Just mathematics? Fostering empowering and inclusive mathematics classrooms with Realistic Mathematics Education

Vinay Kathotia, The Open University, ✉ vinay.kathotia@open.ac.uk
Kate O'Brien, Manchester Metropolitan University
Yvette Solomon, Manchester Metropolitan University

This paper considers how Realistic Mathematics Education (RME) can serve as a platform for developing socially just and inclusive mathematics classrooms, and examines how teachers new to RME interpret and enact this potential. Drawing on exit interviews from a large RME trial in England, we explore teachers' interpretations of RME's potential for transforming students' access to mathematics, and their understanding of the role of the "realistic" element of RME in inclusion. We also study one teacher's classroom practice and reflections to investigate how he endeavours to build on his students' "real life" starting points. Our analysis of how he appropriates RME to achieve his aims underlines the need to identify inclusion as an object of reflection in continuing professional development.

In this paper, we consider the potential of Realistic Mathematics Education (RME) to support a mathematics education that enables us "to do more than tinker with the arrangements in school that contribute to the production of inequities in the lived experiences of learners and educators" (Gutiérrez, 2013, p. 62). Recognising the debate around the concept of inclusion as often assimilative in nature (Ahmed, 2012; Martin, 2019), we examine how RME, as a pedagogical practice, supports teachers in enacting and reflecting on this mathematical mode of inclusivity, in which learners are invited to draw on their own experience to generate mathematics in participative classrooms. By exploring how teachers adopting RME described their interests in inclusion, and using a case study to examine how these interests may not play out in practice, we identify ways in which RME materials and strategies can serve as a catalyst for thinking about the nature of inclusion, and consider how CPD can highlight this potential.

RME as a platform for inclusive mathematics education

RME lends itself to the pursuit of inclusive classrooms in terms of instructional design features and their related pedagogy. A central design principle of RME is that mathematical

Please cite as: Kathotia, V., O'Brien, K., & Solomon, Y. (2021). Just mathematics? Fostering empowering and inclusive mathematics classrooms with Realistic Mathematics Education. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 2, pp. 545–554). Tredition. https://doi.org/10.5281/zenodo.5414524

strategies and models are generated from students' understandings of everyday situations, and that mathematics emerges from these informal models (Gravemeijer & Stephan, 2002). This grounding in students' own understandings of a realisable world emphasises their participation in mathematics at its core, supporting a 'bottom-up' pathway to formal mathematics, which means that students always have a route back to contexts in which they have constructed meaningful models.

RME's design features are supported by a pedagogy of *guided reinvention* (Stephan, Underwood-Gregg & Yackel, 2014). Shifting authority away from the teacher, *guided reinvention* promotes student agency in mathematical discussions where teachers serve primarily as facilitators. Rather than leading conversation, teachers aim to support students' mathematising of context by recognising and working from what is 'everyday' for their particular class, and encouraging students to state, restate and reflect on solutions and strategies in the room (Solomon, Hough & Gough, 2021). In addition to being meaningful to the speaker, students' explanations must be acceptable to other students, in the sense that they too can access and understand them - otherwise an explanation cannot be 'taken-as-shared', so as to become "objects of reflection" for the class (Yackel & Cobb, 1996, pp. 470–471).

In deploying RME, teachers need to be able to work with the hypothetical learning trajectories that underpin RME materials, and to anticipate, analyse and build on students' own informal models of everyday situations. This is a challenging re-direction of their pedagogic practices for many teachers, as illustrated in Wubbels, Korthagen & Broekman's (1997) study of an RME teacher education programme in the Netherlands. Wubbels et al. (1997) found many teacher trainees misunderstood RME as merely a one-way street – simply an innovative avenue toward formal mathematical concepts. Few teachers came to conceptualise mathematics as a more dynamic system of exchange, where mathematical concepts adhere to and live inside of the rhythms of our daily lives and fantasy worlds. Additionally, it was a challenge for teachers to reappropriate the perspectives of mathematical novices, for whom the reality of mathematical concepts can seem doubtful and arbitrary.

