

Application Of Problem-Based Learning To Teaching The Relationships Within A Triangle And Solution Of Triangles

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Article Info	Abstract
<p>Article History</p> <p>Received: April 29, 2021</p> <p>Accepted: July 26, 2021</p> <hr/> <p>Keywords : Problem-Based Learning, Real-World Problems, Relationship Within A Triangle, Teaching approach</p> <p>DOI: 10.5281/zenodo.5137984</p>	<p><i>Problem-based learning (PBL) is an active, student-centered teaching approach that has been interested in many educational institutions worldwide. The effectiveness of applying PBL in teaching subjects in general and Mathematics, in particular, has been clarified by many researchers. In particular, many studies have shown that PBL positively impacts students' practical problem-solving skills and improves students' learning attitudes in mathematics. This study was conducted for eighty-one 10th grade students to test the effectiveness and feasibility of PBL in teaching the relationships within a triangle and the solution of triangles. Students in the experimental class were taught according to a 6-step learning model based on PBL, while students in the control class were guided by the conventional method. Research results show that students in the experimental class achieved higher results than students in the control class about mathematical knowledge and skills in applying knowledge to solve real-world problems related to the interactions within a triangle. Besides, the high level of positivity when participating in group activities, active learning attitude and the desire to continue to experience the PBL learning environment of the students in the experimental class have contributed to confirming the effectiveness of this teaching approach.</i></p>

Introduction

Mathematics has more and more applications in real life; prior mathematical knowledge and skills have helped people resolve real-life problems systematically and accurately, contributing to promoting social development. However, according to the General Education Program in Mathematics, the content of Mathematics is usually logical, abstract and general. Accordingly, in order to understand and learn Math, the Math curriculum in high schools demands to ensure a balance between "learning" knowledge and "applying" knowledge to address specific problems (MoET, 2018). This approach requires teachers to teach according to modern teaching methods to meet the current student-centered perspective and at the same time to link theory with practice to help learners apply the knowledge they have learned in the classroom. The educational objectives of the United Nations Educational, Scientific, and Cultural Organization include learning to know, learning to do, learning to live together, and learning to affirm (UNESCO) (Delors and colleagues, 1996).

A problem-based learning model is a learner-centered approach to teaching that allows students to conduct research, incorporate theory and practice, and apply their newly acquired knowledge and skills to develop possible alternatives to a given problem (Savery, 2015). Many recent studies have shown that, compared with traditional teaching methods, PBL enhances students' ability to transform knowledge to new problems and gain a deeper understanding (Hmelo, 1998; as cited in Lu, Bridges & Hmelo-Silver, 2014). Furthermore, PBL can improve students' attitudes to math through practical problems that at an appropriate age can become a hobby while helping them see the meaning of math study (Westwood, 2011).

The relationships within a triangle are important knowledge areas of the high school math program and appear quite a lot in the national high school graduation exams, the exams for good students. This content is associated with finding angles, sides, length, area in a triangle and expanding to problems associated with the practice. By applying the problem-based teaching method to teaching the content of the quantifiable systems in the triangle, teachers can exploit lesson knowledge with real-life through demanding activities asking students to explore and research to acquire knowledge.

Literature Review

Definition of PBL

PBL was first introduced in medical training at the Health Sciences Center at McMaster University in the 1960s (Barrows & Tamblyn, 1980; as cited in Lu, Bridges & Hmelo-Silver, 2014; de Graaff & Kolmos, 2003). After its inception in medicine, PBL developed in higher education for the health sciences, engineering, architecture and education, and gifted and general education (Walker & Leary, 2009; as cited in Lu, Bridges & Hmelo-Silver, 2014; Dağyar & Demirel, 2015).

PBL is a learner-centered teaching approach because students discuss groups' relevant issues before the preparation and self-study process to activate the student's existing knowledge. Because the student's prior knowledge is not enough for the student to understand the problem deeply, the questions (i.e., learning problem) are constructed for the students to study independently in the study group. Students gather in groups after the self-study period and discuss what they have researched to answer the learning problems. Group discussions are facilitated by teachers and are geared towards gaining knowledge, deeper understanding of the problem, and the skills to deal with it (Barrows, 1996; as cited in Dolmans et al. al., 2016; de Graaff & Kolmos, 2003; Goodman, 2010; Wirkala & Kuhn, 2011).

