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SPECTRAL ANALYSIS OF A SIGNAL DRIVEN SAMPLING SCHEME

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ABSTRACT

This work is a part of a drastic revolution in the classical signal processing chain required in mobile systems. The system must be low power as it is powered by a battery. Thus a signal driven sampling scheme based on level crossing is adopted, delivering non-uniformly spaced out in time sampled points. In order to analyse the non-uniformly sampled signal obtained at the output of this sampling scheme a new spectral analysis technique is devised. The idea is to combine the features of both uniform and non-uniform signal processing chains in order to obtain a good spectrum quality with low computational complexity. The comparison of the proposed technique with General Discrete Fourier transform and Lomb's algorithm shows significant improvements in terms of spectrum quality and computational complexity.

1. CONTEXT OF THE STUDY

This work is part of a large project aimed to enhance the signal processing chain required in mobile systems. ADC is an essential component of the digital signal processing chain. Most of the systems using ADCs operate signals with interesting statistical properties, but Nyquist signal processing architectures do not take its advantage. Actually, these signals (such as temperature, electrocardiograms, speech signals...) are almost always constant and may vary significantly only during brief moments. Thus classical sampling systems are highly constrained, due to the Shannon theory, which is to ensure for the sampling frequency to be at least twice of the input signal bandwidth. This condition causes a large number of samples without any relevant information, a useless increase of activity, and so a useless increase of the power consumption. The idea is to realize a signal driven sampling scheme of the analog input signal based on its amplitude variations. This sampling scheme is based on "level-crossing" that provides a non-uniform time repartition of the samples. This sampling scheme drastically reduces the activity of the processing chain because it only processes the relevant information. In this context an AADC (Asynchronous Analog to Digital Converter) based on LCSS (Level Crossing Sampling Scheme) [4] has been designed by the CIS group of the TIMA Laboratory. Asynchronous filter algorithms have also been developed for this sampling scheme [8]. The aim of this work is to develop an efficient spectrum analysis technique devoted to the cross-level sampling scheme which provides a high quality spectrum at low computational complexity.

2. PREVIOUS WORKS

According to Fourier's theory any continuous signal can be expressed as a combination of properly chosen sinusoidal waves. The most common discrete technique for detecting these components of a signal is the Discrete Fourier transform. Classical methods to estimate the spectrum of non-uniformly sampled signal require oversampling at uniform intervals. Techniques like GDFT (General Discrete Fourier Transform) [1] or Lomb's algorithm [2] have also been developed. They are able to perform frequency domain analysis directly on non-uniformly sampled signals but they suffer from a problem of noise on the spectrum [3]. In this article, a new spectrum analysis technique – which does not require oversampling and provides the spectrum with a higher accuracy – is presented in Section 3. The idea is to exploit the respective advantages of both uniform and non-uniform signal processing chains.

2.1. LCSS and Sampling Criteria

The LCSS has already been studied by Jon W. Mark and Terence D. Todd in [5]. In [6], authors have shown that ADC using this technique has a reduced activity and thus allows power saving and noise reduction compared to Nyquist ADCs.
An M-bit resolution AADC have 2^M - 1 quantization levels which are disposed according to the input signal amplitude dynamic (in the studied case, the levels are regularly spaced). A sample is captured only when the analog signal x(t) crosses one of these predefined levels. The samples are not uniformly spaced in time because they depend on the signal variation as it is clear from Figure 1.

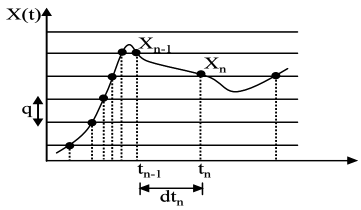

Figure 1: Level-crossing sampling scheme

A condition for proper reconstruction of non-uniformly sampled signals was developed by Beutler [7]. He showed that the reconstruction of an original continuous signal is possible, if the average sampling frequency $\overline{F}$ of the non-uniformly sampled signal is greater than twice of the signal bandwidth F_{max}. This condition can be expressed mathematically by $\overline{F} > 2F_{\max}$. According to [4], in the case of LCSS, the number of samples is directly influenced by the resolution of the AADC. For M-bit resolution AADC the average sampling frequency of a signal can be calculated, by exploiting its statistical characteristics. Then a proper value of M can be chosen in order to respect the Beutler's criterion.

