

Analytical Predictions on Flexural Strengthening of Reinforced Concrete Beams with Hybrid FRP Laminate



K. Pradeeba, A.Rajasekaran

Abstract- This paper presents the predicted regression equation for the study parameters of Reinforced Concrete (RC) beams strengthened with Hybrid Fibre Reinforced Polymer (HyFRP) laminate at the soffit of beam. To study the effectiveness of HyFRP laminate on flexural strengthening a total of five beams were cast and tested. The variable parameters are thickness, elastic modulus and tensile strength of HyFRP laminates. Four combinations of HyFRP laminates precisely, 90% Glass fibre + 10% Basalt fibre of thickness, 80% Glass fibre + 20% Basalt fibre, 70% Glass fibre + 30% Basalt fibre, 60% Glass fibre + 40% Basalt fibre, and their corresponding thickness were 2.78, 3.24, 3.86 and 4.24mm respectively. The test results concluded that reinforced concrete beams strengthened with 70%Glass + 30%Basalt HyFRP laminate enhance the ultimate load carrying capacity of 68.97% with respect to control beam. The values reached through the predicted regression equation showed equitable accuracy with those of experimental values.

Keywords- Basalt, glass, HyFRP, laminate, regression.

I. INTRODUCTION

The concrete structural elements exhibits cipher of distress and deterioration at an premature age of construction due to variety of reasons, leading to repair works. Strengthening systems improve the resistance of an existing structure to internal forces either in a passive or active manner. Passive strengthening systems are typically engaged only with additional loads, beyond those existing at the time of installation are applied to the structure. Bonding steel plates or Fibre Reinforced Polymer (FRP) composites on the structural members are examples of passive strengthening systems. Active strengthening systems typically engage the structure instantaneously and may be accomplished by introducing external forces to the member that counteract the effects of internal forces. Instance of this may embrace the use of external post-tensioning systems or by jacking the member to relieve or transfer the existing load. Whether passive or active, the main challenge is to achieve composite performance between the existing structure and the new strengthening concrete structural elements.

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One of the most common strengthening procedures was external bonding of steel plates to concrete structural members. Disadvantage of using steel is the difficulty in handling and manipulation of steel plates at site. Fibre Reinforced Polymer (FRP) is comparatively an advent technology in advanced composite materials. FRP exhibit several improved properties like high strength-weight ratio, high stiffness-weight ratio, flexibility, high fatigue potential, ease of handling and transportation, very low maintenance cost and corrosion free. Nowadays the application of FRP has been used in almost all fields. With this background the present study was taken to reveal the effectiveness of hybrid FRP on flexural strengthening of RC beams. Multiple linear regression analysis has been carried out to understand the flexural response of HyFRP strengthened reinforced concrete beams. Attariet *et al.*, (2012) conducted an experimental study to enumerate the effectiveness of hybrid FRP with combination of glass fibre and carbon in reinforced concrete beams. The authors reported the test results of seven beams and concluded that double layered glass-carbon FRP fabric strengthening system showed good results on ultimate load carrying capacity, stiffness and ductility. Hawileh *et al.*, (2014) examined the performance of Reinforced Concrete (RC) beams strengthened means of different combinations of externally bonded hybrid Glass and Carbon Fiber Reinforced Polymer (GFRP/CFRP) sheets at the soffit of beams. All the beams were tested under four-point bending until failure. All the beams failed under flexure mode. The ultimate enhancement of load when compared with control beam was ranged from 30% to 90%. The authors proposed regression equation to predict the performance parameters and the findings of those equation are well agreed with those of experimental results. The committee report ACI 440.2R encompassed guidelines for the strengthening of structural elements in in flexure, shear, axial tension and axial compression. Chellapandian *et al.*, (2019) evaluated the efficiency of hybrid fiber reinforced polymer (FRP) plating on the complete behavioral enhancement of reinforced concrete (RC) beams under flexure. A total of 8 RC beams were cast and strengthened using different FRP techniques including (i) near surface mounting (NSM), (ii) external bonding (EB) and (iii) hybrid strengthening using a combination of NSM carbon FRP laminates and EB CFRP fabric. The test specimens were tested under two-point loading until failure.

