

**GRA BASED MULTI CRITERIA DECISION MAKING IN GENERALIZED NEUTROSOPHIC SOFT SET ENVIRONMENT****Surapati Pramanik*, Shyamal Dalapati**

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DOI:**KEYWORDS:** Neutrosophic set, soft set, generalized neutrosophic soft set, multi-attribute group decision making, weighted average operator.**ABSTRACT**

The main objective of the paper is to propose generalized neutrosophic soft multi criteria decision making based on grey relational analysis. The concept of generalized neutrosophic soft sets has been derived from the hybridization of the concepts of neutrosophic set and soft set. In this paper we have defined neutrosophic soft weighted average operator in order to aggregate the individual decision maker's opinion into a common opinion based on choice parameters of the evaluators. In the decision making process, the decision makers provide the rating of alternatives with respect to the parameters in terms of generalized neutrosophic soft set. We determine the order of the alternatives and identify the most suitable alternative based on grey relational coefficient. Finally, in order to demonstrate the effectiveness and applicability of the proposed approach, a numerical example of logistics center location selection problem has been solved.

INTRODUCTION

Evolution of human society evokes complexity in their life and human beings have to deal with uncertainty, imprecise data to solve their real life problems. To deal uncertainty, mathematicians proposed a number of theories such as probability [1], fuzzy sets [2], interval mathematics, etc. Molodtsov [3] described the limitations of these theories in his study and grounded the concept of soft set theory to overcome the difficulties in 1999. Soft set theory has been successfully applied in data analysis [4], optimization [5], etc. The researchers have showed great interest in the theory and they proposed different hybrid soft sets and their applications such as fuzzy soft set [6,7,8], generalized fuzzy soft set [9,10], intuitionistic fuzzy soft set [11,12], possibility intuitionistic fuzzy soft set [13], vague soft set [14], neutrosophic soft set [15], weighted neutrosophic soft set [16], generalized neutrosophic soft set [17, 18]. However, neutrosophic set [19, 20, 21, 22, 23], is the generalization of fuzzy set, and intuitionistic fuzzy set. In 2010, Wang et al. [24] defined single valued neutrosophic set, which is an instance of neutrosophic set. Neutrosophic set and single valued neutrosophic sets have been successfully applied in different research areas such as social sciences [25, 26, 27], conflict resolution [28], artificial intelligence and control systems [29], medical diagnosis [30, 31, 32, 33], decision making [34, 35, 36, 37, 38, 39, 40, 41, 42, 43], image processing [44, 45], decision making in neutrosophic hybrid environment [46, 47, 48, 49, 50, 51, 52, 53, 54]. Neutrosophic sets and soft set sets are two different concepts. Literature review suggests that both are capable of handling uncertainty and incomplete information. It seems that the hybrid system called 'generalized neutrosophic soft set' is capable of dealing with uncertainty, indeterminacy and incomplete information. It seems that generalize neutrosophic soft set is very interesting and applicable in realistic problems. Literature review reveals that only few studies on generalized neutrosophic soft sets [17, 18, 55, 56] have been done.

Deng [57] studied grey relational analysis (GRA). GRA has been applied widely different areas of research such as teacher selection [58], weaver selection [59], brick optimal welding parameter selection [60], failure mode and effects analysis [61], multi attribute decision making (MADM) [62], multi criteria decision making [63, 64, 65], medical diagnosis [66], etc. Biswas et al. [67, 68] at first used the concept of GRA in neutrosophic environment for MADM problems. Pramanik and Mondal [69] studied interval neutrosophic multi-attribute decision-making based on grey relational analysis. Mondal and Pramanik [70] presented a neutrosophic school choice model based



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on modified GRA method. Mondal and Pramanik [71] also introduced rough neutrosophic MADM based on modified GRA. Dey et al. [72] studied neutrosophic soft multi-attribute group decision making based on grey relational analysis. Dey et al. [73] presented neutrosophic soft multi-attribute decision making based on grey relational projection method. Dey et al. [74] studied extended grey relational analysis method for multiple attribute decision making problems under interval neutrosophic uncertain linguistic environment.

GRA based MCDM problem in generalized neutrosophic soft set environment has not yet been addressed in the literature. In this paper, we have presented a generalized neutrosophic soft multi-attribute group decision making model with grey relational analysis.

Rest of the paper is organized in the following way. Section 2 presents some basic definitions of neutrosophic set, generalized neutrosophic set, soft set, generalized neutrosophic soft set and real life example on generalized-neutrosophic soft set. Section 3 presents the grey relational analysis. Section 4 is devoted to present a compact model for a generalized neutrosophic-soft MAGDM based on grey-relational analysis. Section 5 presents an illustrative example to show the applicability of the proposed model. Finally, section 6 presents the conclusion and future direction of research work.

PRELIMINARIES

In this section, we will give the basic concept of neutrosophic set, generalized neutrosophic set, soft set and neutrosophic soft set, generalized neutrosophic soft set.

Definition of neutrosophic set [19, 20]

Let U be a space of points (objects) with generic element in U denoted by u i.e. $u \in U$. A neutrosophic set A in U is denoted by $A = \{ \langle u: T_A(u), I_A(u), F_A(u) \rangle \mid u \in U \}$ where T_A, I_A, F_A represent membership, indeterminacy and non-membership function respectively. T_A, I_A, F_A are defined as follows:

$$T_A : U \rightarrow]^{-} 0, 1^{+}[$$

$$I_A : U \rightarrow]^{-} 0, 1^{+}[$$

$$F_A : U \rightarrow]^{-} 0, 1^{+}[$$

Where $T_A(u), I_A(u), F_A(u)$ are the real standard and non-standard subset of $]^{-} 0, 1^{+}[$ such that $^{-} 0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3^{+}$

Since, $T_A(u), I_A(u), F_A(u)$ assume the values from the subset of $]^{-} 0, 1^{+}[$, so we take $[0, 1]$ instead of $]^{-} 0, 1^{+}[$ due to the application in real life situation because $]^{-} 0, 1^{+}[$ will be complicated to apply the real life problem with neutrosophic nature.

