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μ_N Irresolvable spaces and μ_N Resolvable spaces

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Abstract: In this article, the idea of μ_N irresolvable, μ_N irresolvable, μ_N open hereditarily irresolvable spaces are to be introduced. Some of its characters are to be discussed. Also, the concept of μ_N hyperconnectedness are to be introduced and its properties are to be contemplated.

Key words: μ_N dense, μ_N Resolvable, μ_N Irresolvable, μ_N open hereditarily irresolvable spaces, μ_N submaximal, μ_N hyperconnected

1. Introduction

The idea of fuzzy set[16] which plays a vital role in almost all sectors of mathematics. Belatedly C.L Chang[2] brought out fuzzy topological space and after that several notions in general topology were extended and enhanced in fuzzy topological spaces. K.T Attansov[1] published his idea of intuitionistic set and some research works came into the literature. The concept of neutrosophy and neutrosophic sets were putforth by Samarandache[3],[6],[7],[8],[15] with his idea later on Salama and Albowi[12],[13],[14] introduced neutrosophic crisp sets. The concept of resolvability and irresolvablility in neutrosphic topology was brought out by Dhavaseelan et al[4]. The concept of generalized topological spaces via nutrosophic sets were introduced by N.Raksha Ben[10],[11] and some of its attributes were delineated by them. In this article the concept of μ_N irresolvable, μ_N resolvable, μ_N open hereditarily irresolvable spaces, μ_N submaximal spaces, μ_N connected, μ_N hyperconnected are to be introduced and some of their characters are to be narrated.

2. Necessities

Definition 2.1. [13] Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.1. *[13] Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form $A = \{< \mu_A(x), 1-(\mu_A(x)+\sigma_A(x)), \gamma_A(x) >: x \in X\}$. Since our main purpose is to construct the tools for developing Neutrosophic Set and Neutrosophic topology, we must introduce*

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the neutrosophic sets 0_N *and* 1_N *in* X *as follows:* 0_N *may be defined as follows*
$(0_1)\ 0_N = \{< x, 0, 0, 1 >: x \in X$
1_N *may be defined as follows*
$(1_1)\ 1_N = \{< x, 1, 0, 0 >: x \in X$

Definition 2.2. [13] Let $A = \{< \mu_A, \sigma_A, \gamma_A >\}$ be a NS on X, then the complement of the set A [$C(A)$ for short] may be defined as three kinds of complements :
$(C_1)\ C(A) = A = \{< x, 1-\mu_A(x), 1-\sigma_A(x), 1-\gamma_A(x) >: x \in X\}$

Definition 2.3. [13] Let X be a non-empty set and neutrosophic sets A and B in the form $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$. Then we may consider two possibilities for definitions for subsets $(A \subseteq B)$.
$A \subseteq B$ may be defined as :
$(A \subseteq B) \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x)\ \forall\, x \in X$

Proposition 2.1. *[13] For any neutrosophic set* A*, the following conditions holds:*
$0_N \subseteq A,\ 0_N \subseteq 0_N$
$A \subseteq 1_N,\ 1_N \subseteq 1_N$

Definition 2.4. [13] Let X be a non empty set and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ $B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$ are NSs. Then $A \cap B$ may be defined as :
$(I_1)\ A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$
$A \cup B$ may be defined as :
$(I_1)\ A \cup B = < x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) >$

Definition 2.5. [10] A μ_N topology is a non - empty set X is a family of neutrosophic subsets in X satisfying the following axioms:
$(\mu_{N_1}) 0_N \in \mu_N$
$(\mu_{N_2}) G_1 \cup G_2 \in \mu_N$ for any $G_1, G_2 \in \mu_N$.

Remark 2.2. *[10] The elements of* μ_N *are* μ_N*-open sets and their complement is called* μ_N *closed sets.*

Definition 2.6. [10] Let (X, μ_N) be a μ_N TS and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >\}$ be a neutrosophic set in X. Then the μ_N- Closure of A is the intersection of all μ_N closed sets containing A.

Definition 2.7. [10] Let (X, μ_N) be a μ_N TS and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >\}$ be a neutrosophic set in X. Then the μ_N- Interior of A is the union of all μ_N open sets contained in A.

Definition 2.8. [4] A neutrosophic set A in NTS is called neutrosophic dense if there exists no neutrosophic closed sets B in (X, T) such that $A \subset B \subset 1_N$.

Definition 2.9. [11] The μ_N Topological spaces is said to be μ_N Baire's Space if $\mu_N Int(\cup_{i=1}^{\infty} G_i) = 0_N$ where G_i's are μ_N nowhere dense set in (X, μ_N).

Proposition 2.2. *[11] Let (X, μ_N) be a μ_N TS. Then the following are equivalent.*

(i) (X, μ_N) *is* μ_N *Baire's Space.*

(ii) $\mu_N Int(A) = 0_N$, *for all* μ_N *first category set in* (X, μ_N).

(iii) $\mu_N Cl(A) = 1_N$, *for every* μ_N *Residual set in* (X, μ_N).

Theorem 2.1. *[11] If* A *is a* μ_N *dense set in* (X, μ_N) *and also* μ_N *open set in* (X, μ_N) *then* $\overline{A}$ *is a* μ_N *nowhere dense set in* (X, μ_N).

3. μ_N Dense

Theorem 3.1. *If* (X, μ_N) *is a* μ_N *TS and* A *is* μ_N *dense in* (X, μ_N) *then for any non-empty* μ_N *closed subset* F *in such a way that* $A \subseteq F$ *then* $F = 1_N$.

Proof. Let us guesstimate that A is μ_N dense in (X, μ_N) then for any non-empty μ_N-closed subset F in such a way that $A \subseteq F$. On account of the fact A is μ_N dense, $\mu_N Cl(A) = 1_N$. By our presumption, F is μ_N-closed and $A \subseteq F$ hereinafter we get that $1_N = \mu_N Cl(A) \subseteq \mu_N Cl(F) = F$. As a result of that we get $F = 1_N$. □

Remark 3.1. *The above theorem is fallacious if* F *is not* μ_N *closed.*

Theorem 3.2. *Let* η *be a subset of* (X, μ_N). *If* η *is* μ_N *dense in* (X, μ_N) *then for any non empty* μ_N *open subset* G *in* $(X, \mu_N), G \cap \eta \neq 0_N$.

Proof. Assume that η is μ_N dense in (X, μ_N). Then for every single non-empty μ_N-closed subset F in such a way that $A \subseteq F$ then $F = 1_N$. Suppose $G \cap \eta = 0_N$ for few non-empty μ_N-open subset G of (X, μ_N) precedently we obtain that $\eta \subseteq X - G$ which is μ_N-closed because of that G is μ_N-open subset of (X, μ_N). By the conjucture, $X - G = 1_N$. Hence $G = 0_N$ which is a contradiction to G is a non-empty μ_N-open subset in (X, μ_N). It yields that $G \cap \eta \neq 0_N$. □

Proposition 3.1. *If a neutrosophic subset* η *is* μ_N *dense in* (X, μ_N) *and* $\eta^{'} \subseteq \eta$, *the postiliminary characteristics holds.*

(1) 1_N *is always* μ_N *dense.*

(2) 0_N *is not* μ_N *dense in anyways.*

(3) $\eta \cup \eta^{'}$ *is* μ_N *dense.*

(4) $\mu_N Cl(\eta)$ *is* μ_N *dense.*

(5) Every superset of μ_N *dense set is* μ_N *dense.*

Theorem 3.3. *If* (X, μ_N) *be a* μ_N *TS and* η *is* μ_N *dense and* $G \in \mu_N$ *then,* $G \subset \mu_N Cl(\eta \cap G)$.