The findings concluded that enhancement in load carrying capacity was 85% which was achieved by Near Surface Mounted (NSM) technique. The predicted values through Finite Element Modeling also showed good accuracy with the experimental results. Ibrahim *et al.*, (2020) examined the efficacy of near surface mounted hybrid carbon/glass fibre reinforced polymer (FRP) strips for strengthening of shear-deficient reinforced-concrete (RC) rectangular deep beams. The findings revealed that the NSM-FRP increased the beam shear strength up to 55.8%. The main objective of this study was to propose Regression equations for predicting the performance characteristics of externally bonded HyFRP Reinforced Concrete Beams.

II. EXPERIMENTAL PROGRAMME

A total of 5 beams were cast and tested for the present research work. The beams were 150 mm × 250 mm in cross-section and 3000 mm long. Longitudinal steel ratio adopted for the beam specimens were 0.603%. All beams were casted with M20 grade concrete and tested in four-point bending over a simple span of 2800 mm. The HYSD bars of characteristic strength 545MPa were used for the longitudinal reinforcement. The internal links consisted of mild steel bars of yield strength 310MPa. The specimens were provided with 8mm diameter stirrups at 150 mm spacing. Of the above five beams, one beam served as reference specimen without any strengthening, four beams were strengthened with HyFRP laminates on their soffit. Four combinations of HyFRP laminates were considered in this research, precisely 90% Glass fibre + 10% Basalt fibre of thickness, 80% Glass fibre + 20% Basalt fibre, 70% Glass fibre + 30% Basalt fibre, 60% Glass fibre + 40% Basalt fibre, and 2.78, 3.24, 3.86 and 4.24mm respectively. Measurement on strength, deformation and failure mode of HyFRP laminated as well as the reference beam. The details of test specimens are presented in Table 1. The properties of HyFRP used in this research study are presented in Table 2.

Table 1 Details of Test Beams

Sl. No.	Beam Designation	% Steel Reinforcement	Spacing of Stirrups (mm)	% Combination of Fibre	HYFRP Thickness
1.	RB1	0.603	150	0	0
2.	HyGB1	0.603	150	90%G+10%B	2.78
3.	HyGB2	0.603	150	80%G+20%B	3.24
4.	HyGB3	0.603	150	70%G+30%B	3.86
5.	HyGB4	0.603	150	60%G+40%B	4.24

Table 2 Characteristic Properties of HyFRP laminates

Sl. No.	Type of Fibre in HyFRP	Thickness (mm)	Tensile Strength (MPa)	Elasticity Modulus (GPa)	Composite Ratio
1.	HyGB1	2.78	342.53	24.6	
2.	HyGB2	3.24	372.44	26.3	1.84
3.	HyGB3	3.86	392.38	27.9	2.15
4.	HyGB4	4.24	432.61	29.1	2.81

The beams strengthened with HyFRP and un-strengthened beams were tested under four point-bending in a loading frame of 500kN capacity. The beams were supported on hinge at one end and roller at the other end. The deflection measurement upto ultimate stage was accomplished using a specially designed mechanical dial gauge. The crack width was measured and monitored using crack detection microscope with a least count of 0.02mm accuracy. Crack development and propagation was monitored during the process of testing. The loading was continued until failure and all the measurements were taken at all stages of loading. The detail of the associated instrumentation for testing is presented in Fig. 1.



Fig. 1 Experimental Test Set-up

III. MULTI LINEAR REGRESSION ANALYSIS

Regression analysis is a statistical tool for the exploration of relationships between the variables. Regression analysis with a single explanatory variable is termed simple regression. Multiple linear regression is a method that allows supplementary factors to enter the analysis distinctly. It is valuable for quantifying the impact of various simultaneous influences upon a single dependent variable. Regression analysis is a procedure for relating known input variables and output parameter using statistical ideologies. The general regression technique is to assume a form of relationship for the input parameters and the results, with a number of unknown coefficients. The unknown coefficients are found out using the data available from experiments or other sources using the Legendre's principle of least squared errors. The mathematical technique used for fitting curves, whether linear or non-linear of the predetermined shape. The purpose of regression is to evaluate the unknown coefficients in an equation.

