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COMPLEX MODEL OF THE QUANTUM COSMOLOGY

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Abstract. We study the Hertle-Hawking no boundary proposal in which wave function of the universe is given by a path integral over all compact Euclidean 4-dimensional geometries and mater fields that have the 3-dimensional argument of the wave function on their one and only boundary. We suppose that the original phase transition takes place before the GUT phase transition i.e., in the stage of SUT (Super Unified Theory) phase transition. After the phase transition, there is a fluctuations of the scalar field ϕ, over a smooth average value. These fluctuations result in fluctuations of energy density. In this paper, we choose the potential $V(\phi)$ as a exponential function of time "t" and consider a complex radius of the Friedmann-Robertson-Walker model.

1. Introduction. Until the twentieth century, cosmology, dealing with the subject of the creation of the universe, was mostly a part of metaphysics. In recent decades a lot of the work in quantum cosmology has been done.

There is no consensus yet on how the universe initially came to be, the general assumption is that perhaps an energetic fluctuation caused the universe to tunnel into existence from quantum foam. The spontaneous symmetry breaking of the unified field occurred, thereby separating gravity, matter fields and GUT force field, as well as initiating the expansion of the universe.

Our knowledge of the unification of the fields can guide us to determine how the fields in the vacuum state may be further organized. Although the high temperatures necessary for unification are physically absent in the vacuum, quantum physics indicates their presence in a virtual way. Due to uncertainty principle, as the dimension of space decreases, the equivalent energy per particle (i.e., the temperature) increases. Also an estimate of the increase in equivalent temperature (energy/particle) can be done from the Heisenberg uncertainty relation.

The question of why the large energy is not found in practical, the observed vacuum energy is so small in comparison to the scales of particle physics is known as cosmological constant problem. It is generally thought to be easier to imagine an unknown mechanism which would set vacuum parameter exactly to zero and so it can be considered that there exist another unification SUT (Super Unified Theory) in the very early universe.

The vacuum universe $U(11)$ is thermodynamically equilibrium with the infinite boundary ($R \to \infty$, $R_1 \to \infty$) like a plain white paper. The break-down of the special unitary group $SU(11)$ of $U(11)$ into $SU(6) \times SU(5) \times U(1)$, under the pre-distribution of energy when it is reached below the "critical point" (it is compared with the curie point of the magnet).

The breakdown of SUT symmetry group $SU(11)$, gave two fundamental group like $SU(6) \times SU(5)$ leads to a phase transition and then larger fundamental group $SU(5)$ breaks into a subgroup like $SU(3) \times SU(2)_L \times U(1)$, in which the scalar field ϕ changes. The original vacuum, with false vacuum ($\phi = 0$) is no longer the true vacuum ($\phi = \sigma$). The inflationary stage arises, however, if the true vacuum is not immediately attained.

An analogy will illustrate the scenario. Suppose stream is being cooled through the phase transition temperature of 100°C. Normally we except the steam to condense to water at this temperature. However, it is possible to super-cool the steam to temperature below 100°C, although it is then in an unstable state. The instability sets in when certain parts of the steam condense to droplets of water which then coalesce and eventually the condensation is complete. In the super-cooled state the steam still remains its latent heat, which is released as the droplets form.

Here we suppose that similar like super-cooling takes place past the SUT phase transition. What happens then is somewhat similar to the gas-vapor-liquid analogy. Its details depend on the latent energy group $SU(6)$ like latent heat (its name may be given as 'intelligence', which are responsible for the acceleration of the energy of $SU(5)$, and also the cause for the creation of D.N.A. of the live body etc.) and then on the potential energy function $V(\phi)$.

