OPEN ACCESS

International Journal of Basic Sciences and Applied Computing (IJBSAC)
ISSN: 2394-367X, Volume-3 Issue-3, March 2021

Some Generalized Results to unify Classical Polynomials

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Abstract: *Present work of this paper deals with the unification of classical polynomials in which we have defined a generalized polynomial set analogous to that of associated Legendre polynomial* $P_n^m(x)$ *by taking 5the use of Operator. Also we have derived explicit form, OperationalFormulae generating functions for this function.*

Keywords: *Classical Polynomials,LegendrePolynomials, Rodrigues Formula , Generating Functions.*

I.INTRODUCTION

A special function is a real or complex valued function of one or more real or complex variable which is so completely that it's numerical values be tabulated. The chief organs in the study of special functions have been Rodrigue's type formula, generating functions, recurrence relations, relations with other function, operational folmulae etc. Further many various polynomials have been generalized in different directions with the help of these organs.

Classical polynomials like Legendre, Hermite, Laguerre, Jacobi, Gagenbauer and Bessel functions have been studied to a great extent and these have been generalized in a number of ways see [1]. In an attempt to unify classical polynomials of mathematical physics, Dhillon [4] considered a generalized function.

$z_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k)$ defined by the following Rodrigue's type formula:

(1.1)

$$z_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k)=(Ax+B)^{-\alpha}(1-\delta x^r)^{-\beta/\tau}e^{m+n}[(Ax+B)^{\alpha+qn}(1-\delta x^r)^{\beta/\tau+\delta n}]$$

where

(1.2) $\theta\equiv\alpha^k D_x$ $\qquad D_x\equiv\dfrac{d}{dx}$

In defining (1.1), Dhillon[4] was motivated by Rodrigue's type formula for the associated Legendre polynomials $P_n^m(x)$ and the generalized function $P_n^{(\alpha,\beta,\tau)}(x;r,s,m,A,B)$ of Singh. A.[7] defined respectively as

(1.3) $$P_n^m(x)=\frac{(x^2-1)^{m/2}}{2^n n!}\frac{d^{m+n}}{dx^{m+n}}(x^2-1)^n$$

And

$$P_n^{(\alpha,\beta,\tau)}(x;r,s,m,A,B)=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}D_x^n[(Ax+B)^{\alpha+mn}(1-\tau x^r)^{\beta/\tau+\delta n}]$$

Recently in a similar manner attempt of unifying various classical polynomials of mathematical physics, Joshi and Prajapat [6] considered the operators

(1.5) $T_{k,q}\equiv x^q(k+x\,Dx)$

Where k is a constant and introduced the polynomial set $(M_n^{(\alpha)}(x;r;p;b;k;q):n=0,1,2.......)$ defined by

(1.6)

$$(M_n^{(\alpha)}(x;r,p,b,k,q)=c(b,n)\,x^{-\alpha-nq-n}e^{px^r}T_{k,q}^n\equiv(x^{\alpha+bn}(k+x\,e^{-px^r})$$

Where c(b,n) is a constant such that :

(1.7) $$c(b,n)=\frac{(-1)^{n/2(b-1)(b-2)}}{2^{nb/2(b-1)}(1)_{nb(2-b)}}$$ b, being a non negative integer.

In view of (1.6) we introduce a generalized polynomial set $(S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l),n=0,1,2.......)$

defined by the following Rodrigue's formula:

(1.8)

$$(S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^{n+m}[(Ax+B)^{\alpha+qn}(1-\delta x^r)^{\beta/\tau+\delta n}]$$

Where $T_{k,l}^{n+m}=x^l(k+x\,Dx)$

(1.8) provides an elegant unified representation of the various known extensions of the classical polynomials and includes the polynomials due to Joshi and Singhal [10] and Chatterjea [3] as special cases and reduces to that when k = 0,l = -1 and m=0

II.PRELIMINARIES

THE OPERATOR $T_{k,q}$

Appell and Kampe [2] also describe some results on hypergeometric and hypershperiques function which has main results or polynomial. Gould and Hopper.[5] find some extension on the Operational formulas which are connected through with two generalization of Hermite polynomials. on the otherhand Singhal. and Savita.[8] make truthful results On a unification of generalized Humbert and Laguerre polynomials.

