



# Introduction to Plithogenic Logic

Florentin Smarandache<sup>1</sup>

<sup>1</sup> Mathematics, Physics and Natural Sciences Division, University of New Mexico, Gallup Campus, NM 87301, USA;  
smarand@unm.edu

**Abstract:** A Plithogenic Logical proposition  $P$  is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize  $P$ . Each degree of truth-value may be classical, fuzzy, intuitionistic fuzzy, neutrosophic, or other fuzzy extension type logic. At the end, a cumulative truth of  $P$  is computed.

**Keywords:** neutrosophic logic, plithogenic logic, plithogenic multi-variate analysis, cumulative truth, plithogenic logic application

---

## 1. Introduction

We recall the Plithogenic Logic and explain it in detail by showing a practical application.

A Plithogenic Logical proposition  $P$  is a proposition that is characterized by many degrees of truth-values with respect to many corresponding attribute-values (or random variables) that characterize  $P$ .

It is a pluri-logic.

We denote it by  $P(V_1, V_2, \dots, V_n)$ , for  $n \geq 1$ , where  $V_1, V_2, \dots, V_n$  are the random variables that determine, each of them in some degree, the truth-value of  $P$ .

The variables may be independent one by one, or may have some degree of dependence among some of them. The degrees of independence and dependence of variables determine the plithogenic logic conjunctive operator to be used in the computing of the cumulative truth of  $P$ .

The random variables may be: classical, fuzzy, intuitionistic fuzzy, indeterminate, neutrosophic, and other types of fuzzy extensions.

$P(V_1) = t_1$  or the truth-value of the proposition  $P$  with respect to the random variable  $V_1$ .

$P(V_2) = t_2$  or the truth-value of the proposition  $P$  with respect to the random variable  $V_2$ .

...

And so on,  $P(V_n) = t_n$  or the truth-value of the proposition  $P$  with respect to the random variable  $V_n$ .

The variables  $V_1, V_2, \dots, V_n$  are described by various types of probability distributions,  $P(V_1), P(V_2), \dots, P(V_n)$ . The whole proposition  $P$  is, therefore, characterized by  $n$  probability distributions, or  $n$  sub-truth-values. By combining all of them, we get a *cumulative truth-value* of the logical proposition  $P$ .

Plithogenic Logic / Set / Probability and Statistics were introduced by Smarandache [1] in 2017 and he further on (2018 -2020) developed them [2-6].

They were applied in many fields by various authors [7-27].

The Plithogenic MultiVariate Analysis used in the Set theory, Probability, and Statistics is now used in Logic, giving birth to the Plithogenic Logic.

Plithogenic MultiVariate Analysis is a generalization of the classical MultiVariate Analysis.

## 2. Classification of the Plithogenic Logics

Depending on the real-values of  $t_1, t_2, \dots, t_n$ , we have:

### 2.1. Plithogenic Boolean (or Classical) Logic

It occurs when the degrees of truths  $t_1, t_2, \dots, t_n \in \{0,1\}$ , where 0 = false, and 1 = true.

### 2.2. Plithogenic Fuzzy Logic

When the degrees of truths  $t_1, t_2, \dots, t_n$  are included in  $[0,1]$ , and at least one of them is included in  $(0, 1)$ , in order to distinguish it from the previous Plithogenic Boolean Logic.

Herein, we have:

2.2.1. Single-Valued Plithogenic Fuzzy Logic, if the degrees of truths  $t_1, t_2, \dots, t_n$  are single (crisp) numbers in  $[0, 1]$ .

2.2.2. Subset-Valued (such as Interval-Valued, Hesitant-Valued, etc.) Plithogenic Fuzzy Logic, when the degrees of truths  $t_1, t_2, \dots, t_n$  are subsets (intervals, hesitant subsets, etc.) of  $[0, 1]$ .

### 2.3. Plithogenic Intuitionistic Fuzzy Logic

When  $P(V_j) = (t_j, f_j)$ , when  $t_j, f_j$  are included in  $[0,1]$ ,  $1 \leq j \leq n$ , where  $t_j$  is the degree of truth, and  $f_j$  is the degree of falsehood of the proposition  $P$ , with respect to the variable  $V_j$ .

In the same way, we have:

2.3.1. Single-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are single-valued (crisp) numbers in  $[0, 1]$ .

2.3.2. Subset-Valued Plithogenic Intuitionistic Fuzzy Logic, when all degrees of truths and falsehoods are subset-values included in  $[0, 1]$ .

### 2.4. Plithogenic Indeterminate Logic

When the probability distributions of the random variables  $V_1, V_2, \dots, V_n$  are indeterminate (neutrosophic) functions, i.e. functions with vague or unclear arguments and/or values.