After the separation of two type energies $SU(5)$ and $SU(6)$, they want to interact each other and has a tendency to unify once again, as a result the direction of the energy group $SU(6)$ is opposite to the direction of the energy group $SU(5)$, for remaining the temperature unaltered and then an inflation occurred instantaneously. After the completion, the latent energy $SU(6)$ acts as a field. Again the seed fluctuation can grow to form the large scale structures with a scale invariant spectrum through fluctuations in the scalar fields $\phi(t)$. These fluctuations result in fluctuations of energy density. Through scientific investigations, we have discovered that fields are as real as the material world. In fact, fields represent more of a fundamental reality, because the material world is nothing but a manifestation of the underlying fields. In particular the decoupled of $SU(11)$, gave a several domain compartment along with the domain wall in which the compartment filled either latent energy or matter energy and hence created an inhomogeneous discrete energy universe. Then the latent energy compartments together form a core of a massive universe, while the matter energy compartments together form the surface of the universe (it may be compared with the structure of an atom, the quark form by the group $SU(3)$ is in the centre, while the laptons of the group $SU(2)$ stayed outside the nucleus. Although, at present it may be consider that the actual formation of an atom as the latent energy group $SU(6)$ is in the very centre of the nucleus and then quark made by the strong energy group $SU(3)$, while the leptons form by the weak energy group $SU(2)$ laid outside the nucleous. The whole atom effectively controlled by the latent energy. As the total number of bosons of $SU(6)$ is greater than the total number of bosons of $SU(5)$, so that latent energy of $S(6)$ stayed in the centre of the massive universe and form PBH, while the matter energy of

$SU(5)$, laid out side the centre region of the early universe and hence produce event horizon by the group $SU(6)$ and particle horizon by the group $SU(5)$. The energy group $SU(11)$ do not interact with the energy group $SU(6)$ and $SU(5)$, while $SU(6)$ produces gravitational field with the increase of latent energy and then energy $SU(5)$ form the super-cluster, cluster, galaxy etc. remaining a "Black hole" at the centre.

2. Intelligence : $SU(6)$. In the transformations under the group $SU(6)$, the basic fields here are the latent energy field and we have

$$U = \exp(-iH), \tag{1}$$

Where H is a 6×6 Hermitian matrix of Zero trace. The matrix H now has 35 independent components. In the weak interaction $SU(2)$, we have, H as 2×2 Hermitian matrix of zero trace and the most general form of such matrix is

$$H = \begin{pmatrix} a & b+ic \\ b-ic & -a \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \tag{2}$$

Thus, like above, we have 35 matrix charges $I_1, I_2, I_3, \cdots, I_{35}$ out of which five matrices are diagonal. Corresponding to this, we have 35 bosons. For want of any specific designation, they are referred to simply as J_k. There were no change takes place for exchanging the bosons namely $J_{k3}, J_{k8}, J_{k15}, J_{k24}, J_{k35}$, corresponding to the said five diagonal matrices. We expect the participating interactions of the bosons J_k to have comparable strength. The J_k bosons are expected to generate a latent force. This force is believed to be potentially so large that the exotic matter fluid are expected to transfer into the ordinary matter field, and then everything of the universe.

3. Super Unified Field : $SU(11)$. If we wish to unify all three interactions $SU(6), SU(5), U(1)$ in a Super Grand Unification scheme, we could trivially combine the three into a structure

$$SU(6) \times SU(5) \times U(1) \tag{3}$$

From the symmetry breaking of $SU(11)$, we find $SU(6)$ and $SU(5)$; the subgroups of $SU(11)$, where

$$p(=5) > 1, \quad n - p(= 11 - 5 = 6) > 1$$

So that

$$SU(n) \supset SU(p) \times SU(n-p) \times U(1)$$

$$\text{or, } SU(11) \supset SU(5) \times SU(6) \times U(1) \tag{4}$$

For completeness there are also the orthogonal and sympletic subgroups.