Meanwhile, according to some authors, in a problem-based learning phase (PBL), students work in small groups to cope with a real-world task, here a modeling task. While the teacher serves as a cognitive advisor, the student develops, tests, and adapts models to address the goals established by the teacher (Eric, 2010; Flinkle & Torp, 1995; Ozcan & dan Balim, 2013). Thus, from the above definitions, the problem-based teaching method can be understood to guide students on identifying problems, working in groups, and solving problems together. Learners can gain knowledge and skills relevant to their curricula, such as document analysis, research, and problem-solving techniques.

Characteristics of PBL

Author Barrows (1997) described the characteristics of PBL including: (1) student-centered - students are responsible for self-study; (2) problem-based - the problem used is an ill-structured problem; (3) problem solving; (4) self-directed - students can tailor what needs to be learned and this is based on what students need to do to cope with the problem; (5) repeat - when the student finishes the self-directed process (finding the information needed to solve the problem) the student comes back and applies the new knowledge to resolve the problem; (6) cooperation – students cooperate with each other in problem solving and identifying learning problems; (7) self-reflection – when the problem has been addressed the student will contrast new information with the new (for the future) problem, to generalize a concept or principle and system to be related among element of the problem and of the argument; (8) self-monitoring – students will monitor and evaluate their own progress and achievements; (9) authentic – all learning behaviors required by PBL including all steps taken by students and assessed in real-life situations (Barrows, 1997; as cited in Zabit, 2010). Authors such as Boud and Feletti (1998; as cited in Thao, 2019), Savery and Duffy (2001), de Graaff and Kolmos (2003), Savery (2015), Kardoyo et al. (2019), Jatisunda et al. (2020) also pointed out the characteristics of PBL, but the views are generally similar to those of Barrows (1997).

In a PBL environment, teachers' ability to teach is more demanding than in teacher-centered classrooms. In addition to presenting mathematical knowledge to students, teachers must involve students in gathering information and using knowledge in given situations. First, teachers need to have great math skills to guide students to apply their knowledge in various situations (Prawat, 1997; Smith III, 1997; as cited in Roh, 2003). Teachers in PBL classrooms must also develop pedagogical skills and shift from being the classroom center to guiding students. Knowledge acquisition is challenging for those unfamiliar with PBL (Lewellen & Mikusa, 1999; as cited in Roh, 2003). According to the authors English and Kitsantas (2013), the teacher's role is to organize activities to stimulate students' learning, encourage reflection, and facilitate the student's learning process through instruction, timely feedback, and promotion of independent thinking (English & Kitsantas, 2013). In general, teachers in a PBL environment are no longer the sole source of knowledge but rather a guide and support students in problem-solving and learning new knowledge.

With PBL, students have to work in small groups of 5-8 people (Cerezo, 2004). According to Barrett and Moore (2011), each student in the group should have specific roles such as group leader, secretary, reader, observer, Specifically: (1) The role of the group leader is to encourage all team members to participate, facilitate work within agreed rules and control which members have the upper hand, manage and observe members to assign appropriate tasks well suited; (2) The role of the secretary is to record the unique ideas of the students in the group, to write down methods, problem-solving plans, procedures, minutes of group meetings; (3) The role of the reader is to read aloud to the group any decisions recorded by the secretary; (4) The role of the observer is to help the group manage time, take notes, make suggestions, and evaluate the participation level of the members of a group. There are many different roles within each group, and one person can take on one or more different tasks to suit the circumstances (Barrett & Moore, 2011). Additionally, students need to know how to ask questions, research and use critical thinking in a positive way to solve problems. In a PBL learning environment, students must take responsibility for their learning and bring meaningful knowledge to themselves (English & Kitsantas, 2013).

The goals of PBL are not only to acquire knowledge but also to improve students' self-directed learning and problem-solving skills. Thus, learning (knowledge) and educational (skills) goals are achieved simultaneously in a PBL environment. It also aims to improve student motivation and support meaningful learning experiences using related problems (Alreshidi, 2016). According to Barell (2006), the reasons for implementing PBL are: (1) providing opportunities for students to practice higher-order thinking; (2) equity, where students can learn and

improve their knowledge and skills in any field; (3) improve student motivation by challenging them; (4) active learning support; and (5) reinforce deeper understanding through meaningful experience (Barell, 2006).

