# 8 (Trans)languaging Mathematics as a Source of Meaning in Upper-Secondary School in Sweden 

Åsa Wedin

## Introduction

There is a growing body of research that focuses on the role of linguistic repertoires and students' use of material, linguistic and social resources in meaning-making in mathematics classrooms. However, research has seldom taken a linguistic perspective on the use of language in the negotiation of knowledge of mathematics. In this chapter, the focus is on language, or rather languaging, in a mathematics classroom for recently arrived students (RAS) in upper-secondary school in Sweden where multiple languages were used. Barwell's (2018) concept language as a source of meaning will be used for the analysis by distinguishing three dimensions: multiple languages, multiple discourses and multiple voices. This means that students and teachers will be understood as languaging mathematics rather than as using mathematical language, which is in line with Barwell (2018). Thus, the aim of this chapter is to investigate languaging in the negotiation of meaning in a multilingual mathematics classroom in an upper-secondary school in Sweden.

## Language Introduction for RASs in Sweden

RASs who migrated to Sweden as adolescents face the challenge of having to learn a new language, Swedish, and using it to learn new content. Almost all students in Sweden who finish compulsory school, i.e. Year 9, continue to nonmandatory upper-secondary school. RASs who do not have the
entry requirements to one of the mainstream programs in upper-secondary school before the age of 16 are instead admitted to the Language Introduction Program (språkintroduktionsprogrammet, here LIP), where they take Swedish as a Second Language and other content subjects to meet the entry requirements. The challenge of attaining the required level of Swedish is in itself demanding, and both international (Cummins, 2000; Thomas \& Collier, 1997) and national research (Axelsson, 2013) show that this can take several years. In addition to learning Swedish, most students need to complete courses in other subjects in order to meet the entry requirements. Current regulations stipulate that they need to do so by the age of 19. LIP is thus a transitional program, and students try to transfer to mainstream programs as quickly as possible.

According to the Education Act (2010:800: Chapter 6), LIP courses should be planned individually for each student following careful mapping procedures. Students also have the right (Ordinance for UpperSecondary School 2010:2039: Chapter $9 \$ 9$ ) to mother tongue tuition and study guidance in the mother tongue (SGMT). Students who need SGMT should be given support in a language that they master (not Swedish, however).

In a situation that is demanding for schools as well as students (there being a high number of students who need SGMT and a considerable number of languages involved), there is a need for qualified SGMT tutors as well as teachers who are proficient in some of the languages used by students. In the classroom selected for this study, the teacher, here named Khaled, had himself emigrated from Iran and had a sound command of Farsi and English, as well as Swedish. As some of his students were of Afghani origin and spoke Dari, a language that is close to Farsi, Khaled used all three languages - Swedish, English and Farsi - in class. An SGMT tutor who was present for parts of the lessons spoke Somali and some Arabic. In the two classes selected for this chapter, most students understood one or two of the languages mentioned here.

## From Using Language in Mathematics to Languaging Mathematics

The main body of research on languaging in mathematics among second language (L2) students focuses on their use of mathematical language. Mathematics is a challenging subject for L 2 students as it requires language development from everyday ways of talking about mathematical phenomena to a more technical register, which Halliday (1975) calls the mathematics register. According to Prediger et al. (2018: 1), 'language proficiency is the background factor with the strongest connection to mathematics achievement among all social and linguistic background factors'. Schleppegrell (2007: 139) describes challenges as including 'the multi-semiotic formations of mathematics, its dense noun phrases that
participate in relational processes, the precise meanings of conjunctions and implicit logical relationships that link elements in mathematics discourse'. Developing language skills in mathematics includes learning not only new vocabulary but also new 'styles of meanings and modes of argument [...] and of combining existing elements into new combinations' (Halliday, 1978: 195-196). It is widely known that academic language is explicit and precise, and this is specifically the case when we consider the natural sciences (see, for example, Lemke, 1989, 2003; Wedin \& Bomström Aho, 2019). However, Schleppegrell (2007) stresses that in mathematics, the precision lies not in language itself but in the way it is used and that mathematical knowledge is constructed through language in ways that differ from what is the case with other academic subjects.

Mathematics is often talked about as being a language in itself, and through the multisemiotic systems that mathematics draws on to construe knowledge, features such as order, position, relative size and orientation are used (Pimm, 1987). These multisemiotic and meaning-creating systems include symbols, oral language, written language, graphs and diagrams. Thus, through mathematical language, meanings may be expressed beyond what ordinary verbal language can express (Schleppegrell, 2007). The following characteristics are suggested by Halliday (1975, in Cuevas, 1984: 136) for the mathematics register:

- Natural language words reinterpreted in the context of mathematics, such as set, point, field, column, sum, even (number) and random.
- Locutions, such as square on the bypotenuse and least common multiple.
- Terms created from combinations of natural language words, such as feedback and output.
- Terms formed from combining elements of Greek and Latin vocabulary, such as parabola, denominator, coefficient and asymptotic.

According to Meaney et al. (2011), the register of mathematics includes styles of meaning and ways of presenting arguments that require language structures that are often borrowed from specialized forms in what they call natural language. They exemplify this with the phrase the area under the given curve. Lemke (2003) too stresses the importance of teaching mathematics in parallel with language while also highlighting the role of visual representation.

Two key features in the mathematics register are highlighted by Schleppegrell (2007):

- Multiple semiotic systems (mathematics symbolic notations, oral language, written language, graphs and visual displays) and
- Grammatical patterns (technical vocabulary, dense noun phrases, being and having verbs, conjunctions with technical meanings and implicit logical relationships).