Definition: Single valued neutrosophic set [24]

Let U be a space of points with generic element in U denoted by u i. e. $u \in U$. A single valued neutrosophic set G in U is characterized by a truth-membership function $T_G(u)$, an indeterminacy-membership function $I_G(u)$ and a falsity-membership function $F_G(u)$, for each point u in U , $T_G(u), I_G(u), F_G(u) \in [0, 1]$, when U is continuous then single-valued neutrosophic set G can be written as

$$G = \int_U \langle T_G(u), I_G(u), F_G(u) \rangle / u, u \in U$$

When U is discrete, single-valued neutrosophic set can be written as

$$\sum_{i=1}^n \langle T_G(u_i), I_G(u_i), F_G(u_i) \rangle / u_i, u_i \in U$$

Complement of neutrosophic set [19, 20]

The complement of a neutrosophic set A is denoted by A' and defined as

$$A' = \{ \langle u: T_{A'}(u), I_{A'}(u), F_{A'}(u) \rangle, u \in U \}$$

$$T_{A'}(u) = \{ 1^{+} \} - T_A(u)$$

$$I_{A'}(u) = \{ 1^{+} \} - I_A(u)$$



$$F_A(u) = \{1^+\} - F_A(u)$$

Definition: 2.4 [19, 20]

A neutrosophic set M is contained in another neutrosophic set L i.e. $M \subseteq L$ if for all $a \in U$,

$$T_M(u) \leq T_L(u)$$

$$I_M(u) \leq I(u)$$

$$\text{And } F_M(u) \leq F_L(u)$$

Definition: 2.5 Generalized neutrosophic set [75]

Let U be a space of point with generic element in U denoted by u . Let M be a neutrosophic set in U denoted by $M = \langle u, T_M(u), I_M(u), F_M(u), u \in U \rangle$ is said to be generalized neutrosophic set if

$$T_M(u) \wedge I_M(u) \wedge F_M(u) \leq 0.5$$

Where $T_M(u), I_M(u), F_M(u)$ represent degree of membership function, indeterminacy function and non-membership function respectively.

Definition: 2.6 Soft set [3]

Let U be an initial universe set P is the set of parameters. Let B be non-empty subset of P i.e. $B \subset P$. Let $\mathbb{P}(U)$ be the power set of U . Then the order pair (S, B) is called soft set over U , where S is the mapping from B to $\mathbb{P}(U)$ i. e. $S: B \subset P \rightarrow \mathbb{P}(U)$

Definition: 2.7 Neutrosophic soft set [15]

Let U be the universe set and $N(U)$ denote the set of all neutrosophic subset of U . Let P be the set of all parameter and B is the non-empty sub-set of P i.e. $B \subset P$, then the order pair (S, B) is said to be neutrosophic soft set if $S: B \rightarrow N(U)$.

Definition: 2.8 Generalized neutrosophic soft set [17]

Let U be an initial universe set and $N(U)$ denote the set of all neutrosophic subset of U , Let P be the set of parameters and B be the non-empty subset of P i.e. $B \subset P$ then the order pair (S, μ) is said to be generalized neutrosophic soft set over U if

$$S: P \rightarrow N(U)$$

$$\mu: P \rightarrow [0, 1] = I \text{ i.e. } \mu \text{ is the fuzzy set}$$

Combining this two mapping we represent generalized neutrosophic soft set as

$$S^\mu: P \rightarrow N(U) \times I$$

$$S^\mu = \{(S(p_1), \mu(p_1)): p_1 \in P, S(p_1) \in N(U), \mu(p_1) \in [0,1] = I\}$$

For each $p_1 \in P$, $S(p_1)$ denotes the neutrosophic value of the parameter p_1 . $S(p_1)$ is presented as follows:

$$S(p_1) = \langle u, T_{S(p_1)}(u), I_{S(p_1)}(u), F_{S(p_1)}(u), u \in U \rangle \text{ where } T, I, F: U \rightarrow [0, 1]$$

Where T, I, F are the truth, indeterminacy and falsity membership function of the element $u \in U$ such that $0 \leq T_{S(p)}(u) + I_{S(p)}(u) + F_{S(p)}(u) \leq 3$.

Here, S^μ is the parameterized family of neutrosophic sets over U , which has the degree of possibility of the approximate value set, which is denoted by $\mu(p)$ for any parameter $p \in P$. Here, $\mu(p)$ also represents the importance of parameter p . The importance of the parameter p is provided by the decision maker. So S^μ can be defined as follows:

$$S^\mu(p) = \left\{ \frac{u_1}{S(p)(u_1)}, \frac{u_2}{S(p)(u_2)}, \dots, \frac{u_n}{S(p)(u_n)}, \mu(p), u_1, u_2, \dots, u_n \in U \right\}$$

Example:

Consider a generalized neutrosophic soft set S^μ , where U is the set of location. We select a location for logistic center on the basis of the parameters (P), namely cost, distance to suppliers, distance to customers, conformance to governmental regulation and law, quality of service and environmental impacts i.e.

$$U = \{l_1, l_2, l_3, l_4\} \text{ and } P = \{p_1, p_2, p_3, p_4, p_5, p_6\}$$

p_1 (stand for) = Cost

p_2 (stand for) = Distance to suppliers

p_3 (stand for) = Distance to customers

p_4 (stand for) = Conformance to governmental regulation and law

p_5 (stand for) = Quality of service



$$S^\mu(p_1) = \left\{ \frac{l_1}{(.6,.4,.3)}, \frac{l_2}{(.5,.3,.2)}, \frac{l_3}{(.3,.5,.3)}, \frac{l_4}{(.7,.5,.3)}, (.4) \right\}$$

$$S^\mu(p_2) = \left\{ \frac{l_1}{(.5,.3,.4)}, \frac{l_2}{(.3,.5,.6)}, \frac{l_3}{(.6,.7,.2)}, \frac{l_4}{(.5,.2,.4)}, (.2) \right\}$$

$$S^\mu(p_3) = \left\{ \frac{l_1}{(.7,.3,.4)}, \frac{l_2}{(.8,.2,.4)}, \frac{l_3}{(.6,.1,.2)}, \frac{l_4}{(.6,.5,.4)}, (.6) \right\}$$

$$S^\mu(p_4) = \left\{ \frac{l_1}{(.3,.5,.5)}, \frac{l_2}{(.5,.5,.5)}, \frac{l_3}{(.7,.8,.2)}, \frac{l_4}{(.3,.5,.6)}, (.3) \right\}$$

$$S^\mu(p_5) = \left\{ \frac{l_1}{(.5,.3,.2)}, \frac{l_2}{(.6,.3,.4)}, \frac{l_3}{(.7,.3,.2)}, \frac{l_4}{(.4,.6,.3)}, (.5) \right\}$$

$$S^\mu(p_6) = \left\{ \frac{l_1}{(.7,.2,.3)}, \frac{l_2}{(.8,.3,.2)}, \frac{l_3}{(.5,.4,.3)}, \frac{l_4}{(.6,.2,.3)}, (.5) \right\}$$