What is real? RME teaching, student empowerment and social justice

This struggle to stitch together the different mathematical realities of teachers and students is of particular interest in this paper, as it cues critical questions about *whose* reality matters in mathematics classrooms. The nature of reality is, of course, what is at stake in social justice efforts that seek to foreground the brilliance of black and brown, female, differently-abled, and poor students. As Delpit (2006) reminded us:

> We all carry worlds in our heads, and those worlds are decidedly different. We educators set out to teach, but how can we reach the worlds of others when we don't even know they exist? Indeed, many of us don't even realize that our own worlds exist only in our heads and in the cultural institutions we have built to support them. (p. xxiv)

US projects, like the Algebra Project (Moses & Cobb, 2001) or Civil's Bridge Project (Gonzalez et al., 2001), have set out to bridge the violent erasures enacted by white supremacy and patriarchy, while also strategically navigating the way in which a white, male, middle-class mathematics serves as a gatekeeper for students' access to university education or other ambitions.

Working to tackle the complex cohesion between multiple real world*s* and mathematical concepts, RME's focus on students' understanding of 'realistic' contexts seems poised to take on similar issues. However, Gutstein (2006) argues that the 'realisable' contexts of RME were sometimes in tension with his vision of teaching for social justice. Working with *Mathematics in Context* (MiC), a US middle school implementation of RME (NCRMSE & FI, 1997–1998), Gutstein (2006) recognised the features of RME that "support teaching mathematics for social justice" (p. 103). He praises the MiC materials for presenting mathematics "as a sensemaking activity" and positioning students "as arbitrators of knowledge" (p. 103). However, while MiC helped his students to develop the necessary "mathematical power" (p. 107), "MiC by itself … does not challenge students to analyze injustice or see themselves as potential social change agents" (p. 104). While 'realisable' contexts of white middle class pursuits – a school camping and canoe trip for example – served to generate an emergent mathematics of ratio and ratio tables, students reported that this context had no experiential resonance for them. Nor were such contexts relevant to the political and social critique that Gutstein felt was most important for his students' agency.

For Gutstein therefore, there is a tension between teaching for social justice and teaching for mathematical power. In the Algebra and Bridge projects, there is also a tension between full ownership of mathematics and meeting the gatekeeping practices of our educational systems, where test scores are the main tokens of exchange value. How does one balance the need for an equitable and inclusive classroom and meaningful discussions which are 'real' for the students, against pressures to take shortcuts to mathematical models and debased but 'functional' ends? How teachers navigate these tensions as they interpret and implement RME is a core concern of this paper. Given these issues concerning the potential of RME pedagogy to serve as a vehicle for empowering or inclusive mathematical activity, we ask the following research questions: How do RME teachers describe their aims for inclusion? How do they describe and enact RME's 'realistic' component?

Methodology

The wider context of this study is our role as researchers, material designers and trainers in a randomised controlled trial of RME in England between 2018 and 2020, with Year 7 and 8 students (ages 11–13), involving over 120 teachers located in 60 intervention schools. Ten modules on number, proportion, geometry, algebra and data were developed by the team, building on our long-term development work with RME, including adaptation of MiC materials for use in the UK (Dickinson, Eade, Gough & Hough, 2010). Teachers attended 7 face-to-face training days, with on-line training sessions provided to replace a planned 8th training day during the COVID-19 school closures in England. The CPD programme exposed teachers to RME pedagogy, introducing the mathematical landscapes underpinning our materials, and emphasising guided-reinvention strategies for orchestrating classroom discussion that sought to build on students' informal models. It is worth noting that we did not make inclusion an explicit part of the training, as we return to this issue below.