The operator $T_{k,q}$ has been defined as

$T_{k,q}\equiv x^q(k+xD)$

Manuscript received on March20, 2021.
Revised Manuscript received on March03, 2021.
Manuscript published on March30, 2021.
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Some of the well known properties of this operator ,which we shall require in the investigations are listed below:

(2.1) $T_{k,q}^{n}(x^{\alpha+m}) = q^{n}(\frac{\alpha+m+k}{q})_{n}\, x^{\alpha+m+nq}$

(2.2) $F(T_{k,q})[x^{\alpha} f(x)] = x^{\alpha} F(T_{k,q} + x^{q}\alpha) f(x);$

(2.3) $F(T_{k,q})[e^{g(x)} f(x)] = e^{g(x)} F[T_{k,q} + x^{q+1} g(x)] f(x);$

(2.4) $(T_{k,q})^{n}[x^{j}uv] = x^{j}\sum_{i=0}^{n}\binom{n}{i}(T_{k,q}^{n-i}u)(T_{q}^{i}v)$

(2.5) $(T_{k,q})^{n}(u.v) = \sum_{i=0}^{n}\binom{n}{i}(T_{k,q}^{n-i}u)(T_{q}^{i}v)$

Where

(2.6) $T_{q} \equiv x^{q+1}D_{x};$

(2.7) $(T_{k,q})^{n}(xuv) = x\sum_{i=0}^{n}\binom{n}{i}(T_{k,q}^{n-i}v)(T_{1,q}^{i}u)$,Where

(2.8) $T_{1,q} = x^{q}(1+xD);$

(2.9) $(T_{k,q})^{n} f(x) = x^{-k}T_{q}^{n}(x^{k} f(x);$

(2.10) $e^{t}(T_{k,q})[x^{\alpha} f(x)] = \frac{x^{\alpha}}{(1-x^{q}qt)^{\alpha+k/q}} f[\frac{x}{(1-x^{q}qt)^{1/q}}]$

(2.11) $_{T}F_{\mu}\left[\begin{matrix}(a);\\ T\quad tT_{k,q}\\ (b\mu);\end{matrix}\right] x^{\alpha}e^{px^{r}} = \sum_{j=0}^{\infty}\frac{p_{j}}{j!}x^{\alpha+rj}$

$*$

$_{T+1}F_{\mu}[(a_{T}),(\frac{\alpha+rj+k}{q}); {}_{x}q_{qt}\, j;$

Where (a_T) stands for the sequence of parameters namely a_1, a_2,….. a_T with similar interpretation for (b_μ).

(2.12) $T_{q}^{n}[f(z(x)] = \sum_{i=0}^{n}\frac{(-1)^{i}}{i!}(\frac{d^{i}}{dz})f(z)\sum_{j=0}^{i}(-1)^{j}\binom{i}{j}(z(x))^{i-j}T_{q}^{n}(z(x))^{j}$

III.OPERATIONAL EQUATION

Consider

$$\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

$$=\sum_{i=0}^{m+n}\binom{m+n}{i}\left(T_{k,l}^{m+n-i}(Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}\right)\left(T_{1}^{i}(Y)\right)$$

(3.1) $\sum_{i=0}^{m+n}\binom{m+n}{i}(Ax+B)^{\alpha+qi}(1-\tau x^{r})^{\beta/\tau+sn}S_{n-i}^{(\alpha+qi,s+\tau si,\tau)}(x;r,s,A,B,m,k,l)(T_{1}^{i}Y)$

Using(1.8)

Where Y is sufficiently differentiable function of x

Also

$$\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

$$=\underset{k,l}{\overset{m+n-1}{T}}T_{k,l}[(Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

$$=\underset{k,l}{\overset{m+n-1}{T}}(kx^{l}+x^{l+1}d/dx)(Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