Proof. Suppose $\nu \in G$ but $\nu \notin \mu_N\ Cl\ (\eta \cap G)$ then $\nu \in \overline{(\mu_N\ Cl\ (\eta \cap G))}$ $\Rightarrow \nu \in \mu_N\ Int\ \overline{((\eta \cap G))} \subseteq \overline{\eta} \cup \overline{G}$ which shows that either ν belongs to $\overline{\eta}$ or ν belongs to $\overline{G}$.

case (i) : Assume ν belongs to $\overline{\eta} \Rightarrow G \subseteq \overline{\eta}$ that provides us that $G \cap \eta = 0_N$ which is contrary to Theorem 3.2.3. Hence $\nu \in \mu_N\ Cl\ (\eta \cap G)$.

case (ii) : Assume ν belongs to $\overline{G}$, $G \subset \overline{\eta}$ which is contradiction. Hence $\nu \in \mu_N\ Cl\ (\eta \cap G)$. Thus, $G \subset \mu_N\ Cl\ (\eta \cap G)$. □

Theorem 3.4. *If (X, μ_N) be a μ_N TS and η is μ_N dense and μ_N open in (X, μ_N) then $\mu_N Fr(\eta) = \overline{\eta}$.*

Proof. Suppose η is mu_N dense and μ_N open in (X, μ_N), $\mu_N Cl(\eta) = 1_N$ and $\mu_N Int(\eta) = \eta$. Now $\mu_N Fr(\eta) = \mu_N Cl(\eta) - \mu_N Int(\eta) = \mu_N Cl(\overline{\eta}) = \overline{\eta}$. □

Theorem 3.5. *If (X, μ_N) be a μ_N TS and η is μ_N dense subset of (X, μ_N) then $\mu_N Fr(\eta) = \mu_N Cl(\overline{\eta})$.*

Proof. $\mu_N Fr(\eta) = \mu_N Cl(\eta) - \mu_N Int(\eta) = \mu_N Cl(\overline{\eta})$. □

Remark 3.2. *The back and forth statement of above statement need not be true.*

Example 3.1. *Let $X = \{a\}$. We define neutrosophic sets $A = \{< 0.3, 0.4, 0.5 >\}, B = \{< 0.3, 0, 0.1 >\}, C = \{< 0.4, 0.6, 0.8 >\}, D = \{< 0.4, 0, 0.1 >\}, E = \{< 0.4, 0.4, 0.5 >\}$. Here the μ_N Dense sets are $\{B, \overline{D}, 1_N\}$. Now $\mu_N Fr(A) = \mu_N Cl(\overline{A})$. But A is not μ_N dense subset of (X, μ_N).*

Theorem 3.6. *If a neutrosophic subset η is μ_N dense in (X, μ_N) if and only if $\mu_N Ext(\eta) = 0_N$.*

Proof. Suppose η is μ_N dense, $\mu_N Cl(\eta) = 1_N$. Now, $\mu_N Ext(\eta) = \mu_N Int(\overline{\eta}) = \overline{(\mu_N Cl(\eta))} = 0_N$. Conversely assume $\mu_N Ext(\eta) = 0_N$ then $\mu_N Cl(\eta) = \overline{(\mu_N Int(\overline{\eta}))} = 1_N$. □

4. μ_N Irresolvable and μ_N Resolvable

Definition 4.1. A neutrosophic set A in μ_N TS (X, μ_N) is called μ_N Resolvable if there exists a μ_N dense set A in (X, μ_N) such that $\mu_N Cl(\overline{A}) = 1_N$. Otherwise, it is μ_N Irresolvable.

Example 4.1. *Let $X = \{a\}$. We define neutrosophic sets A, B, C, D and E as follows: $P = \{< 0.3, 0.3, 0.5 >\}, Q = \{< 0.1, 0.2, 0.3 >\}, R = \{< 0.3, 0.2, 0.3 >\}, S = \{< 0.3, 0.6, 0.2 >\}, T = \{< 0.3, 0.8, 0.5 >\}$ under $\mu_N = \{0_N, P, Q, R\}$ where (X, μ_N) form a μ_N TS. Now, $\mu_N Int P = P, \mu_N Int Q = Q, \mu_N Int R = R, \mu_N Int S = O_N, \mu_N Int T = O_N, \mu_N Int 1_N = C$, and $\mu_N Cl 0_N = \{< 0.3, 0.8, 0.3 >\}, \mu_N Cl(P) = 1_N, \mu_N Cl(Q) = 1_N$, $\mu_N Cl(R) = 1_N, \mu_N Cl(S) = 1_N, \mu_N Cl(T) = \{< 0.3, 0.8, 0.3 >\}, \mu_N Cl(1_N) = 1_N, \mu_N Cl(1_N^c) = 1_N, \mu_N Cl(\overline{P}) = \{< 0.5, 0.7, 0.3 >\}, \mu_N Cl(\overline{Q}) = \{< 0.3, 0.8, 0.1 >\}, \mu_N Cl(\overline{R}) = \{< 0.3, 0.8, 0.3 >\}, \mu_N Cl(\overline{S}) = 1_N, \mu_N Cl(\overline{T}) = 1_N, \mu_N Cl(\overline{1_N}) = \{< 0.3, 0.8, 0.3 >\}$. Here, P, Q, R, S, T^c are μ_N Dense sets and $\mu_N Cl(\overline{S}) = 1_N$. Hence (X, μ_N) is μ_N Resolvable.*

Theorem 4.1. *If (X, μ_N) is μ_N irresolvable iff $\mu_N Int(A) \neq O_N$ for all μ_N dense set A in (X, μ_N).*

Proof. Since (X, μ_N) is μ_N irresolvable space for all μ_N dense set A we get $\mu_N Cl(\overline{A}) \neq 1_N$. From this we deduce $\overline{(\mu_N Int(A))} \neq 1_N$ that yields us that $\mu_N Int(A) \neq O_N$. Conversely we assume that $\mu_N Int(A) \neq O_N$ for all μ_N dense set A in (X, μ_N). Suppose that (X, μ_N) is μ_N resolvable then there exists a μ_N dense set A in (X, μ_N) such that $\mu_N Cl(\overline{A}) = 1_N$ which implies us that $\overline{(\mu_N Int(A))} = 1_N$. From this we get $\mu_N Int(A) = O_N$ which is a contradiction to our assumption. Hence (X, μ_N) is μ_N irresolvable. □

Definition 4.2. A μ_N TS is called μ_N submaximal space if for each neutrosophic set $A \neq 1_N$ in (X, μ_N) such that $\mu_N Cl(A) = 1_N$, then $A \in \mu_N$.