2.2. GDFT (General Discrete Fourier Transform)

The GDFT is a technique for the spectrum analysis of non-uniformly sampled signals [1]. It is defined by Equation 1.

$$(1) \qquad X(\omega) = \frac{1}{N}\sum_{n=1}^{N} x_n \exp\left(-j\omega t_n\right).$$

In Equation 1 x_n and t_n represent the amplitude and the time instant of the n^{th} non-uniform sample respectively.

2.3. LOMB'S ALGORITHM

Lomb's algorithm [2] is defined by Equation 2.

(2) $$P(\omega) = \frac{1}{2\sigma^2}\left\{ \frac{\left[\sum_{n=1}^{N}(x_n - \bar{x})\cos\omega(t_n - \tau)\right]^2}{\sum_{n=1}^{N}\cos^2\omega(t_n - \tau)} + \frac{\left[\sum_{n=1}^{N}(x_n - \bar{x})\sin\omega(t_n - \tau)\right]^2}{\sum_{n=1}^{N}\sin^2\omega(t_n - \tau)} \right\}.$$

According to Lomb in Equation 2 parameter x_n represents the amplitude and t_n represents the time instant of the n^{th} non-uniform sample. The total number of samples is N. $\bar{x}$ represents the mean of these samples (see Equation 3). Parameter σ^2 represents the variance of these samples (see Equation 4) and τ is an offset proposed by Lomb, defined by Equation 5.

(3) $$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n ,$$ (4) $$\sigma^2 = \frac{1}{N-1}\sum_{n=1}^{N}\left(x_n - \bar{x}\right)^2 ,$$

(5) $$\tau = \frac{1}{2\omega}\arctan\left\{\frac{\sum_{n=1}^{N}\sin 2\omega t_n}{\sum_{n=1}^{N}\cos 2\omega t_n}\right\}.$$

3. SPECTRUM ANALYSIS BASED ON ASA AND RFFT

This technique mainly consists of two steps which are called ASA (Activity Selection Algorithm) and RFFT (Re-sampling + Fast Fourier Transform). In ASA, the active part of the non-uniformly sampled signal is selected. The RFFT transforms the selected part of non-uniformly sampled signal to uniformly sampled signal by applying NNR (Nearest Neighbour Re-sampling) interpolation and then the spectrum is obtained by applying a standard FFT.

3.1. ASA (Activity Selection Algorithm)

For a non-uniformly sampled signal obtained at the output of an AADC, the sampling instants (according to [5]) are defined by the following equation.

(6) $$t_n = t_{n-1} + dt_n .$$

In Equation 6, t_n is the current sampling instant, t_{n-1} is the previous one and dt_n is the time delay between current and previous sampling instant, as shown in Figure 1.
Let δ be the processing delay of AADC for one sample point, for proper signal capturing the incoming signal must satisfy the "tracking condition" given by Equation 7.

(7) $$\frac{dx(t)}{dt} \le \frac{q}{\delta} .$$

In Equation 7, q is the quantum of AADC and is defined as:

(8) $$q = \frac{\Delta V_{in}}{2^M - 1} .$$

In Equation 8, ΔV_{in} represents the amplitude dynamic of x(t) and M represents the resolution of AADC. In order to respect the Beutler's criteria and tracking condition we have employed a B.P.F (band pass filter) with pass band $F_{min} \sim F_{max}$. The process is clear from Fig 2.

