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A Study of n-Refined Neutrosophic Complex Numbers and Their Properties

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Abstract: This paper is dedicated to define for the first time the concept of complex n-refined neutrosophic number as a direct application of n-refined neutrosophic sets and as a new generalization of neutrosophic complex numbers. Also, it presents some of their elementary properties such as, conjugates, absolute values, invertibility, and algebraic operations.

Keywords: Refined neutrosophic complex number, refined neutrosophic real number, invertible number

1. Introduction

Neutrosophy is a new branch of philosophy founded by Smarandache [6,36], to study the indeterminacy in the real world problems. It has a huge effect in many areas such as topology [7,27,29], equations [3,30], decision making [8], abstract algebra [25,26,39,41], and number theory [35]. As an extension of these concepts, Salama introduced the Neutrosophic crisp sets Theory as a generalization of crisp sets theory [53] and developed, inserted and formulated new concepts in the fields of mathematics, statistics, computer science and information systems through neutrosophics [54-59]. Neutrosophic algebraic studies began with the definitions of neutrosophic groups [9,17], and rings [13]. The neutrosophic rings and their generalizations such as refined neutrosophic rings [19], and n-refined neutrosophic rings [11,12], were very useful in the study of neutrosophic algebraic structures. Neutrosophic algebraic structures were defined basing on neutrosophic rings and fields, where we find many concepts from classical algebra were generalized into neutrosophic algebraic systems such as neutrosophic matrices over neutrosophic fields [1,42], refined neutrosophic matrices over refined neutrosophic fields [24], n-refined neutrosophic spaces over n-refined neutrosophic fields [21,32], linear modules and ideals [4,5,20,22]. Neutrosophic complex numbers were firstly studied in [43]. Recently, many of their properties were discussed in [44], especially their invertibility, absolute values, and complex functions. Also, they were generalized into refined neutrosophic complex numbers in [46].

Through this paper, we study n-refined neutrosophic complex numbers for the first time. On the other hand, we study many related properties of these numbers such as the invertibles, conjugates, and absolute values.

2. Neutrosophic complex number

Definition 2.1.Neutrosophic Real Number:

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where$0. I = 0$ and $I^n = I$ for all positive integers n.

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

Definition 2.2. Neutrosophic Complex Number:

Suppose that z is a neutrosophic complex number, then it takes the following standard form: $z = a + bI + i(c + dI)$ where a, b, c, d are real coefficients, and $I$is the indeterminacy element, where$i^2 = -1$ i.e.$i = \sqrt{-1}$.

We recall $a + bI$ the real part, then it takes the following standard form $Re(z) = a + bI$.

We recall $c + dI$ the imagine part, then it takes the following standard form $Im(z) = c + dI$.

For example:

$$z = 4 + I + i(2 + 2I)$$

Note: we can say that any real number can be considered a neutrosophic number.

For example: $z = 3 = 3 + 0. I + i(0 + 0. I)$

Definition 2.3. Division of neutrosophic real numbers:

Suppose that w_1, w_2 are two neutrosophic number,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

3. n-Refined neutrosophic complex numbers.

Definition 3.1.

We define a n- refine neutrosophiccomplex number by the following form:

$$z = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n) \qquad , \qquad \text{where}$$

$a_o, a_1, a_2, \ldots, a_n, b_o, b_1, b_2, \ldots, b_n$are real coeffients. For example:

$$z = (-1 + I_1 + 2I_2 - I_3) + i(1 - 2I_1 - I_2 + 2I_3)$$

We recall $a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n$ the real part, then it takes the following standard form $Re(z) = a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n$We recall $b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n$ the imagine part, then it takes the following standard form $Im(z) = b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n$.

