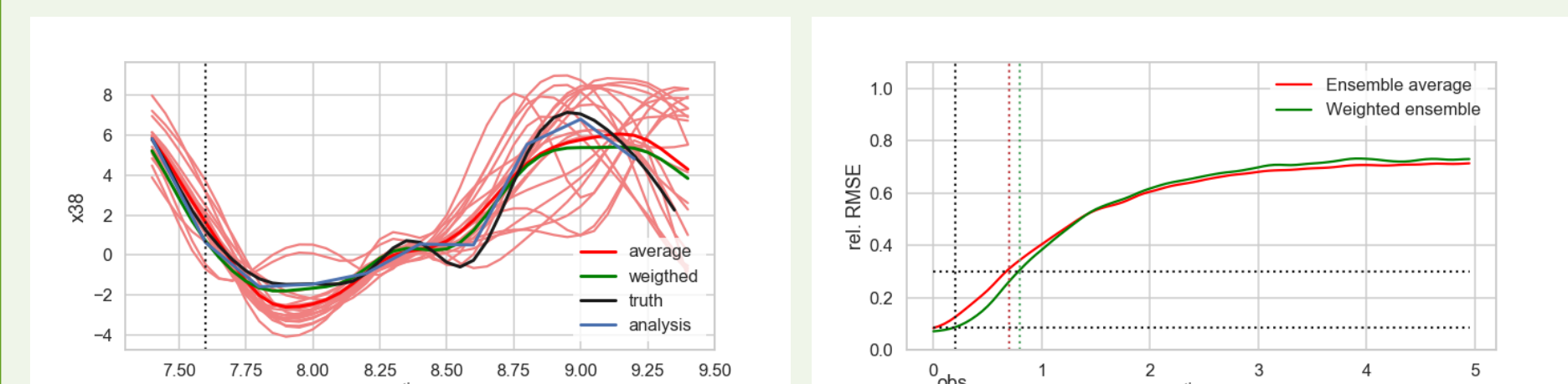
$$w_{n,i} \propto \exp\left(-\frac{1}{2}(\boldsymbol{\rho}_i \circ \mathbf{d})^T \mathbf{R}^{-1}(\boldsymbol{\rho}_i \circ \mathbf{d})\right),$$

where

- $\mathbf{d} = (\mathbf{y} - \mathbf{x}^n)$
- $\boldsymbol{\rho}_i$  is a localization vector whose elements are:  $\rho_{ik} = f(d(i,k)/L)$ ,  $\rho_{ik} = 0$  if  $d(i,k) > L$
- $\mathbf{R} = (\lambda^\circ)^2 \mathbf{E}^{\text{obs}}$ , is the observation error.  $\mathbf{E}^{\text{obs}}$  is the diagonal error provided with the NOAA product and  $\lambda^\circ$  is an inflation factor.

### Illustration with a Lorenz model

The algorithm has been tested using a Lorenz 96 model with a observation at 0.2 time after the analysis time.