Regression coefficient is an unknown parameter presented into the equation, to modify the combination of input variables. On solving the regression problem using the code of least squared errors, all the regression coefficients are estimated. Legendre's principle of least squared errors attempts to solve the problem of regression using the condition that the square of the difference between the actual value and the value predicted by the equation should be a minimum. This is accomplished by finding the derivative of the square of the error with reference to each one of the unknown coefficients in the assumed equation. Root Mean Squared Error (RMSE) is the square root of the Mean Squared Error (MSE). This indicates the extent of deviation from the expected value to the higher side or to the lower side. Hence, the RMSE is a better measure of error when compared to the MSE. Multivariate linear regression helps to construct first order equations involving more than one independent variable. The basic formulation for multivariate linear regression is,

$$\begin{pmatrix} \frac{\partial}{\partial a_0} \\ \frac{\partial}{\partial a_1} \\ \frac{\partial}{\partial a_2} \\ \frac{\partial}{\partial a_3} \\ \vdots \\ \frac{\partial}{\partial a_n} \end{pmatrix} \sum_{i=1}^K (P_i - (a_0 + a_1 x_{1i} + a_2 x_{2i} + a_3 x_{3i} + \dots + a_n x_{ni})) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (1)$$

where, $a_0 \dots a_n$ are the coefficients to be determined, $x_1 \dots x_n$ are the independent variables, P is the dependent variable or the actual result value for the set of i^{th} input data and K is the number data sets available for regression. The above Equation 1 reduces to the following equation which can be solved by summing up the values of independent and dependent variables after carrying out the required operations.

$$\sum_{i=1}^K \begin{bmatrix} 1 & x_{1i} & x_{2i} & x_{3i} & \dots & x_{ni} \\ x_{1i} & x_{1i}^2 & x_{1i}x_{2i} & x_{1i}x_{3i} & \dots & x_{1i}x_{ni} \\ x_{2i} & x_{2i}x_{1i} & x_{2i}^2 & x_{2i}x_{3i} & \dots & x_{2i}x_{ni} \\ x_{3i} & x_{3i}x_{1i} & x_{3i}x_{2i} & x_{3i}^2 & \dots & x_{3i}x_{ni} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{ni} & x_{ni}x_{1i} & x_{ni}x_{2i} & x_{ni}x_{3i} & \dots & x_{ni}^2 \end{bmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} = \sum_{i=1}^K \begin{pmatrix} P_i \\ P_i x_1 \\ P_i x_2 \\ P_i x_3 \\ \vdots \\ P_i x_n \end{pmatrix} \quad (2)$$

The data used for the regression analysis to predict the regression equation is given in Table 3 and the predicted regression equations are presented in Table 4. In this analysis the independent variables are thickness of HyFRP, elastic Modulus of HyFRP and tensile strength of HyFRP. The dependent variables considered are load and deflection at various stages, deflection ductility, deflection ductility ratio, energy ductility, energy ductility ratio and maximum crack width.

Table 3 Data Used for the Regression Analysis

Designation	First Crack Load (kN)	First Crack deflection (mm)	Yield Load (kN)	Yield deflection (mm)	Service Load (kN)	Service deflection (mm)	Ultimate Load (kN)	Ultimate deflection (mm)	Deflection Ductility	Deflection Ductility Ratio	Energy Ductility	Energy Ductility Ratio	Maximum Crack width
RB1	22.03	1.86	49.05	6.71	46.09	6.03	69.14	34.29	5.11	1	6.53	1	1.22

1HyG _{R1}	26.97	3.85	53.95	8.15	57.62	9.50	86.43	50.61	6.20	1.21	9.15	1.40	1.14
1HyG ₂	29.43	2.31	66.21	8.46	69.42	11.61	104.14	53.29	6.29	1.23	10.83	1.65	0.98
1HyG _{R3}	34.33	2.08	95.64	8.77	77.88	6.93	116.83	56.48	6.44	1.26	11.78	1.80	0.88
1HyG ₄	29.43	3.56	56.40	9.38	53.66	8.33	80.49	48.67	5.18	1.01	8.02	1.22	1.12

IV. RESULTS OF REGRESSION ANALYSIS

The results of regression analysis in the form of equation for various study parameters related with HyFRPstrengthened reinforced concrete beams for flexural strengthening are presented in Table 4. The proposed regression equations are enabled to predict analytical results so as to compare with those experimental results. The predicted results indicated a reasonable accuracy.