$$SU(11) \supset 0(11) \tag{5}$$

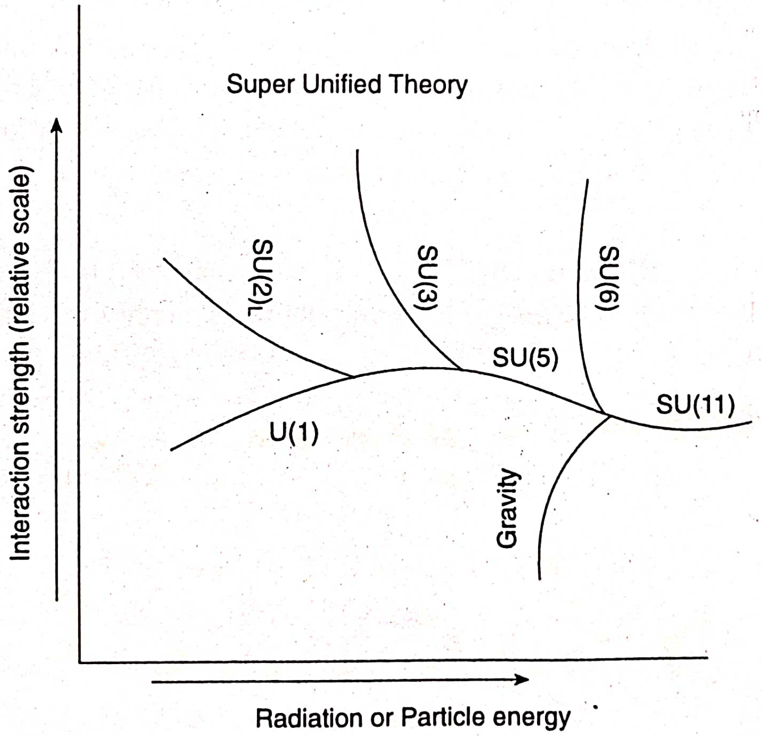
$$SU(22) \supset US_p(22) \tag{6}$$

Since the rank of $SU(11)$ is 10 and $U(1)$ is 1, a useful check is that the sum of the ranks of the subgroup $SU(5)$ and $SU(6)$ is less than or equal to the rank of the original group.

Thus, however, it was realized that such a structure (3) can form part of a single larger structure denoted by $SU(11)$. Again, if we go back of equation (1) and apply it to 11×11 matrices, the matrix H has 120 arbitrary constant. Thus there are 120 bosons that now medidate between the different type of energy fields. Of these we already have 24 from $SU(5)$ and 35 from $SU(6)$ and 1 from $U(1)$.

Thus, $120 - (24 + 35 + 1) = 60$ more bosons are needed to make up the list of 120. For want of any specific designation, they are referred to simply as the J bosons. The J bosons are expected to link the participants of $SU(6)$ with $SU(5)$ i.e, $SU(2)$ and $SU(3)$ and $U(1)$. There are emitted and absorbed $\bar{J}$ particles (an anti-J particles).

As the energy group $SU(5)$ advanced for unification with $SU(6)$, the strength of weak force gradually increases and the strength of strong forces decreases, ultimately the unification occurred at the extreme situation.



It can be expected, that for the symmetry breaking of $SU(11)$, created an amount of positive energy, negative energy and an equivalent amount of latent energy.

4. Excess production of J_k bosons in the early universe. Let us denoted the mass of the J-bosons by M_J, and its coupling strength by α_J. The coupling strength depending on what type of particle J is, let us denoted by $\Gamma_{c'}$ the rate of collisions that do not conserve the number of J_k bosons, i.e., collisions in which the J-boson is involved. Denote the characteristic decay rate of the J-boson by Γ_J, we thus have three time scales to play with:

$$\Gamma_J^{-1}, \Gamma_{c'}^{-1} \quad \text{and} \quad H_I^{-1}$$

At the earliest epochs, with constant temperature $> 10^{19}$ GeV, the latent energy was the strongest force between the various constituents of the universe. Other interactions were unimportant under the hypothesis of asymptotic freedom. As the universe continued to changing phase and its constant temperature dropped there was a phase when gravity as well as latent force become weaker while the other interactions still remained unimportant. Thus for $T \leq 10^{19}$ GeV, the particles remained essentially free for some time.