(3.2)= $\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$

Where

(3.3) $Y_{1} = \left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{l+r}(\beta+\tau sn)}{(1-\tau x^{r}} + T_{k,l}\right)Y$

Similarly from equation (3.2) we see that :

(3.4) $\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$

$$=\underset{k,l}{\overset{m+n-2}{T}}[(Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y_{2}]$$

Where

(3.5) $Y_{2} = \left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{1+r}(\beta+\tau sn)}{(1-\tau x^{r})} + T_{k,l}\right)Y_{1}$

Or using equation (3.3):

$$Y_{2} = \left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{1+r}(\beta+\tau sn)}{(1-\tau x^{r})} + T_{k,l}\right)^{2}Y$$

Repeating the above process (m+n-2) times, from (3..4)we obtain

(3.6)

$$\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

$$=(Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}\left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{1+r}(\beta+\tau sn)}{(1-\tau x^{r})} + T_{k,l}\right)^{m+n}Y$$

From equation (3.1) and (3.6) we arrive at following important operational formula:

(3.7)

$$\left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{1+r}(\beta+\tau sn)}{(1-\tau x^{r})} + T_{k,l}\right)^{m+n}Y$$

$$=\sum_{i=0}^{m+n}\binom{m+n}{i}(Ax+B)^{-q(n-i)}(1-\tau x^{r})^{\beta/\tau+sn}S_{n-i}^{(\alpha+qi,s+\tau si,\tau)}(x;r,s,A,B,m,k,l)(T_{1}^{i}Y)$$

When y=1 from equation (3.7), we get

(3.8) $\left(\frac{Ax^{l+1}(\alpha+qn)}{(Ax+B} - \frac{rx^{1+r}(\beta+\tau sn)}{(1-\tau x^{r})} + T_{k,l}\right)^{m+n}1$

$$=(Ax+B)^{-qn}(1-\tau x^{r})^{-sn}S_{n}^{\alpha,\beta,\tau}(x;r,s,A,B,m,k,l)$$

When k=0,l=-1and m=0, relation (3.8) reduces to the operational formula for the function of Singh [7]:

(3.9) $\left(D+\frac{A(\alpha+qn)}{(Ax+B)} - \frac{rx^{r-1}-1(\beta+\tau sn)n}{(1-\tau x^{r})}\right).1$

$$=(Ax+B)^{-qn}(1-\tau x^{r})^{-sn}P_{n}^{\alpha,\beta,\tau}(x;r,s,q,A,B)$$

Srivastava. and Singhal.[9] modified some basic polynolmial those are connected with class of polynomials defined by generalized Rodrigue’s formula as similaras other formulae for the polynomials of this class can be obtained as the particular case of (3.8)

Again consider

$$\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^{r})^{\beta/\tau+sn}Y]$$

$$=T_{k,l}^{m+n}[(Ax+B)^{\alpha-1+qn}(1-\tau x^{r})^{\beta/\tau-1+sn}(kx^{l}(Ax+B)(1-\tau x^{r})$$

$$+A(\alpha+qn)x^{l+1}(1-\tau x^{r})r(\beta+\tau sn)x^{l+r}(Ax+B)$$

$$+(Ax+B)(1-\tau x^{r})\theta Y];\theta = x^{l+1}D$$

$$= T_{k,l}^{m+n-2}[(Ax+B)^{\alpha+2+qn}(1-\tau x^r)^{\beta/\tau-1+sn}(kx^l(Ax+B)(1-\tau x^r)$$
$$+A(\alpha+qn)x^{l+1}(1-\tau x^r)-r(\beta+\tau sn)x^{1+r}(Ax+B)$$
$$+(Ax+B)(1-\tau x^r)\theta]x(kx^l(Ax+B)(1-\tau x^r)+A(\alpha-1+qn)x^{l+1}(1-\tau x^r)$$
$$-r(\beta-\tau+\tau sn)x^{1+r}(Ax+B)+(Ax+B)(1-\tau x^r)\theta Y]$$