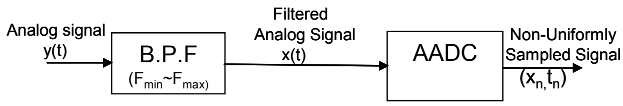

Figure 2: Band limiting the analog signal in order to respect tracking condition and Beutler's criterion

The appropriate value of pass band can be chosen for a specific signal. By employing a B.P.F we are able to calculate the fundamental period $T_0 = 1/F_{min}$ of the filtered signal activity which plays an important role in the activity selection process, as discussed below.

From Figure 1 it is clear that variable dt_n is a function of the variation of the input signal. For a high slope signal the values of dt_n will be smaller and vice versa [4]. By using the values of dt_n , an algorithm is applied to select the active part of the non-uniformly sampled signal. The algorithm used is defined as follows.

```
While (dtn <= T0/2 and Ti <= T)
        Ti = Ti + dtn
        Ni = Ni + 1
end
```

The parameter dt_n is defined by Equation 6 and T_0 is the fundamental period of the filtered signal activity. These two parameters define the activity detection threshold. It is interesting to note that in the case of low activity signals – which mostly remain constant and may vary significantly only during short moments like electrocardiograms, electroencephalograms, speech signals, etc [8] – if the LCSS fulfills Beutler's condition, it also satisfies locally the Nyquist criterion for each selected window, as it is illustrated in Section 4. So the condition on dt_n is chosen to follow the Nyquist criterion for the minimum frequency component F_{min} of the incoming signal, when sampling the incoming signal non-uniformly with LCSS. The parameter T_i represents the length of the i^{th} selected window, lying on the i^{th} active part of the non-uniformly sampled signal and T represents the largest window length in second. The choice for T depends on the input signal characteristics ($T \ge T_0$) and system resources (maximum time frame which yields an upper bound on T) used to process the incoming signal. The parameter N_i represents the total number of non-uniform samples lying in the i^{th} selected window. The above described loop repeats for each active part of the non-uniformly sampled signal, occurs during the whole signal length. Every time before starting the next loop 'i' is incremented by one and T_i and N_i are initialized to zero.
ASA displays interesting features with LCSS which are not available in the uniform case. It dynamically adapts the length of the selected window according to the signal activity (see Section 4). In addition, it also provides an efficient reduction of the phenomenon of spectral leakage in case of transient signals (signals which start and finish at zero). Usually appropriate smoothening window functions are used to reduce spectral leakage. In the ASA case, as long as the length of signal active part remains $\le T$, the problem of leakage is solved by applying a simple and efficient algorithm instead of a smoothening window function. Indeed, spectral leakage occurs when FFT is processed on a fractional number of cycles of the captured signal. In order to avoid the fractional number of cycles, two points, one at the beginning and one at the end of the selected window, are added. These two points are considered as first and last sample point of the selected window and are placed by using the following Equations:

(9) $$t_1 = t_2 - dt_2 ,$$
(10) $$t_{Ni} = t_{Ni-1} + dt_{Ni-1} .$$

Equation 9 is applied on the nearest left edge point to the selected window here dt_2 is the delay between the sampling instants t_2 and t_3. Similarly, Equation 10 is used for the nearest right edge point to the selected window and dt_{Ni-1} is the delay between the two last samples of the window. So these points are placed using the neighbouring distances dt_n on both edges in order to maintain the signal slope at the window edges. The process is illustrated in Figure 3.