Theorem 4.2. *If the μ_N TS (X, μ_N) is μ_N submaximal then (X, μ_N) is μ_N irresolvable.*

Proof. Let (X, μ_N) be a μ_N submaximal space. Assume that (X, μ_N) is μ_N resolvable space. Let A be a μ_N dense set in (X, μ_N) then $\mu_N Cl(\overline{A}) = 1_N$. From this we get that $\overline{(\mu_N Int(A))} = 1_N$ which implies $\mu_N Int(A) = O_N$. This concludes that $A \notin \mu_N$ which is a contradiction. Hence (X, μ_N) is μ_N irresolvable space. □

Remark 4.1. *The contrary statement of the above theorem need not be true. That is, if the μ_N TS (X, μ_N) is μ_N irresolvable then (X, μ_N) need not be μ_N submaximal. On assumimg that (X, μ_N) is μ_N irresolvable space we obtain that there is no μ_N dense set in (X, μ_N) such that $\mu_N Cl(\overline{A}) = 1_N$. From this we cannot conclude that every μ_N dense set A in (X, μ_N) is μ_N-open in (X, μ_N). Hence, (X, μ_N) need not be μ_N submaximal.*

Definition 4.3. A μ_N TS is called μ_N maximal irresolvable space if (X, μ_N) is μ_N irresolvable and every μ_N dense set $A \neq 1_N$ of (X, μ_N) is μ_N open.

Definition 4.4. The μ_N TS (X, μ_N) is said to be μ_N open hereditarily irresolvable if $\mu_N Int(\mu_N Cl A) \neq 0_N$ then $\mu_N Int(A) \neq 0_N$, for any non zero neutrosophic set A in (X, μ_N).

Theorem 4.3. *Let (X, μ_N) be a μ_N TS. If (X, μ_N) is μ_N-open hereditarily irresolvable space then (X, μ_N) is μ_N Irresolvable.*

Proof. Let A be a μ_N dense set in (X, μ_N) then $\mu_N Cl(A) = 1_N$ which implies us that $\mu_N Int(\mu_N Cl A) \neq 0_N$ because we have $\mu_N Int(1_N) \neq 1_N$. Since (X, μ_N) is μ_N-open hereditarily irresolvable, $\mu_N Int(A) \neq 0_N$. Now by making use of " If (X, μ_N) is μ_N Irresolvable iff $\mu_N Int(A) = 0_N$ for all μ_N dense sets A in (X, μ_N)". Thus we conclude that (X, μ_N) is μ_N Irresolvable. □

Remark 4.2. *The reversal concept of the theorem need not be true. That is "Let (X, μ_N) be a μ_N TS. If (X, μ_N) is μ_N Irresolvable then (X, μ_N) need not be μ_N-open hereditarily irresolvable space". This can be explained with the help of th e upcoming example.*

Example 4.2. *Let (X, μ_N) be a μ_N TS. We define $\mu_N = \{0_N, A, B, C, D\}$ where $A = \{< 0.7, 0.3, 0.8 >< 0.5, 0.8, 0.9 >\}, B = \{< 0.4, 0.9, 0.9 >< 0.3, 0.9, 0.9 >\}, C = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.8 >\}, D = \{< 0.5, 0.8, 0.8 >< 0.5, 0.8, 0.7 >\}, E = \{< 0.3, 0.9, 0.9 >< 0.4, 0.9, 0.9 >\}$. Here, $\overline{E}$ is μ_N dense set but $\mu_N Cl(E) \neq 1_N$. Hence it is μ_N Irresolvable. $\mu_N Int(\mu_N Cl A) = \{< 0.7, 0.3, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$ and $\mu_N Int(A) = \{< 0.7, 0.3, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$, $\mu_N Int(\mu_N Cl B) = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$ and $\mu_N Int(B) = \{< 0.4, 0.9, 0.9 >< 0.3, 0.9, 0.9 >\} \neq 0_N$ and $\mu_N Int(\mu_N Cl C) = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$ and $\mu_N Int(C) = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.8 >\} \neq 0_N$, $\mu_N Int(\mu_N Cl D) = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$ and $\mu_N Int(D) = \{< 0.5, 0.8, 0.8 >< 0.5, 0.8, 0.7 >\} \neq 0_N$, $\mu_N Int(\mu_N Cl E) = \{< 0.5, 0.8, 0.7 >< 0.5, 0.8, 0.7 >\} \neq 0_N$ and $\mu_N Int(E) = \{< 0, 1, 1 >< 0, 1, 1 >\} = 0_N$. In this example $\mu_N Int(\mu_N Cl E) \neq 0_N$ but $\mu_N Int(E) = 0_N$ which implies us that "If (X, μ_N) is μ_N Irresolvable then (X, μ_N) need not be μ_N-open hereditarily irresolvable space."*

Theorem 4.4. *Let (X, μ_N) be a μ_N TS. If (X, μ_N) is μ_N-open hereditarily irresolvable space, then $\mu_N Cl(A) = 1_N$ for any non zero μ_N dense set A in (X, μ_N) which implies that $\mu_N Cl(\mu_N Int A) = 1_N$.*

Proof. Let A be a neutrosophic set in (X, μ_N) such that $\mu_N Cl(A) = 1_N$. From this we obtain that $\overline{(\mu_N Cl(A))} = 0_N$ which gives us that $\mu_N Int(\overline{A}) = 0_N$. Since (X, μ_N) is μ_N-open hereditarily irresolvable by using above theorem we have $\mu_N Int(\overline{\mu_N Cl A}) = 0_N$. Therefore $\overline{(\mu_N Cl(\mu_N Int A))} = 0_N$ that yields us that $\mu_N Cl(\mu_N Int A) = 1_N$. □

Theorem 4.5. *If $\mu_N Cl(\cap_{i=1}^{\infty} \omega_i) = 1_N$ where ω_i's are μ_N dense sets in a μ_N-open hereditarily irresolvable space then (X, μ_N) is a μ_N Baire space.*

Proof. On considering $\mu_N Cl(\cap_{i=1}^{\infty} \omega_i) = 1_N$ where $\mu_N Cl(\omega_i) = 1_N$ we get that $\mu_N Int(\cup_{i=1}^{\infty}(\overline{\omega_i})) = 0_N$, where $\mu_N Int(\overline{(\omega_i)}) = 0_N$. Let $\vartheta_i = \overline{(\omega_i)}$. Then, $\mu_N Int(\cup_{i=1}^{\infty} \vartheta_i) = 0_N$ where $\mu_N Int(\vartheta_i) = 0_N$. Since (X, μ_N) is a μ_N-open hereditarily irresolvable space, $\mu_N Int(\vartheta_i) = 0_N$ that yields us that $\mu_N Int(\mu_N Cl(\vartheta_i)) = 0_N$. Hence ϑ_i is μ_N nowhere dense set in (X, μ_N). Hence, $\mu_N Int(\cup_{i=1}^{\infty} \vartheta_i) = 0_N$ where ϑ_i's are μ_N nowhere dense sets in (X, μ_N) that provides us that (X, μ_N) is a μ_N Baire space. □