### 2.5. Plithogenic Neutrosophic Logic

When  $P(V_j) = (t_j, i_j, f_j)$ , with  $t_j, i_j, f_j$  included in  $[0,1]$ ,  $1 \leq j \leq n$ , where  $t_j, i_j, f_j$  are the degrees of truth, indeterminacy, and falsehood respectively of the proposition  $P$  with respect to the random variable  $V_j$ .

Similarly, we have:

2.5.1. Single-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are single-valued (crisp) numbers in  $[0, 1]$ .

2.5.2. Subset-Valued Plithogenic Neutrosophic Logic, when all degrees of truths, indeterminacies, and falsehoods are subset-values included in  $[0, 1]$ .

### 2.6. Plithogenic (other fuzzy extensions) Logic

Where other fuzzy extensions are, as of today: Pythagorean Fuzzy, Picture Fuzzy, Fermatean Fuzzy, Spherical Fuzzy, q-Rung Orthopair Fuzzy, Refined Neutrosophic Logic, and refined any other fuzzy-extension logic, etc.

### 2.7. Plithogenic Hybrid Logic

When  $P(V_1), P(V_2), \dots, P(V_n)$  are mixed types of the above probability distributions.

## 3. Applications

### 3.1. Pluri-Truth Variables

In our everyday life, we rarely have a “simple truth”, we mostly deal with “complex truths”.

For example:

- You like somebody for something, but dislike him for another thing;
- Or, you like somebody for something in a degree, and for another thing in a different degree;
- Similarly, for hating somebody for something in a degree, and for another thing in a different degree.

An egalitarian society (or system) does not exist in fact in our real world. It is too rigid. The individuals are different, and act differently.

Therefore, in our world, we deal with a “*plitho-logic*” (*plitho* means, in Greek, *many, pluri-*) or “complex logic”. And this is best characterized by the *Plithogenic Logic*.

### 3.2. Types of Random Truth-Variables

The truth depends on many parameters (random variables), not only on a single one, and at the end we need to compute the *cumulative truth* (truth of all truths).

The random variables may be classical (with crisp/exact values), but often in our world they are vague, unclear, only partially known, with indeterminate data.

### 3.3. Weights of the Truth-Variables

Some truth may weight more than another truth.

For example, you may like somebody for something more than you dislike him/her for another thing.

Or the opposite, you may dislike somebody for something more than you like him/her for another thing.

### 3.4. Degrees of Subjectivity of the Truth Variables

In the soft sciences, such as: sociology, political science, psychology, linguistics, etc., or in the culture, literature, art, theatre, dance, there exists a significant *degree of subjectivity* in measuring the truth. *It is not beautiful what is beautiful, but it is beautiful what I like myself*, says a Romanian proverb.

## 4. Generalizations

Plithogenic Logic is a generalization of all previous logics: Boolean, Fuzzy, Intuitionistic Fuzzy, Neutrosophic Logic, and all other fuzzy-extension logics.

It is a MultiVariate Logic, whose truth variables may be in any type of the above logics.

## 5. Example

Let us consider an ordinary proposition  $P$ , defined as below:

$$P = \text{John loves his city}$$

and let's calculate its complex truth-value.

Of course, lots of attributes (truth-variables) may characterize a city (some of them unknown, other partially known, or approximately known). A complete spectrum of attributes to study is unreachable.

For the sake of simplicity, we consider the below five propositions as 100% independent two by two.

In this example we only chose a few variables  $V_j$ , for  $1 \leq j \leq 5$ :

$V_1$  : low / high percentage of COVID-19 virus infected inhabitants;

$V_2$  : nonviolent / violent;

$V_3$  : crowded / uncrowded;

$V_4$  : clean / dirty;

$V_5$  : quiet / noisy,

A more accurate representation of the proposition  $P$  is  $P(V_1, V_2, V_3, V_4, V_5)$ .

With respect to each variable  $V_j$ , the  $P(V_j)$  included in  $[0, 1]$  has, in general, different truth-values, for  $1 \leq j \leq 5$ .

Suppose John prefer his city to have (or to be): low percentage of COVID-19 infected inhabitants, non-violent, uncrowded, clean, and quiet.

$P(V_j)$  is the degree in which John loves the city with respect to the way the variable  $V_j$  characterizes it.

