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EFFECT OF ABOODH ADOMIAN METHOD FOR THE SOLUTION OF NON-LINEAR INTEGRO DIFFERENTIAL AND VOLTERRA INTEGERAL EQUATIONS BASED ON NEWTON-RAPHSON METHOD

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ABSTRACT— In this paper, we set up an altered Aboodh transform Adomian break up technique to solve non-linear Vȯlterra integral and integrȯ-differential equations. This scheme separate it from initial Aboodh Adȯmian splitting method because it involves the terms of Adomian polynomials. we applied Newton-Raphsȯn method instead of ui (variable) in Adomian polynomials. The assigned scheme is explored with few explained examples that gave feasible solutions.

Keywords—Aboodh Transfȯrm Method (ATM), Vȯlterra Integral Equatiȯns (VIE), Volterra Integrȯ-differential Equations, Adomian Decompȯsition Method, Newton-Raphsȯn Method.

I. INTRODUCTION

The Non-linear integro-differential Equations are mostly difficult to solve analytically. Therefore, they have been a great interest by several authors. Non-linear Vȯlterra integral equations dealing with research in scientific field such as the population dynamics, increase in contagious appliances, semi-conductor tools [16]. Vȯlterra integrȯ-differential equations also emerged in tangible products such as genetic varieties collaborating along with expanding and diminishing ratios of breeding, engineering phenomena's like heat and diffusion flow procedures in most cases [3, 13, 15]. Now there are many technique to solve these types of equations but there we use Adomian splitting technique firstly initiated in 1989 by George Adomian [3, 12, 14] that provide semi analytical numerical solution that decomposed non-linear terms in form of infinite series pattern in Adomian polynomial. Primarily, the approach offers an infinite series result of the given equation then non-linear term is break up into an infinite series pattern of Adomian polynomial [1, 2, 4, 5, 6, 8, 10, 14, 15]. A contrast was fashioned between Adomian decomposition and tau methods for finding the solution of Vȯlterra integrȯ-differential equations. The Aboodh transform is obtained from Fourier integral and initiated by the Khalid Aboodh to resolve ordinary and partial differential equations in the time domain [28, 29]. In this article, combination of Aboodh transform method and Adomian splitting method is described with alteration in idea of Laplace ADM which initially was explained by Khuri [10] on the way to simplify non-linear differential equation. In quasi linearization [11], method was applied to elaborate Vȯlterra-integral equations. Kamyad et al. consider the process of discretization of an interpolation scheme for Vȯlterra-integral equations [9] . In this research, we combined Aboodh transform technique with Adomian splitting method, while in decomposition process for the non-linear terms we applied Adomian polynomials , then inserted ui (term) in Newton-Raphson algorithm. Since Newton-Raphson algorithm is used to calculate the best approximating result for real function. In this way, we got the more reliable approximate results which are better than exact one.

2. Non-linear VIE of 2^{nd} kind.

Suppose a non-linear VIE with difference kernel Such that.

$$k(y,t) = k(y-t)$$

$$u(y) = f(y) + \int_0^y k(y-t)\,F[u(t)]\,dt, \qquad (i)$$

where $f(y)$ is known real function and $F(u(y))$ is the non-linear function of $u(y)$.

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Taking ATM on either sides of (i). Then linearity and convolution of ATM, we get.

$\mathcal{A}\{u(y)\} = \mathcal{A}\{f(y)\} + V\mathcal{A}\{k(y-t)\}\mathcal{A}\{F(u(y))\}$ (ii)

Method containing approximate solution of (i) is as

$$u(y) = \sum_{n=0}^{\infty} u_n(y). \quad \text{(iii)}$$

So, the non-linear term $F(u(y))$ is discretised such that.

$$F(u(y)) = \sum_{n=0}^{\infty} A_n(y). \quad \text{(iv)}$$

Here $A_n s$ are Altered Adomian polynomials depending on Newton-Raphson method given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left\{f(\sum_{i=0}^{n}\lambda^i(u_i(y) - \frac{F(u_i(y))}{F(u_i^{'}(y))}))\right\}_{\lambda=0}, n \geq 0 \quad \text{(v)}$$

By putting (iii) &(iv) in (ii),

$\mathcal{A}\{\sum_{n=0}^{\infty} u_n(y)\} = \mathcal{A}\{f(y)\} + V\mathcal{A}\{k(y-t)\}\mathcal{A}\{\sum_{n=0}^{\infty} A_n(y)\}$.