Remark 3.1. An n-refined neutrosophic complex number can be defined as follows:

$$z = a + bI_1 + cI_2 + \cdots + fI_3, \text{ where } a, b, c, \ldots, f \text{are complex number. For example:}$$

$$z = (-1 + i) + iI_1 + (3 - 2i)I_2 - 2iI_3.$$

Remark 3.2.

$$I_n. I_n = I_n, \quad I_i. I_j = I_{\min (i,j)}.$$

Definition 3.2. The conjugate of a n-refined neutrosophiccomplex number:

Let $z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$ a refined neutrosophic complex number. We demote the conjugate of arefindneutrosophic complex number by $\bar{z}$ and define it by the following form:

$$\bar{z} = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) - i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$$

For example:

$z = (-1 - I_1 + I_2 + 4\boldsymbol{I_3}) + i(2 + I_1 + I_2 - \boldsymbol{I_3})$, Then $\bar{z} = (-1 - I_1 + I_2 + 4\boldsymbol{I_3}) - i(2 + I_1 + I_2 - \boldsymbol{I_3})$.

Definition 3.3. The absolute value of an n-refined neutrosophiccomplex number:

Suppose that $z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$ is a refined neutrosophic complex number. The absolute value of a z can be defined by the following form:

$$|z| = \sqrt{(a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n)^2 + (b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)^2}$$

Remark3.3.

$$(\mathbf{1}).\overline{(\bar{z})} = z.$$

Proof: Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})$, then

$$\bar{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) - i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n}).$$

Now.

$$\overline{(\bar{z})} = \overline{\left((a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) - \iota(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)\right)}$$
$$= (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n) = z$$

$$(\mathbf{2}).z + \bar{z} = \boldsymbol{2Re(z)}$$

Proof: Let$\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})$, then

$$\bar{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) - i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n}).$$

Now.

$$\boldsymbol{z} + \bar{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})$$
$$+ (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) - i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})$$

$$\boldsymbol{z} + \bar{z} = 2[(\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n})] = \boldsymbol{2Re(z)}$$

$$(3).z - \bar{z} = 2Im(z)$$

Proof: Let $\boldsymbol{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) + i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})$, then

$$\bar{z} = (\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) - i(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n}).$$

Now.

$$z - \bar{z} = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$$
$$- [(\boldsymbol{a_o} + \boldsymbol{a_1I_1} + \boldsymbol{a_2I_2} + \cdots + \boldsymbol{a_nI_n}) - \boldsymbol{i}(\boldsymbol{b_o} + \boldsymbol{b_1I_1} + \boldsymbol{b_2I_2} + \cdots + \boldsymbol{b_nI_n})]$$
$$z - \bar{z} = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$$
$$- (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$$

$$z - \bar{z} = 2i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n) = 2Im(z)$$

$$(4). \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

Proof:

Let $z_1 = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)$,

$$z_2 = (c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n) + i(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n).$$

Now.

$$z_1 + z_2 = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2 + \cdots + (a_n + c_n)I_n]$$
$$+ i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2 + \cdots + (b_n + d_n)I_n]$$

Then.

$$\overline{z_1 + z_2} = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2 + \cdots + (a_n + c_n)I_n]$$
$$- i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2 + \cdots + (b_n + d_n)I_n]$$

$$\overline{z_1 + z_2} = [(a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) - i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)]$$
$$+ [(c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n) - i(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n)] = \overline{z_1} + \overline{z_2}$$

Definition 3.4. The multiplication of two n-refined neutrosophicreal numbers:

Let $w_1 = a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n$, $w_2 = b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n$ are two n-refined neutrosophic real number, and we put:

$N_0 = a_0$, $N_j = a_0 + a_j + a_{j+1} + \cdots + a_n$; $1 \leq j \leq n$

$M_0 = b_0$, $M_j = b_0 + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n$; $1 \leq j \leq n$

Then a product $w_1 . w_2$ is defined by form:

$$w_1 . w_2 = N_0 M_0 + \sum_{i=1}^{n}[N_i M_i - N_{i+1} M_{i+1}] I_i + [N_n M_n - N_0 M_0] I_n$$

Definition 3.5. The multiplication of two n-refined neutrosophic complex numbers:

Let $z_1 = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n) + i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)$,

$$z_2 = (c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n) + i(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n).$$