We express the above generalized neutrosophic soft set in matrix form as follows:

$$\begin{pmatrix} (.6,.4,.3)(.5,.3,.2)(.3,.5,.3)(.7,.5,.3)(.4) \\ (.5,.3,.4)(.3,.5,.6)(.6,.7,.2)(.5,.2,.4)(.2) \\ (.7,.3,.4)(.8,.2,.4)(.6,.1,.2)(.6,.5,.4)(.6) \\ (.3,.5,.5)(.5,.5,.5)(.7,.8,.2)(.3,.5,.6)(.3) \\ (.5,.3,.2)(.6,.3,.4)(.7,.3,.2)(.4,.6,.3)(.5) \\ (.7,.2,.3)(.8,.3,.2)(.5,.4,.3)(.6,.2,.3)(.5) \end{pmatrix}$$

The above matrix has been constructed only for one generalized neutrosophic soft set i.e. for only one decision maker. If the problem consists of D decision makers and L locations/ objects and each location has p parameters, then we can obtain D no. of generalized neutrosophic soft set i.e. D number of matrix having p number of rows and L+1 number of columns. Last column of the matrix represents the degree of possibility of each parameter to the decision makers.

Definition: 2.9 Null or empty generalized neutrosophic soft set [17]

A generalized neutrosophic soft set S^μ over U is said to be a null generalized neutrosophic soft set if $\mu(p) = 0$ and $S(p) = \langle u; T_{S(p)}(u), I_{S(p)}(u), F_{S(p)}(u), u \in U \rangle = \langle u, 0, 0, 0; u \in U \rangle$

i.e. $T_{S(p)}(u) = 0, I_{S(p)}(u) = 0, F_{S(p)}(u) = 0 \quad p \in P = \text{parameter and } \forall u \in U.$

Null or empty generalized neutrosophic soft set can be denoted by \emptyset^μ and defined by

$\emptyset^\mu: P \rightarrow N(U) \times I$

$\mu: P \rightarrow I = [0,1]$

$\emptyset^\mu = \langle \emptyset(p), \mu(p) \rangle$

Definition: 2.10 Absolute generalized neutrosophic soft set [17]



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Absolute or extreme generalized neutrosophic soft set is the generalized neutrosophic soft set which obtain extreme value of the neutrosophic components for all $p \in P$ and $u \in U$ over U , which can be denoted by \tilde{A}^η and defined by $\tilde{A}^\eta = \langle u, \tilde{A}(p), \eta(p) : p \in P, \tilde{A}(p) \in N(U) \ u \in U \ \eta(p) \in [0,1] \rangle$

$\tilde{A}: P \rightarrow N(U)$

And $\eta: P \rightarrow I = [0,1]$

Such that $\eta(p) = 1$,

$\tilde{A}(p) = \langle u, T_{\tilde{A}(p)}(u), I_{\tilde{A}(p)}(u), F_{\tilde{A}(p)}(u), u \in U \rangle$

$= \langle U: 1, 1, 1 \rangle$

i.e. $\forall u \in U, T_{\tilde{A}(p)}(u) = 1, I_{\tilde{A}(p)}(u) = 1, F_{\tilde{A}(p)}(u) = 1$

Definition: 2.11 Generalized neutrosophic soft subset [18]

Let M^μ and N^η be two generalized neutrosophic soft set over U .

M^μ is said to be generalized neutrosophic soft subset of N^η if $\mu \subseteq \eta$ and M is the neutrosophic subset of N .

i.e. $T_M(u) \leq T_N(u), I_M(u) \leq I_N(u)$ and $F_M(u) \geq F_N(u)$ for any $u \in U$ and for any $p \in P$ denoted by $M^\mu \subseteq N^\eta$.

Again if $N^\eta \subseteq M^\mu$ then $M^\mu = N^\eta$

Definition: 2.12 Complement of generalized neutrosophic soft set [17]

Complement of a generalized neutrosophic soft set S^μ over U is denoted by $S^{\mu'}$ and defined by

$S^{\mu'} = \{S'(p), \mu'(p), p \in P, S: P \rightarrow N(U), \mu: P \rightarrow [0, 1]\}$

Such that $\mu'(p) = 1 - \mu(p)$ and

$S'(p) = \langle u, T_{S'(p)}(u), I_{S'(p)}(u), F_{S'(p)}(u) \rangle, u \in U, p \in P$

$= \langle u, F_{S(p)}(u), 1 - I_{S(p)}(u), T_{S(p)}(u) \rangle, u \in U, p \in P$

Definition: 2.13 Union of two generalized neutrosophic soft set [17]

Suppose M_1^μ and M_2^η are two generalized neutrosophic soft set over U . The union of two generalized neutrosophic soft sets denoted by $M^\xi = M_1^\mu \cup M_2^\eta$, has been defined by

$M^\xi = \{(M(p), \xi(p)), p \in P, M(p) \in N(U), \xi(p) \in [0,1]\}$

Where $M: P \rightarrow N(U)$

$\xi: P \rightarrow [0,1]$

$M^\xi: P \rightarrow N(U) \times I$

$p \in P, M(p)$ expressed as $M(p) = \langle u, T_{M(p)}(u), I_{M(p)}(u), F_{M(p)}(u) \rangle$

where $T_{M(p)}(u) = \max \{T_{M_1(p)}(u), T_{M_2(p)}(u)\}$

$I_{M(p)}(u) = \max \{I_{M_1(p)}(u), I_{M_2(p)}(u)\}$

$F_{M(p)}(u) = \min \{F_{M_1(p)}(u), F_{M_2(p)}(u)\}$

$\xi(p) = \max \{\mu(p), \eta(p)\}, \forall p \in P = \text{parameters}$

Definition: 2.14 Intersection of two generalized neutrosophic soft set [17]

Assume that M_1^μ and M_2^η be two generalized neutrosophic soft set over the same universe U . The intersection of two sets denoted by $M^{\hat{h}} = \{(M_3(p), \hat{h}(p)): p \in P, M_3(p) \in N(u), \hat{h}(p) \in [0,1]\}$

$M_3(p)$ can be expressed as $M_3(p) = \langle u, T_{M_3(p)}(u), I_{M_3(p)}(u), F_{M_3(p)}(u) \rangle$

Where $T_{M_3(p)}(u) = \min \{T_{M_1(p)}(u), T_{M_2(p)}(u)\}$

$I_{M_3(p)}(u) = \min \{I_{M_1(p)}(u), I_{M_2(p)}(u)\}$

$F_{M_3(p)}(u) = \max \{F_{M_1(p)}(u), F_{M_2(p)}(u)\}$

$\hat{h}(p) = \min \{\mu(p), \eta(p)\} \ \forall p \in P$

Conversion between linguistic variables and Single valued neutrosophic numbers



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A linguistic variable refers to a variable whose values are represented by words or sentences in natural or artificial languages. Importance of the decision makers in the decision making process may not be equal. It can be expressed using linguistic variables such as very important, important, medium important, unimportant, very unimportant, etc. We have presented a conversion method between linguistic variables and single valued neutrosophic number (see the Table- 1).