Table 4 Proposed Regression Equations

Sl. No	Predictio n Parameter	Equation	Fitness	RMS E
1.	First Crack Load	$22.02 + 10.49t_{k_{frp}} - .32f_{y_{frp}} + 3.56E_{frp}$	0.9888	0.4229
2.	Deflection at First Crack Load	$1.88 - 2.44t_{k_{frp}} + 0.08f_{y_{frp}} - 0.80E_{frp}$	0.6445	0.4843
3.	Yield Load	$48.96 + 65.80t_{k_{frp}} - 2.34f_{y_{frp}} + 25.54E_{frp}$	0.9712	2.8253
4.	Deflection at Yield Load	$6.71 + 0.53t_{k_{frp}} + 0.02f_{y_{frp}} - 0.24E_{frp}$	0.9997	0.0126
5.	Service Load	$45.97 + 26.74t_{k_{frp}} - 1.31f_{y_{frp}} + 15.83E_{frp}$	0.8877	3.8002
6.	Deflection at Service Load	$6.00 - 6.65t_{k_{frp}} + 0.13f_{y_{frp}} - 0.84E_{frp}$	0.7873	0.9052
7.	Ultimate Load	$68.96 + 40.12t_{k_{frp}} - 1.96f_{y_{frp}} + 23.75E_{frp}$	0.8876	5.7037
8.	Deflection at Ultimate Load	$34.26 + 6.05t_{k_{frp}} - 0.42f_{y_{frp}} + 5.92E_{frp}$	0.9886	0.8151
9.	Deflection Ductility	$5.11 + 0.28t_{k_{frp}} - 0.06f_{y_{frp}} + 0.83E_{frp}$	0.9723	0.0958
10.	Deflection Ductility Ratio	$1.00 + 0.06t_{k_{frp}} - 0.01f_{y_{frp}} + 0.16E_{frp}$	0.9700	0.0195
11.	Energy Ductility	$6.51 + 3.23t_{k_{frp}} - 0.19f_{y_{frp}} + 2.49E_{frp}$	0.9034	0.5865
12.	Energy Ductility Ratio	$1.00 + 0.50t_{k_{frp}} - 0.03f_{y_{frp}} + 0.38E_{frp}$	0.9067	0.0878
13.	Maximum Crack Width	$1.22 - 0.34t_{k_{frp}} + 0.01f_{y_{frp}} - 0.16E_{frp}$	0.8520	0.0468

Analytical Predictions on Flexural Strengthening of Reinforced Concrete Beams with Hybrid FRP Laminate

Note: t_{frp} - Thickness of HyFRP; E_{frp} - Elastic Modulus of HyFRP;
 f_{yfrp} - Tensile strength of HyFRP

The proposed regression predictions were compared with the experimental test results and depicted through Figs. 2 to 14 exhibit well agreed.