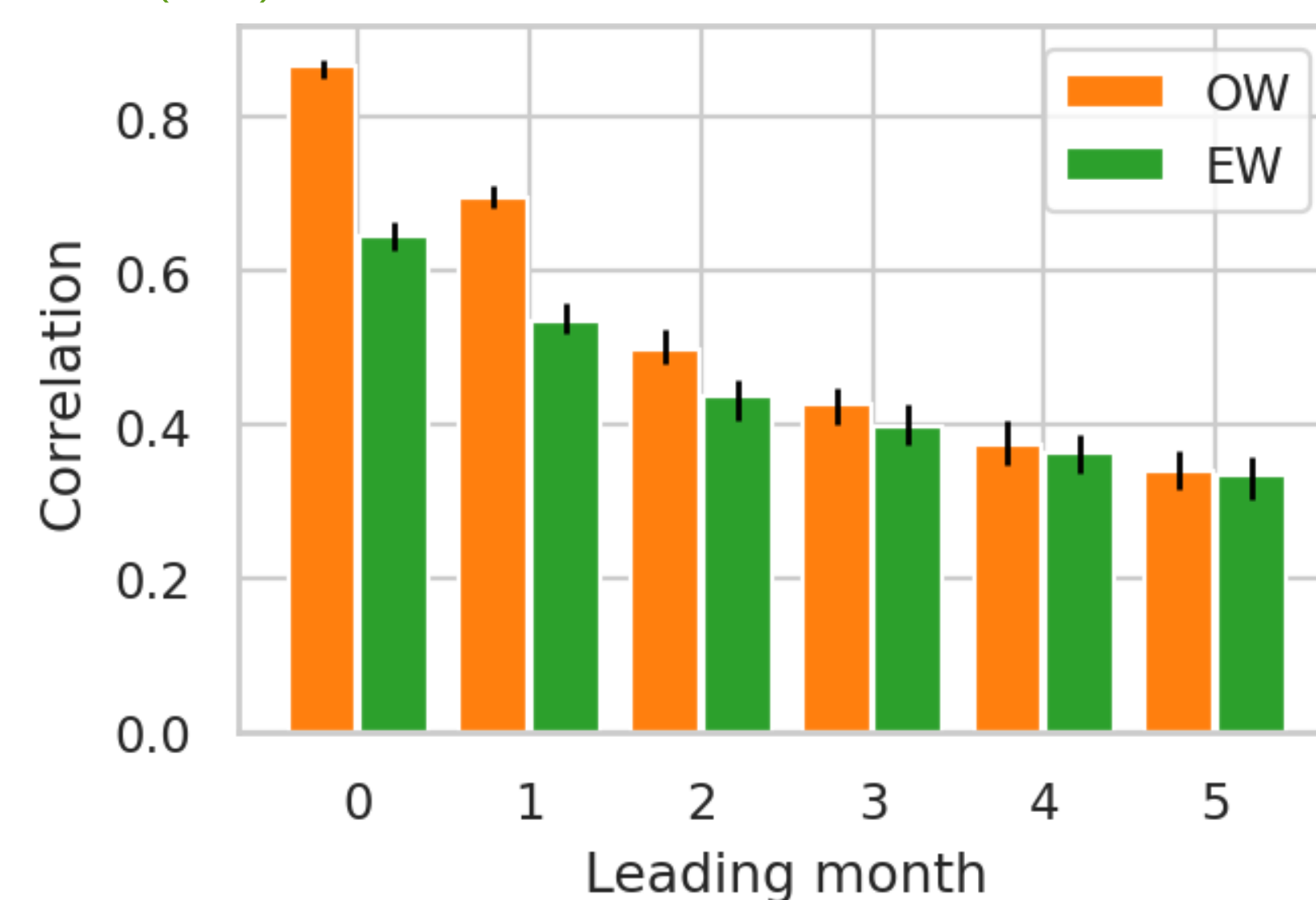


- The forecast skill (in RMSE) is improved until a time of 1 ( $> 3$  Lyapunov time)
- The error of the OW mean at the observation time is comparable to the error of the analysis (initial time)