Table 1. Conversion between linguistic variables and single valued neutrosophic numbers

Linguistic variables	Single valued neutrosophic numbers
Very important(VI)	(0.90, 0.10, 0.10)
Important(I)	(0.80, 0.20, 0.20)
Medium important(MI)	(0.50, 0.25, 0.50)
Unimportant(UI)	(0.20, 0.20, .80)
Very unimportant(VUI)	(0.10, 0.10, 0.90)

GREY-RELATIONAL ANALYSIS [57]

We now present the process for finding the grey relational co-efficient to ranking the alternatives according the largest degree of grey relation coefficient. Let Y_0 be the referential sequence and Y_i be the comparative sequence at point t. Then grey relation co-efficient $\sigma(Y_0(t), Y_i(t))$ satisfies the four conditions

Normal interval

$$0 < \sigma(Y_0, Y_i) \leq 1$$

$$\sigma(Y_0, Y_i) = 1 \Leftrightarrow y_0 = y_i$$

$$\sigma(Y_0, Y_i) = 0 \Leftrightarrow y_0, y_i \in \Phi \text{ where } \Phi \text{ is empty set}$$

Dual symmetry

$$Y_0, Y_i \in Y$$

$$\sigma(Y_0, Y_i) = \sigma(Y_i, Y_0) \Leftrightarrow \{Y_0, Y_i\}$$

Wholeness:

$$\sigma(y_0, y_i) \stackrel{\text{often}}{\neq} \sigma(y_i, y_0)$$

Approachability

If $|Y_0(t) - Y_i(t)|$ approaching larger then σ reduces to smaller. The grey relational co-efficient [57] of the referential sequences and comparative sequence at point t, can be expressed as follows:

$$\sigma(y_0(t), y_i(t)) = \frac{\min_i \min_t |y_0(t) - y_i(t)| + \rho \max_i \max_t |y_0(t) - y_i(t)|}{|y_0(t) - y_i(t)| + \rho \max_i \max_t |y_0(t) - y_i(t)|} \quad (1)$$

$\rho \in [0, 1]$ refers to the distinguishable co-efficient used to adjust the range of the comparison environmental and to control level of differences of the relation co-efficient. When $\rho = 0$ comparison environment disappears and when $\rho = 1$, the compassion environment is unaltered. Generally, $\rho = .5$ is considered for decision making environment.

A GENERALIZED NEUTROSOPHIC SOFT MAGDM BASED ON GREY RELATIONAL

Assume that $L = \{L_1, L_2, \dots, L_m\}$ ($m \geq 2$) be the discrete set of alternatives, $A = \{a_1, a_2, \dots, a_n\}$ ($n \geq 2$) be the set of decision makers and $P = \{p_1, p_2, p_3, \dots, p_r\}$ be the set of choice parameters provided by the decision makers.



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The weights of the decision makers are unknown but the weights of the parameters are known from definition of generalized neutrosophic soft set. The ratings of the alternatives and importance of the choice-parameters are provided by the decision makers in the form of generalized neutrosophic soft sets. The steps for solving MAGDM by proposed approach have been presented below.

Step: 1 Formation of generalized neutrosophic soft decision matrix

Selection of key parameters is regarded as one of the important issue in a MAGDM problem. The key parameters are generally provided by the evaluator. Assume that the rating of alternative L_i ($i = 1, 2, \dots, m$) with respect to the parameter p_i ($i = 1, 2, \dots, r$) provided by the k -th ($k = 1, 2, \dots, n$) DM is represented by GNSSs ($M_k^{\lambda_k}$) ($k = 1, 2, \dots, n$) and they can be presented in the matrix form $\langle a_{ij}^k; \lambda_k(p_i) \rangle$ ($k = 1, 2, \dots, n; i = 1, 2, \dots, r; j = 1, 2, \dots, m$). Therefore, the decision matrix of k -th decision maker can be represented as follows:

$$A^k = \langle a_{ij}^k; \lambda_k(p_i) \rangle \quad (2)$$

Here $a_{ij}^k = \langle T_{ij}^k, I_{ij}^k, F_{ij}^k \rangle$, $T_{ij}^k, I_{ij}^k, F_{ij}^k \in [0, 1]$ and $0 \leq T_{ij}^k + I_{ij}^k + F_{ij}^k \leq 3$; $\lambda_k(p_i) \in [0, 1]$ $i = 1, 2, \dots, r; j = 1, 2, \dots, m; k = 1, 2, \dots, n$.

Step: 2 Determination of the weight of the decision makers

In the group decision making process the weights of the decision makers are very crucial for decision making [58]. Assume that the group decision making unit consists of n decision makers. The importance of the decision makers in the group decision making process may not be equal. The importance of the decision makers may be expressed as linguistic variables and the linguistic variables can be converted into single valued neutrosophic numbers (see table 1). Assume that $D_q = (\alpha_q, \beta_q, \delta_q)$ be a single valued neutrosophic number that represents the rating of the q -th decision maker. Then the weight of the q -th decision maker [76] can be presented as follows:

$$\Psi_q = \frac{\alpha_q + \beta_q \left(\frac{\alpha_q}{\alpha_q + \delta_q} \right)}{\sum_{q=1}^n \alpha_q + \beta_q \left(\frac{\alpha_q}{\alpha_q + \delta_q} \right)} \quad (3)$$

$$\text{And } \sum_{q=1}^n \Psi_q = 1$$

This expression is the extension of the work of Boran et al. [77] in intuitionistic fuzzy number.

If we consider the importance of the all decision makers is same, then the weight of the decision makers will be $(1/n)$.