Theorem 4.6. *If (X, μ_N) is a μ_N Baire irresolvable space, then $\mu_N Cl(\cup_{i=1}^{\infty} \sigma_i) \neq 1_N$ where σ_i's are μ_N nowhere dense sets in (X, μ_N).*

Proof. Let σ_i be μ_N first category set in (X, μ_N) there upon $\kappa = \cup_{i=1}^{\infty}(\sigma_i)$, where σ_i's are μ_N nowhere dense sets in (X, μ_N). By the reason of (X, μ_N) is a μ_N Baire space, $\mu_N Int(\kappa) = 0_N$ thereupon we get $\overline{(\mu_N Int(\kappa))} = 1_N$ which entails us that $\mu_N Cl(\overline{\kappa}) = 1_N$. Already we have that (X, μ_N) is irresolvable space, $\mu_N Cl(\overline{\overline{\kappa}}) \neq 1_N$. Hence, $\mu_N Cl(\kappa) \neq 1_N$ and so we obtain that $\mu_N Cl \cup_{i=1}^{\infty}(\sigma_i) \neq 1_N$ where σ_i's are μ_N nowhere dense sets in (X, μ_N). □

Definition 4.5. A μ_N TS (X, μ_N) is said to be μ_N strongly irresolvable if $\mu_N Cl(\mu_N Int A) = 1_N$ for every dense set except 1_N in (X, μ_N).

Theorem 4.7. *A μ_N TS (X, μ_N) is μ_N submaximal space then (X, μ_N) is a μ_N Strongly irresolvable space.*

Proof. Assume (X, μ_N) is μ_N submaximal space. By the definition of μ_N submaximal spaces, we obtain that every μ_N dense set except $1_N \in \mu_N$ is μ_N-open in (X, μ_N). Now $mu_N Cl(A) = 1_N \Rightarrow \mu_N Cl(\mu_N Int A) = 1_N$. Hence, (X, μ_N) is μ_N Strongly irresolvable space.

The converse of the above theorem need not be true. □

Remark 4.3. *A μ_N TS (X, μ_N) is a μ_N Strongly irresolvable space then (X, μ_N) need not be μ_N submaximal. On assuming that (X, μ_N) is μ_N Strongly irresolvable space. We obtain that, $\mu_N Cl(A) = 1_N \Rightarrow \mu_N Cl(\mu_N Int A) = 1_N$ but we cannot obtain that $\mu_N Int A = A$ which leads us to μ_N submaximal space. Hence, we conclude that (X, μ_N) is a μ_N Strongly irresolvable space then (X, μ_N) need not be μ_N submaximal.*

Theorem 4.8. *If the μ_N TS (X, μ_N) is a μ_N strongly irresolvable Baire space and ν is a μ_N first category set in (X, μ_N) then ν is a μ_N nowhere dense set in (X, μ_N).*

Proof. Let ν be a μ_N first category set in (X, μ_N). Then $\gamma = \cup_{i=1}^{\infty} \nu_i$ where ν's are μ_N nowhere dense sets in (X, μ_N). Since (X, μ_N) is a μ_N Baire space by theorem 2.15, $\mu_N Int(\nu) = 0_N$ in (X, μ_N). Thereupon we get $\overline{(\mu_N Int(\nu))} = 1_N$ that entails us that $\mu_N Cl(\overline{\nu}) = 1_N$. By the cause of (X, μ_N) is a μ_N strongly irresolvable space for the μ_N dense set except $1_N \in \mu_N$ in (X, μ_N). Hence we retrieve that $\mu_N Cl(\mu_N Int\overline{\nu}) = 1_N$. Thus we get that $\overline{(\mu_N Int(\mu_N Cl\nu))} = 1_N$ and so we get $\mu_N Int(\nu_N Cl\nu) = 0_N$. Thus we obtain ν is a μ_N nowhere dense set in (X, μ_N). □

Theorem 4.9. *If the μ_N TS is a μ_N submaximal Baire space and ν is a μ_N first category set in (X, μ_N), then ν is a μ_N nowhere dense set in (X, μ_N).*

Proof. Let ν be a μ_N first category set in (X, μ_N). Since (X, μ_N) is μ_N submaximal Baire space by theorem 4.17 (X, μ_N) is μ_N Strongly irresolvable space. Then (X, μ_N) is μ_N Strongly irresolvable baire space. Since ν is a μ_N first category set in (X, μ_N). By Theorem 4.19 we get that ν is a μ_N nowhere dense set in (X, μ_N). □

Theorem 4.10. *If $\mu_N Cl(\cap_{i=1}^{\infty} \delta_i) = 1_N$ where δ_i's are μ_N dense sets in a μ_N submaximal space (X, μ_N), then (X, μ_N) is a μ_N Baire space.*

Proof. Let δ_i's be μ_N dense sets in μ_N submaximal space. Then $\delta_i \in \mu_N$. Now $\mu_N Cl(\delta_i) = 1_N$ and $\mu_N Int(\delta_i) = \delta_i$ that entails us that $\mu_N Cl(\mu_N Int\delta_i) = 1_N$. From this we obtain that $\overline{(\mu_N Cl(\mu_N Int\delta_i))} = 0_N \Rightarrow \mu_N Int(\mu_N Cl(\overline{\delta_i})) = 0_N$. Hence, $\overline{\delta_i}$'s are μ_N nowhere dense sets in (X, μ_N). Now we consider that $\mu_N Cl(\cap_{i=1}^{\infty} \delta_i) = 1_N$ by taking complement we obtain that $\mu_N Int(\cup_{i=1}^{\infty} \overline{\delta_i}) = 0_N$. Now by making use of theorem 2.15 we obtain that (X, μ_N) is a μ_N Baire space. □

Theorem 4.11. *If $\mu_N Cl(\mu_N Int(A)) \neq 1_N$, for every neutrosophic set A in μ_N strongly irresolvable space (X, μ_N), then $\mu_N Cl(A) \neq 1_N$ in (X, μ_N).*

Proof. Let A be a neutrosophic set in (X, μ_N) such that $\mu_N Cl(\mu_N Int(A)) \neq 1_N$. Now we are in a need to prove that $\mu_N Cl(A) \neq 1_N$ in (X, μ_N). We are assuming that $\mu_N Cl(A) = 1_N$ in (X, μ_N). Already its given that (X, μ_N) is μ_N strongly irresolvable space $\Rightarrow$ if $\mu_N Cl(A) = 1_N$ then $\mu_N Cl(\mu_N Int(A)) = 1_N$ which is a contradiction. Henceforth we obtain $\mu_N Cl(A) \neq 1_N$ in (X, μ_N). □

Theorem 4.12. *If A is a neutrosophic set in a μ_N strongly irresolvable space (X, μ_N) such that $\mu_N Int(\mu_N ClA) \neq 0_N$, then $\mu_N Int(A) \neq 0_N$ in (X, μ_N).*