The multiple characteristics of the mathematics register make the concept of translanguaging particularly relevant for the use of multiple languages in mathematics classrooms. Among common grammatical features in mathematics language, Schleppegrell (2007) mentions verbs, conjunctions and nouns. The verbs be and have (and related verbs such as equals, means) are types of vocabulary whose functions vary between languages. The verb be is often a function word, as is the Swedish copula verb vara. As such, they are not easily translated between languages. Schleppegrell describes a type of noun phrase that is common in mathematics, consisting of three components: a pre-numerative phrase, the head noun and a qualifier. The pre-numerative phrase may be an abstract, quantifiable mathematical attribute of the head noun, such as the volume of, the difference between, the angle of, while the noun may be, for example, a geometrical form such as a cube or a rhomboid, and the qualifier may be a qualifying phrase (which can be divided by 3). Examples of such noun phrases are the volume of a cube with the sides 4 cm , the square of 5 . Schleppegrell also shows that conjunctions have specific, technical meanings in the register of mathematics, such as if, when and therefore. Other verb forms that are used in a register-specific sense include given and assume.

Schleppegrell (2007) also highlights the fact that mathematics problems express processes but become 'a thing' when represented through language. She exemplifies this with an equation, which in mathematics is expressed as a process, such as $(1+x)^{2}+x^{2}=25$, but is represented in language as a noun: The sum of the square of one plus $x$ and the square of $x$ is 25 . Thus, the student needs to understand the relation between 'the things' of grammar and the processes of reasoning in mathematics.

For science, Gibbons (2003) describes students' linguistic development as a mode continuum, where students move from visual contextualization through everyday language to formal, scientific language. She also reveals the important role that teachers have in guiding students in this development. Furthermore, she stresses that teachers' spoken language is an important link between symbolic and visual representations in mathematics, with its multisemiotic nature, as support for students to draw on different meaning-making modes for understanding. However, Hansson (2012) has shown in her research that mathematics education in Sweden is characterized by a low level of teacher instruction, and as a result, a great deal of responsibility for their own understanding is placed onto the students themselves. According to Hansson, this is even more, so the case in classes with a high proportion of migrant students or a high rate of students from homes with low socioeconomic status, which, she claims, results in pedagogical segregation in Sweden when it comes to mathematics education.

The importance of building on and including students' prior linguistic repertoires, as well as their prior subject knowledge, has been stressed by
researchers such as Thomas and Collier (1997), Cummins (2000), Gibbons (2006) and García (2009). As shown in other chapters in this volume, this includes translanguaging, which Lindahl (2015), from her studies in science classrooms that are based on sign language, describes as 'seamless shuttling' between different linguistic resources.

In research on mathematics education, Gwee and Saravanan (2018) studied code-switching between Singapore Colloquial English and Standard English among teachers in Singapore classrooms, finding that the former was used mainly for curriculum access. Dahm and de Angelis (2019) also found mother tongue literacy to have a positive impact on mathematics learning among students with multilingual backgrounds in Grade 9 in France. In a study of mathematics education in Grade 8 in Los Angeles, Abedi and Lord (2001) showed that linguistic modification had a positive impact on the performance of students in tests. They found that students who especially benefited were English language learners, students from low socioeconomic backgrounds and students at a low level and in average-level mathematics classes. For L2 students, it is important to remember that not all languages have developed mathematics terminology. In Arabic, for example, there is highly specific mathematical terminology. Thus, Arabic or concepts in Arabic are often used in the teaching of mathematics in the Middle East. However, for students who have limited education, these concepts may be unknown, and there may not be translation equivalents in their mother tongues. Thus, they may be encountering this terminology in Swedish for the first time ever.

In Sweden, Norén and Svensson Källberg (2018) argue that RASs are constructed in official policy documents as mathematics learners in need of rescue while lacking the asset that is most valued, namely the Swedish language. Mother tongues other than Swedish are, according to Norén and Svensson Källberg, seen as assets by teachers, while these students are at the same time thought of as having deficiencies and needing to improve their skills in Swedish and to progress in mathematics if they are to become desirable citizens. They found that when the mapping of students' prior knowledge is carried out with the Swedish curriculum as the norm, some of the earlier mathematical knowledge of the students is made invisible or at least not valued. As in other studies, they found a strong focus on the need to have a command of Swedish, the highest valued language.

Moschkovich (2015) argues for more complex perspectives on multilingualism and mathematics, and for viewing language as a resource, based on Ruiz's (1984) categorization of language as a problem, right or resource. Also, Planas and Setati-Phakeng (2014) draw on Ruiz in their study from Catalonia (Spain) and South Africa when analyzing monolingual norms and multilingual classroom practices. Barwell (2018), however, argues that this reasoning has its limitations. First, he finds the categorization made by Ruiz imprecise, as right and resource work together and are used in parallel. Second, he argues that language as a
resource is of limited analytic value as it lacks a definition of resource. Thus, he sees a risk that the concept of language as resource may contribute to a pedagogical approach that supports existing language ideologies rather than challenging them. He also argues that language as a resource carries implicit assumptions about the nature of language as a neutral substance.

Referring to students' use of multiple material, linguistic and social resources, Barwell (2018) draws on the shift in sociolinguistics from describing languages, varieties, dialects and registers to a more complex view of language as fluid and changing, a shift that is the result of the work of researchers such as Hornberger (1989, 2003), García (2009), Blackledge and Creese (2010), Blommaert (2010) and Blommaert and Rampton (2011). Based on a view of language as socially loaded linguistic resources (Blommaert, 2005), Barwell proposes sources of meaning as an analytic concept. He argues for a need to consider participants' repertoires of multiple sources of meaning, organized along three dimensions: languages, discourses and voices. Thus, he argues that this 'framework involves several principles: Mathematical meaning-making is relational, language is agentive, language is diverse and involves multiple languages, multiple discourses and multiple voices, and language is stratified and stratifying' (Barwell, 2018: 166). Language is thus perceived as being in constant tension, between a centripetal force (uniformity) and an opposing tendency toward novelty and variation (a centrifugal force). In mathematics, this means that some forms of mathematical expression are seen as less desirable, such as those that are closer to everyday language, while others are seen as better, namely those that conform to what has been called the mathematics register.