Step: 3 Aggregation of the weights of the parameters

The importance of parameter depends on decision maker's choice. In this paper, we have defined generalized neutrosophic soft weighted aggregate operator for aggregation of the weights of the parameters as follows:

$$\lambda(p_i) = \left(1 - \prod_{k=1}^n (1 - \lambda_{(k)}(p_i))^{\Psi_k} \right) \quad (4)$$

Step: 4 Construction of the aggregated generalized neutrosophic soft decision matrix

In the group decision making situation, all the individual assessments require to be combined into a group opinion based on neutrosophic soft weighted average operator. Let A be the aggregate decision matrix, then A has been defined as follows:

$$A = \langle a_{ij}; \lambda \rangle = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m}, \lambda(p_1) \\ a_{21} & a_{22} & \dots & a_{2m}, \lambda(p_2) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{r1} & a_{r2} & \dots & a_{rm}, \lambda(p_r) \end{pmatrix} \quad (5)$$

Here, $a_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ where $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$ and $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$, $i = 1, 2, \dots, r; j = 1, 2, \dots, m$.



$$\text{and } T_{ij} = 1 - \prod_{k=1}^n (1 - T_{ij}^{(k)})^{\psi_k}, \quad I_{ij} = \prod_{k=1}^n (I_{ij}^{(k)})^{\psi_k},$$

$$F_{ij} = \prod_{k=1}^n (F_{ij}^{(k)})^{\psi_k}, \quad (6)$$

$$\lambda(p_i) = (1 - \prod_{k=1}^n (1 - \lambda_{(k)}(p_i))^{\psi_k}) \quad (7)$$

Step: 5 Determination of the reference sequence based on generalized neutrosophic soft set

$$\text{Let } a^* = ((T_1^*, I_1^*, F_1^*), (T_2^*, I_2^*, F_2^*), \dots, (T_r^*, I_r^*, F_r^*))^T$$

$$\text{Where } a_{ij}^* = (T_i^*, I_i^*, F_i^*) = (\max_j T_{ij}, \min_j I_{ij}, \min_j F_{ij}), \text{ where } i = 1, 2, 3, \dots, r;$$

Reference sequence should be characterized by the optimal sequence of the criteria values. 1, 0, 0 are the values of the aspired levels of the membership function, indeterminacy functions, falsity (non-membership) function, respectively. Therefore, the point consisting of highest membership value, minimum indeterminacy, minimum falsity (non-membership) value would represent the reference value or ideal point or utopia point. For generalized neutrosophic soft decision matrix the maximum value $a_i^* = (1, 0, 0)$ can be used as the reference value, then the reference sequence can be represented as follows:

$$a^* = [(1, 0, 0), (1, 0, 0), \dots, (1, 0, 0)]^T$$

Step: 6 Calculation of the grey relational coefficient

The calculation of the grey relational coefficient for each alternative can be defined as follows:

$$\sigma_{ij}(y_i(t), y_j(t)) = \frac{\min_i \min_j |a_{ij} - a_{ij}^*| + \rho \max_i \max_j |a_{ij} - a_{ij}^*|}{|a_{ij} - a_{ij}^*| + \rho \max_i \max_j |a_{ij} - a_{ij}^*|} \quad (8)$$

σ_{ij} is the grey relational coefficient and $\rho \in [0, 1]$ is the distinguishing coefficient.

Step: 7 Calculation of degree of grey relational coefficient

We calculate the degree of the grey relation coefficient of each alternative using grey relational coefficient and aggregate parameter weights by the equation (9).

$$\Omega_j = \sum_{i=1}^r \sigma_{ij} \lambda(p_i); j = 1, 2, \dots, m \quad (9)$$

Step: 8 Ranking all the alternatives

We arrange all alternatives according to their degree of grey relational coefficient and the best alternative corresponds to the greatest degree of grey relational coefficient.

ILLUSTRATIVE EXAMPLE

Suppose that a new modern logistic center is required in a town. There are four locations L_1, L_2, L_3, L_4 . A committee of four decision makers or experts, namely, $\Theta_1, \Theta_2, \Theta_3, \Theta_4$ is formed to select the most appropriate location on the basis of six parameters are adapted from the study [78], namely, cost (P_1), distance to suppliers (P_2), distance to customers (P_3), conformance to government regulation and law (P_4), quality of service (P_5) and environmental impact (P_6) are considered for selecting parameters. Since, there are four decision makers we obtained four generalized neutrosophic soft set i.e. $M_1^{\lambda_1}, M_2^{\lambda_2}, M_3^{\lambda_3}$ and $M_4^{\lambda_4}$. Let U be the set of locations i.e. $U = \{L_1, L_2, L_3, L_4\}$ and P is the set of parameters i.e. $P = \{P_1, P_2, P_3, P_4, P_5, P_6\}$. The four generalized neutrosophic soft sets in matrix form for four decision makers are given below respectively.



$$\left. \begin{aligned} M_1^{\lambda_1}(P_1) &= \left\{ L_1 / \langle .6, .4, .8 \rangle, L_2 / \langle .7, .5, .6 \rangle, L_3 / \langle .8, .3, .4 \rangle, L_4 / \langle .8, .6, .7 \rangle \right\}, (6) \\ M_1^{\lambda_1}(P_2) &= \left\{ L_1 / \langle .5, .8, .7 \rangle, L_2 / \langle .6, .7, .8 \rangle, L_3 / \langle .8, .4, .5 \rangle, L_4 / \langle .7, .8, .5 \rangle \right\}, (5) \\ M_1^{\lambda_1}(P_3) &= \left\{ L_1 / \langle .3, .5, .6 \rangle, L_2 / \langle .7, .8, .3 \rangle, L_3 / \langle .7, .8, .4 \rangle, L_4 / \langle .4, .7, .4 \rangle \right\}, (5) \\ M_1^{\lambda_1}(P_4) &= \left\{ L_1 / \langle .6, .7, .3 \rangle, L_2 / \langle .8, .2, .7 \rangle, L_3 / \langle .8, .1, .4 \rangle, L_4 / \langle .7, .8, .6 \rangle \right\}, (4) \\ M_1^{\lambda_1}(P_5) &= \left\{ L_1 / \langle .3, .6, .7 \rangle, L_2 / \langle .7, .2, .3 \rangle, L_3 / \langle .8, .6, .7 \rangle, L_4 / \langle .8, .4, .5 \rangle \right\}, (6) \\ M_1^{\lambda_1}(P_6) &= \left\{ L_1 / \langle .2, .5, .8 \rangle, L_2 / \langle .7, .8, .8 \rangle, L_3 / \langle .7, .7, .8 \rangle, L_4 / \langle .8, .2, .5 \rangle \right\}, (3) \end{aligned} \right\}$$