## Results

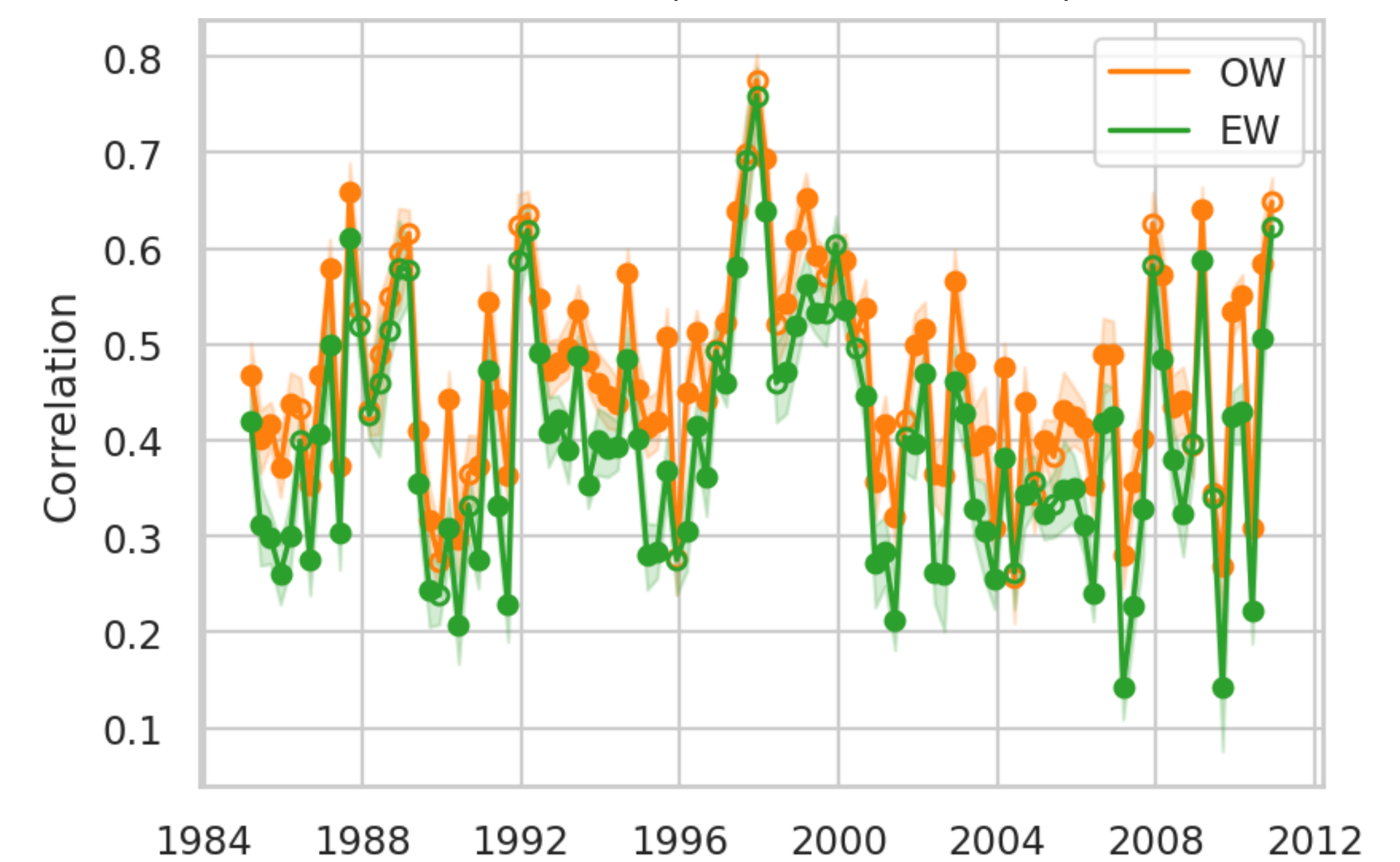
The validation is performed on the sea-surface temperature by comparing with future observations. All the confidence intervals and significance of the results are computed by bootstrapping with 100 samples.

Figure: Global correlation space- and time-wise as a function of the lead month for the **optimal weights (OW)** and the **equal weights (EW)** hindcast.