Proof. Let A be a neutrosophic set in (X, μ_N) such that $\mu_N Int(\mu_N ClA) \neq 0_N$ in (X, μ_N). Now we have to obtain that $\mu_N Int(A) \neq 0_N$. Suppose that $\mu_N Int(A) = 0_N$ in (X, μ_N). Then $\mu_N Cl(\overline{A}) = \overline{(\mu_N Int(A))} = 1_N$. We have that (X, μ_N) is μ_N strongly irresolvable space, $\mu_N Cl(A) = 1_N$ yields us $\mu_N Cl(\mu_N IntA) = 1_N$. Thus, we get $\overline{(\mu_N Int(\mu_N ClA))} = 1_N \Rightarrow \mu_N Int(\mu_N ClA) = 0_N$ in (X, μ_N) which is a contradiction. Thus we obtain $\mu_N Int(A) \neq 0_N$ in (X, μ_N). □

Theorem 4.13. *$\eta_i \subseteq \overline{\eta_j}, i \neq j$ where η_i's are μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then η_j is a μ_N nowhere dense set in (X, μ_N).*

Proof. Let η_i and η_j, $i \neq j$ be neutrosophic sets in (X, μ_N) such that $\eta_i \subseteq \overline{(\eta_j)}$ and $\mu_N Cl(\eta_i) = 1_N$ in (X, μ_N). Since, (X, μ_N) is μ_N strongly irresolvable space, for the μ_N dense sets η_i, $\mu_N Cl(\mu_N Int \eta_i) = 1_N$ in (X, μ_N). We have that $\eta_i \subseteq \overline{(\eta_j)}$ that implies us that $\mu_N Cl(\mu_N Int \eta_i) \subseteq \mu_N Cl(\mu_N Int \overline{(\eta_j)})$ thereupon we obtain $1_N \subseteq \mu_N Cl(\mu_N Int \overline{(\eta_j)}) \Rightarrow \mu_N Cl(\mu_N Int \overline{(\eta_j)}) = 1_N$. Thus we get $\overline{(\mu_N Int(\mu_N Cl \eta_j))} = 1_N$. Henceforth, we obtain that $\mu_N Int(\mu_N Cl \eta_j) = 0_N \Rightarrow \eta_j$ is a μ_N nowhere dense set in (X, μ_N). □

Theorem 4.14. *If each μ_N dense sets η is a μ_N first category set in a μ_N strongly irresolvable space (X, μ_N), then $\mu_N Int(\mu_N Cl(\cap_{i=1}^{\infty} \overline{(\eta_i)})) = 0_N$, where η_i's are μ_N nowhere dense sets in (X, μ_N).*

Proof. Let η be μ_N dense set in (X, μ_N). Since (X, μ_N) is μ_N strongly irresolvable space, $\mu_N Cl(\eta) = 1_N$ that entails us that $\mu_N Cl(\mu_N Int \eta) = 1_N$ in (X, μ_N). Suppose that η is a μ_N first category set in (X, mu_N). Then $\eta = \cup_{i=1}^{\infty} \eta_i$, where η_i's are μ_N nowhere dense sets in (X, μ_N). Thus we obtain that $\mu_N Cl(\mu_N Int \eta) = 1_N \Rightarrow \mu_N Cl(\mu_N Int \cup_{i=1}^{\infty} \eta_i) = 1_N$ in (X, μ_N). From this we retrieve that $\overline{(\mu_N Cl(\mu_N Int \cup_{i=1}^{\infty} \eta_i))} = 0_N$. Hence $\mu_N Int(\mu_N Cl(\cap_{i=1}^{\infty}(\overline{(\eta_i)}))) = 0_N$, where η_i's are μ_N nowhere dense sets in (X, μ_N). □

Theorem 4.15. *If $\eta \subseteq \nu$, where ν is a neutrosophic set and ν is a μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then $\overline{\nu}$ is a μ_N nowhere dense sets in (X, μ_N).*

Proof. Let ν be a μ_N dense set in (X, μ_N) such that $\nu \subseteq \nu$. Since (X, μ_N) is a μ_N strongly irresolvable space (X, μ_N), for the μ_N dense sets $\mu_N Cl(\mu_N Int \eta) = 1_N$ in (X, μ_N). Now $\eta \subseteq \nu$ implies us that $\mu_N Cl(\mu_N Int \eta) \subseteq \mu_N Cl(\mu_N Int \nu)$. Henceforth $1_N \subseteq \mu_N Cl(\mu_N Int \nu)$ which leads us into $\mu_N Cl(\mu_N Int \nu) = 1_N$ in (X, μ_N). From this we deduce that $\overline{(\mu_N Cl(\mu_N Int \nu))} = 0_N$ that yields us that $\mu_N Int(\mu_N Cl(\overline{\nu})) = 0_N$ in (X, μ_N). Thus we obtain that $\overline{\nu}$ is a μ_N nowhere dense sets in (X, μ_N). □

Theorem 4.16. *If $\eta \subseteq \nu$, where ν is a neutrosophic set and η is a μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then $\overline{\nu}$ is not a ν_N open set in (X, μ_N).*

Proof. Let η be a μ_N dense set in (X, μ_N) in a manner that $\eta \subseteq \nu$. Since (X, μ_N) is a μ_N strongly irresolvable space, By theorem 4.25 we obtain that $\overline{\nu}$ is a μ_N nowhere dense sets in (X, μ_N). Thereupon $\mu_N Int(\mu_N Cl(\overline{\nu})) = 0_N$. But we know that $\mu_N Int(\overline{\nu}) \subseteq \mu_N Int(\mu_N Cl(\overline{\nu})) \Rightarrow \mu_N Int(\overline{\nu}) \subseteq 0_N$ that leads us into that $\mu_N Int(\overline{\nu}) = 0_N$. Henceforth $\overline{\nu}$ is not a μ_N-open set in (X, μ_N). □

Theorem 4.17. *If $\eta \subseteq \nu$, where ν is a neutrosophic set and η is a μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then $\overline{\nu}$ is a μ_N Rare set in (X, μ_N).*

Proof. Let η be a μ_N dense set in (X, μ_N) in a manner that $\eta \subseteq \nu$. Since (X, μ_N) is a μ_N strongly irresolvable space, By theorem 4.26 we obtain that $\overline{\nu}$ is a μ_N nowhere dense sets in (X, μ_N). Thereupon $\mu_N Int(\mu_N Cl(\overline{\nu})) = 0_N$. But we know that $\mu_N Int(\overline{\nu}) \subseteq \mu_N Int(\mu_N Cl(\overline{\nu})) \Rightarrow \mu_N Int(\overline{\nu}) \subseteq 0_N$ that leads us into that $\mu_N Int(\overline{\nu}) = 0_N$. Henceforth $\overline{\nu}$ is a μ_N Rare set in (X, μ_N) □

Theorem 4.18. *If η is a μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then $1_N - \eta$ is a μ_N nowhere dense set and μ_N-semi closed in (X, μ_N).*