$$\left. \begin{aligned} M_2^{\lambda_2}(P_1) &= \left\{ L_1 / \langle .6, .8, .7 \rangle, L_2 / \langle .6, .7, .3 \rangle, L_3 / \langle .6, .2, .4 \rangle, L_4 / \langle .2, .7, .8 \rangle \right\}, (4) \\ M_2^{\lambda_2}(P_2) &= \left\{ L_1 / \langle .8, .5, .6 \rangle, L_2 / \langle .8, .2, .8 \rangle, L_3 / \langle .8, .5, .4 \rangle, L_4 / \langle .5, .6, .8 \rangle \right\}, (5) \\ M_2^{\lambda_2}(P_3) &= \left\{ L_1 / \langle .8, .6, .7 \rangle, L_2 / \langle .7, .5, .4 \rangle, L_3 / \langle .7, .3, .4 \rangle, L_4 / \langle .6, .2, .7 \rangle \right\}, (3) \\ M_2^{\lambda_2}(P_4) &= \left\{ L_1 / \langle .7, .3, .8 \rangle, L_2 / \langle .7, .6, .4 \rangle, L_3 / \langle .5, .6, .7 \rangle, L_4 / \langle .7, .8, .3 \rangle \right\}, (3) \\ M_2^{\lambda_2}(P_5) &= \left\{ L_1 / \langle .6, .4, .3 \rangle, L_2 / \langle .8, .3, .1 \rangle, L_3 / \langle .8, .5, .8 \rangle, L_4 / \langle .8, .2, .8 \rangle \right\}, (5) \\ M_2^{\lambda_2}(P_6) &= \left\{ L_1 / \langle .8, .4, .5 \rangle, L_2 / \langle .8, .3, .5 \rangle, L_3 / \langle .8, .1, .5 \rangle, L_4 / \langle .6, .4, .8 \rangle \right\}, (6) \end{aligned} \right\}$$

$$\left. \begin{aligned} M_3^{\lambda_3}(P_1) &= \left\{ L_1 / \langle .4, .6, .4 \rangle, L_2 / \langle .8, .3, .8 \rangle, L_3 / \langle .8, .2, .7 \rangle, L_4 / \langle .7, .2, .8 \rangle \right\}, (5) \\ M_3^{\lambda_3}(P_2) &= \left\{ L_1 / \langle .8, .5, .6 \rangle, L_2 / \langle .7, .4, .8 \rangle, L_3 / \langle .6, .3, .4 \rangle, L_4 / \langle .6, .3, .7 \rangle \right\}, (2) \\ M_3^{\lambda_3}(P_3) &= \left\{ L_1 / \langle .7, .7, .6 \rangle, L_2 / \langle .8, .2, .7 \rangle, L_3 / \langle .7, .2, .8 \rangle, L_4 / \langle .8, .2, .8 \rangle \right\}, (4) \\ M_3^{\lambda_3}(P_4) &= \left\{ L_1 / \langle .8, .3, .8 \rangle, L_2 / \langle .7, .8, .3 \rangle, L_3 / \langle .7, .3, .8 \rangle, L_4 / \langle .6, .4, .8 \rangle \right\}, (3) \\ M_3^{\lambda_3}(P_5) &= \left\{ L_1 / \langle .7, .4, .7 \rangle, L_2 / \langle .3, .5, .7 \rangle, L_3 / \langle .6, .5, .7 \rangle, L_4 / \langle .7, .6, .7 \rangle \right\}, (6) \\ M_3^{\lambda_3}(P_6) &= \left\{ L_1 / \langle .7, .6, .5 \rangle, L_2 / \langle .7, .2, .6 \rangle, L_3 / \langle .8, .2, .8 \rangle, L_4 / \langle .3, .5, .7 \rangle \right\}, (5) \end{aligned} \right\}$$



$$\left. \begin{aligned}
 M_4^{i_4}(P_1) &= \left\{ \frac{L_1}{\langle .6, .3, .8 \rangle}, \frac{L_2}{\langle .5, .8, .8 \rangle}, \frac{L_3}{\langle .3, .7, .7 \rangle}, \frac{L_4}{\langle .7, .2, .7 \rangle} \right\}, (3) \\
 M_4^{i_4}(P_2) &= \left\{ \frac{L_1}{\langle .7, .5, .6 \rangle}, \frac{L_2}{\langle .6, .6, .4 \rangle}, \frac{L_3}{\langle .7, .5, .6 \rangle}, \frac{L_4}{\langle .8, .3, .4 \rangle} \right\}, (2) \\
 M_4^{i_4}(P_3) &= \left\{ \frac{L_1}{\langle .7, .3, .8 \rangle}, \frac{L_2}{\langle .8, .5, .7 \rangle}, \frac{L_3}{\langle .8, .3, .7 \rangle}, \frac{L_4}{\langle .3, .7, .7 \rangle} \right\}, (4) \\
 M_4^{i_4}(P_4) &= \left\{ \frac{L_1}{\langle .6, .6, .8 \rangle}, \frac{L_2}{\langle .7, .4, .8 \rangle}, \frac{L_3}{\langle .6, .4, .7 \rangle}, \frac{L_4}{\langle .7, .3, .6 \rangle} \right\}, (5) \\
 M_4^{i_4}(P_5) &= \left\{ \frac{L_1}{\langle .8, .5, .7 \rangle}, \frac{L_2}{\langle .6, .5, .7 \rangle}, \frac{L_3}{\langle .4, .4, .3 \rangle}, \frac{L_4}{\langle .7, .5, .7 \rangle} \right\}, (6) \\
 M_4^{i_4}(P_6) &= \left\{ \frac{L_1}{\langle .7, .4, .6 \rangle}, \frac{L_{25}}{\langle .5, .6, .7 \rangle}, \frac{L_3}{\langle .6, .4, .6 \rangle}, \frac{L_4}{\langle .7, .3, .8 \rangle} \right\}, (4)
 \end{aligned} \right.$$

Step: 1 Formation of generalized neutrosophic soft matrix

The matrix form of above four generalized neutrosophic soft set in the form of (2) defined above have been presented as follows.