Which by iteration gives

(3.10)

$$\underset{k,l}{\overset{m+n}{T}}[Ax+B)^{\alpha+qn}(1-\tau x^r)^{\beta/\tau+sn}Y]$$
$$=(Ax+B)^{\alpha+(m+n)+qn}(1-\tau x^r)^{\beta/\tau-(m+n)+sn}$$
$$\prod_{i=0}^{m+n-1}(kx^l(Ax+B)(1-\tau x^r)+A(\alpha-1+qn)x^{l+1}(1-\tau x^r)-r(\beta-\tau+\tau sn)x^{1+r}(Ax+B)+(Ax+B)(1-\tau x^r)\theta]Y$$

From (3.1) and (3.10) we obtain the following product type operational formula:

(3.11)

$$\prod_{i=0}^{m+n-1}(kx^l(Ax+B)(1-\tau x^r)+A(\alpha-i+qn)x^{l+1}(1-\tau x^r)-r(\beta-\tau+\tau sn)x^{1+r}(Ax+B)+(Ax+B)(1-\tau x^r)x^{l+1}D]Y$$

$$=\sum_{i=0}^{m+n}\left(\frac{m+n}{i}\right)(Ax+B)^{m+n-q(n-1)}(1-\tau x^r)^{m+n-s(n-i)}$$
$$*S_{n-i}^{(\alpha+qi,\beta+\tau si,\tau)}(x;r,s,q,A,B,m,k,l)(T_1^i(Y))$$

(3.11) gives various operational formulae for the polynomials of the class $S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)$ as the special cases.

IV.GENERATING FUNCTIONS:

Using property (2.9) equation (1.8) can be written as :

$$S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)=(Ax+B)^{-\alpha}(1-\tau x^r)^{\beta/\tau}$$
$$*x^{-k}e^{m+n}[x^k(Ax+B)^{\alpha+qn}(1-\tau x^r)^{\beta/\tau+sn};\theta=x^{l+1}D_x$$
$$=x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{\beta/\tau}$$
$$*\left(\frac{d}{du}\right)^{m+n}[(-ul)^{-k/l}(A(-ul)^{-i/l}+B)^{\alpha+qn}(1-\tau(-ul)^{-r/l})^{\beta/\tau+sn}]$$

Where

(4.2) $u=-\frac{x^{-1}}{l}$ gives

(4.3) $\frac{d}{du}=x^{l+1}D_x$

The modified form of the Lagrange's expansion theorem is given by:

(4.4) $\frac{F(p)}{1-t\phi(p)}=\sum_{n=0}^{\infty}\frac{t^n}{n!}D^n[(\phi(x))^n F(x)]$

Where

(4.5) p= x+t $\phi(p)$

And $\phi(p)$ is derivable at p=x and $\phi(x)\ =0$

Using (4.3), equation (4.1) can be written as :

$$\sum_{n=0}^{\infty}\frac{t^n}{n!}S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)=x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{\beta/\tau}$$
$$*\sum_{n=0}^{\infty}\frac{t^n}{n!}(x^{l+1}D_x)^{m+n}[\{(A(-ul)^{-i/l}+B)^q(1-\tau(-ul)^{-r/l})^s\}^n]$$
$$*[(-ul)^{-k/l}(A(-ul)^{-i/l}+B)^\alpha(1-\tau(-ul)^{-r/l})^{\beta/\tau}]$$

Or

(4.6) $\sum_{n=0}^{\infty}\frac{t^n}{n!}S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)*x^k(Ax+B)^\alpha(1-\tau x^r)^{\beta/\tau}$

$$=(x^{l+1}D_x)^m*\sum_{n=0}^{\infty}\frac{t^n}{n!}(x^{l+1}D)[(\phi(u))^n.F(u)]$$