We collected a variety of data in the CPD sessions, making video, audio or photographic records of teachers' discussions and work. Six schools were asked to participate in the trial as Design Schools, in order to enable the team to gain some insights into how teachers were incorporating the RME materials and pedagogic strategies in the classroom. We visited these schools throughout the project, observing lessons and talking to teachers afterwards. We also interviewed 30 teachers in exit interviews at the end of their Year 8 teaching in 2020 about their aims in mathematics teaching and in joining the trial, their interpretation of RME and their experience of working with the materials. We did not explicitly ask teachers to comment on inclusion. Full ethics approval was sought and gained for this study from Manchester Metropolitan University. Participation in data collection was voluntary and all participants are anonymised.

For this paper, we selected a teacher from one Design School as a case study. Peter was responsible for a class of the lowest-attaining students in a large, suburban, co-educational, non-selective school in central England with a diverse intake (over 1500 students). Although the school's special educational needs and free school meals pupil numbers were below the national average, these students were over-represented in Peter's class. We visited Peter and his RME teaching partner three times over the course of the trial when the students were in Year 7 (March and June 2019) and then Year 8 (October 2019). On each visit, we observed one lesson with each teacher and we talked to them together at the end of the school day, with a focus on the lessons we had observed. Peter also participated in an exit interview after the trial had ended.

We analysed interviews thematically, focusing on teachers' accounts of the connection between features of RME and their global goals as mathematics teachers, paying particular attention to references to socially just or inclusive mathematics. We inspected the classroom data for the use of RME pedagogic strategies, and analysed Peter's post-lesson reflections on these events in terms of the relationships between his aims as a mathematics teacher and his appropriation and interpretations of RME.

Findings

Teachers' aims in developing inclusive classrooms

Before we turn to our discussion of Peter, we present a brief analysis of teachers' reasons for engaging with RME as expressed in the exit interviews. Of particular interest to us here are teachers' spontaneous comments on their inclusive aims. A majority described their aim in mathematics teaching in terms of emotional or nurturing themes. Many talked about building resilience and confidence, for example, "I try to create in my classroom ... a safe space to fail" (T1), or instilling their own love of mathematics into what they know is an unpopular subject: "wanting them to get to … a curiosity … to actually challenge think, and wonder" (T2). There were also teachers who recognised their own privileges in life and wanted to "give something back": "I'd like a purpose … I thought well I can do maths ... and then I can just kind of make a difference that way" (T3); "everyone gets the same opportunity but it's not always the same based on kind of what you've got going on at home ... that made me just really want to help" (T4).

Some teachers extended these ideas to broader claims about the importance of access to mathematics:

> Maths is a worthwhile subject... it helps you with everything–it helps you with your life... And I don't care what you're going to be – you're going to be a hairdresser, you're going to need some maths (T5).

These aims fed into many teachers' interest in RME in delivering a reality-based curriculum and pedagogy, which their students needed because it related to their world: "some of them, they need something tangible to look at, don't they?" (T1); "trying to put the learning in a context that the kids care about and that makes sense to them" (T6). RME provided "that element that I think was missing from my maths learning when I was at school, this ownership of why we use the formulas that we do" (T7).

There were a few more forensic responses about how RME worked to build knowledge, as in T8's comment that his previous starting point of presenting a bar model to students had missed the point of emergent mathematics:

> I love bar models ... but I never got why kids don't like it. ... [But with RME] we didn't actually start with bar modelling, we started with a sandwich being shared... how we would simplify it and how that model was the bar model. I always thought the bar model is where you start ... but there was a step beforehand that helped pupils get there.

Thus, we see a spectrum of concerns about inclusion and the role of context in achieving this. These issues are also reflected in Peter's exit interview. To explore how these connections play out in the classroom, we next take a closer look at Peter's appropriation of RME.

Case study: Peter's aims as an RME teacher

In his exit interview, Peter emphasises his desire to nurture the "weakest" students, "doing the right thing by them". The role of relationships is critical in this:

> In the training year, I realised that teaching was not just solely about your subject, but… the whole child element of it... If you can get that relationships element working, then they will be far more perceptive to what you want to do.