$$A_1 = \left(\begin{array}{l}
 \langle .6, .4, .8 \rangle \langle .7, .5, .6 \rangle \langle .8, .3, .4 \rangle \langle .8, .6, .7 \rangle (6) \\
 \langle .5, .8, .7 \rangle \langle .6, .7, .8 \rangle \langle .8, .4, .5 \rangle \langle .7, .8, .5 \rangle (5) \\
 \langle .3, .5, .6 \rangle \langle .7, .8, .3 \rangle \langle .7, .8, .4 \rangle \langle .4, .7, .4 \rangle (5) \\
 \langle .6, .7, .3 \rangle \langle .8, .2, .7 \rangle \langle .8, .1, .4 \rangle \langle .7, .8, .6 \rangle (4) \\
 \langle .3, .6, .7 \rangle \langle .7, .2, .3 \rangle \langle .8, .6, .7 \rangle \langle .8, .4, .5 \rangle (6) \\
 \langle .2, .5, .8 \rangle \langle .7, .8, .7 \rangle \langle .7, .7, .8 \rangle \langle .8, .2, .5 \rangle (3)
 \end{array} \right)$$

$$A_2 = \left(\begin{array}{l}
 \langle .6, .8, .7 \rangle \langle .6, .7, .3 \rangle \langle .6, .2, .4 \rangle \langle .2, .7, .8 \rangle (4) \\
 \langle .8, .5, .6 \rangle \langle .8, .2, .8 \rangle \langle .8, .5, .4 \rangle \langle .5, .6, .8 \rangle (5) \\
 \langle .8, .6, .7 \rangle \langle .7, .5, .4 \rangle \langle .7, .3, .4 \rangle \langle .6, .2, .7 \rangle (3) \\
 \langle .7, .3, .8 \rangle \langle .7, .6, .4 \rangle \langle .5, .6, .7 \rangle \langle .7, .8, .3 \rangle (3) \\
 \langle .6, .4, .3 \rangle \langle .8, .3, .1 \rangle \langle .8, .5, .8 \rangle \langle .8, .2, .8 \rangle (5) \\
 \langle .8, .4, .5 \rangle \langle .8, .3, .5 \rangle \langle .8, .1, .5 \rangle \langle .6, .8, .4 \rangle (6)
 \end{array} \right)$$



$$A_3 = \begin{pmatrix} \langle .4, .6, .4 \rangle \langle .8, .3, .8 \rangle \langle .8, .2, .7 \rangle \langle .7, .2, .8 \rangle (.5) \\ \langle .8, .5, .6 \rangle \langle .7, .4, .8 \rangle \langle .6, .3, .4 \rangle \langle .6, .3, .7 \rangle (.2) \\ \langle .7, .7, .6 \rangle \langle .8, .2, .6 \rangle \langle .7, .2, .8 \rangle \langle .8, .2, .8 \rangle (.4) \\ \langle .8, .3, .8 \rangle \langle .7, .8, .3 \rangle \langle .7, .3, .8 \rangle \langle .6, .4, .8 \rangle (.3) \\ \langle .7, .4, .7 \rangle \langle .3, .5, .7 \rangle \langle .6, .5, .7 \rangle \langle .7, .6, .7 \rangle (.6) \\ \langle .7, .6, .5 \rangle \langle .7, .2, .6 \rangle \langle .8, .2, .8 \rangle \langle .3, .5, .7 \rangle (.5) \end{pmatrix}$$

$$A_4 = \begin{pmatrix} \langle .6, .3, .8 \rangle \langle .5, .8, .8 \rangle \langle .3, .7, .7 \rangle \langle .7, .2, .7 \rangle (.3) \\ \langle .7, .5, .6 \rangle \langle .6, .6, .4 \rangle \langle .7, .5, .6 \rangle \langle .8, .3, .4 \rangle (.2) \\ \langle .7, .3, .8 \rangle \langle .8, .5, .7 \rangle \langle .8, .3, .7 \rangle \langle .3, .7, .7 \rangle (.4) \\ \langle .6, .6, .8 \rangle \langle .7, .4, .8 \rangle \langle .6, .4, .7 \rangle \langle .7, .3, .6 \rangle (.5) \\ \langle .8, .5, .7 \rangle \langle .6, .5, .7 \rangle \langle .4, .4, .3 \rangle \langle .7, .5, .7 \rangle (.6) \\ \langle .7, .4, .6 \rangle \langle .5, .6, .7 \rangle \langle .6, .4, .6 \rangle \langle .7, .3, .8 \rangle (.4) \end{pmatrix}$$

Step: 2 Determination of the weight of the decision makers

The weights of the decision makers have presented in the Table 2.

Table 2. The weights of the decision makers

Decision maker's	Θ_1	Θ_2	Θ_3	Θ_4
Linguistic variable	VI	I	MI	UI
Weights	.352	.341	.222	.085

Step: 3 Aggregated weights of the parameters

Using the equation (4), aggregated weights of the parameters have been obtained as follows:

$$\lambda(p_1) = .49, \lambda(p_2) = .42, \lambda(p_3) = .41, \lambda(p_4) = .36, \lambda(p_5) = .57, \lambda(p_6) = .47$$

Step: 4 Construction of the aggregated generalized neutrosophic soft matrix

Using generalized neutrosophic soft weighted average operator given by the equation (6), (7), the aggregated matrix can be constructed as follows:

$$A = \begin{pmatrix} \langle .56, .54, .66 \rangle \langle .68, .52, .52 \rangle \langle .72, .26, .47 \rangle \langle .63, .45, .75 \rangle (.49) \\ \langle .71, .59, .63 \rangle \langle .70, .40, .78 \rangle \langle .76, .41, .45 \rangle \langle .63, .54, .62 \rangle (.42) \\ \langle .65, .55, .65 \rangle \langle .74, .48, .41 \rangle \langle .71, .39, .49 \rangle \langle .58, .34, .59 \rangle (.41) \\ \langle .69, .43, .57 \rangle \langle .74, .42, .48 \rangle \langle .68, .26, .59 \rangle \langle .68, .63, .50 \rangle (.36) \\ \langle .57, .47, .52 \rangle \langle .68, .30, .27 \rangle \langle .74, .52, .68 \rangle \langle .77, .35, .65 \rangle (.57) \\ \langle .41, .47, .60 \rangle \langle .73, .41, .60 \rangle \langle .76, .26, .66 \rangle \langle .35, .41, .52 \rangle (.47) \end{pmatrix}$$

Step: 5 Determination of the reference sequence

The reference sequence based on generalized neutrosophic soft set can be constructed as follows:

$$a^* = [(1, 0, 0), (1, 0, 0), \dots, (1, 0, 0)]^T$$

Step: 6 Calculation of grey relational coefficient

Table 3: Calculation of $\min \delta_{ij}$ and $\max \delta_{ij}$ without considering the last column of the aggregated matrix in step

4

	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{\min}	δ_{\max}
P ₁	.44,	.32,	.28,	.37		
P ₂	.29,	.30,	.24,	.37		



P ₃	.35	.26,	.29,	.42	
P ₄	.31,	.26,	.32,	.32	
P ₅	.43,	.32,	.26,	.23	
P ₆	.59,	.27,	.24,	.65	
Min δ_{ij}	.29	.26	.24	.23	.23
Max δ_{ij}	.59	.32	.32	.65	.65