Thus, classroom practices that are in focus in this chapter will be studied with a focus on the development from informal language resources about mathematics to the register of mathematics, which construes more technical and precise meanings, with attention paid to multiple discourses, multiple voices and multiple languages, which is referred to here as (trans) languaging in mathematics. In this case, for some of the students who had not studied mathematics at higher levels, this also included developing varied types of mathematical thinking.

## Methodology

The data that are analyzed in this chapter was created as part of a research project on RASs in upper-secondary school in Sweden ${ }^{1}$. Linguistic ethnography has been used as the methodological frame (Creese, 2008; Copland \& Creese, 2015; Martin-Jones \& Martin, 2017). Linguistic ethnography links 'the micro to the macro, the small to the large, the varied to the routine, the individual to the social, the creative to the constraining, and the historical to the present and to the future' (Copland \& Creese,

2015: 26). In this case, the use of linguistic ethnography is particularly relevant when a complex phenomenon is being analyzed such as how language is used in meaning-making in multilingual classrooms.

The data were created in two classes, A at the beginner level and B at the advanced level, consisting of $10-15$ students each, taught by the same teacher. However, only four to nine students were present during the observed lessons. Due to the vulnerability of these students, ethical issues were carefully considered throughout, and the students and teacher were carefully informed about the study prior to their consent being requested. Pseudonyms are used, and presentations are such that recognition is avoided.

The data consist of fieldnotes; photographs and audio recordings; artifacts such as handouts and textbooks from eight 60-minute lessons, two of which are analyzed here and formal and informal teacher interviews. The content of the lessons may be understood as corresponding to Grade 4 and Grade 9 in Swedish compulsory school. The languages used by students were Dari, Somali, Arabic, Tigrinya, Kurdish (Kurmanji), English and Swedish. As I myself master only a few of these, the analysis is based on what may be understood from the interaction as a whole, with features such as body language, gaze, engagement and the solutions to the mathematical tasks as important.

To identify situations where (multiple) language(s) is (are) used as a source(s) of meaning, a modification of Gibbons' (2006) model of episode summary (see Figure 8.1) was used to create analytic units. The theoretical base for Gibbons' model is a combination of Sociocultural Theory and Systemic Functional Linguistics, with field (what is the topic and content), tenor (who are involved) and mode (which modalities are used). Here, tenor will include multiple discourses; interaction pattern will include multiple voices and mode will include multiple languages. Thus, the three dimensions of Barwell's language as sources of meaning - multiple languages, multiple discourses and multiple voices - were combined with Gibbons' episode summaries.

| How (Tenor, interaction, relation) discourses |  |  | What (Field, representation) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Teaching and learning processes | Dominant <br> participant <br> and <br> interaction <br> structures <br> voices | Mode, degree of contextembedding languages | Knowledge constructed about the subject | Knowledge constructed about language | Knowledge constructed about being a student |

Figure 8.1 Episode summary modified from Gibbons (2006: 96) including Barwell's sources of meaning

The analytic unit episode is defined by Gibbons as 'a bonded unit that roughly correlates with a single teaching activity’ (2006: 96). Gibbons’ episode builds on Lemke (1990) and is linguistically marked by 'realizations of frames and markers', such as 'well' and 'now', and its opening and closing is marked by three nonlinguistic features: (1) participant structure is likely to change, as are (2) physical seating arrangements, and (3) each episode has its own particular purpose or function. The analysis involves the kinds of meanings created within and across episodes and the intertextual relationships that exist between them. In this case, this means that the sequences of activities in the lessons, the episodes, were analyzed with a focus on languaging in the negotiation of meaning in mathematics. Thus, understanding was constructed as part of what goes on in the classroom to contextualize what 'being in the classroom was like: what sorts of things that were said and done' (Gibbons, 2006: 97). Thus, a thick description was created in the form of an analysis of the episodes in the cultural and linguistic context of the classroom by analyzing aspects of discourse, voice and language in the learning of the second language and acquisition of subject knowledge to show language patterns of negotiation of meaning. The main focus was on language, while voice was used in relation to who gets to talk and who is listened to, and discourse was here mainly used for the roles that individuals had in classroom interaction.

Different episodes are linked to each other, not only during a particular lesson but also between lessons, for example, by having related a topic. The teacher may link a certain episode to an earlier one through repetition or reference ('Do you remember X?') or to a future lesson by referring to a next step or an upcoming test. Thus, individual episodes are linked to and nested in each other.

Observations of eight lessons revealed a three-stage pattern in the organization of each lesson: (I) Opening of lesson, (II) Mathematical meaning-making and (III) Termination of lesson. For this chapter, two of the eight lessons were selected, one from Group A and one from Group B, to represent the eight lessons. The range of oral interaction varies between the groups with students in the more advanced group (Group B) working mainly on their own on exercises.

## Negotiation of Meaning in Group A

In the first lesson, with Group A, the teacher (Khaled) has five students in the classroom, here called Gulan, Maxamed, Osman, Sarwar and Hani. All five had received very little schooling prior to arriving in Sweden. During the first half of the lesson, Suleymaan, who works as an SGMT tutor in Somali, is also present. The five students speak different languages (as well as Swedish): Gulan speaks Kurdish (Kormanji); Maxamed, Osman and Hani speak Somali and Sarwar speaks Dari. Suleymaan, the SGMT tutor, speaks Somali and Arabic. This means that while Hani,

Maxamed, Osman and Sarwar may receive support in both Swedish and another language (Somali and Farsi/Dari), Gulan is helped only in Swedish. Maxamed and Osman are brothers, and as Maxamed has difficulties with mathematics, his brother helps him in the lessons, also correcting his mistakes.