Where

(4.7)

$$\phi(u)=[(A(-ul)^{-i/l}+B)^q(1-\tau(-ul)^{-r/l})^s]$$

And

(4.8) $F(u)=[(-ul)^{-k/l}(A(-ul)^{-i/l}+B)^\alpha(1-\tau(-ul)^{-r/l})^s]$

Now applying Lagrange's theorem(4.4)on R.H.S of equation (4.6) and by use of the equation (4.7) and (4.8) we obtain the required generating relation as:

$$x^k(Ax+B)^{-\alpha}(1-\tau x^r)^{\beta/\tau}\sum_{n=0}^{\infty}(S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)\frac{t^n}{n!}$$
$$=(x^{l+1}D_x)^m\frac{[(-pl)^{-k/l}(A(-pl)^{-i/l}+B)^\alpha(1-\tau(-pl)^{-r/l})^{\beta/\tau}]}{1+\tau rst(-pl)^{(-r+1)/l}(A(-pl)^{-1/l}+B)^\alpha(1-\tau(-pl)^{-r/l})^{s-1}-Aqt(-pl)^{-(1+l)/l}}$$
$$*\frac{1}{(A(-pl)^{-i/l}+B)^{q-1}(1-\tau(-pl)^{-r/l})^s}$$

Where

$$p=u+t(A(-pl)^{-i/l}+B)^q(1-\tau(-pl)^{-r/l})^s$$

Or

(4.9) $x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{\beta/\tau}$

$$*(x^{l+1}D_\alpha)^m[\frac{(-pl)^{-k/l}x^\alpha y^{\beta/\tau}}{1+\tau rst(-pl)^{(-r+1)/l}x^q y^{s-1}-Aqt(-pl)^{-(1+l)/l}x^{q-1}y^s}]$$
$$=\sum_{n=0}^{\infty}\frac{t^n}{n!}S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)$$

Where,

(4.10) $p=u+x^q y^s t$;

(4.11) $x=(A(-pl)^{-i/l}+B)$

And

(4.12) $y=(1-\tau(-pl)^{-r/l})$

V.GENERALISED RESULTS

The generating relation (4.9)-(4.12) is a generalization of so many well known generating relations .For example taking $\alpha=0,\beta=0,\tau=1,r=1,s=1,q=1,A=1,B=1,k=0,l=-1$ from equation (4.9) and (4.10) we get;

(5.1) $D^m\left[\frac{1}{1+2pt}\right]=\sum_{n=0}^{\infty}S_n^{(0,0,1)}(x;1,1,1,1,1,m,o,-1)\frac{t^n}{n!}$

Where

(5.2) $p=x+t(1-p^2)$

Equation (5.2) on solving for p gives

(5.3) $1+2pt=\sqrt{(4t^2+4tx+1)}$

Using (5.3) and (4.12) equation (5.1) takes the form:

$$D^m(4t^2+4tx+1)^{-1/2}]=\sum_{n=0}^{\infty}(-2)^n n!(x-1)^{-m/2}P_n^m(x)\frac{t^n}{n!}$$

Or

$$(x^2-1)^{m/2}D^m[(1+w^2+2wx)^{-1/2}]=\sum_{n=0}^{\infty}P_n^m(x)(-w)^n$$

Or,

(5,4)

$$(x^2-1)^{m/2}(-1)^m(/2)_m(2w)^m(1+w^2+2wx)^{-1/2-m}=\sum_{n=0}^{\infty}P_n^m(x)(-w)^n$$

Where 2w =t

(5.4) is the generating relation for associated Legendre function (5.1).

Similarly taking

$\alpha=a,\beta=p,\tau=0,s=0,q=0,A=1,B=0,k=0,l=-1$

From equation (4.9) we obtain the following generating relation for the function of Gould and Hopper[5];

(5.5)

$$(p/x)^a\exp[vx^r(1-(1+t/x)^r)]=\sum_{n=0}^{\infty}H_n^r(x,a,p)\frac{t^n}{n!}$$

We can write

$$e^{tS}f(x)=\sum_{n=0}^{\infty}\frac{t^n}{n!}S^n f(x)$$

$$=\sum_{n=0}^{\infty}\frac{t^n}{n!}\sum_{i=0}^{n}\binom{n}{i}(Ax+B)^{-q(n-i)}(1-\tau x^r)^{-s(n-i)}$$

$$=\sum_{n=0}^{\infty}\frac{t^n}{n!}(Ax+B)^{-qn}(1-\tau x^r)^{-sn}S_n^{(\alpha-qn,\beta-\tau sn,\tau)}(x;r,s,q,A,B,m,k,l)$$

$$*\sum_{i=0}^{\infty}\frac{t^i}{i!}T_1^i f(x)$$

[Using(1.8)]