The grey relational coefficient matrix has been obtained for $\rho = 0.5$ using the table 2 as follows:

$$\sigma_{ij} = \begin{pmatrix} .72, .86, .92, .80 \\ .90, .89, .98, .80 \\ .82, .95, .90, .74 \\ .87, .95, .86, .86 \\ .74, .86, .95, 1.0 \\ .61, .93, .98, .57 \end{pmatrix}$$

Step: 7 Calculation of the degree of grey relational coefficient

The calculation of the degree of grey relational coefficient using the equation (9) has been performed as follows:

$$\Omega_j = \sum_{i=1}^6 \sigma_{ij} \lambda(p_i), j=1, 2, 3, 4 ;$$

$$\Omega_1 = 2.09, \Omega_2 = 2.45, \Omega_3 = 2.54, \Omega_4 = 1.66.$$

Step: 8 Ranking the alternatives

Arrange the alternative according to the degree of grey relational coefficient (Ω_j) ($j=1, 2, 3, 4$) in descending order. Greater value of Ω_j implies the better alternative L_j .

Here $\Omega_3 > \Omega_2 > \Omega_1 > \Omega_4$ then ranks of the four locations are as follows:

$$L_3 > L_2 > L_1 > L_4$$

Therefore, L_3 is the best logistic center.

Determination of ranking order when equal weights of the decision makers are considered

We present the ranking of logistics center location when weights of the decision makers are equal. The first two steps are same as above. Equal weights of the decision makers imply

$$\psi_1 = \psi_2 = \psi_3 = \psi_4 = (1/4) = .25$$

Step: 3 Aggregated weights of the parameters

Using the equation (4), aggregated weights of the parameters have been obtained as follows:

$$\lambda(p_1) = .46, \lambda(p_2) = .31, \lambda(p_3) = .60, \lambda(p_4) = .62, \lambda(p_5) = .42, \lambda(p_6) = .54$$

Step: 4 Construction of the aggregated generalized neutrosophic soft matrix

Using generalized neutrosophic soft weighted average operator given by the equation (6), (7), the aggregated matrix in the form of (5) can be formed as:

$$A = \begin{pmatrix} \langle .56, .49, .65 \rangle \langle .67, .54, .58 \rangle \langle .68, .30, .53 \rangle \langle .65, .36, .75 \rangle (.46) \\ \langle .72, .56, .62 \rangle \langle .69, .43, .67 \rangle \langle .74, .42, .47 \rangle \langle .67, .46, .58 \rangle (.31) \\ \langle .67, .50, .67 \rangle \langle .76, .45, .47 \rangle \langle .73, .35, .55 \rangle \langle .57, .37, .63 \rangle (.60) \\ \langle .69, .44, .63 \rangle \langle .73, .44, .51 \rangle \langle .67, .29, .63 \rangle \langle .68, .53, .54 \rangle (.62) \\ \langle .63, .47, .57 \rangle \langle .64, .35, .35 \rangle \langle .69, .50, .58 \rangle \langle .76, .39, .54 \rangle (.42) \\ \langle .65, .47, .59 \rangle \langle .69, .41, .62 \rangle \langle .74, .27, .66 \rangle \langle .64, .31, .69 \rangle (.54) \end{pmatrix}$$



Step: 5 Determination of the reference sequence

The reference sequence based on generalized neutrosophic soft set is

$$a^* = [(1, 0, 0), (1, 0, 0), \dots, (1, 0, 0)]^T,$$

Step: 6 Calculation of grey relational coefficient

Calculation of grey relational coefficient has been provided in the Table 4.

Table 4: Calculation of $\min \delta_{ij}$ and $\max \delta_{ij}$ without considering the last column of the aggregated matrix in step 4

	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{\min}	δ_{\max}
P ₁	.44,	.33,	.32,	.35		
P ₂	.28,	.31,	.26,	.33		
P ₃	.33	.24,	.27,	.43		
P ₄	.31,	.27,	.33,	.32		
P ₅	.37,	.36,	.31,	.24		
P ₆	.35,	.31,	.26,	.36		
Min δ_{ij}	.28	.24	.26	.24	.24	
Max δ_{ij}	.44	.36	.33	.43		.44

The grey relational coefficient matrix has been constructed for $\rho = 0.5$, using the table 2 as follows:

$$\sigma_{ij} = \begin{pmatrix} .70, .84, .85, .81 \\ .92, .87, .96, .84 \\ .84, 1.0, .94, .71 \\ .87, .94, .84, .85 \\ .78, .79, .87, 1.0 \\ .81, .87, .96, .79 \end{pmatrix}$$

Step: 7 Calculation of the degree of grey relational coefficient

The calculation of the degree of grey relational coefficient using the above equation (9) has been performed as follows:

$$\Omega_j = \sum_{i=1}^6 \sigma_{ij} \lambda(p_i), j = 1, 2, 3, 4;$$

$$\Omega_1 = 2.42, \Omega_2 = 2.64, \Omega_3 = 2.66, \Omega_4 = 2.43.$$

Step: 8 Ranking the alternatives

Arrange the alternative according to the degree of grey relational coefficient (Ω_j) ($j=1, 2, 3, 4$) in descending order. Greater value of Ω_j implies the better alternative L_j .

Here we have obtained $\Omega_3 > \Omega_2 > \Omega_4 > \Omega_1$.

Then ranks of the four locations are as follows:

$$L_3 > L_2 > L_4 > L_1$$

Therefore, L_3 is the best logistic center.

Note 1. Comparison of ranking order with weights factors of decision makers.

- i. The ranking order for unequal weights of the decision makers is $L_3 > L_2 > L_1 > L_4$.
- ii. The ranking order for the equal weight of the decision makers is $L_3 > L_2 > L_4 > L_1$.

The ranks of the first two location centers i.e. $L_3 > L_2$ remain the same. But the ranks of L_1 and L_4 change due to weights factor of decision makers. Therefore, the ranking order depends on the weights of the decision maker.



CONCLUSION

Firstly, we have defined generalized neutrosophic soft weighted average operator to aggregate all individual opinions. We have also developed multi-attribute group decision making (MAGDM) model in generalized neutrosophic soft environment based on grey relational analysis. We have also presented an illustrative example of logistic center location selection problem. We have also presented the sensitivity analysis for the weights factor of decision makers in decision making process. We hope that the proposed MAGDM model will assist to solve various types of MAGDM problems such as medical diagnosis, engineering problems and different kind of practical real life group decision making problems.

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