Proof. Let eta be a μ_N-dense sets in (X, μ_N). Since (X, μ_N) is μ_N strongly irresolvable space, for the μ_N dense sets η, $\mu_N Cl(\mu_N Int\eta) = 1_N$ in (X, μ_N). Thereupon $\overline{(\mu_N Cl(\mu_N Int\eta))} = 0_N$. From this we retrieve that $\mu_N Int(\mu_N Cl(\overline{\eta})) = 0_N$ that leads us into $\overline{\eta}$ is a μ_N nowhere dense set in (X, μ_N). On considering $\mu_N Int(\mu_N Cl(\overline{\eta})) \subseteq \overline{\eta}$. This clearly shows that $\overline{\eta}$ is μ_N-semi closed in (X, μ_N). Henceforth $\overline{\eta}$ is a μ_N nowhere dense set and μ_N-semi closed in (X, μ_N). □

Theorem 4.19. *If $\eta = \cap_{i=1}^{\infty}\eta_i$ is a μ_N dense sets, where η_i's are neutrosophic sets in μ_N strongly irresolvable space (X, μ_N), then $\overline{\eta}$ is a μ_N first category set in (X, μ_N).*

Proof. Let $\eta = \cap_{i=1}^{\infty}\eta_i$ is a μ_N dense set in (X, μ_N). Thereupon we get $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) = 1_N$. At the same time we know that $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) \subseteq \cap_{i=1}^{\infty}\mu_N Cl(\eta_i)$ in (X, μ_N). Thus we obtain that $1_N \subseteq \cap_{i=1}^{\infty}\mu_N Cl(\eta_i)$. That is, $\cap_{i=1}^{\infty}\mu_N Cl(\eta_i) = 1_N$. So, $\mu_N Cl(\eta_i) = 1_N$ in (X, μ_N). Since (X, μ_N) is a μ_N strongly irresolvable space, by theorem 4.28, $\overline{\eta}_i$'s are μ_N nowhere dense sets in (X, μ_N). Now $\eta = \cap_{i=1}^{\infty}\eta_i$ implies us that $\overline{\eta} = \overline{(\cap_{i=1}^{\infty}\eta_i)} = \cup_{i=1}^{\infty}(\overline{(\eta_i)})$. Thus we obtain $\overline{\eta} = \cup_{i=1}^{\infty}(\overline{(\eta_i)})$ that implies us that $\overline{\eta}$ is a μ_N nowhere dense set in (X, μ_N). Hence, $\overline{\eta}$ is a μ_N first category set in (X, μ_N). □

Theorem 4.20. *If each μ_N dense set η is a μ_N-open set in a μ_N-topological space (X, μ_N), then (X, μ_N) is a μ_N strongly irresolvable space.*

Proof. let η be a μ_N dense set in (X, μ_N) in a manner that $\mu_N Int(\eta) = \eta$. Thereupon by the theorem 2.16, $\overline{\eta}$ is a μ_N nowhere dense set in (X, μ_N). Thus we obtain that $\mu_N Int(\mu_N Cl\overline{\eta}) = 0_N$. Hence we get that $\overline{(\mu_N Cl(\mu_N Int\eta)))} = 0_N$ from this we deduce that $\mu_N Cl(\mu_N Int\eta) = 1_N$ in (X, μ_N). Hence for the μ_N dense set η in (X, μ_N), $\mu_N Cl(\mu_N Int\eta) = 1_N$ in (X, μ_N). Therefore, (X, μ_N) is a μ_N strongly irresolvable space. □

Theorem 4.21. *If $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) = 1_N$, where η_i's are μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N), then (X, μ_N) is a μ_N Baire space.*

Proof. Let η_i's be μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N) in a manner that $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) = 1_N$. Since (X, μ_N) is a μ_N strongly irresolvable space, $\mu_N Cl(\mu_N Int\eta_i) = 1_N$ in (X, μ_N). Then, $\overline{(\mu_N Cl(\mu_N Int\eta_i))} = 0_N$ in (X, μ_N). Hence we retrieve that $\mu_N Int(\mu_N Cl(\overline{(\eta_i)})) = 0_N$ in (X, μ_N). From this we clearly get that $\overline{\eta}$'s are μ_N nowhere dense sets in (X, μ_N). Now, $\mu_N Cl(cap_{i=1}^{\infty}\eta_i) = 1_N \Rightarrow \overline{(\mu_N Cl(\cap_{i=1}^{\infty}\eta_i))} = 0_N \Rightarrow \mu_N Int(\cup_{i=1}^{\infty}\overline{\eta_i}) = 0_N$. Thus we deduce that $\mu_N Int(cup_{i=1}^{\infty}(\overline{eta_i})) = 0_N$ where $\overline{\eta}$'s are μ_N nowhere dense sets in (X, μ_N). Henceforth, (X, μ_N) is a μ_N Baire space. □

Theorem 4.22. *If η_i's where i ranges from 1 to ∞ are μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N) and μ_N Baire space then $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) = 1_N$ in (X, μ_N).*

Proof. Let η_i's are μ_N dense sets in a μ_N strongly irresolvable space (X, μ_N) then we obtain that for the μ_N dense sets, $\mu_N Cl(\mu_N Int\eta_i) = 1_N$ in (X, μ_N). From this we obtain that $\overline{(\mu_N Cl(\mu_N Int\eta_i))} = 0_N \Rightarrow \mu_N Int(\mu_N Cl\overline{\eta})) = 0_N$. From this we clearly get that $\overline{\eta}$'s are μ_N nowhere dense sets in (X, μ_N). Since (X, μ_N) is a μ_N Baire space, $\mu_N Int(\cup_{i=1}^{\infty}(\overline{\eta_i})) = 0_N \Rightarrow \mu_N Int(\overline{(\cap_{i=1}^{\infty}\eta_i)}) = 0_N \Rightarrow \overline{(\mu_N Cl(cap_{i=1}^{\infty}\eta_i))} = 0_N$. Henceforth $\mu_N Cl(\cap_{i=1}^{\infty}\eta_i) = 1_N$ in (X, μ_N). □

5. μ_N Connectedness & μ_N Disconnectedness

Definition 5.1. A μ_N-topological spaces (X, μ_N) is said to be μ_N disconnected if there exists μ_N-open sets $A \neq 0_N, B \neq 0_N$ in (X, μ_N) such that $A \vee B = 1_N$ and $A \wedge B = 0_N$.

If (X, μ_N) is not μ_N disconnected then it is μ_N connected.