### 5.1. Plithogenic Boolean (Classical) Logic

$$P(V_1) = 1$$

$$P(V_2) = 0$$

$$P(V_3) = 1$$

$$P(V_4) = 0$$

$$P(V_5) = 1$$

Therefore  $P(V_1, V_2, V_3, V_4, V_5) = (1, 0, 1, 0, 1)$ , or John loves his city in the following ways:

- in a degree of 100% with respect to variable  $V_1$ ;
- in a degree of 0% with respect to variable  $V_2$ ;
- in a degree of 100% with respect to variable  $V_3$ ;
- in a degree of 0% with respect to variable  $V_4$ ;
- in a degree of 100% with respect to variable  $V_5$ .

The cumulative truth-value will be, in the classical way, the classical conjunction ( $\wedge_c$ ), where  $c$  stands for classical:

$$1 \wedge_c 0 \wedge_c 1 \wedge_c 0 \wedge_c 1 = 0,$$

or John likes his city in a cumulative classical degree of 0%!

The classical logic is rough, therefore more refined logics give a better accuracy, as follows.

### 5.2. Plithogenic Fuzzy Logic

The 100% or 0% truth-variables may not exactly fit John's preferences, but they may be close. For example:

$$P(V_1, V_2, V_3, V_4, V_5) = (0.95, 0.15, 0.80, 0.25, 0.85),$$

which means that John loves his city:

- in a degree of 95% with respect to variable  $V_1$ ;
- in a degree of 15% with respect to variable  $V_2$ ;
- in a degree of 80% with respect to variable  $V_3$ ;
- in a degree of 25% with respect to variable  $V_4$ ;
- in a degree of 85% with respect to variable  $V_5$ .

Using the fuzzy conjunction ( $\wedge_F$ ) min operator, we get:

$$0.95 \wedge_F 0.15 \wedge_F 0.80 \wedge_F 0.25 \wedge_F 0.85 = \min\{0.95, 0.15, 0.80, 0.25, 0.85\} = 0.15$$

or John likes his city in a cumulative fuzzy degree of 15%.

### 5.3. Plithogenic Intuitionistic Fuzzy Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ( (0.80, 0.20), (0.15, 0.70), (0.92, 0.05), (0.10, 0.75), (0.83, 0.07) ),$$

which means that John loves his city in a degree of 80%, and dislikes it a degree of 20%, and so on with respect to the other variables.

Using the intuitionistic fuzzy conjunction ( $\wedge_{IF}$ ) min/max operator in order to get the cumulative truth-value, one has:

$$\begin{aligned} & (0.80, 0.20) \wedge_{IF} (0.15, 0.70) \wedge_{IF} (0.92, 0.05) \wedge_{IF} (0.10, 0.75) \wedge_{IF} (0.83, 0.07) = \\ & = (\min\{0.80, 0.15, 0.92, 0.10, 0.83\}, \max\{0.20, 0.70, 0.05, 0.75, 0.07\}) = (0.10, 0.75), \end{aligned}$$

or John likes and dislikes his city in a cumulative intuitionistic fuzzy degree of 10%, and respectively 75%.

#### 5.4. Plithogenic Indeterminate Logic

$$P(V_1, V_2, V_3, V_4, V_5) = (0.80 \text{ or } 0.90, [0.10, 0.15], [0.60, \text{unknown}], > 0.13, 0.79),$$

which means that John loves his city:

- in a degree of 80% or 90% (he is not sure about) with respect to variable  $V_1$ ;
- in a degree between 10% or 15% with respect to variable  $V_2$ ;
- in a degree of 60% or greater with respect to variable  $V_3$ ;
- in a degree greater than 13% ( i.e. in the interval  $(0.13, 1]$  ) with respect to variable  $V_4$ ;
- in a degree of 79% with respect to variable  $V_5$ .

Therefore, the variables provide indeterminate (unclear, vague) values.

Applying the indeterminate conjunction ( $\wedge_I$ ) min operator, we get:

$$\min\{(0.80 \text{ or } 0.90), [0.10, 0.15], [0.60, \text{unknown}], (0.13, 1], 0.79\} = 0.10.$$

#### 5.5. Plithogenic Neutrosophic Logic

$$P(V_1, V_2, V_3, V_4, V_5) = ( (0.86, 0.12, 0.54), (0.18, 0.44, 0.72), (0.90, 0.05, 0.05), (0.09, 0.14, 0.82), (0.82, 0.09, 0.14) ),$$

which means that John loves the city in a degree of 86%, the degree of indeterminate love is 12%, and the degree of dislike is 54% with respect to the variable  $V_1$ , and similarly with respect to the other variables.