During this phase it becomes necessary to examine the nature of distribution, functions are as follows. Assuming ideal gas approximation and thermodynamic equilibrium, it is then possible to write down the distribution functions of any given species of particles. Let us use the symbol L to denote typical species ($L = 1, 2, \cdots$). Thus $n_L(P)dp$ denotes the number density of species in the momentum range $(P, P + dP)$, where

$$n_L(P) = \frac{g_L}{2\pi^2\hbar^3} P^2 \left[\text{EXP}\, \frac{E_L(P) - \mu_L}{kT} - 1\right]^{-1}$$

Where T = the temperature of the distribution, g_L = the number of spin states of the species, k = the Boltzmann constant and $E_L^2 = c'^2p^2 + m_L^2c'^2$is the energy corresponding to rest mass m_L of a typical particle. The quantity μ_L is the chemical potential of the species L. We get set $\mu_L = 0$, $g_L = 1$, $m_L = 0$, for J_k bosons. Since particles and antiparticles annihilate in pairs and produce J_k bosons their chemical potentials are equal and opposite. Again we saw that for $T \leq T_J$, the distribution function cannot preserve its form under changing phase. Thus it may get distorted from its equilibrium form. Now of the various species in the very early universe, the J-bosons are probably the most massive. Thus, provided they have a high enough value T_J, there is a chance that the J bosons will first dropout of equilibrium. For this to happen, however, it is also necessary that they have not all decayed by then. The collision rate.

A comparison of the three rats shows that

$$\Gamma_{c'} \approx \alpha_J \ll \Gamma_J$$

Soon after gravity became weak that means the amount of equivalent energy is not adequate and then the changing phase of the universe with the essentially no interaction between the species.

5. Quantum Cosmology of Zero Loop. We consider the Hartle-Hawking path integral for no boundary' proposal, by taking the wave function,

$$\psi[g_{ij}(y^k), \phi^A(y^k)] \approx \psi_0 - \text{loop}\,[g_{ij}, \phi^A] = \sum_{\text{some extrema}} e^{-I[g_{\mu\nu}, \phi^\Omega]} \tag{7}$$

Where we summing over a small set of extrema of the Euclidean action I, generally complex classical solutions of the field equations.

The zero-loop approximation gives

$$\psi(a_b, \phi_b) \approx \psi_{0-\text{loop}}(a_b, \phi_b) = \sum_{\text{Some extrema}} e^{-I[a_b, \phi_b]} \tag{8}$$

where $I(a_b, \phi_b)$ is the Euclidean action of a classical solution that is compact and has the S^3 geometry and homogeneous scalar field as its one and only boundary.

One boundary FRW-scalar histories have a time parameter 't' that can be taken to run from 0 (at a regular 'centre') to 1 (at the boundary), and then to have $\phi = \phi(t)$ and four metric.

Consider the metric

$$ds^2 = \left[\sqrt{\frac{2G}{3\pi}} N(t)\right]^2 dt^2 + \left[\sqrt{\frac{2G}{3\pi}} a(t)\right]^2 d\Omega_3^2 \tag{9}$$

Where $N(t)$ is the lapse function and $d\Omega_3^2$ is the metric on a unit 3-sphere S^3.

If the scalar field potential is

$$\left[\frac{9}{16G^2}\right] V(\phi)$$

[with the coefficient again chosen to simplify the formulas below, in terms of the rescaled potential $[V(\phi)]$, then the Euclidean action fo the history is

$$I = \int dt \left(\frac{1}{2N}(-a\dot{a}^2 + a^3\dot{\phi}^2) + \frac{1}{2}N(-a + a^3 V) \right) \tag{10}$$

$$\text{i.e., } \frac{dI}{dt} = 0 = \frac{1}{2N}(-a\dot{a}^2 + a^3\dot{\phi}^2) + \frac{1}{2}N(-a + a^3 V) = L = H \tag{11}$$

Here ϕ is the scalar field.