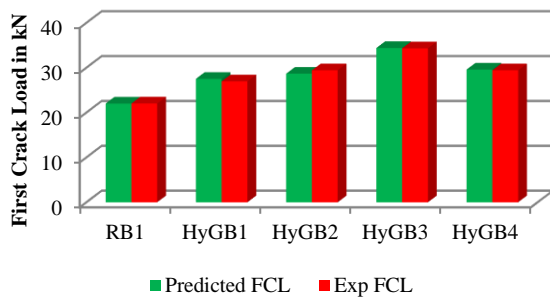


Fig. 2 Experimental vs Predicted results for First Crack Load

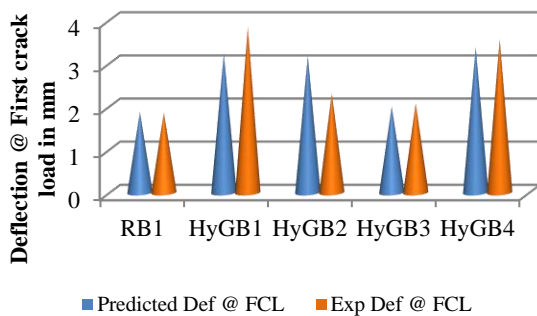


Fig. 3 Experimental vs Predicted results for Deflection at First Crack Load

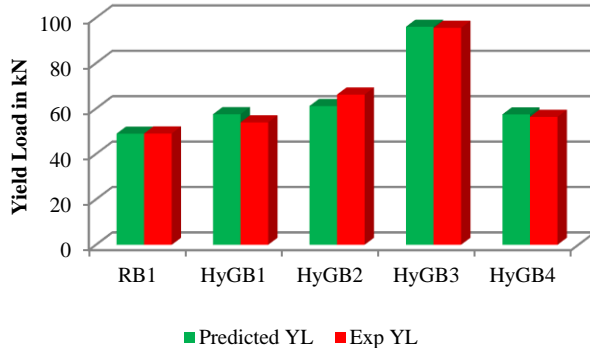


Fig. 4 Experimental vs Predicted results for Yield Load

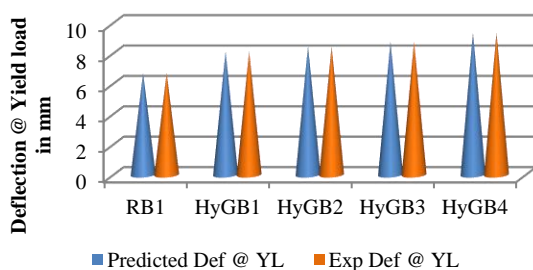


Fig. 5 Experimental vs Predicted results for Deflection at Yield Load

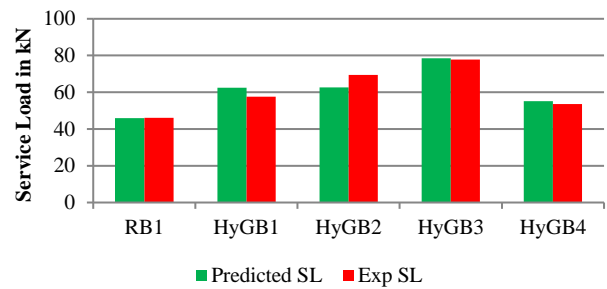


Fig. 6 Experimental vs Predicted results for Service Load

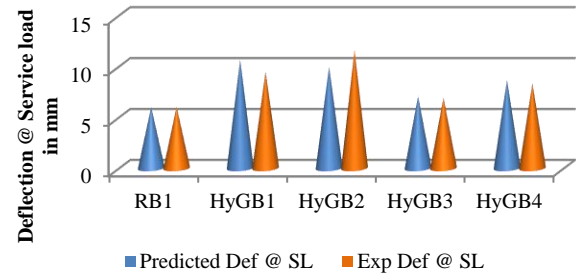


Fig. 7 Experimental vs Predicted results for Deflection at Service Load

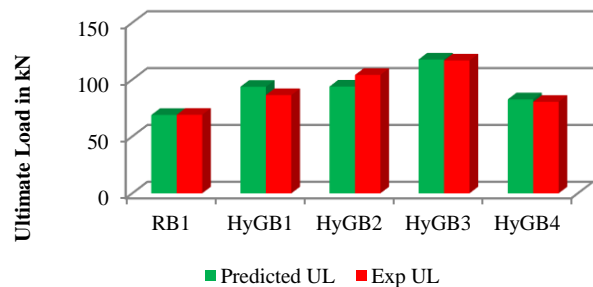


Fig. 8 Experimental vs Predicted results for Ultimate Load

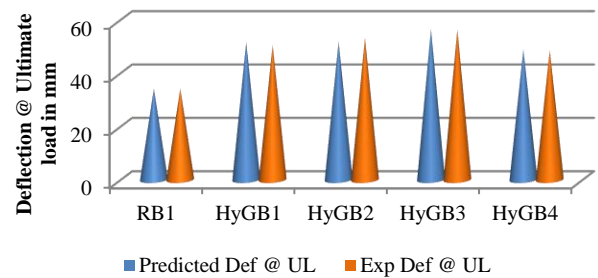


Fig. 9 Experimental vs Predicted results for Deflection at Ultimate Load

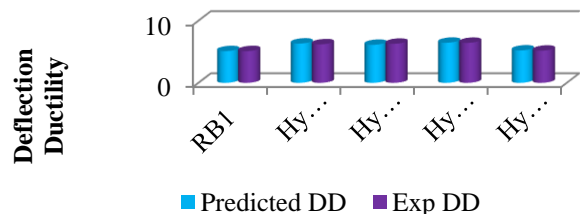


Fig. 10 Experimental vs Predicted results for Deflection Ductility

V. CONCLUSIONS