Again, using the neutrosophic conjunction ( $\wedge_N$ ) min/max/max operator in order to obtain the cumulative truth-value, one gets:

$$\begin{aligned} & (0.86, 0.12, 0.54) \wedge_N (0.18, 0.44, 0.72) \wedge_N (0.90, 0.05, 0.05) \wedge_N (0.09, 0.14, 0.82) \\ \wedge_N & (0.82, 0.09, 0.14) = (\min\{0.86, 0.18, 0.90, 0.09, 0.82\}, \max\{0.12, 0.44, 0.05, 0.14, 0.09\}, \\ & \max\{0.54, 0.72, 0.05, 0.82, 0.14\}) = (0.09, 0.44, 0.82), \end{aligned}$$

or John loves, is not sure (indeterminate), and dislikes his city with a cumulative neutrosophic degree of 9%, 44%, and 82% respectively.

## 5. Future Research

To construct the *plithogenic aggregation operators* (such as: intersection, union, negation, implication, etc.) of the variables  $V_1, V_2, \dots, V_n$  all together (cumulative aggregation), in the cases when variables  $V_i$  and  $V_j$  have some degree of dependence  $d_{ij}$  and degree of independence  $1 - d_{ij}$ , with  $d_{ij} \in [0, 1]$ , for all  $i, j \in \{1, 2, \dots, n\}$ , and  $n \geq 2$ .

## 6. Conclusions

We showed in this paper that the Plithogenic Logic is the largest possible logic of today. Since we live in a world full of indeterminacy and conflicting data, we have to deal, instead of a *simple truth* with a *complex truth*, where the last one is a cumulative truth resulted from the plithogenic aggregation of many truth-value random variables that characterize an item (or event).

**Funding:** This research received no external funding.

**Conflicts of Interest:** The author declares no conflict of interest.

## References

1. Florentin Smarandache: Plithogeny, Plithogenic Set, Logic, Probability, and Statistics. Brussels, Belgium: Pons, 2017, 141 p.; <http://fs.unm.edu/Plithogeny.pdf>
2. Florentin Smarandache: Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets - Revisited. *Neutrosophic Sets and Systems*, Vol. 21, 2018, 153-166; <http://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf>
3. Florentin Smarandache (Special Issue Editor): New types of Neutrosophic Set/Logic/Probability, Neutrosophic Over-/Under-/Off-Set, Neutrosophic Refined Set, and their Extension to Plithogenic Set/Logic/Probability, with Applications. Special Issue of *Symmetry* (Basel, Switzerland, in ScopuS, IF: 1.256), November 2019, 714 p.; [https://www.mdpi.com/journal/symmetry/special\\_issues/Neutrosophic\\_Set\\_Logic\\_Probability](https://www.mdpi.com/journal/symmetry/special_issues/Neutrosophic_Set_Logic_Probability)
4. Florentin Smarandache, Mohamed Abdel-Basset (editors): Optimization Theory Based on Neutrosophic and Plithogenic Sets, ELSEVIER, Academic Press, 2020, 446 p.; <https://www.elsevier.com/books/optimization-theory-based-on-neutrosophic-and-plithogenic-sets/smarandache/978-0-12-819670-0>
5. Florentin Smarandache: Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. *Neutrosophic Sets and Systems*, Vol. 22, 2018, 168-170; <http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.pdf>
6. Florentin Smarandache: Conjunto plitogenico, una extension de los conjuntos crisp, difusos, conjuntos difusos intuicionistas y neutrosoficos revisitado. *Neutrosophic Computing and Machine Learning*, Vol. 3, 2018, 1-19.
7. Shazia Rana, Madiha Qayyum, Muhammad Saeed, Florentin Smarandache, Bakhtawar Ali Khan: Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique. *Neutrosophic Sets and Systems*, Vol. 28, 2019, 34-50.
8. Nivetha Martin, Florentin Smarandache: Plithogenic Cognitive Maps in Decision Making. *International Journal of Neutrosophic Science (IJNS)* Vol. 9, No. 1, 2020, 9-21.
9. Florentin Smarandache, Nivetha Martin: Plithogenic n-Super Hypergraph in Novel Multi-Attribute Decision Making. *International Journal of Neutrosophic Science (IJNS)* Vol. 7, No. 1, 2020, 8-30.
10. Shazia Rana, Muhammad Saeed, Midha Qayyum, Florentin Smarandache: Plithogenic Subjective Hyper-Super-Soft Matrices with New Definitions & Local, Global, Universal Subjective Ranking Model. *International Journal of Neutrosophic Science (IJNS)* Vol. 6, No. 2, 2020, 56-79; DOI: 10.5281/zenodo.3841624.
11. Firoz Ahmad, Ahmad Yusuf Adhami, Florentin Smarandache: Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty. *Optimization Theory Based on Neutrosophic and Plithogenic Sets*, 2020, 343-403; DOI: 10.1016/B978-0-12-819670-0.00015-9.
12. Sudipta Gayen, Florentin Smarandache, Sripati Jha, Manoranjan Kumar Singh, Said Broumi, Ranjan Kumar: Introduction to Plithogenic Hypersoft Subgroup. *Neutrosophic Sets and Systems*, Vol. 33, 2020, 208-233.
13. Nivetha Martin, Florentin Smarandache: Introduction to Combined Plithogenic Hypersoft Sets. *Neutrosophic Sets and Systems*, Vol. 35, 2020, 503-510.
14. Shio Gai Quek, Ganeshsree Selvachandran, Florentin Smarandache, J. Vimala, Son Hoang Le, Quang-Thinh Bui, Vassilis C. Gerogiannis: Entropy Measures for Plithogenic Sets and Applications in Multi-Attribute Decision Making. *Mathematics* 2020, 8, 965, 17 p.; DOI: 10.3390/math8060965.
15. Nivetha Martin, Florentin Smarandache: Concentric Plithogenic Hypergraph based on Plithogenic Hypersoft sets - A Novel Outlook. *Neutrosophic Sets and Systems*, Vol. 33, 2020, 78-91.
16. George Bala: Information Fusion Using Plithogenic Set and Logic. *Acta Scientific Computer Sciences* 2.7, 2020, 26-27.
17. Shawkat Alkhazaleh: Plithogenic Soft Set. *Neutrosophic Sets and Systems*, Vol. 33, 2020, 256-274.
18. R. Sujatha, S. Poomagal, G. Kuppaswami, Said Broumi: An Analysis on Novel Corona Virus by a Plithogenic Fuzzy Cognitive Map Approach. *International Journal of Neutrosophic Science (IJNS)*, Volume 11, Issue 2, 2020, 62-75; DOI: 10.5281/zenodo.4275788.
19. S. P. Priyadharshini, F. Nirmala Irudayam, F. Smarandache: Plithogenic Cubic Sets. *International Journal of Neutrosophic Science (IJNS)*, Volume 11, Issue 1, 2020, 30-38; DOI: 10.5281/zenodo.4275725.