6. FRW-Scalar Model with Complex Scale Factor. We have from the equation (11),

$$-\dot{a}^2 + a^2\dot{\phi}^2 + N^2(-1 + a^2 V) = 0 \tag{12}$$

We consider $a = R + iR_I$ as complex radius. Now substituting 'a' in the equation (12), and separating real and imaginary part, we have,

$$-(\dot{R}^2 - \dot{R}_I^2) + (R^2 - R_I^2)\dot{\phi}^2 - N^2 V + N^2 V(R^2 - R_I^2) = 0 \tag{13}$$

$$\text{and} \quad \dot{\phi}^2 + N^2 V - \frac{\dot{R}}{R}\frac{\dot{R}_I}{R_I} = 0 \tag{14}$$

Now we consider,

$$H_R = \frac{\dot{R}}{R} \quad \text{and} \quad H_I = \frac{\dot{R}_I}{R_I};$$

$$R(t) = \ell_p e^{H_R t} \quad \text{and} \quad R_I(t) = \ell_p e^{H_I t}$$

where $R(0) = R_I(0) = \ell_p$ and consider $N(t) = 1$.

Thus the equation (14) becomes

$$\dot{\phi}^2 = H_R H_I - V \tag{15}$$

6.1 FRW-Scalar Models with An Exponential Potential $V = e^{2\alpha\phi}$. To illustrate some of these ideas quantitatively, it is helpful to consider the case of an exponential potential

$$V(\phi) = e^{2\alpha\phi} \tag{16}$$

where α is a real parameter that characterizes how fast the potential varies as a function of ϕ.

We have from the equation (15) and (16),

$$\frac{d\phi}{dt} = \pm\sqrt{H_R H_I - e^{2\alpha\phi}}$$

Integrating, we have

$$e^{2\alpha\phi}\left(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1\right)^2 = \pm 4H_R H_I\left(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1\right) - 4H_R H_I \tag{17}$$

[where A is integration constant]

$$\text{i.e.,} \quad e^{2\alpha\phi} = \frac{4H_R H_I A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t}}{(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1)^2} = V \quad \text{[taking +Ve Sign]} \tag{18}$$

Hence

$$\phi = \frac{1}{\alpha}\log(2\sqrt{H_R H_I}A) - \frac{1}{\alpha}\log\left[A^2 e^{\mp\alpha\sqrt{H_R H_I}t} + e^{\pm\alpha\sqrt{H_R H_I}t}\right] \tag{19}$$

Differentiating both side with respect to time t, we get

$$\dot{\phi} = \pm\sqrt{H_R H_I}\frac{A^2 e^{\mp\alpha\sqrt{H_R H_I}t} - e^{\pm\alpha\sqrt{H_R H_I}t}}{A^2 e^{\mp\alpha\sqrt{H_R H_I}t} + e^{\pm\alpha\sqrt{H_R H_I}t}} \tag{20}$$

Again, if we take negative sign, then from the equation (17), we have

$$V = e^{2\alpha\phi} = \frac{-4H_R H_I(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 2)}{(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1)^2} \tag{21}$$

$$\phi = \frac{1}{2\alpha}\log\left\{\frac{4i^2 H_R H_I (A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 2)}{(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1)^2}\right\}. \tag{22}$$

When $A = 1$, then

$$V = H_R H_I \left[\frac{e^{\mp\alpha\sqrt{H_R H_I}t} + e^{\pm\alpha\sqrt{H_R H_I}t} + 2}{2}\right]^2 \tag{23}$$

$$\text{and} \quad \phi = \frac{1}{\alpha}\log\left[\sqrt{H_R H_I}\left\{\frac{e^{\pm\alpha\sqrt{H_R H_I}t} + e^{\mp\alpha\sqrt{H_R H_I}t}}{2}\right\}^{-1}\right. \tag{24}$$