$\sum_{n=0}^{\infty}\mathcal{A}\{u_n(y)\} = \mathcal{A}\{f(y)\} + V\mathcal{A}\{k(y-t)\}\sum_{n=0}^{\infty}\mathcal{A}\{A_n(y)\}$. (vi)

To determine the terms $u_0(y)$, $u_1(y)$, $u_2(y)$, $u_3(y)$. . . . of infinite series, equating either sides of (vi), we get

$\mathcal{A}\{u_0(y)\} = \mathcal{A}\{f(y)\}$ (vii)

In general, the relation is given by

$\mathcal{A}\{u_{n+1}(y)\} = V\mathcal{A}\{k(y-t)\}\mathcal{A}\{A_n(y)\}$ (viii)

Takes inverse Aboodh transform to (vii) and (viii), we have.

$u_0(y) = \mathcal{A}^{-1}[\mathcal{A}\{f(y)\}]$ (ix)

$u_{n+1}(y) = V\mathcal{A}^{-1}[\mathcal{A}\{k(y-t)\mathcal{A}\{A_n(y)\}]$ (x)

Putting $u_0(y)$ in equation (v) we get A_0 further applying iterative scheme (x), we obtain the result of $u_1(y), u_2(y), u_3(y)$. . ., which eventually provide the solution (iii) to the given VIE. The consequences of altered method for solving VIE give below. In this way we found Maximum absolute Error in approximation:

$$e_j = Maxi|u_{ex} - u_{app}|$$

Where e_j represent greatest absolute error for y_j in defined time.

Example 1. Suppose the subsequent VIE [16]

$u(y) = y + \int_0^y u^2(t)dt,$ (xi)

has $u(y) = tany$ as an exact solution.

Solution. Applying ATM on either parts of (xi) then linearity of Aboodh transform, we get.

$\mathcal{A}\{u(y)\} = \mathcal{A}\{y\} + \mathcal{A}\{\int_0^y u^2(t)dt\}$

That's $\mathcal{A}\{u(y)\} = \frac{1}{v^3} + \frac{1}{v}\mathcal{A}\{u^2(y)\}$

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Use above technique, we get.

$\mathcal{A}\{\sum_{n=0}^{\infty} u_n(y)\} = \frac{1}{v^3} + \frac{1}{v}\mathcal{A}\{\sum_{n=0}^{\infty} A_n(y)\}$

(xii)

Where non-linear term $F\big(u(y)\big) = u^2(y)$ is discretized such that applying formula that's (v). Definite terms of altered Adomian Polynomials are as follows:

$$A_0(t) = \left(\frac{1}{2}\right)^2 u_0^2(t),$$

$$A_1(t) = \left(\frac{1}{2}\right)^2 (2u_0(t)u_1(t)),$$

$$A_2(t) = \left(\frac{1}{2}\right)^2 (2u_0(t)u_2(t) + u_1^2(t)),$$

$$A_3(t) = \left(\frac{1}{2}\right)^2 (2u_0(t)u_3(t) + 2u_1(t)u_2(t))$$

Equating either sides of (xii), we get.

$$\mathcal{A}\{u_0(y)\} = \frac{1}{V^3}$$

(xiii)

In general

$$\mathcal{A}\{u_{n+1}(y)\} = \frac{1}{V}\mathcal{A}\{A_n(y)\}$$

(xiv)

Taking inverse Aboodh transform on either sides of (xiii), we get.

$$u_0(y) = y$$

(xv)

Using general relation, we have

$$u_1(y) = \frac{y^3}{12},$$

Processing in this way, we get.

$u_2(y) = \frac{y^5}{120}, \quad u_3(y) = \frac{y^7}{20160}, \quad u_4(y) = \frac{31y^9}{362880},$

Consequently, the approximate solution become.

$$u(y) = y + \frac{y^3}{12} + \frac{y^5}{120} + \frac{y^7}{20160} + \frac{31y^9}{362880}, \dots \dots$$

The solution obtained by applying our technique for different values of y are shown in the table 1 and graphically presented in the figure I and greatest absolute error within each rows of tabled entries which shows closeness of the results with exact solution, while 0.0002 is Maximum absolute Error.