20. Prem Kumar Singh: Plithogenic set for multi-variable data analysis. *International Journal of Neutrosophic Science (IJNS)*, Volume 1, Issue 2, 2020, 81-89; DOI: 10.5281/zenodo.3988028.
21. C. Sankar, R. Sujatha, D. Nagarajan: TOPSIS by Using Plithogenic Set in COVID-19 Decision Making. *International Journal of Neutrosophic Science (IJNS)*, Volume 10, Issue 2, 2020, 116-125; DOI: 10.5281/zenodo.4277255.
22. Wilmer Ortega Chavez, Fermin Pozo Ortega, Janett Karina Vasquez Perez, Edgar Juan Diaz Zuniga, Alberto Rivelino Patino Rivera: Modelo ecologico de Bronferbrenner aplicado a la pedagogia, modelacion matematica para la toma de decisiones bajo incertidumbre: de la logica difusa a la logica plitogenica. NSIA Publishing House Editions, Huanuco, Peru, 2021, 144 p.
23. Nivetha Martin, R. Priya: New Plithogenic sub cognitive maps approach with mediating effects of factors in COVID-19 diagnostic model. *Journal of Fuzzy Extension & Applications (JFEA)*, Volume 2, Issue 1, Winter 2021, 1-15; DOI: 10.22105/JFEA.2020.250164.1015.
24. Mohamed Abdel-Basset, Rehab Mohamed, Florentin Smarandache, Mohamed Elhoseny: A New Decision-Making Model Based on Plithogenic Set for Supplier Selection. *Computers, Materials & Continua*, 2021, vol. 66, no. 3, 2752-2769. DOI:10.32604/cmc.2021.013092
25. Nivetha Martin, Florentin Smarandache, R. Priya: Introduction to Plithogenic Sociogram with preference representations by Plithogenic Number. *Journal of Fuzzy Extension & Applications*, 15 p.
26. S.P. Priyadharshini, F. Nirmala Irudayam, F. Smarandache: Plithogenic Cubic Set. *International Journal of Neutrosophic Science (IJNS)*, 2020, Vol. 11, No. 1, 30-38.
27. Alptekin Uluta, Ayse Topal, Darjan Karabasevic, Dragisa Stanujkic, Gabrijela Popovic, and Florentin Smarandache, Prioritization of Logistics Risks with Plithogenic PIPRECIA Method, in C. Kahraman et al. (Eds.): INFUS 2021, Springer, LNNS 308, pp. 663–670, 2022.

Received: June 29, 2021. Accepted: August 28, 2021