Thus,

$$\dot{\phi} = \pm\sqrt{H_R H_I}\frac{e^{\mp\alpha\sqrt{H_R H_I}t} - e^{\pm\alpha\sqrt{H_R H_I}t}}{e^{\mp\alpha\sqrt{H_R H_I}t} + e^{\pm\alpha\sqrt{H_R H_I}t}} \tag{25}$$

$$\text{i.e.,} \quad \rho \approx \pm\frac{\sqrt{H_R H_I}}{2c^2}\frac{\frac{e^{\mp\alpha\sqrt{H_R H_I}t} - e^{\pm\alpha\sqrt{H_R H_I}t}}{2}}{\frac{e^{\mp\alpha\sqrt{H_R H_I}t} + e^{\pm\alpha\sqrt{H_R H_I}t}}{2}} \tag{26}$$

6.2 $\alpha = 0$ de-sitter Example, $V =$ constant. When $\alpha = 0$, the potential is independent of ϕ. For the exponential potential $V(\phi) = e^{2\alpha\phi}$ given (17) and (21), this would give $V = 1$, but one can easily generalize the result to any constant V.

For example from equation (17)

$$V = \frac{4H_R H_I A^2}{(A^2+1)^2}, \tag{27}$$

and from equation (21)

$$V = \frac{-4H_R H_I (A^2+2)}{(A^2+1)^2} \tag{28}$$

Now in the Einstein Universe, we consider $H_R = H_I$, so we have from equation (27)

$$H_R = \frac{\dot{R}}{R} = \pm\left(\frac{A^2+1}{2A}\right)$$

Integrating, we get

$$R = Be^{\pm\left(\frac{A^2+1}{2A}\right)t} \tag{29}$$

[where B, is the integration constant]

Also, we have from the equation (28)

$$H_I = \frac{\dot{R}_I}{R_I} = \pm\frac{i(A^2+1)}{2\sqrt{A^2+2}}$$

$$\text{i.e., } R_I = Ce^{\mp i\left(\frac{A^2+1}{2\sqrt{A^2+2}}\right)t} \tag{30}$$

[where, C is the integration constant]

Now, $B = C = \ell_p$, thus equation (29) and (30) becomes

$$R = \ell_p e^{\pm\left(\frac{A^2+1}{2A}\right)t} \tag{31}$$

$$\text{and } R_I = \ell_p e^{\pm\left(\frac{A^2+1}{2\sqrt{A^2+2}}\right)(it)} \tag{32}$$

We have from (22)

$$\frac{4i^2 H_R H_I(A^2+2)}{(A^2+1)^2} = 1$$

And taking $H_R = H_I$ then

$$H_I = \pm\frac{i}{2}\frac{A^2+1}{\sqrt{A^2+2}}$$

$$R_I = \ell_p e^{\pm\frac{A^2+1}{2\sqrt{A^2+2}}(it)}$$

6.3 $\phi = 0$ Example, $V = 1$. We have from the equation (18)

$$2\sqrt{H_R H_1} A e^{\pm\alpha\sqrt{H_R H_I}t} = 1 + A^2 e^{\pm 2\alpha\sqrt{H_R H_I}t} \quad \text{[taking positive sign.]}$$

$$\text{i.e., } t = \pm\frac{1}{\alpha\sqrt{H_R H_I}}\log\left\{\frac{1}{A}\left(\sqrt{H_R H_I} \pm \sqrt{H_R H_I - 1}\right)\right\} \tag{33}$$

If $H_R = H_I$ then (33) becomes

$$t = \pm\frac{1}{\alpha H_R}\log\left\{\frac{H_R}{A} \pm \frac{\sqrt{H_R^2 - 1}}{A}\right\} \tag{34}$$

6.4 The Energy Density of ϕ-Field. We have from the equation (17)

$$\phi = \frac{1}{2\alpha}\log\left\{4H_R H_I(\pm A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} \pm 1 - 1) - \frac{1}{\alpha}\log(A^2 e^{\mp 2\alpha\sqrt{H_R H_I}t} + 1)\right. \tag{35}$$

the inflationary model seems capable of producing the spectrum, through fluctuations in the scalar field $\phi(t)$.