A product $z_1 . z_2$ is defined by form:

$$z_1 . z_2 = [(a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n)$$
$$+ i(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)][(c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n)$$
$$+ i(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n)]$$

$$z_1 . z_2 = (a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n)(c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n)$$
$$- (b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n)$$
$$+ i[(a_o + a_1 I_1 + a_2 I_2 + \cdots + a_n I_n)(d_o + d_1 I_1 + d_2 I_2 + \cdots + d_n I_n)$$
$$+ (c_o + c_1 I_1 + c_2 I_2 + \cdots + c_n I_n)(b_o + b_1 I_1 + b_2 I_2 + \cdots + b_n I_n)]$$

By using Definition 3.4, we get the product $z_1.z_2$.

Remark3.1.

$$(1).\overline{z_1.z_2} = \overline{z_1}.\overline{z_2}$$

$$(1).z.\bar{z} = |z|^2.$$

Definition 3.6. Let $w = a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n$ be a n-refinedneutrosophicreal number, then the invertible of w defined as follows:

$$w^{-1} = \frac{1}{w} = \frac{1}{a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n} = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n)^{-1}$$

$\frac{1}{w} = (a_0)^{-1} + \sum_{i=1}^{n}[M_i - M_{i+1}]^{-1}I_i + [(M_n)^{-1} - (M_0)^{-1}]I_n$, where:

$$M_0 = a_0, M_j = a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n \; ; \; 1 \le j \le n$$

Definition 3.7. The invertible a n-refined neutrosophic complex number.

Let $z = (a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$, then the invertible of z defined as follows:

$$z^{-1} = \frac{1}{z} = \frac{1}{(a_o + a_1I_1 + a_2I_2 + \cdots + a_nI_n) + i(b_o + b_1I_1 + b_2I_2 + \cdots + b_nI_n)} = \frac{\bar{z}}{z.\bar{z}} = \frac{\bar{z}}{|z|^2}$$

Example3.1. Let $z = (1 + I_1 - I_2 + 2I_3) + i(2 + 2I_1 - I_2 - I_3)$, $\bar{z} = (1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)$.

then.

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)}{(1 + I_1 - I_2 + 2I_3)^2 + (2 + 2I_1 - I_2 - I_3)^2} = \frac{(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)}{5 + 9I_1 + 2I_2 + 7I_3}$$

$$z^{-1} = [(1 + I_1 - I_2 + 2I_3) - i(2 + 2I_1 - I_2 - I_3)](5 + 9I_1 + 2I_2 + 7I_3)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2 + 2I_3)(5 + 9I_1 + 2I_2 + 7I_3)^{-1} - i(2 + 2I_1 - I_2 - I_3)(5 + 9I_1 + 2I_2 + 7I_3)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2 + 2I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) - i(2 + 2I_1 - I_2 - I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right)$$

By using the Definition 3.4 we get:

$$(1 + I_1 - I_2 + 2I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) = \frac{1}{5} + \frac{55}{42}I_1 - \frac{44}{210}I_2 + \frac{29}{35}I_3$$

$$(2 + 2I_1 - I_2 - I_3)\left(\frac{1}{5} + \frac{1}{5}I_1 + \frac{1}{6}I_2 + \frac{1}{7}I_3\right) = \frac{2}{5} + \frac{298}{5}I_1 - \frac{12}{35}I_2 - \frac{2}{35}I_3$$

Hence,

$$z^{-1} = \left(\frac{1}{5} + \frac{55}{42}I_1 - \frac{44}{210}I_2 + \frac{29}{35}I_3\right) - i\left(\frac{2}{5} + \frac{298}{5}I_1 - \frac{12}{35}I_2 - \frac{2}{35}I_3\right).$$

The condition of invertibility can be found in [41].

5. Conclusions

In this paper, we have defined for the first time the concept of n-refined neutrosophic complex numbers. Also, we have discussed some of their elementary properties such as the conjugate, the multiplication, absolute values and other related topics.

As a future research direction, we aim to study the natural generalization of those numbers by n-cyclic refined neutrosophic complex numbers.

Conflicts of Interest: Authors declare that there is no conflict of interest.

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