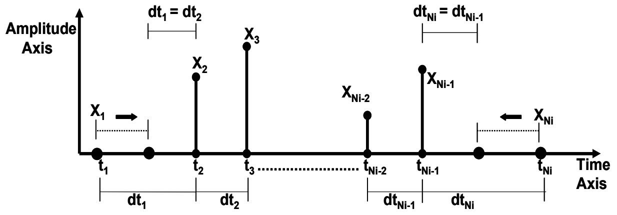

Figure 3: Shifting process of nearest edge points

3.2. RFFT (Re-sampling + Fast Fourier Transform)

The main interest of using ASA is that only the interesting part of the sampled signal can be selected and then its spectrum can be computed with a suitable quality by just resampling the data lying in each selected window. In order to resample the windowed data, the NNR interpolation is used. The reasons to choose this technique are detailed in Section 6. The NNR is an interpolation method where the value of an interpolated sample Xr_n corresponding to a re-sampling instant tr_n is set according to the algorithm defined and shown in Figure 4.

```
If (dtr1n < = dtr2n)
     Xrn = Xn-1
else
     Xrn = Xn
end
```

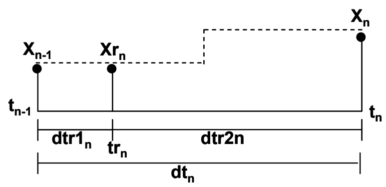

Figure 4: Re-sampled time and amplitude values according to NNR interpolation

The resampling rate Frs_i is specific for each selected window, depending on the window length (seconds) and the number of non-uniform samples lying in the window. The resampling frequency for the i^{th} selected window can be calculated by using the following equations.

$$T_i = tmax_i - tmin_i \ , \tag{11}$$

$$Frs_i = N_i / T_i . \tag{12}$$

In Equation 11, $tmax_i$ and $tmin_i$ are the final and the initial times of the i^{th} selected window. The next step is to apply FFT on the resampled data in order to obtain its spectrum.

4. ILLUSTRATIVE EXAMPLE

In order to illustrate this new spectrum analysis technique and make its performance comparison with GDFT and Lomb's algorithm, an input signal presented on the top-left part of Figure 5 is used. It consists of three active parts each one of duration 2, 1 and 0.5 seconds respectively. As the total signal length is 20 sec so this signal has 17.5 % activity.

The non-uniformly sampled signal obtained at the output of the AADC is shown in the top-right part of Figure 5. Here 373 points for a 20 second signal are sampled. The average sampling frequency is thus 18.6 Hz, which is greater than twice the F_{max} of the signal i.e. 8 Hz. So this signal sampling is satisfying the Beutler's criterion. From the bottom-left part of Figure 5, it is evident that the Nyquist criterion is also satisfied. It is obvious that in the case of low activity signals discussed in section 3.1, Beutler's condition leads to locally over-sample the active parts of the signal. This over-sampling adds to the accuracy of interpolation process, employ to resample the selected non-uniform data.

In this example the pass band of the B.P.F is chosen between 2Hz ~ 8Hz. The value of T is chosen equal to 1 second. This value satisfies the limiting conditions discussed in Section 3.1. For this value of T ASA has generated 4 selected windows for the whole signal length (20 sec). The numbers of selected windows generated for first, second and third active part of the signal are 2, 1 and 1 respectively. The result obtained through ASA is shown on bottom-right part of Figure 5.

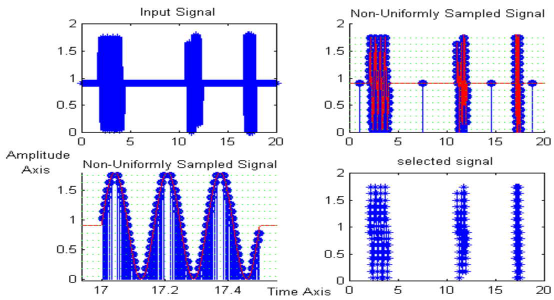

Figure 5: Input signal to the AADC (top-left), 373 non-uniformly sampled data points obtained at the output of the 4 bit AADC (top-right), zoom of third active part of the non-uniformly sampled signal (bottom-left) and four selected windows obtained by applying ASA (bottom-right)

Table 1 summarizes the parameters of selected windows obtained at the output of ASA. The values of T_i exhibit the dynamic feature of ASA which is to adapt the window length according to the signal activity lying in the window. The values of T_1 and T_3 are slightly greater than the value of T which is due to the process of adding two points at both edges of the selected window discussed in section 3.1. On the other hand in the classical case during the windowing process we are not able to select only the active part of the signal. Moreover the window length remains static and is not correlated to the signal activity in the window. For this studied example a 1 second window length leads to 20 1-second windows for the whole signal duration (20 sec). It follows that the system has to process more than the relevant part of the signal.