Example 5.1. *Let* $X = \{a\}$ *define neutrosophic sets* $0_N = \{< 0, 1, 1 >\}, \delta_1 = \{< 0.3, 0.3, 0.5 >\}, \delta_2 = \{< 0.1, 0.2, 0.3 >\}, \delta_3 = \{< 0.3, 0.2, 0.3 >\}, \delta_4 = \{< 0.3, 0.6, 0.2 >\}, \delta_5 = \{< 0.3, 0.8, 0.5 >\}, 1_N = \{< 1, 0, 0 >\}$ *and we define a* μ_N *TS* $\mu_N = \{0_N, \delta_1, \delta_2, \delta_3\}$. *Here* $\delta_1 \neq 0_N, \delta_2 \neq 0_N, \delta_1 \vee \delta_2 = \{< 0.3, 0.2, 0.3 >\} \neq 1_N$ *and* $\delta_1 \wedge \delta_2 = \{< 0.1, 0.3, 0.5 >\}$. *Hence* (X, μ_N) *is* μ_N *connected.*

Example 5.2. *Let* $X = \{a, b\}$ *define neutrosophic sets* $0_N = \{< 0, 1, 1 >< 0, 1, 1 >\}, \alpha_1 = \{< 1, 0, 0 >< 0, 0, 1 >\}, \alpha_2 = \{< 0, 1, 1 >< 1, 1, 0 >\}, \alpha_3 = \{< 1, 0, 0 >< 1, 1, 1 >\}$ *and we define a* μ_N *TS as* $\{0_N, \alpha_1, \alpha_2, \alpha_3\}$. *Here,* $\alpha_1 \neq 0_N, \alpha_2 \neq 0_N, \alpha_1 \vee \alpha_2 = \{< 1, 0, 0 >< 1, 0, 0 >\} = 1_N, \alpha_1 \wedge \alpha_2 = \{< 0, 1, 1 >< 0, 1, 1 >\} = 0_N$. *Hence* (X, μ_N) *is* μ_N *disconnected.*

Definition 5.2. Let (X, μ_N) be a μ_N-topological spaces. If there exists μ_N-open sets A and B in X satisfying the following properties, then N is said to be $\mu_N C_n$ disconnected.

(a) $\mu_N C_1 : N \lesseqgtr A \vee B, A \wedge B \lesseqgtr N^c, N \wedge A \neq 0_N, N \wedge B \neq 0_N$

(b) $\mu_N C_2 : N \lesseqgtr A \vee B, N \wedge A \wedge B = 0_N, N \wedge A \neq 0_N, N \wedge B \neq 0_N$

(c) $\mu_N C_3 : N \lesseqgtr A \vee B, A \wedge B \lesseqgtr N^c, A \nleq N^c, B \nleq N^c$

(d) $\mu_N C_4 : N \lesseqgtr A \vee B, N \wedge A \wedge B = 0_N, A \nleq N^c, B \nleq N^c$

N is said to be $\mu_N C_n$ connected $(n = 1, 2, 3, 4)$ if N is not $\mu_N C_n$ disconnected.

Remark 5.1. *Clearly, we obtain the following implications between the four types of* $\mu_N C_n$ *connected* $(n = 1, 2, 3, 4)$.

(a) Every $\mu_N C_1$ *connected implies* $\mu_N C_2$ *connected.*

(b) Every $\mu_N C_1$ *connected implies* $\mu_N C_3$ *connected.*

(c) Every $\mu_N C_3$ *connected implies* $\mu_N C_4$ *connected.*

(d) Every $\mu_N C_2$ *connected implies* $\mu_N C_4$ *connected.*

The following examples shows us that the converse implications need not be true

Example 5.3. *Let* $X = \{a, b\}, \mu_N = \{0_N, A, B, C\}$ *where* $A = \{< 0.4, 0.6, 0.6 >< 0.1, 0.9, 0.9 >\}, B = \{< 0.5, 0.5, 0.5 >< 0.3, 0.9, 0.7 >\}, C = \{< 0.3, 0.7, 0.7 >< 0.1, 0.9, 0.9 >\}$. *Here,* C *is* $\mu_N C_2$ *connected,* $\mu_N C_3$ *connected,* $\mu_N C_4$ *connected but* $\mu_N C_1$ *disconnected.*

Example 5.4. *Let* $X = \{a, b\}, \mu_N = \{0_N, P, Q, R, S\}$ *where* $P = \{< 0.2, 0.6, 0.6 >< 0.8, 0.2, 0.2 >\}, Q = \{< 0.8, 0.2, 0.2 >< 0.6, 0.2, 0.2 >\}, R = \{< 0.8, 0.2, 0.2 >< 0.8, 0.2, 0.2 >\}, S = \{< 0.1, 0.9, 0.9 >< 0.1, 0.9, 0.9 >\}$. *Here,* S *is* $\mu_N C_4$ *connected but* $\mu_N C_3$ *disconnected.*

Definition 5.3. A μ_N-topological spaces (X, μ_N) is said to be $\mu_N C_5$ disconnected if there exists μ_N subset A in X which is both μ_N-closed and μ_N-open in (X, μ_N) such that $A \neq 0_N, A \neq 1_N$. If X is not $\mu_N C_5$ disconnected then it is said to be $\mu_N C_5$ connected.

Example 5.5. *Let* $X = \{a, b\}, \mu_N = \{0_N, A_1, A_2\}$ *where* $A_1 = \{< 0.3, 0.5, 0.9 >< 0.6, 0.5, 0.4 >\}, A_2 = \{< 0.9, 0.5, 0.3 >< 0.4, 0.5, 0.6 >\}$. *Here,* $A_1 \neq 0_N, A_1 \neq 1_N$ *is both* μ_N*-closed and* μ_N*-open in* (X, μ_N). *Thus* (X, μ_N) *is* $\mu_N C_5$ *disconnected.*

Theorem 5.1. μ_N *disconnectedness implies* $\mu_N C_5$ *disconnected. Equivalently,* $\mu_N C_5$ *connectedness implies* μ_N *connectedness.*

Proof. Suppose there exists a non-empty μ_N-open sets A and B such that $A \vee B = 1_N$ and $A \wedge B = 0_N$. Then $\mu_A \vee \mu_B = 1_N, \sigma_A \wedge \sigma_B = 0_N, \gamma_A \wedge \gamma_B = 0_N$ and $\mu_A \vee \mu_B = 0_N, \sigma_A \wedge \sigma_B = 1_N, \gamma_A \wedge \gamma_B = 1_N$. In otherwords, $B^c = A$. Hence, A is $\mu_N clopen$ which yields us that X is $\mu_N C_5$ disconnected.

But the reversal statement of the above theorem need not be true. □

Example 5.6. *Let* $X = \{a, b\}, \mu_N = \{0_N, A_1, A_2\}$ *where* $A_1 = \{< 0.3, 0.5, 0.9 >< 0.6, 0.5, 0.4 >\}, A_2 = \{< 0.9, 0.5, 0.3 >< 0.4, 0.5, 0.6 >\}$. *Here,* $A_1 \neq 0_N, A_1 \neq 1_N$ *is both* μ_N*-closed and* μ_N*-open in* (X, μ_N). *Thus* (X, μ_N) *is* $\mu_N C_5$ *disconnected but not* μ_N *disconnected.*

Theorem 5.2. *A* μ_N*-topological spaces* (X, μ_N) *is* $\mu_N C_5$ *connected if and only if there exists no non-empty* μ_N*-open sets* U *and* V *in* (X, μ_N) *such that* $U = V^c$.