$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}[T_{k,l}^m\{e^tT_{k,l}(Ax+B)^{\alpha}(1-\tau x^r)^{\beta/\tau}\}]$$

$$*e^tT_l f(x)$$

(5.6)

$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^m[(1-1x^1t)^{-k/l}(Ax(1-x^l lt)^{-1/l}+B)^{\alpha}$$

$$*(1-\tau x(1-x^1lt)^{-r/l})^{\beta/\tau}]*f(x(1-x^l lt)^{-1/l}]$$

$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau-k}(x^{l+1}D_x)^m[x^k(1-x^1lt)^{-k/l}]$$

$$*(Ax(1-x^l lt)^{-1/l}+B)^{\alpha}(1-\tau x^r(1-x^1lt)^{-r/l})^{\beta/\tau}]*f(x(1-x^l lt)^{-1/l}]$$

(5.7)

$$=x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}(x^{l+1}D_x)^m[F(x)]f(x(1-x^1 1t)^{-l/1}]$$

Where

(5.8) $$F(x)=x^k c^k(Axc+B)^{\alpha}(1-\tau c^r x^r)^{\beta/\tau}$$

And

(5,9) $$c=(1-x^l lt)^{-1/l}$$

To find the value of $(x^{l+1}D_x)[F(x)]$ in (5,7) we use the following formula given by Srivastava and Singhal.[9]

(5.10)

$$D_{g(x)}^m\{F(x)\}=D_w^m\{F(x)\}g^1(x)\left(\left(\frac{x-v}{g(x)-g(v)}\right)^{m+1}\right)_{v=x}$$

Where

(5.11) $$D_{g(x)}=\frac{d}{dg(x)}$$

Hence

(5.12) $$x^{l+1}D_x=\frac{1}{x^{-l-1}}\left(\frac{d}{dx}\right)=\frac{d}{d(-\frac{x^{-1}}{1})}$$

Hence

(5.13) $$g(x)=-\frac{1}{l}x^{-1}$$

Therefore from (5,11):

$$(x^{l+1}D_x)^m[F(x)]=D_w^m(x^kc^k(Axc+B)^{\alpha}(1-\tau c^rx^r)^{\beta/\tau}x^{-l-1}(\frac{x-v}{\frac{-1}{l}x^{-1}+\frac{1}{l}v^{-1}})^{m+n}$$

$$=D_w^m(x^{k-1-l}c^k(Axc+B)^{\alpha}(1-\tau c^rx^r)^{\beta/\tau}l^{m+1}(\frac{u}{x^{-1}-(u+x)^{-1}})^{m+1})_{u=0}$$

[On substituting v= u+x]

(5.14)

$$=D_w^m(l^{m+1}x^{k-1-l}c^k(Axc+B)^{\alpha}(1-\tau c^rx^r)^{\beta/\tau}l(\frac{ux^1}{1-(1+u/x)^{-1}})^{m+1})_{u=0}$$

With the help of (5.14), from equation (5.7) we get:

$$e^{tS}f(x)=x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}$$

$$=[D_w^m(l^{m+1}x^{k-1-l}c^k(Axc+B)^{\alpha}(1-\tau c^rx^r)^{\beta/\tau}l(\frac{ux^1}{1-(1+u/x)^{-1}})^{m+1})_{u=0}]xf(xc)$$

Or,

(5.15)

$$e^{tS}f(x)=x^{-k}(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}[D_w^m\psi(x,w,u)]_{u=0}f(xc)$$

Where

(5.16)

$$(x,u,w)=l^{m+1}x^{k-1-l}c^k(Axc+B)^{\alpha}(1-\tau c^rx^r)^{\beta/\tau}l(\frac{ux^1}{1-(1+u/x)^{-1}})^{m+1}$$