In the lesson, seven episodes were identified in Stage I: (1) Welcome, (2) Take out the book and (3) Organize work; in Stage II: (4) Repetition and individual work and (5) Handout and in Stage III: (6) Termination of the lesson. Stage I, including the first three episodes, Introduction, Take out the book and Organize work, covers the first 11 minutes of the lesson. The teacher starts by welcoming the students and asking them how they are. He introduces the lesson by saying that two of them, Hani and Sarwar, are to work on their own using their books, while he will go through fractions and decimal numbers with the other three, Gulan, Osman and Maxamed. He then asks all students to take out their books and makes sure that Hani and Sarwar know what they are expected to do. Hani says that she forgot her book at home, so Khaled gives her his book. Khaled shows concern for students' well-being and says to Gulan, ${ }^{2}$ for example, 'If you are not tired and if you feel well enough, you can answer the questions'. His interest in his students' well-being is obvious in the other lessons as well, and in interviews with him, he explains that he is aware that some of them have a tough time outside school. In the third episode, Khaled divides the class by asking Gulan to move to where Maxamed and Osman are sitting, close to the whiteboard. While Gulan is moving, Hani asks Suleymaan in Somali for help, so he sits down at her desk and starts helping her.

At this first stage, the discourse is that of a traditional classroom, with interaction dominated by the teacher's directives, while the students' role is to listen and do as they are told. Khaled is the one holding the floor and students listen, as does Suleymaan, the SGMT tutor. Khaled uses mainly Swedish at this initial stage and exchanges only a few words in Dari with Sarwar but tells him in Swedish that the reason why he is to work on his own is that he has studied geometry before and that the other students are to work with fractions and decimal numbers. The focus is on why individual students should do particular activities rather than on learning mathematics. At this stage, social relations play an important role, and Khaled demonstrates his interest in ensuring that students are at ease.

At Stage II, and Episodes 4 and 5, Khaled turns his focus to the group of Gulan, Maxamed and Osman. In Episode 4, he is involved with these three students, and when Suleymaan is not helping Hani, he is also involved with this group and sometimes provides explanations to Maxamed and Osman in Somali. Khaled starts by revising the decimal system from earlier lessons about units, tens, hundreds and thousands. He
takes out a sack of teaching material and empties its contents onto the desks in front of the three students. The teaching material consists of fake money and magnetic figures resembling units, tens, hundreds and thousands. He uses these artifacts, mainly the magnetic figures that he arranges on the whiteboard (pictures), to talk about the decimal system. He starts by demonstrating the use. On one occasion, he pretends to make a mistake and questions the students (nonitalics is said in Swedish).

```
Excerpt 8.1
Khaled: Är det rätt?
    Is this right?
Gulan: Ja
    Yes
Osman: Nej, det är fel
    No,it's wrong
Khaled: Efter tusental kommer ...
    After thousand comes ...
Students: Tiotusental
    Ten thousands
Khaled: efter tiotusental
        After ten thousands
Osman: Miljoner
        Millions
Khaled: Aaa innan miljoner ... hundratusental eller hur? Sen som
        Osman säger miljon.
        Abh before millions ... hundred thousands, right? Then, as
        Osman says, million.
Osman: Därför att jag gillar mycket pengar (skrattar)
        Because I like a lot of money (laughs)
```

Here, Khaled invites students to correct him and to perform the role of expert while he himself takes on the role of novice. He uses pause (...) to encourage students to fill in, and when they do not do so, he gives the answer himself. Khaled then hands out the fake money and says: 'A lot of money here' and laughs, 'but not real money, this helps us to understand', thus connecting to Omar's statement that he likes money. He then writes ' 2785 ' on the whiteboard and invites students to help him show this by using the material. He asks: 'How many tens? Eight or five?' He starts to arrange tens on the board and has students help him. Then, he asks:

## Excerpt 8.2

Khaled: Hur många hundratal? How many hundreds?
Osman: Nästan tio Nearly ten
Khaled: (points to the Figure, 7)
Osman: Sju
Seven

He asks the three students to show numbers individually on the desk using the fake money. While they are trying to work out the correct way to represent the numbers, he and Suleymaan help them individually. They continue to represent numbers, led by Khaled, using numbers and magnetic figures on the whiteboard, money on the desks and talk. When Maxamed has problems completing the task to represent a number in magnetic figures on the bench, Osman explains to him, mainly in Somali, and provides corrective feedback on what he has done. He asks Khaled to also explain to Maxamed, making sure that what he said to Maxamed was correct. Khaled then asks each of the students to represent a four-digit number on the whiteboard in magnetic figures.

In Figure 8.2, we can see how meaning is negotiated through interaction in collaboration between the teacher and students using digits, material and oral number-naming. The pictures visualize how the body is part of the negotiation, particularly through pointing (Picture 1) and gaze (Picture 3). Picture 2 shows how Osman (to the right) monitors the work of his brother (in black at the whiteboard) and walks up to help him represent the number correctly using the material. In Episode 4, knowledge is negotiated in terms of the relation between numbers in digits (2785), oral number names in Swedish 'tvåtusensjuhundraåttiofem' (two thousand seven hundred and eighty-five) and their representation mainly in magnetic figures as well as to some extent in fake money. For Maxamed and Osman, this is also expressed in Somali in their interaction with each other and with Suleymaan.