Proof. Assume (X, μ_N) is $\mu_N C_5$ connected. Suppose U and V are μ_N-open sets in (X, μ_N) such that $U \neq 0_N, V \neq 0_N$ and $U = V^c$. Since $U = V^c, V^c$ is μ_N-open in (X, μ_N) which implies that V is μ_N-closed in (X, μ_N) and $U \neq 0_N$ implies $V \neq 1_N$. Hence, V is both μ_N-open and μ_N-closed in (X, μ_N) such that $V \neq 0_N$ and $V \neq 1_N$ that implies us that V is $\mu_N C_5$ disconnected which is a contradiction to (X, mu_N) is $\mu_N C_5$ connected. Hence there is no non empty μ_N-open sets U and V in (X, μ_N) such that $U = V^c$.

Conversely we assume that there is no non empty μ_N-open sets U and V in (X, μ_N) such that $U = V^c$. Let V be μ_N-closed in (X, μ_N) and U be both μ_N-open and μ_N-closed in (X, μ_N) such that $U \neq 0_N, U \neq 1_N$. Now take $U^c = V$ is a μ_N-open set and $V \neq 1_N$ which implies us that that $U^c = V \neq 0_N$. Hence we get $V \neq 1_N$ which is a contradiction to our assumption. Hence (X, μ_N) is $\mu_N C_5$ connected. □

Theorem 5.3. *A* μ_N*-topological spaces* (X, μ_N) *is* μ_N *connected space if and only if there exists no non-empty* μ_N*-open sets* U *and* V *in* (X, μ_N) *such that* $U = V^c$.

Proof. Assume (X, μ_N) is μ_N connected. Suppose U and V are μ_N-open sets in (X, μ_N) such that $U \neq 0_N, V \neq 0_N$ and $U = V^c$. Since $U = V^c, V^c$ is μ_N-open in (X, μ_N) which implies that V is μ_N-closed in (X, μ_N) and $U \neq 0_N$ implies $V^c \neq 1_N$. Hence, V is a proper μ_N subset which is both μ_N-open and μ_N-closed in (X, μ_N) that implies us that X is μ_N disconnected which is a contradiction to (X, μ_N) is μ_N connected. Hence there is no non zero μ_N-open sets U and V in (X, μ_N) such that $U = V^c$.

Conversely we assume that there is no non zero μ_N-open sets U and V in (X, μ_N) such that $U = V^c$. Let V be μ_N-closed in (X, μ_N) and U be both μ_N-open and μ_N-closed in (X, μ_N) such that $U \neq 0_N, U \neq 1_N$. Now take $U^c = V$ is a μ_N-open set and $V \neq 1_N$ which implies us that that $U^c = V \neq 0_N$. Hence we get that there is non-empty μ_N-open sets U and V such that $U = V^c$ which is a contradiction to our assumption. Hence (X, μ_N) is μ_N connected. □

Theorem 5.4. *A μ_N-topological spaces (X, μ_N) is μ_N connected space if and only if there exist no non-zero μ_N subsets U and V in (X, μ_N) such that $U = V^c, V = (\mu_N ClU)^c$ and $U = (\mu_N ClV)^c$.*

Proof. Let U and V be two μ_N subsets in (X, μ_N) such that $U \neq 0_N, V \neq 0_N$ and $U = V^c, V = (\mu_N ClU)^c$ and $U = (\mu_N ClV)^c$. Since, $(\mu_N ClU)^c$ and $(\mu_N ClV)^c$ are μ_N-open sets in X. U and V are μ_N-open sets in X. This implies X is not μ_N connected which is a contradiction. Therefore there exists no μ_N-open sets in X such that $U = V^c, V = (\mu_N ClU)^c$ and $U = (\mu_N ClV)^c$.
Suffiency:Let U be both μ_N-open and μ_N-closed sets in X such that $U \neq 0_N$, $U \neq 1_N$. By taking $V = U^c$ which is a contradiction to our hypothesis. Hence, (X, μ_N) is μ_N connected. □

6. μ_N Hyperconnected

Definition 6.1. A μ_N TS is said to be μ_N hyperconnected if every non-empty μ_N-open subset of (X, μ_N) is μ_N dense in (X, μ_N).

Example 6.1. *Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}, \delta_1 = \{< 0.3, 0.3, 0.5 >\}, \delta_2 = \{< 0.1, 0.2, 0.3 >\}, \delta_3 = \{< 0.3, 0.2, 0.3 >\}, \delta_4 = \{< 0.3, 0.6, 0.2 >\}, \delta_5 = \{< 0.3, 0.8, 0.5 >\}, 1_N = \{< 1, 0, 0 >\}$ and we define a μ_N TS $\mu_N = \{0_N, \delta_1, \delta_2, \delta_3\}$. Here $\delta_1, \delta_2, \delta_3, \overline{\delta_4}, \overline{\delta_5}, 1_N$ are μ_N dense sets in (X, μ_N). Here every μ_N-open subset of (X, μ_N) is mu_N dense in (X, μ_N). Thus, (X, μ_N) is μ_N hyperconnected.*

Theorem 6.1. *Every μ_N hyperconnected is μ_N connected.*

Proof. Assume that (X, μ_N) is not μ_N connected that entails us that there exists two non-empty proper sets $A \in \mu_N$ and $B \in \mu_N$ such that $A \cap B = 0_N$ and $A \cup B = 1_N$ from this we deduce $A \cup B \in \mu_N$ and $\mu_N Cl(A \cup B) = \mu_N Cl(1_N) \neq 1_N$. Here we obtained that $A \cup B$ is μ_N-open but not μ_N dense which is a contradiction. Henceforth (X, μ_N) is μ_N connected. □

Remark 6.1. *The contrary statement of the above theorem need not be true.*

Example 6.2. *Let $X = \{a\}$ define neutrosophic sets $0_N = \{< 0, 1, 1 >\}, \vartheta_1 = \{< 0.7, 0.8, 0.9 >\}, \vartheta_2 = \{< 0.3, 0.4, 0.6 >\}, \vartheta_3 = \{< 0.9, 0.7, 0.6 >\}, 1_N = \{< 1, 0, 0 >\}$ and we define a μ_N TS $\mu_N = \{0_N, \vartheta_1, \vartheta_3\}$. Here, ϑ_1 and ϑ_3 are μ_N-open sets in (X, μ_N) but they are not μ_N dense in (X, μ_N). Hence (X, μ_N) is not μ_N hyperconnected. But (X, μ_N) is μ_N connected.*

Theorem 6.2. *(X, μ_N) is μ_N hyperconnected if and only if every μ_N subset of (X, μ_N) is either μ_N dense or μ_N nowhere dense.*

Proof. Let (X, μ_N) be a μ_N hyperconnected space. Let A be any μ_N subsets such that $A \subseteq 1_N$. Suppose A is not μ_N nowhere dense. Then $\mu_N Cl(X - \mu_N ClA) = X - \mu_N Int(\mu_N ClA) \neq 1_N$. Since $\mu_N Int(\mu_N ClA) \neq 1_N$. By our assumption we get A is μ_N dense.

Conversely, Let A be non-empty μ_N-open in X. Now for any non-empty μ_N-open set we have $A \subseteq \mu_N Int(\mu_N ClA)$ which implies that A is not μ_N nowhere dense but by hypothesis we have A is μ_N dense. Hence the theorem. □

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