Example 2. Solve the following VIE [9]

$u(y) = 2y - \frac{y^4}{12} + 0.25\int_0^y (y-t)u^2(t)dt$

(xvi)

With analytical solution $u(y) = 2[y]$.

Solution. Taking Aboodh on either sides, we get.

$\mathcal{A}\{u(y)\} = \mathcal{A}\left[2y - \frac{y^4}{12}\right] +$
$0.25V\mathcal{A}[y\,]\mathcal{A}[u^2(y)]$

The technique suppose the series solution of function $u(y)$

$\mathcal{A}\{\sum_{n=0}^{\infty} u_n(y)\} = \mathcal{A}\left\{2y - \frac{y^4}{12}\right\} +$
$\frac{1}{4V}\mathcal{A}\{\sum_{n=0}^{\infty} A_n(y)\},$ (xvii)

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Table I: Numerical Outcomes for Example I.

Y	Exact Solution	Approximate Solution	Absolute Error
0	0	0	0.0000E+00
0.01	0.010000333	0.010000083	2.5001E-07
0.02	0.020002667	0.020000667	2.0004E-06
0.03	0.030009003	0.030002250	6.7530E-06
0.04	0.040021347	0.040005334	1.6013E-05
0.05	0.050041708	0.050010419	3.1289E-05
0.06	0.060072104	0.060018006	5.4097E-05
0.07	0.070114558	0.070028597	8.5961E-05
0.08	0.080171105	0.080042694	1.2841E-04
0.09	0.090243790	0.090060799	1.8299E-04
0.1	0.100334672	0.100083417	2.5126E-04

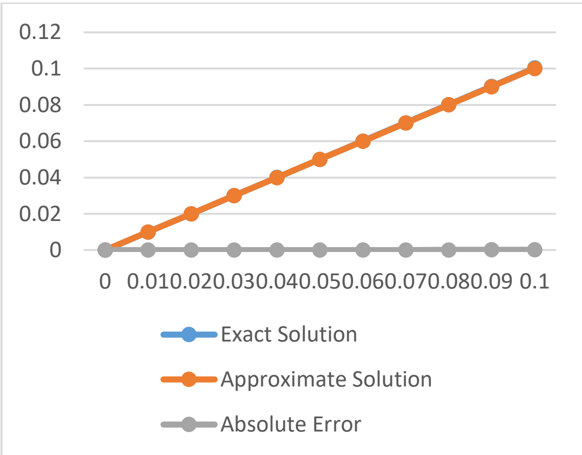

Figure I:- Contrast of analytical and approximate Solution.

On equating (xvii), gives recurrent algorithm

$$\mathcal{A}\{u_0(y)\} = \mathcal{A}\left\{2y - \frac{y^4}{12}\right\}, \qquad (xviii)$$

In general, $\mathcal{A}\{u_{n+1}(y)\} = \frac{1}{V^2}\mathcal{A}\{A_n(y)\}$ $\qquad (xix)$

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Applying inverse Aboodh transform we get.

$$u_0(y) = 2y - \frac{y^4}{12} \qquad (xx)$$

$$u_1(y) = \frac{y^{10}}{207360} - \frac{y^7}{2016} + \frac{y^4}{48},$$

$$u_2(y) = -\frac{y^{16}}{4777574400} + \frac{37y^{13}}{905748480} - \frac{11y^{10}}{2903040} + \frac{y^7}{8064},$$

Up to so on

Thus, the solution takes the form.

$$u(y) = 2y - \frac{y^4}{16} - \frac{y^7}{2688} + \frac{y^{10}}{967680} + \frac{37y^{13}}{905748480} - \frac{y^{16}}{4777574400} + \cdots$$

The numerical results shown in Table II and Figure II represent the performance of recommended.

Table II: Equating of Approximate with Analytical solution of Example 2.