In Episode 5, Khaled hands out exercises on the decimal system to these three students. At this point, Suleymaan leaves the room. Khaled tells them to read the instructions carefully, and as they start working, Sarwan asks for help, and Khaled goes to his desk and explains to him in Farsi, reading his text in Swedish, translating and explaining. When Khaled returns from Sarwan, Gulan, who is working on exercises in the handout where she is required to write a row of numbers in order of magnitude, addresses him: 'I don't understand order of magnitude'. Khaled starts explaining to Gulan, while Osman explains in Somali to Maxamed what Khaled is saying, but soon Khaled includes the whole class in his explanations (the words in bold are said in English):


Figure 8.2 Pictures 1, 2 and 3

## Excerpt 8.3

| K: | Kolla på mig, jag tror han har förklarat (hänvisar till Osman) men störst, mindre, minst förklara störst, minst, vilken är minst? <br> Look at me, I think he has explained (referring to Osman) but biggest, smaller, smallest explain biggest, smallest, which one is smallest? |
| :---: | :---: |
| Gulan: | Ental |
|  | Units |
| K: | Vi pratar inte om ental, vi pratar om vilken är minst (vänder sig mot |
|  | Maxamed) förstår du? |
|  | We are not talking about units, we're talking about which is smallest (he |
|  | turns toward Maxamed): do you understand? (nods) |
| Maxamed: K: | Storleksordning betyder den kommer först sen den sen den eller hur? (visar med kroppen) Size small eller large eller (tittar i Gulans uppgiftspapper) |
|  | Gulan börja med minst ... men här började vi med störst Order of magnitude, means this is first then this, then this, doesn't it? (demonstrates with his body) Size small or large or (looks in Gulan's |
|  | hand out) Gulan, start with smallest ... but here we started with biggest |
| Osman: | Till exempel Large, X Large, Small på kläder |
|  | For example, Large, X Large, Small in clothes |
| K: | (laughs and starts to show on his own shirt in the neck where the marking for size is, and turns to Osman) Large, X Large kanske din pappa är X Large jag vet inte (vänder sig till Gulan) du måste skriva ett tal ett belopp |
|  | Large, X Large perhaps your father is X Large I don't know (turns toward Gulan) you have to write a number a sum |

In this example, Khaled talks about size order, and his comment on Gulan's suggested answer 'Units' is that they are not talking about units but about which is smallest. By way of negotiation, initiated by Osman, exemplifying with $\mathrm{L}, \mathrm{XL}$ and so on, he turns to the task of selecting the greatest sum. Pictures 4,5 and 6 in Figure 8.3 show how the body is part of the negotiation of mathematical meaning. Here, Khaled's explanation is initiated by Gulan's question from the handout with exercises, but he turns to explain the words biggest/smaller/smallest, while Osman turns to size in clothes, and Khaled uses his shirt to demonstrate. After his explanation to everyone in Swedish, he turns toward Sarwan and explains in Farsi, while Osman explains to his brother, Maxamed, in Somali. Students then continue to work on their exercises for another 20 minutes with Khaled walking around assisting those who ask for help and with Osman helping Maxamed while also working on his own questions.

In Episodes 4 and 5, mathematical knowledge is negotiated by the teacher, Khaled, and mainly the three students in the small group, Gulan, Maxamed and Osman, while Sarwan and Hani (to a lesser extent) are included in the last part about size order. In Episode 4, Suleymaan, the SGMT tutor, is also included in the interaction. The main topic for Gulan, Maxamad and Osman is the decimal system, which they work on through talk, digits and objects in the form of fake


Figure 8.3 Pictures 4, 5 and 6
money and magnetic figures, and order of magnitude. Negotiation involves shuttling between different modes (orality, mathematics numbers on the whiteboard, textbooks and handouts, gestures and materials including the inside collar of the teacher's shirt) and different languages (Swedish, Somali, Farsi and Dari). For students, different modes and languages are important when negotiating meaning, and collaboration is important for the meaning-making. During these episodes, Gibbons' (2006) mode continuum regarding science, where students start with visual contextualization moving through everyday language to formal scientific language, is not reconstructed. Rather, Khaled moves between written numbers, oral expressions and the material, starting and ending with the written representation, using written mathematics language, anchored in everyday language and thinking using gestures and materials. As in Gibbons' mode continuum, this includes going between varied language registers (mathematics register and everyday language) and varied ways of reasoning (mathematical reasoning and everyday thinking about mathematical matters).

Finally, at Stage III, Episode 6, when about 10 minutes of the lesson remain, Khaled ends the lesson. He addresses all five students, saying: 'Bra jobbat, mitt förslag, på kvällen fortsätt jobba, under helgen. Jag känner mig lugn, alla' (Well done, my suggestion, continue to work in the evening, over the weekend. I feel calm, everybody.) Just like at the beginning of the lesson, he talks about being a student and working at home and expresses confidence in the students' performance.

To sum up, at the beginning and closing stages, the teacher addresses issues of being a student, making sure that they are feeling confident and also encouraging them to study at home. At the second stage, meaningmaking, which constitutes the main part of the lesson, the focus is on negotiation of meaning - in this lesson with the teacher focusing on three students and the topic of the decimal system and the student-initiated topic of size. Swedish is the main language used here, particularly at the start and end of the lesson, but for the negotiation of meaning, various linguistic resources are used - both oral and written - together with embodied expressions. Written digits are combined with written text in handouts and textbooks, and some students write comments in different languages. Swedish is combined with Somali, Farsi and Dari, which
makes the fluidity of languaging visible. It is interesting to note how Osman takes on the role of mediator between the teacher and his brother, checking with the teacher that he is right. This process includes translanguaging between mainly Swedish and Somali, including the multiple semiotic systems of mathematics. In the negotiation of mathematical meaning, the teacher and students use the linguistic resources that are at hand without visible borders or restrictions. While Swedish is used more than other languages, the fluidity and 'seamlessness' (Lindahl, 2015) of their translanguaging is obvious. Thus, the analysis shows that the teacher's talk dominates, as he initiates activities, addresses students and poses questions to them, while the students answer questions and ask for clarification or explanation. Swedish predominates along with mathematical expressions, while other languages are used as sources of meaning through translanguaging.