Peter is quite conscious about the markers of academic success (in school and beyond) but he underscores the importance of the "real world" for his "weaker" students: "it's not about a GCSE grade. It's not about making them superb mathematicians. It's getting them to a point where they can function in the real world." Thus, despite it pushing him out of his "comfort zone", Peter was keen to "get realistic elements into maths education because" it is a means of "making sure that with every student you're doing right by them" and to "get the weakest to come along on the journey." How do these aims play out in practice?

Achieving inclusion? Peter's practice and reflections on working with RME

To delve into the interplay of Peter's inclusion-related goals and see how these play out in his classroom, we zoom in on part of an algebra lesson, observed during our second visit to Peter's class. The lesson started with two questions, introduced the day before, on symbolic representations of brick patterns (S = standing or short, L = lying or long). Peter deployed a number of RME strategies – drawing students into discussion, remaining neutral (i.e. not responding either positively or negatively to contributions), probing explanations to elicit justification (e.g., asking "because?"). We pick up the thread five minutes into the lesson, as the class moves from one question to the next. Peter flags up his assessment that the students will find the ideas challenging:

Teacher: Okay. So, this is where it can get a little more confusing. [Reading out question on the board.] Mike [a fictional character in the question] also says
Gina: [softly but audibly] Oh, not Mike!
Teacher: "SLSLSLSL could be written as: length of row = 4(S+L)". Thirty seconds with your partner, can you explain this?

[Students embark on 2 minutes of discussion, Gina can be heard saying "BODMAS, that's BODMAS." BODMAS is an acronym used in the UK for order of operations in arithmetic: first Brackets, then Order (exponentiation), then Division, ...]

Teacher: Gina, you like talking, what are you thinking? What's Mike thinking?
Gina: I don't know ...
Teacher: Why ... can you try and explain?
Gina: Why don't you ask Mike?
Teacher: Well, I can't. Mike's not here. That's why I'm asking you.
Gina: Does he go to a different school? [other student starts giggling]
Teacher: Yes.
Gina: What school does he go to? [breaking into giggle]
Teacher: School in Manchester. [a few students cry out "MAN CHES TUH!"]
Gina: How do you know?
Teacher: So ... because I do. What's he trying to do?
Gina: He's trying to do BODMAS.
Teacher: Why do you say that?
Gina: BODMAS? Because it looks similar to BODMAS. Because he has to use BODMAS. [in the background, a student asks, "is that true?"]
Teacher: Okay, yeah. You are true. The ... the ... we will have to do … brackets and ... so there is an element of that. Can you ask someone else to carry on for me?
Gina: [tentatively] Ask someone else …
Teacher: In the room. Pick someone else in the room. Don't have to have their hand up. [Gina nominates Anna ... giggling ...]
Teacher: Anna. Surprise, surprise ... Anna, can you explain what Mike is thinking?
Anna: No, I can't. ... I don't understand it.

[The lesson opens into a wider discussion with a number of students, including one working at the board who takes it back to the context of the bricks, but Peter isn't satisfied with the responses. We rejoin the lesson as Peter writes expressions on the board.]

Gina: [excited] They are all patterns. A pattern with brackets, a pattern with numbers, a pattern with letters.

Teacher: I know you are excited and loving maths but we need to stop shouting and raise your hand.

From the start, we hear that Gina is unsympathetic to the presentation of this question, perhaps critical of the use of the unknown but all-knowing "Mike" ("Oh, not Mike!"). We hear her talking about BODMAS, in response to the fictitious Mike's suggestion that the letters could be written as 4(S+L). Peter calls on her to share her thinking, but undermines any inclusive aspect of this move by positioning her as inappropriately talkative ("You like talking"). Gina appears to resist the doubled-edged message here (talk/be silent) by suggesting that Peter should ask Mike what he is thinking, forcing him to admit Mike's fictional nature, and to comment on his teaching strategy ("That's why I'm asking you"). As Peter pursues his request that Gina should tell the class what Mike is thinking, she neatly uses this as an opening to reintroduce her initial idea, now co-opting Mike as her mouthpiece: "He's trying to do BODMAS". Peter's ultimate dismissal of Gina's contribution is protracted, in his reluctant acceptance that BODMAS is indeed involved ("there is an element of that") and his invitation to pass on to someone else, which leads to some sarcasm about Gina's choice ("Anna. Surprise surprise"), to his later explicit request to "stop shouting and raise your hand".