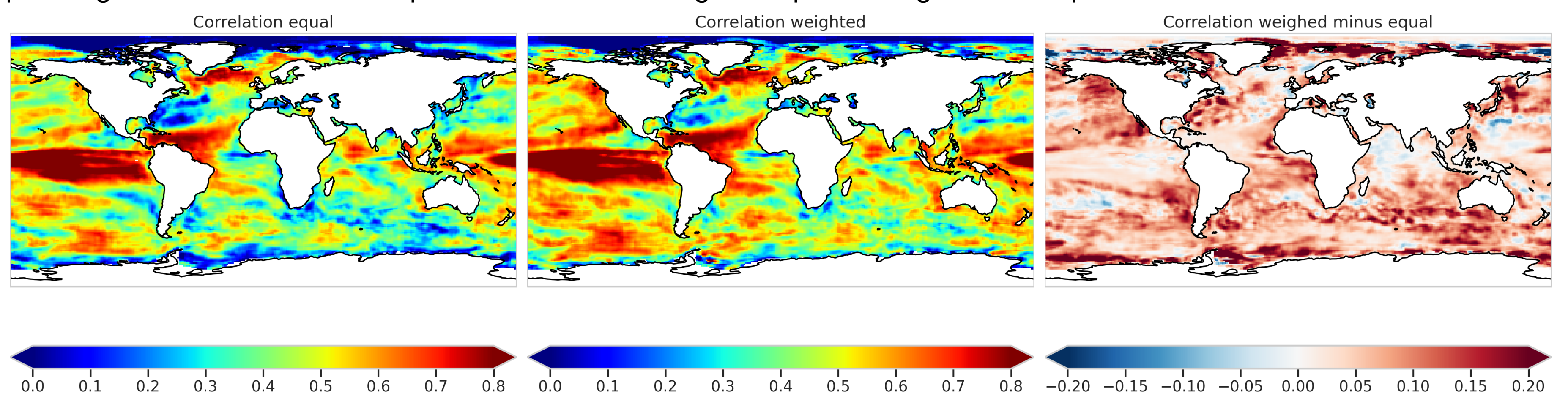
- The optimal weights hindcast has higher skill up to 2-3 leading months.

Figure: Global correlation space-wise for a two months leading time as a function of the date for the **optimal weights (OW)** and the **equal weights (EW)** hindcast. The filled circles (resp. unfilled) correspond to the dates where the differences are significant (resp. not significant).



- The improvement is found for the whole period.

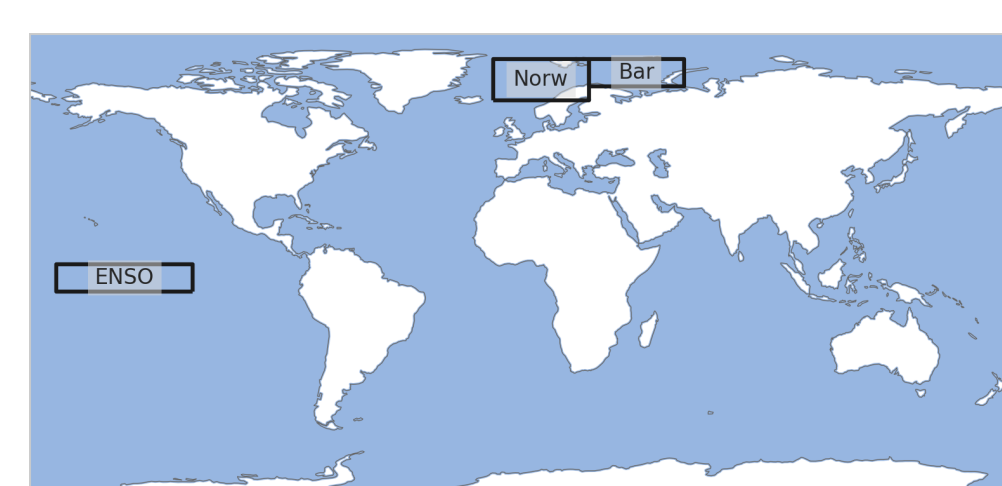
Figure: Time-wise correlation 1985-2010 at a two months leading time for the hindcast computed with equal weights (the baseline on the left panel) and with optimal weights (the center panel). The right panel shows the difference between the weighted hindcast correlation and the equal weights hindcast correlation, positive values indicating that optimal weights have improved the correlation.