## Intensive Work in Group B

The characteristics of the selected lesson in Group B differ from the previous lesson. Group B includes students who are close to meeting the requirements of mainstream programs at upper-secondary school, and as the lesson takes place at the end of the term, the national tests are approaching, only a week way. In this lesson too, only five students are present, and these are the five who are confident they will pass the test: Kifle, who speaks Tigrinya, English and Arabic, and Mehran, Ali, Baqer and Hamid, who are all speakers of Dari. As Khaled speaks Farsi (as well as Swedish and English), he uses Farsi with the Dari-speaking students, while Suleymaan, the SGMT tutor who is present, places himself close to Kifle as they share Arabic. During the lesson, a teaching assistant, who speaks English and Swedish, is also present. It is his first day at the school, and in the first part of the lesson, he sits down at an empty desk.

As in the other lesson, this lesson involves three stages: (I) Introduction, Episode 1, (II) Meaning-making, Episodes 2 and 3 and (III) Termination, Episode 4. The first stage is only about 5 minutes long, with one episode where Khaled welcomes the students and introduces the lesson. As the national tests are approaching, the plan is that students will work on a topic that he finds important and relevant, the equation of a line. He starts the next stage, meaning-making, and Episode 2, preparation for the national test, by walking up to the whiteboard and drawing a coordinate system (Figure 8.4). He briefly repeats the rules for the relation between different equations and their graphs, focusing on the slope of the line and where it cuts the $y$-axis.

Thus, the topic itself includes going between different representations of the same mathematical knowledge, something that Khaled demonstrates by pointing to the equations and their corresponding lines, explaining their relations. Khaled speaks mainly Swedish, with


Figure 8.4 Picture 7, illustrations for the equation of a line
translations into Farsi, while Suleymaan makes sure that Kifle has understood. As Kifle's mathematical skills are good (he passed the test the week after), Suleymaan does not have to explain much and mainly makes sure that he understands Khaled's Swedish. Khaled does not ask many questions, and although students listen attentively, it is not possible to tell from the interaction whether they all understand. When Khaled says in Swedish, 'Linjen skär y-axeln' (the line cuts the $y$-axis), Baqer comments, 'Ibland på dari det betyder åsna' (Sometimes in Dari it means donkey). Then, Khaled laughs and comments that it is true because ' $\tau$ ', which is pronounced similar to the Swedish skär (pronounced like share in English) means donkey in Dari as well as in Farsi. This short comment makes students' navigation between languages in their meaning-making visible. This episode is the short introduction and during the next episode (Episode 3), students work individually with assigned pages in their books for the most part of the lesson, more than 40 minutes. Some students find the exercises difficult, and both Khaled and Suleymaan help the students individually. Because Kifle manages to complete the exercises well, Suleymaan mainly helps the other students who are using Swedish. The teaching assistant, who is new to the position, sits close to two students and tells them in Swedish and English how to complete the exercises, but after a while, Khaled asks him to let the students think on their own. The students work through the whole lesson until 5 minutes remain, at which point Khaled
ends the lesson by telling them to work on exercises at home in preparation for the test.

While this lesson includes less talk and less work with materials than the previous lesson, it follows the same pattern, with an introduction and end mainly focused on social relations and on what the students are expected to do. At Stage II, the meaning-making part, the teacher and students negotiate meaning focusing on the mathematics topic for the lesson, going between the representations. In this lesson, the teacher's instructions are shorter than in the previous lesson, and he does not involve many modalities except for drawings and writing on the whiteboard to illustrate the relation between equations and the coordinate system with relevant lines and explanations in everyday language with few questions to students. Nonetheless, several languages are involved, mainly Swedish, Farsi, Dari and Arabic, and the one student's comment on the similarities between Dari and Swedish is an example of the role of multiple languages in student engagement.

## Languages as Sources of Meaning

The combination of Barwell's languages as sources of meaning with its focus on languages, discourses and voices, and Gibbons' episode summaries provided analytical tools for these lessons, to reveal patterns of discourse, voice and language. The discourse may be understood as a discourse of mathematics education. The shuttling between modes is common in mathematics classrooms, although here this includes different verbal linguistic resources. The teacher's three-stage plan for the lessons included mainly social relations in the first and last stages, where he opened and closed the lesson by making sure that students were well and urging them to study outside school, while Stage II, the meaning-making, constituted the part where the main part of the negotiation of meaning in mathematics took place. In both lessons, the teacher, the students and the SGMT tutor made use of varied resources for meaning-making, navigating between linguistic resources. Particularly in the first lesson, varied resources were at play, and the negotiation of meaning took place using oral and written language in combination with illustrations, body language and objects such as fake money and magnetic figures. Furthermore, in the other lesson, in Group B, the teacher's strategy may be characterized as anchoring mathematical knowledge, this time starting with a mathematical expression in the form of an equation related to the coordinate system drawn on the whiteboard, followed by an explanation using everyday language in combination with body language and the representations on the board, and ending with the expected answer.

That mathematical knowledge represented through various modes in Group A is relevant, as students had not yet developed advanced mathematical knowledge and were still at a basic level. The strategy used by the
teacher (negotiating mathematical knowledge by starting and ending with the most typical form of representing knowledge in mathematics, written digits, explaining in between by using materials and oral talk) is common in mathematics education. This does not straightforwardly represent what Gibbons calls a mode continuum referring to science, but for mathematics, this could be explained as anchoring mathematics language and thinking in everyday talk and thinking through material while shuttling between languages and modalities. The anchoring may be exemplified by Khaled writing ' 2785 ', pronouncing it and supporting students to represent it using teaching material, and then finally referring back to the digits. He thus anchored the language of mathematics not only in everyday language but also in everyday ways of thinking mathematically. Another example is the student-initiated topic order of magnitude that Khaled related to clothes sizes, and his use of his own clothes and body, starting and finishing with the given mathematics task. Thus, this became a link between students' living sense and mathematical thinking. In this process, language played a central part, and Khaled and the students negotiated mathematical meaning by shuttling between diverse modalities and languages.