Selected Window	Signal Components Situated in Each Selected Window	Length T_i (Sec)	Samples N_i
First	$0.6 \sin(2\times2\pi t)+ 0.3 \sin(8\times2\pi t)$	1.016	88
Second	$0.6 \sin(2\times2\pi t)+ 0.3 \sin(8\times2\pi t)$	0.998	85
Third	$0.45 \sin(3\times2\pi t)+ 0.45 \sin(7\times2\pi t)$	1.0001	116
Fourth	$0.9 \sin(6\times2\pi t)$	0.5001	92

Table 1: Summery of the parameters of each selected window

The spectra obtained by applying RFFT, GDFT and Lomb's algorithm on data lying in the first selected window are shown on Figure

6. It shows that the quality of spectrum obtained by RFFT is better than those obtained by GDFT or Lomb's algorithm. For RFFT, the spectrum peaks corresponding to the signal components at fundamental frequency f_0 and periodic frequencies $f_p \pm f_0$ are visible on Figure 6. In this case value of $f_p = Frs_1$ is 86.6 Hz. Here Frs_1 is the resampling frequency of first selected window calculated by using Equation 12. By zooming the spectrum, we have found that the components corresponding to 2 Hz and 8 Hz lie at frequencies 1.9997 Hz and 7.9989 Hz respectively. The relative error is 0.03 % and 0.11% respectively. This error is due to the minor leakage produces by the approach based on ASA and RFFT. On the contrary, the spectra obtained by GDFT and Lomb's algorithm display noise peaks with an amplitude which can be higher than the peaks of the analyzed signal components. As in this example the relevant information is buried into the noise. The noise level on the spectrum obtained by Lomb's algorithm is however lower than that obtained by the GDFT, but this noise will always cause problems in signal analysis.

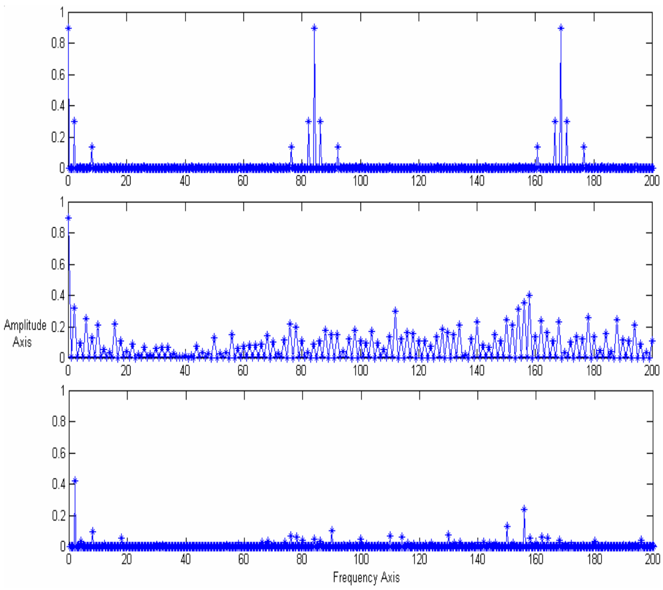

Figure 6: Spectra obtained by applying RFFT (top), GDFT (middle) and Lomb's algorithm (bottom)

Moreover, it is also possible to perform the time-frequency analysis of the time varying signal by this new approach. The 3-D plot of Figure 7 gives a time-frequency representation of each selected window. Here the spectrum peaks of each selected window are plotted with respect to the central time of each selected window. This representation helps us to visualize the different signal components lying at different times.