Harrison in 1970 and Zeldovich in 1972 has argued independently, from theoretical considerations that at the time of entering the horizon, the amplitude of a typical density perturbations should have the form $F(k)\alpha k^{-3}$ where $F(k) \equiv |\delta(k,t)|^2$ and k, the wave number.

$$t = t(k).$$

The fact that $\Delta T/T$ in the microwave background radiation is $< 10^{-5}$, implies that $|\delta(k,t)|^2$ was $\ll 1$ in the radiation dominated phase of expansion.

It can be shown that the root—mean-square fluctuation of mass M as a fraction of average mass contained in a region of size R is proportional to $k^3|\delta|^2$ at $k = R^{-1}$. Therefore, for the above $F(k)$, $\langle(\delta M/M)^2\rangle$ will be independent of the scale R at $t = t(k)_{\text{enter}}$, thus giving equal power at all scales at the time they enter the horizon.

A scale-invariant spectrum is indicated by the distribution of discrete large-scale structures.

We write the equation (35), the fluctuation as $F(t)$ over a smoothed average value $\phi_0(t)$.

Thus

$$\phi(t) = \phi_0(t) + F(t) \tag{36}$$

These fluctuations result in fluctuations of energy density.

The energy density of a scalar field is

$$\rho c^2 = \frac{1}{2}\dot{\phi}^2(t) \tag{37}$$

The average energy density during inflation being dominated by the constant term V_0 of the Coleman-Weinberg-potential. We have the density constract

$$\delta_k(t) = \frac{\delta \rho c^2}{V_0} \tag{38}$$

We use $\dot{\phi}_0$ the mean evolution of ϕ in the slow roll-over phase; but what is $F(t)$? Now in actually the fluctuations in ϕ are of quantum origin but here, in a classical approximation, we are using $F(t)$ to mimic them classically. In quantum field theory the field would be an operator $\hat{\phi}(t)$ whose Fourier coefficient $\hat{q}_k(t)$ are also operators. In a quantum state specified by the wave function Ψ_k, the fluctuations of $\hat{q}_k$ are given by the dispersion relation

$$\sigma^2(t) = \langle \Psi_k | q_k^2(t) | \Psi_k \rangle \tag{39}$$

The mean value (in $k = 0$ mode) of Ψ_k being zero. This is because ϕ_0, the average of ϕ, is homogeneous. Since $\sigma_k^2(t)$ appears to be a good measure of quantum fluctuations.

We may write

$$\delta_k(t) = \frac{\dot{\phi}_0(t)}{V_0}\sigma_k(t) \tag{40}$$

Thus we have taken a semi-classical approximation to estimate the fluctuations in the energy density of the ϕ field which act as the seed fluctuation of density during the inflationary phase $t_i < t < t_f$.

7. Concluding Remarks. In the paper, we consider a path integral over all compact Euclidean 4-dimensional geometries and matter fields by considering a complex scale factor. There were a stage of *SUT* of Phase transition before *GUT*. We think field created by the latent energy group. We consider a scalar ϕ-field. The fluctuation of the scalar field ϕ result in fluctuations of energy density.

We find the value of the scale factor, when potential $V = 1$ and also the scale factor, related with imaginary time. It has been showed that the energy density changes exponentially with time. We found the time, when $\phi = 0$, in terms of the parameter H_R and H_I.

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