In Group B, where content and students' thinking were at a higher, more abstract mathematical level, Khaled and the students built on students' prior knowledge, and as a result, much more took place invisibly in their minds, and what could be observed (and recorded) was more fragmented, and consisted mainly of talk in relation to mathematical constructs, such as the coordinate system and relations between lines with varying slopes and equations. The talk was close to the task in the textbook, and the focus was on solving given problems.

In the lessons, students' prior knowledge was linked to different languages, and in the case of students who had already mastered the mathematics content, such as Kifle, their main task was to learn how these relations are expressed in Swedish. Thus, the task for the participants in both lessons may be understood as being a combination of negotiating knowledge (about the equation of a line and the decimal system) and developing Swedish mathematics language. This is an abstract process, taking place in the minds of individual students and being difficult to capture, but the case of the comparison between Dari and Swedish is one example of how the negotiation of meaning in mathematics included navigation between languages. Thus, the shuttling between different modalities commonly used to express mathematical meaning, using numbers and formula in combination with material and oral expressions in these two classrooms, also included navigation between varied linguistic resources.

Through the analysis, varied aspects of languages, discourses and voices were made visible. While the discourse in the first lesson included
more visible shuttling between languages and modalities, and meaning was more openly negotiated in collaboration between the participants, in the second lesson, students worked hard and most work took place in silence and visibly through writing. With a high-stakes test approaching, it is not surprising that the students were quiet in this lesson. The teacher-student roles were mainly traditional, although there were exceptions: for example, the change in roles initiated by the teacher pretending to be ignorant and several student initiatives, such as when one student related the Swedish skär to dari خر, and when Osman changed roles in his mediating. One example of pragmatic language use was when Suleymaan, whose official task was to support students through Somali, found there was no Somali-speaking student present. His first choice was to support Kifle because they both knew Arabic, and when he realized that Kifle was managing on his own, his next choice was to support other students whose only language they had in common was Swedish.

## Discussion

In these two lessons, translanguaging plays an important role as a source of meaning in ways where language is not fixed but is negotiated. Similar to Gibbons' mode continuum from science classrooms, this classroom also shows a shuttling between registers, modes and languages in the negotiation of meaning and anchoring mathematical knowledge and mathematical language, in written and oral form, through everyday language and in relation to students' everyday experiences and thinking.

The analysis of these two lessons shows that there is much more going on than simply the negotiation of meaning in mathematics. The patterns of translanguaging show the importance of making mathematics teachers aware of the role of (trans)languaging in mathematics education. The teacher's anchoring of knowledge through translanguaging in this case gives the impression of it taking place without deliberate planning by the teacher but rather gives a spontaneous impression. This shows the importance of considering not only the negotiation of mathematical knowledge but also aspects of (trans)languaging in mathematics teacher education and research on mathematics education. Using an explicit translanguaging pedagogy, the development of both knowledge and language may be more deliberately planned. Teachers need to be able to assess students' mathematical understanding, for which language is crucial. While students need to be given space to express process and product in mathematical knowledge, teachers need to understand the role of (trans)languaging as a source of meaning in mathematics.

## Notes

(1) The project Recently arrived students in Swedish upper-secondary school - a multidisciplinary study on language development, disciplinary literacy and social inclusion, 2018-2021 - was financed by the Swedish Research Council, Grant No. 2017-03566.
(2) Transcripts were translated from Swedish by the author.

## References

Abedi, J. and Lord, C. (2001) The language factor in mathematics tests. Applied Measurement in Education 14 (3), 219-234.
Axelsson, M. (2013) Flerspråkiga barn utvecklar litteracitet. In H.P. Laursen (ed.) Literacy og sproglig diversitet. [Literacy and Linguistic Diversity] (pp. 249-274). Århus: Aarhus Universitetsforlag.
Barwell, R. (2018) From language as a resource to sources of meaning in multilingual mathematics classrooms. Journal of Mathematical Behavior 50, 155-168.
Blackledge, A. and Creese, A. (2010) Multilingualism: A Critical Perspective. London: Continuum.
Blommaert, J. (2005) Situating language rights: English and Swahili in Tanzania revisited. Journal of Sociolinguistics 9 (3), 390-417.
Blommaert, J. (2010) The Sociolinguistics of Globalization. Cambridge: Cambridge University Press.
Blommaert, J. and Rampton, B. (2011) Language and superdiversity. Diversities 13 (2), 1-21.
Compulsory School Ordinance (2011) Svensk författningssamling. [Swedish Code of Statutes] 2011:185. Available at: https://www.riksdagen.se/sv/dokument-lagar/doku-ment/svensk-forfattningssamling/skollag-2010800_sfs-2010-800
Copland, F. and Creese, A. (2015) Linguistic Ethnography: Collecting Analysing and Presenting Data. Los Angeles: SAGE.
Creese, A. (2008) Linguistic ethnography. In K. King and N. Hornberger (eds) Encyclopedia of Language and Education (2nd edn) (pp. 229-241). New York: Springer.
Cuevas, G.J. (1984) Mathematics learning in English as a second language. Journal for Research in Mathematics Education 15 (2), 134-144.
Cummins, J. (2000) Language, Power and Pedagogy: Bilingual Children in the Crossfire. Clevedon: Multilingual Matters.
Dahm, R. and De Angelis, G. (2019) The role of mother tongue literacy in language learning and mathematical learning: Is there a multilingual benefit for both? International Journal of Multilingualism 15 (2), 194-213.
Education Act (2010) 2010:800. Stockholm: Skolverket.
Eikset, A. and Meaney, T. (2018) When does a difference make a difference? Teaching about language diversity in mathematics teacher education. Nordic Studies in Mathematics Education 23 (3-4), 225-246.
García, O. (2009) Bilingual Education in the 21st Century: A Global Perspective. Oxford: Wiley-Blackwell.
Gibbons, P. (2003) Mediating language learning: Teacher interactions with ESL students in a content-based classroom. TESOL Quarterly 37 (2), 247-73.
Gibbons, P. (2006) Bridging Discourses in the ESL Classroom: Students, Teachers and Researchers. London: Continuum.
Gweea, S. and Saravanan, V. (2018) Use of code-switching in multilingual content subject and language classrooms. International Journal of Multilingualism 15 (2), 117-134.