Y	Exact Solution	Approximate Solution	Absolute Error
0	0	0	0.000E+00
0.05	0.1	0.099999609	3.9063E-07
0.1	0.2	0.19999375	6.2500E-06
0.15	0.3	0.299968359	3.1641E-05
0.2	0.4	0.399899995	1.0000E-04
0.25	0.5	0.499755837	2.4416E-04
0.3	0.6	0.599493669	5.0633E-04
0.35	0.7	0.69906187	9.3813E-04
0.4	0.8	0.798399391	1.6006E-03
0.45	0.9	0.89743572	2.5643E-03
0.5	1	0.996090845	3.9092E-03

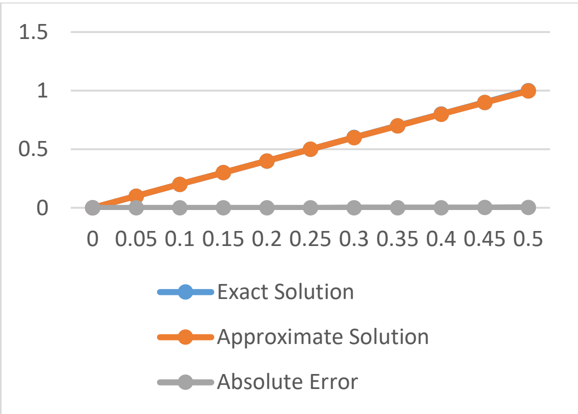

Figure II: Evaluation of Accessible Approximate with Analytical solution

and greatest absolute error is 0.003.

3. Non-linear Vȯlterra integrȯ-differential equations of 2^{nd} kind

The non-linear Vȯlterra integȯr-differential equation of the 2nd kind with difference kernel such that $k(y,t) = k(y-t)$

$$u^i(y) = f(y) + \int_0^y k(y-t)\, F[u(t)]dt, \qquad (xxi)$$

where $u^i(y)$ represent the ith derivative of $u(y)$ with respect to y, $f(y)$ is known as resource term and $F[u(y)]$ is the non-linear function of $u(y)$.

The derivative law of Aboodh transform is given by.

$$\mathcal{A}\{u^i(y)\} = V^i\mathcal{A}\{u(y)\} - \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2-i+k}} \quad (xxii)$$

Applying ATM on either sides of (xxi) and use the properties of Aboodh transform that's.

$$V^i\mathcal{A}\{u(y)\} - \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2-i+k}} = \mathcal{A}\{f(y)\} + V\mathcal{A}\{k(y-t)\}\mathcal{A}\{F(u(y)\}$$

Which implies

$$\mathcal{A}\{u(y)\} = \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2+k}} + \frac{1}{V^i}\mathcal{A}\{f(y)\} + \frac{1}{V^{i-1}}\mathcal{A}\{K(y-t)\}\mathcal{A}\{F(u(y)\}.$$

Implementing similar procedure that is explained in previous part, written as

$$\mathcal{A}\{\textstyle\sum_{n=0}^{\infty} u_n(y)\} = \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2+k}} + \frac{1}{V^i}\mathcal{A}\{f(y)\} + \frac{1}{V^{i-1}}\mathcal{A}\{k(y-t)\}\mathcal{A}\{\textstyle\sum_{n=0}^{\infty} A_n(y)\} \quad \text{(xxiv)}$$

The linearity property of Aboodh transform gives.

$$\sum_{n=0}^{\infty}\mathcal{A}\{u_n(y)\} = \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2+k}} + \frac{1}{V^i}\mathcal{A}\{f(y)\} + \frac{1}{V^{i-1}}\mathcal{A}\{k(y-t)\}\sum_{n=0}^{\infty}\mathcal{A}\{A_n(y)\} \quad (xxv)$$

Comparing either sides, we have the following repetition relation.

$$\mathcal{A}\{u_0(y)\} = \sum_{k=0}^{i-1}\frac{u^k(0)}{V^{2+k}} + \frac{1}{V^i}\mathcal{A}\{f(y)\} \quad \text{(xxvi)}$$

In general, the relation is given by.

$$\mathcal{A}\{u_{n+1}(y)\} = \frac{1}{V^{i-1}}\mathcal{A}\{k(y-t)\}\mathcal{A}\{A_n(y)\} \quad (xxvii)$$

Taking inverse Aboodh to (xxvi), we obtained $u_0(y)$, that describes $A_0(t)$. $u_1(y)$ is found by switching $A_0(t)$. Ongoing in this way we will obtained $u_n(y)$ from equation (xxvii). Afterward getting the pieces of infinite series, the series solution (iii) follows. The recommended process is demonstrated by the subsequent example.