Recalling this episode in the post-lesson interview, and in response to a question about whether the student's introduction of formal mathematics 'got in the way', Peter says:

> So – I pre-empted it. I knew that – I was surprised that it was the girl in question, because actually she's one of my weakest. So, for her to be shouting that out – and she always does – was actually good. I didn't ignore it but I sort of brushed it aside, and said "Oh, it's BODMAS, you did the brackets first. Oh, brilliant! OK, so how does that work in this context then?"

Peter would like his students to see beyond the procedural bracket operations and use the context of the bricks to notice that SLSLSLSL and 4(S+L) both represent "4 lots of S and L". He sees RME as delivering on making algebra meaningful: "What I think is really good in the RME is it ... brings in the concepts ... where the model is going to come from". But, we also see here that Peter's representation of events suggests praise for Gina, as a girl marked as "one of my weakest" who is "always" "shouting out", but this time has contributed something good - BODMAS. But there is a chasm to be bridged and, after working with several students, Peter invokes a mythical student, George, who 'provides' the bridging explanation. Peter explains this strategy:

> Sometimes, I have to use George and Finn and different ways and it's mainly when the students aren't bringing them up themselves. And I know ... it will start a different discussion and hopefully unlock a few more doors for certain students.

While the student discussion may not have generated the desired mathematics, we see Peter persisting in his endeavour to achieve his "inclusive" purposes: raising student engagement and enabling at least some students to understand the material. As the lesson progresses, students are able to connect with the initial context, with varying degrees of success, and have opportunities to exercise choice – which strategies they employ, which problems they work on – and to relate to the open-ended nature of the problems. Given the nature of some of the RME contexts (fair sharing, air travel, working in a fish and chip shop, national elections, ...), there were opportunities for discussions related to social justice but these fledgling discussions, when they did arise, were not connected to the mathematics. Ultimately, Peter's focus was on providing students with access to methods and knowledge by "giving them that stepping stone of realism" rather than building on what is real for them in order to make sense of both mathematics and the 'real world'.

What does RME achieve for Peter?

For Peter, RME provides a mechanism for students to participate and become more confident in mathematics, and his own role as 'facilitator':

> I really love teaching RME because, actually, it's given me a way into them, to show that they can be enthusiastic and try different things. ... I think the more I get them talking, the less I have to talk. So, I become sort of that facilitator that I want to be for RME.

Peter says that RME has "massively changed my approach to teaching maths ... opening my eyes to lots of realistic informal models, which I could then go in and use." He prioritises providing an accessible entry point to formal mathematics and developing functional mathematical skills over following through on his students' responses. Yet, Peter does not simply use RME for its "realistic informal models". He deploys a number of RME strategies to elicit student explanations and foster discussion, and recognises the value of giving time to discussion - to strengthen both his relationship with the students and their relationship with mathematics. Indeed, the students take these opportunities to engage with contexts, exercise their creative/resistant agency, and try to connect their various mathematical understandings to the materials. Given this mixed and nuanced picture, and the partially tapped potential of RME, how could we have better supported our teachers' reading, appropriation and enactment of RME? We pick this up in our concluding remarks.

Discussion: Just mathematics?

Inclusion is a complex concept and its relationship to ostensibly wider aims of mathematics teaching for social justice remains up for debate. In their exit interviews, teachers connected to the inclusion components of RME on a broad spectrum. Some approached inclusion from a nurturance stand-point, while others invoked the importance of finding ways to inspire. Some, however, were very precise in naming the twin force of emergent mathematics, identifying the way in which students' experiences can reverberate across mathematical thinking.