Halliday, M.A.K. (1975) Some aspects of sociolinguistics. In E. Jacobsen (ed.) Interactions Between Linguistics and Mathematical Education: Final Report of the Symposium Sponsored UNESCO, CEDO and ICMI, Nairobi, Kenya, September 1-11, 1974 (UNESCO ED-74/CONF.808, pp. 64-73). Paris: UNESCO.
Halliday, M.A.K. (1978) Language as Social Semiotic. London: Edward Arnold.
Hansson, $\AA$. (2012) The meaning of mathematics instruction in multilingual classrooms: analyzing the importance of responsibility for learning. Educational Studies in Mathematics 81, 103-125.
Hornberger, N.H. (1989) Continua of biliteracy. Review of Educational Research 59 (3), 271-296.
Hornberger, N.H. (ed.) (2003) Continua of Biliteracy: An Ecological Framework for Educational Policy, Research, and Practice in Multilingual Settings. Clevedon: Multilingual Matters.
Lemke, J.L. (1989) Making text talk. Theory into Practice 28 (2), 136-141. https://www. jstor.org/stable/1477000
Lemke, J.L. (1990) Talking Science: Language, Learning, and Values. Norwood: NJ: Ablex.
Lemke, J.L. (2003) Mathematics in the middle: Measure, picture, gesture, sign, and word. In M. Anderson, A. Sáenz-Ludlow, S. Zellweger and V.V. Cifarelli (eds) Educational Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing (pp. 215-234). Brooklyn, NY, and Ottawa, Ontario: Legas.
Lindahl, C. (2015) Tecken av betydelse: En studie av dialog i ett multimodalt, teckenspråkigt tvåspråkigt NO-klassrum. [Signs with Meaning: A Study of Dialogue in a Multimodal Sign Language natural Sciences Classroom.] PhD Thesis, Stockholm: Stockholms universitet.
Martin-Jones, M. and Martin, D. (2017) Researching Multilingualism: Critical and Ethnographic Perspectives. London: Routledge.
Meaney, T., Trinick, T. and Fairhall, U. (2011) Teaching Mathematics in a Second Language. Charles Sturt University, University of Auckland, Te Kura o Te Koutu Conference paper.
Moschkovich, J.N. (2015) Academic literacy in mathematics for English learners. The Journal of Mathematical Behavior 40, 43-62.
Norén, E. and Svensson Källberg, P. (2018) Fabrication of newly-arrived students as mathematical learners. Nordic Studies in Mathematics Education 23 (3-4), 15-37.
Petersson, J. and Norén, E. (2017) To halve a fraction: An issue for second language learners. Education Inquiry 8 (3), 173-191.
Pimm, D. (1987) Speaking Mathematically: Communication in Mathematics Classrooms. London: Routledge \& Kegan Paul.
Planas, N. and Setati-Phakeng, M. (2014) On the process of gaining language as a resource in mathematics education. ZDM The International Journal of Mathematics Education 46 (6), 883-893.
Prediger, S., Wilhelm, N., Büchter, A., Gürsoy, E. and Benholz, C. (2018) Language proficiency and mathematics achievement: Empirical study of language-induced obstacles in a high stakes test, the central exam ZP10. Journal für Mathematik-Didaktik 39 (3). doi: 10.1007/s13138-018-0126-3
Ordinance for Upper-Secondary School (2010) 2010:2039. Available at https://www.riks-dagen.se/sv/dokument-lagar/dokument/svensk-forfattningssamling/gymnasieforordning-20102039_sfs-2010-2039
Ruiz, R. (1984) Orientations in language planning. NABE Journal 8 (2), 15-34.
Schleppegrell, M.J. (2007) The linguistic challenges of mathematics teaching and learning: A research review. Reading \& Writing Quarterly 23, 139-159.
Svensson, P., Meaney, T. and Norén, E. (2014) Immigrant students' perceptions of their possibilities to learn mathematics: The case of homework. For the Learning of Mathematics 34 (3), 32-37.

The School Inspectorate (2014) Utbildningen för nyanlända elever. [The Education for Recently Arrived Students] Rapport 2014:03. Stockholm: Skolinspektionen.
The School Inspectorate (2017) Skolhuvudmäns mottagande av nyanlända elever i grundskolan. [Reception among school authorities of recently arrived students in primary school]. Stockholm: Skolinspektionen.
Thomas, W.P. and Collier, V.P. (1997) School Effectiveness for Language Minority Students. Washington: National Clearinghouse for Bilingual Education.
Wedin, $\AA$. and Bomström Aho, E. (2019) Agency in science learning in a second language setting: Multimodal and multilingual strategies and practices among recently arrived students in upper-secondary school in Sweden. International Electronic Journal of Elementary Education 12 (1), 67-74. DOI: 10.26822/iejee. 2019553339