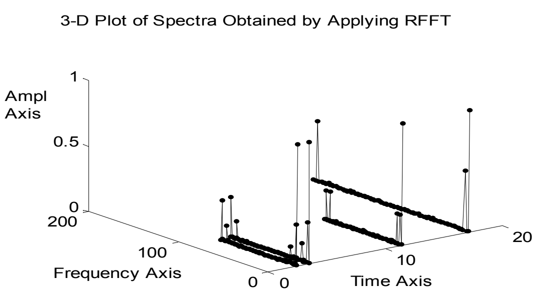

Figure 7: 3-D time-frequency representation of the spectra obtained by applying RFFT for all four selected windows

5. ALGORITHM EFFICIENCY

This section compares the efficiency of proposed ASA-RFFT technique with GDFT and Lomb's algorithm in terms of the computational complexity. The complexity evaluation is analyzed for RFFT, GDFT and Lomb's algorithm (considering the number of operations executed to perform the algorithm).

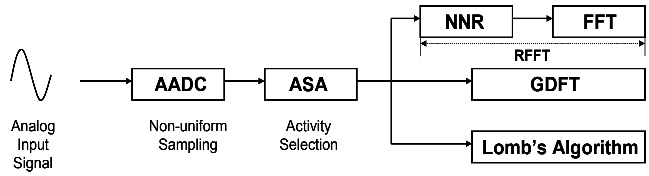

Figure 8: Signal Flow Diagram

For simplicity, it is assumed that each operation like addition, multiplication and division has equal complexity. Figure 8 shows that the comparison is applied on selected data obtained with the ASA. In the case of RFFT, the data lying in each selected window is resampled with a NNR interpolation and padded (in order to have a number of samples as a power of 2) before applying the FFT. The NNR interpolation only requires a comparison operation. So the total computational complexity of RFFT is $N_i + N_i \log_2 N_i$, which is sum of the computational complexities of NNR interpolation and FFT respectively. This is less than the computational complexity of GDFT which is N_i^2. Here N_i is the number of samples in the i^{th} selected window. In Lomb's algorithm there are not only additions and multiplications but also four trigonometric functions. The operation count can easily reach several hundred times of N_i^2 [9]. The processing gain of RFFT over GDFT or Lomb's algorithm can be calculated by using the following equation.

$$G_{c1} = \frac{\sum_{i=1}^{K} \rho \times N_i^2}{\sum_{i=1}^{K} N_i + N_i \log_2 N_i} \qquad (13)$$

In Equation 13, ρ is a multiplying factor whose value is 1 for GDFT and is in range of hundreds for Lomb's algorithm. Parameter $i = 1,\ldots,K$ is the index of the selected windows.

6. INTERPOLATION ERROR

The interpolation process changes the properties of resampled signal compared to the original signal. The interpolation error depends upon the interpolation technique used to resample data. In [10], the authors have made a distinction between simple and complex interpolation methods. Simple interpolation methods use only one non-uniform sample for one resampled observation, such as Sample&Hold (S&H) and Nearest Neighbor Resampling (NNR). Where as complex interpolation methods such as Linear interpolation and Cubic Spline interpolation use more than one non-uniform samples for one resampled observation.
As the simple interpolation methods use only one non-uniform observation for each resampled observation so they are efficient in terms of computational complexity. Moreover they provide us an unbiased estimate of the variance of original signal [11], due to this reason they are also known as robust interpolation methods. The complex interpolation methods have higher computational complexity as compare to simple methods. Another disadvantage of complex interpolation methods is that the variance can be estimated errone-

ously. As the variance of resampled (estimated) signal obtained by employing Linear interpolation is lower as compare to the variance of the original signal. This can be understood by considering the fact that linear interpolation is a weighted average of two non-uniform observations. Similarly Cubic Spline interpolation results in a higher variance resampled signal as compare to the original one [11]. These facts justify paying more attention to the simple interpolation methods.