Taking

$$f(x)=S_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)$$

From equation (5.5) we get :

$$\sum_{j=0}^{\infty}\frac{t^j}{j!}S^jS_n^{(\alpha,\beta,\tau)}(x;r,s,q,A,B,m,k,l)$$

$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^m[(1-1x^1t)^{-k/l}(Ax(1-x^l lt)^{-1/l}+B)^{\alpha}$$

$$(1-\tau x^r)(1-x^1lt)^{-r/l})^{\beta/\tau}]$$

$$*S_n^{(\alpha,\beta,\tau)}(x(1-x^1lt)^{-1/1};r,s,q,A,B,m,k,l) \quad (5.17)$$

Retrieval Number: 100.1/ijbsac.B0201033321
DOI:10.35940/ijbsac.B0201.033321
Journal Website: www.ijbsac.org

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication

$$\sum_{j=0}^{\infty}\frac{t^j}{j!}(Ax+B)^{-qj}(1-\tau x^r)^{-sj}S_{n+j}^{(\alpha-qj,\beta-\tau sj,\tau)}(x;r,s,q,A,B,m,k,l)$$
$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^{m}[(1-1x^1t)^{-k/l}(Ax(1-x^1lt)^{-1/l}+B)^{\alpha}$$
$$*(1-\tau x^r)(1-x^1lt)^{-r/l})^{\beta/\tau}]$$
$$*S_n^{(\alpha,\beta,\tau)}(x(1-x^1lt)^{-1/1};r,s,q,A,B,m,k,l)$$

(5.17) gives another generating function for the polynomial set in particular for associated Legendre function Now(5.17) reduces to:

$$\sum_{j=0}^{\infty}\frac{t^j}{j!}(x+i)^j(1-x)^j(-2)^{n+j}(n+j)!(x^2-1)^{-m/2}P_{n+j}^m(x)$$
$$=D^m[(-2)^m n!((x+t)-1)^{-m/2}P_n^m(x+t)]$$

Or

(5.18) $$\sum_{j=0}^{\infty}\binom{n+j}{j}w^jP_{n+j}^m(x)$$
$$=(x^2-1)^{-m/2}D^m[((x+w)(1-x^2))^2-1)^{-m/2}P_n^m(x+w(1-x^2))]$$

Where

(5.19) $$\frac{t}{(1-x^2)}=w$$

Relations (5.6) and (5.17) are to bethe new generating relations.

VI.RESULT

The further generalized result is given by equation (5.6) which is shown as:

$$\sum_{j=0}^{\infty}\frac{t^j}{j!}(Ax+B)^{-qj}(1-\tau x^r)^{-sj}S_{n+j}^{(\alpha-qj,\beta-\tau sj,\tau)}(x;r,s,q,A,B,m,k,l)$$
$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^{m}[(1-1x^1t)^{-k/l}(Ax(1-x^1lt)^{-1/l}+B)^{\alpha}$$
$$*(1-\tau x(1-x^1lt)^{-r/l})^{\beta/\tau}]*f(x(1-x^1lt)^{-1/l}]$$
$$\sum_{j=0}^{\infty}\frac{t^j}{j!}(Ax+B)^{-qj}(1-\tau x^r)^{-sj}S_{n+j}^{(\alpha-qj,\beta-\tau sj,\tau)}(x;r,s,q,A,B,m,k,l)$$
$$=(Ax+B)^{-\alpha}(1-\tau x^r)^{-\beta/\tau}T_{k,l}^{m}[(1-1x^1t)^{-k/l}(Ax(1-x^1lt)^{-1/l}+B)^{\alpha}$$
$$(1-\tau x^r)(1-x^1lt)^{-r/l})^{\beta/\tau}]$$
$$*S_n^{(\alpha,\beta,\tau)}(x(1-x^1lt)^{-1/1};r,s,q,A,B,m,k,l)$$

VII.CONCLUSION

The present paper deals with the unification of classical polynomial in which we have define generalized polynomial set analogous to rate of associated Legendre polynomials. In which we have drive explicit form, operational formula and generating function for this function.

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