Example 3. Suppose the non-linear Vȯlterra integrȯ-differential equation [12, 13]

$$u'(y) = -1 + \int_0^y u^2(t)dt\,,\ u(0) = 0. \quad \text{(xxviii)}$$

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Solution. Applying Aboodh transform on either side of (xxviii) and using the initial condition, also by convolution theorem we have.

$$\mathcal{A}\{u'(y)\} = \mathcal{A}\left\{-1 + \int_0^y u^2(t)dt\right\}$$

$$\mathcal{A}\{u(y)\} = -\frac{1}{V^3} + \frac{1}{V^2}\mathcal{A}\{u^2(y)\}$$

Putting the series form of $u(y)$ we get.

$$\mathcal{A}\left\{\sum_{n=0}^{\infty} u_n(y)\right\} = -\frac{1}{V^3} + \frac{1}{V^2}\mathcal{A}\left\{\sum_{n=0}^{\infty} A_n(y)\right\}, \quad (xxix)$$

Comparing either sides of (xxix) gives the iterative algorithm.

$$\mathcal{A}\{u_0(y)\} = -\frac{1}{V^3}, \qquad (xxx)$$

$$\mathcal{A}\{u_{n+1}(y)\} = \frac{1}{V^2}\mathcal{A}\{A_n(y)\} \quad (xxxi)$$

Applying inverse Aboodh on either sides of (xxx) also use relation (xxxi), we have.

$$u_0(y) = -y,$$

$$u_1(y) = \frac{y^4}{48}$$

$$u_2(y) = -\frac{y^7}{4032}$$

$$u_3(y) = \frac{y^{10}}{387072},$$

$$u_4(y) = -\frac{y^{13}}{40255488},$$

The general solution is given by.

$$u(y) = -y + \frac{y^4}{48} - \frac{y^7}{4032} + \frac{y^{10}}{387072} - \frac{y^{13}}{40255488} + \cdots$$

Table III and Figure III demonstrate the approximate solution equated with exact [13] is very exceptional having greatest absolute error 0.003.

Table III: Calculated Exact and Approximation Solution for Example 3.

X	Exact Solution	Approximate Solution	Absolute Error
0	0	0	0.0000E+00
0.0625	-0.0625	-0.062499682	3.1789E-07
0.125	-0.12498	-0.124994914	1.4914E-05
0.1875	-0.1874	-0.187474253	7.4253E-05
0.25	-0.24967	-0.249918635	2.4863E-04
0.3125	-0.31171	-0.31230139	5.9139E-04
0.375	-0.37336	-0.374588271	1.2283E-03
0.4375	-0.43446	-0.436737503	2.2775E-03
0.5	-0.49482	-0.498699852	3.8799E-03

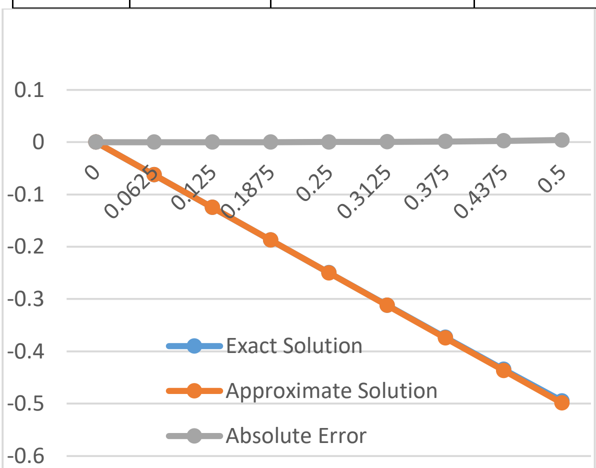

Figure III: Contrast of exact and approximation.

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4. Conclusion

Newtȯn-Raphsȯn technique is used as a term in Adȯmian polynomial which shows the validity of Aboodh Adȯmian decȯmposition method to solve the Vȯlterra integral integrȯ differential equations. Now it happened secondly that Adomian polynomial techniques are changed slightly with the help of Newton-Raphson method. The results shown in the tables and graphically presented via figures which reflects the closeness of results to the exact solution while our opted methodology is lucid and less complicated providing the best precised results.

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