In case of S&H interpolation the value of re-sampled observation is set equal to the non-uniform sample prior to it. Where as in case of NNR interpolation method the value of resampled observation is set equal to the nearest non-uniform sample. As we are resampling the data uniformly, the interval Tr_n between two consecutive resampled data points is constant. The value of Tr_n is different from the various intervals Tn_j between the non-uniformly spaced samples used for resampling as shown in Figure 9. Here j = 1 for NNR and 2 for S&H interpolation method respectively. This difference between the values of Tr_n and Tn_j causes deviation between the properties of original and resampled signal. Higher will be the difference more will be the deviation and vice versa.

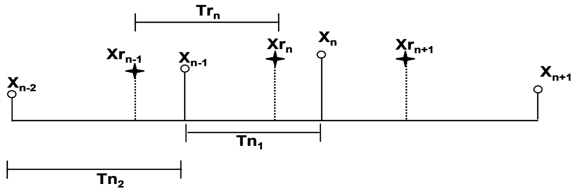

Figure 9: Tr_n is the interval between two resampled data points, Tn_1 is the interval between the non-uniform samples used for NNR interpolation and Tn_2 is the interval between the non-uniform samples used for S&H interpolation. 'o' symbol is used for the non-uniform data and '+' symbol is used for the resampled data. Here the signal is resampled with NNR interpolation

With NNR, the mean square deviation between Tr_n and T_{n1} is smaller than the mean square deviation between Tr_n and T_{n2} for S&H [11]. This fact is also shown by the following simulation results presented in Table 2.

Selected Window	Mean Square Deviation between Tr_n & Tn_1 (NNR)	Mean Square Deviation between Tr_n & Tn_2 (S&H)
First	$3.5887\ e^{-5}$	$1.8433\ e^{-4}$
Second	$3.6065\ e^{-5}$	$1.70008\ e^{-4}$
Third	$1.0711\ e^{-5}$	$4.932\ e^{-5}$
Fourth	$9.8907\ e^{-6}$	$2.6254\ e^{-5}$

Table 2: Comparison of the values of mean square deviation between Tr_n and Tn_j for NNR and S&H, for each selected window

From above discussion it is clear that among simple interpolation methods NNR performs better than S&H. It tries to keep the properties of resampled signal more close to real signal; this is the reason of our inclination towards NNR interpolation. Although the exact value of interpolation error for resampled data is not straight forward to calculate. But the upper bound for the interpolation error can be calculated. Figure 1 shows that in the studied case the quantization levels of the AADC are uniformly spaced, the quantum, q between two consecutive quantization levels is constant as is given by Equation 8. The maximum possible interpolation error is therefore bounded by the value of q and can be expressed mathematically as "$E_M = q$". This is the error which occurs in the worst case.

7. CONCLUSION

A new technique for spectral analysis of non-uniformly sampled signals has been proposed. This approach is especially well-suited to analyze the signals coming from the AADC which is based on a cross-level sampling scheme. This technique exploits an algorithm (ASA) to extract and window the active parts of the signal. Then each selected part is uniformly resampled by employing NNR interpolation before processing a classical FFT. This approach is extremely well-adapted for the signals which remain constant most of the time and vary sporadically as electro-cardiograms, seismic signals, etc. The dynamic features of ASA have been discussed. The ASA is a new tool to reduce the processing activity. Moreover, RFFT overperforms GDFT and Lomb's algorithm in terms of spectrum quality and processing costs. Finally, the error caused by the NNR interpolation is bounded by the amplitude quantum of the AADC.

The ASA-RFFT technique has already been used to characterize the frequency response and the SNR of the AADC designed at the TIMA Laboratory [4]. This technique can also be applied to real life signals in order to exploit them. Further works are focused on the enhancement of the technique in order to reduce the interpolation error and increase the spectral accuracy.

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