As we see in the example of Peter, putting these good intentions into practice is not straightforward. Peter appropriates and deploys certain elements of RME (e.g., informal models and strategies for fostering discussion), but, in doing so, he voices potentially damaging microaggressions (Ball, 2021) in the name of inclusive discussion. This points to the need for inclusion to become an "object of reflection" in future RME CPD projects. More explicit discussions about the social construction of mathematical ability and the way that mathematics *lives inside* of problems and contexts might have gone some way in nuancing Peter's rhetorical style. If possible, CPD should also support teachers in reworking RME materials to harness contexts most salient to their students' lives. These solutions, however, do not address Gutstein's (2006) rightful complaint that students should understand mathematics as a tool for "reading and writing the world" – reframing and uncovering injustice. As in Peter's lessons, there were opportunities during our training sessions for deepening discussion around the social reality of certain contexts. For example, one activity – ranking British Prime Ministers by age – elicited active discussion on elitism and colonial governance. But, we didn't always build on this. Just as RME teachers responded to pressures in their classrooms, or missed building on some student responses, so did we.

This brings us to the question of what counts in and as mathematics, what counts as worthy of mathematical measurement: Whose everyday activities take precedence in mathematising our shared world? While Peter may not have implemented an 'ideal' RME, he has evidence that overall his students are doing better – on assessments, in making sense of mathematics, and at 'having a go'. For him, this is a vindication of investing in RME. Mathematical empowerment today could open up mathematics for social justice tomorrow. In which case, is just focusing on 'the mathematics' a viable starting point? As Martin, Gholson and Leonard (2010) point out, one cannot separate mathematical practice from social justice, equity and inclusion. Our experience of supporting teachers to become adept at using RME materials in their desire to enhance students' access to mathematics has highlighted the need to address inclusion more explicitly as an object of reflection with respect to both curriculum and pedagogy.

References

Ahmed, S. (2012). *On being included.* Durham, NC: Duke University Press.

Ball, D. (2021, January 27). Making mathematics teaching work: Raising its power to disrupt white supremacy [Video]. YouTube. https://www.youtube.com/watch?v=6hgFxAWe0I8&t=11s

Delpit, L. D. (2006). *Other people's children: Cultural conflict in the classroom.* London, New York, NY: New Press.

Dickinson, P., Eade, F., Gough, S. & Hough, S. (2010). Using Realistic Mathematics Education with low to middle attaining pupils in secondary schools, in M. Joubert & P. Andrews (Eds.). *Proceedings of the British Congress for Mathematics Education* (pp. 73–80).

González, N., Andrade, R., Civil, M., & Moll, L. C. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, 6(1-2), 115–132.

Gravemeijer, K. & Stephan, M. (2002). Emergent Models as an instructional design heuristic. In K. P. E. Gravemeijer, R. Lehrer, B. v. Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 145–169). Dordrecht: Kluwer.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice.* New York: Routledge.

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race, Ethnicity and Education, 22*(4), 459–478.

Martin, D. B., Gholson, M. L., & Leonard, J. (2010). Mathematics as gatekeeper: Power and privilege in the production of knowledge. *Journal of Urban Mathematics Education, 3*(2), 12–24.

Moses, R. & Cobb, C. (2001). *Radical equations: Civil rights from Mississippi to the Algebra Project.* Boston, MA: Beacon Press.

NCRMSE & FI (National Center for Research in Mathematical Sciences Education & Freudenthal Institute). (1997–1998). *Mathematics in context: A connected curriculum for grades 5–8.* Chicago: Encyclopedia Britannica.

Stephan, M., Underwood-Gregg, D. & Yackel, E. (2014). Guided reinvention: What is it and how do teachers learn this teaching approach? in Y. Li, E. A. Silver, & S. Li (Eds.). *Transforming mathematics instruction* (pp. 37–57). Dordrecht, NL: Springer.

Solomon, Y., Hough, S., Gough, S. (2021). The role of appropriation in guided reinvention: Establishing and preserving devolved authority with low-attaining students, *Educational Studies in Mathematics, 106*(2), 171–188.

Wubbels, T., Korthagen, F., & Broekman, H. (1997). Preparing teachers for Realistic Mathematics Education. *Educational Studies in Mathematics, 32*, 1–28.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.