An observation of the measures of fitness of regression shows that the multivariate linear regression can estimate the prediction values with reasonable levels of accuracy for all study parameters of reinforced concrete beams strengthened with HyFRP laminates. Multiple linear regression proved as a best statistical tool to study the various parameters for the flexural performance of HyFRP laminated RC beams. The analysis results inferred that the overall significance of the regression model was best fitted. The root mean square error values varied from 0.012 to 5.703. The errors were within a sensible limit and hence the prediction of regression model was well agreed with those of experimental results.

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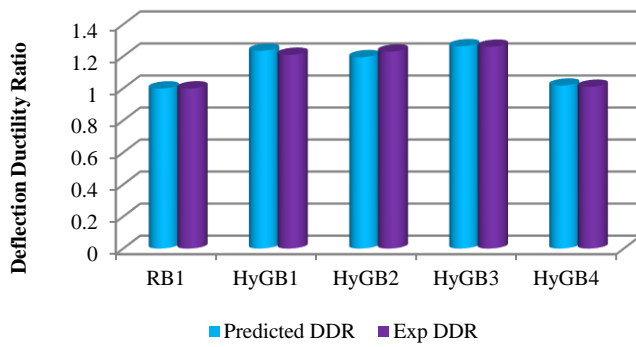


Fig. 11 Experimental vs Predicted results for Deflection Ductility Ratio

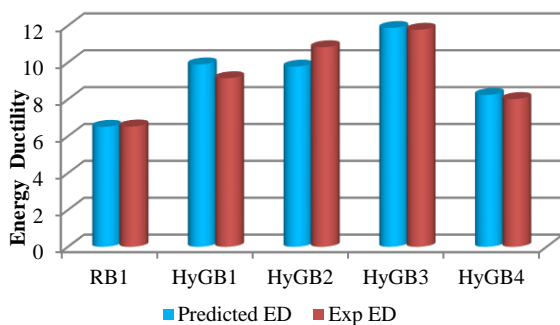


Fig. 12 Experimental vs Predicted results for Energy Ductility

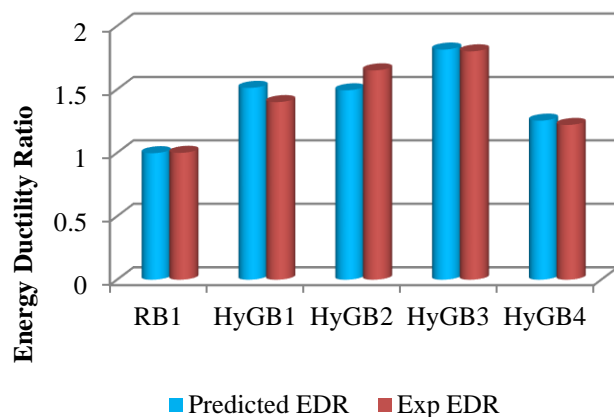


Fig. 13 Experimental vs Predicted results for Energy Ductility Ratio

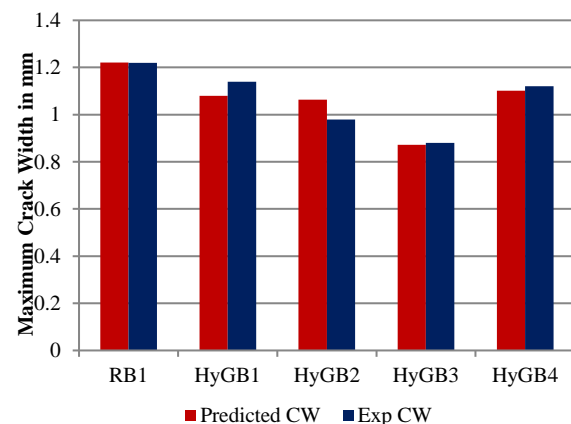


Fig. 12 Experimental vs Predicted results for Maximum Crack Width

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