- The improvement is consistent on the whole domain but has different magnitude depending on the region.

### Regional improvement

Correlation at a two month leading time globally and for different regions



Region	Correlation	
	OW	EW
Global	<b>0.50</b>	0.44
ENSO	<b>0.93</b>	0.90
Bar	<b>0.50</b>	0.33
Norw	0.63	0.63

- Some regions with low skill using EW have better skill with the optimal weights (OW), e.g. the Barents Sea.

### Conclusion

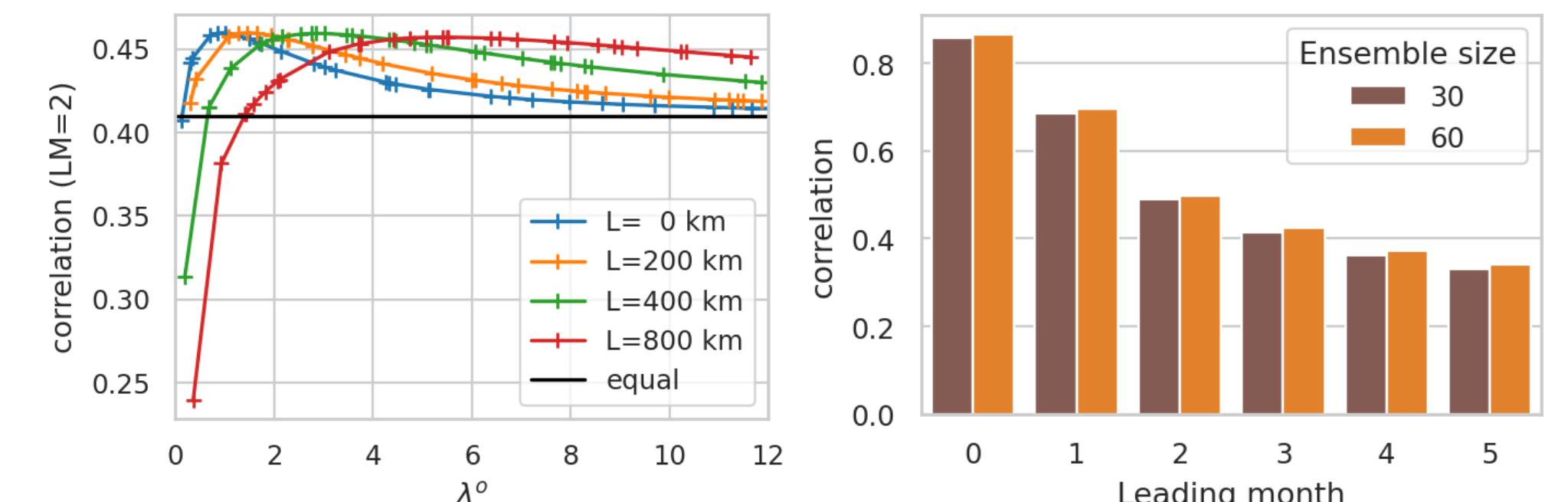
- Seamless update of the forecast between two operational forecast releases with the arrival of fresh data
- Improvement up to a 3 months lead-time
- No need to recompute the forecast
- Little sensitivity to the hyperparameter tuning

### Sensitivity

Two hyperparameters need to be tuned to determine the optimal weights:

- The localization radius  $L$ : represents the spatial radius around a grid point where the observations have an impact on the weight.
- The inflation factor  $\lambda^\circ$ : represents the confidence in the new observation.

The algorithm can also be sensitive to the size of the ensemble  $N$ .



- The algorithm is sensitive to each parameter individually but the increasing of the localization  $L$  can be compensated by the increasing of the inflation factor  $\lambda^\circ$  leading to similar skill.
- The algorithm skill is slightly degraded with a smaller ensemble.

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