Problems in PBL

According to Lu, Bridges and Hmelo-Silver (2014), PBL presents students in different fields with different types of problems, such as diagnostic problems, design problems, strategic performance problems, and decision-making problems. These issues are key to the successful design and implementation of PBL activities. According to Newell and Simon (1972), these problems are often divided into two categories, well-structured and ill-structured (Newell & Simon, 1972; as cited in Lu, Bridges & Hmelo-Silver, 2014). Ill-structured problems are common in daily life and can often be solved by different solutions or alternative solutions (Shin, Jonassen & McGee, 2003; as cited in Temel, 2014; Jatisunda et al., 2020). In a study by Walker and Leary (2009), statistics show that design problems and strategic performance problems achieve the best performance in PBL. In particular, ill-structured problems used in PBL serve as a foundation for high-level collaborative problem-solving interactions (Van Berkel & Schmidt, 2000; as cited in Lu, Bridges & Hmelo-Silver, 2014; Finkle & Torp, 1995; Ronis, 2008; Wirkala & Kuhn, 2011). In general, author Ronis (2008) believes that problems in PBL should be ill-structured problems, practical and appropriate for learning groups. Issues should be logical and authentic (real-world) and not ambiguous; The goal of problem-solving is the discovery process and aspects of research rather than finding a solution, which is expected to uncover mathematical principles from real-world problems (Ronis, 2008).

PBL processes

There is no specific model for teaching strategies by PBL method; many authors have studied models to concretize the teaching process according to the PBL approach, such as Savery and Duffy (2001); Busfield and Peijs (2003); Ronis (2008); Lu, Bridges and Hmelo-Silver (2014); Dolmans et al. (2016); Hendriana, Johanto and Sumarmo (2018); Thao (2019); Jatisunda et al. (2020). The authors' PBL models are student-centered, student-centered, and focus on teamwork to deal with problems. Nevertheless, each different model will be suitable for each classroom condition and in different circumstances. The model proposed by Busfield and Peijs (2003) consists of seven steps as follows:

- Step 1: Explain expressions, sentences, concepts
- Step 2: Identify the problem
- Step 3: Make a plan to solve the problem
- Step 4: List the solutions systematically
- Step 5: Identify self-study exercises
- Step 6: Practice individual exercises
- Step 7: Report and evaluate

In step 1, each student examines the problem given to them, identifying unclear words and terms and concepts to discuss. In step 2, the groups discuss and analyze what the problem is to be resolved, consider the issues from different perspectives and present the problem; The teacher guides and give feedback to the students. In step 3, each student finds his/her solution to the problem and is discussed in the group. Groups discuss solutions and choose the best solution to handle the problem in step 4. In step 5, each student will have individual tasks assigned to each group, determining deadlines. Then, each student self-practices the tasks assigned in step 5. In the final step, the groups report, discuss, share information and determine the next assignment (Busfield & Peijs, 2003).

Advantages of PBL

The effectiveness of PBL is often assessed on students' learning outcomes, cognition, metacognition, attitudes and behaviors under different teaching methods (Lu, Bridges & Hmelo-Silver, 2018). Many studies show that using PBL in math class helps students develop mathematical and general skills. Among them, the ability to overcome problems can be mentioned (Dolmans et al., 2016; Eric 2010; Hendriana, Johanto & Sumarmo, 2018; Roh, 2003; Surya & Syaputra, 2017; Thao, 2019), thinking power (Eric, 2010; MacMath, Wallace & Chi, 2009; Surya & Syaputra, 2017), critical thinking (Dolmans et al., 2016; Kardoyo et al., 2019; Surya & Syaputra, 2017); Savery & Duffy, 2001), creative thinking (Kardoyo et al., 2019; Surya & Syaputra, 2017; MacMath, Wallace & Chi, 2009; Savery & Duffy, 2001), modeling competence (Eric, 2010; Roh, 2003) and communication capacity (Roh, 2003; Surya & Syaputra, 2017). Besides, students who study with PBL have a higher motivation, learning attitude, and self-confidence than traditional learning (Hendriana, Johanto & Sumarmo, 2018; Jatisunda et al. al, 2020; Westwood, 2011).

Constraints in implementing PBL

Although PBL has many benefits to the teaching and learning process, the practical application of PBL presents significant challenges for both teachers and students. The results of the survey of teachers in Ingram's research (2018) have shown that most teachers said that they have difficulties in the role of the instructor, and the problems of time, program and resources are also challenges for teachers. Furthermore, in terms of the role of the instructor, some studies indicate that teachers may face difficulties due to not orienting students well, not

identifying student progress and not correcting their mistakes. Like many teachers, children struggle to develop a practical problem (MacMath, Wallace & Chi, 2009).

The opposite is true for students who are involved in PBL. At the beginning of PBL, students may feel uncomfortable because they are used to conventional teaching and learning methods. In PBL, students are held accountable for their learning, which requires students to be more active and perform tasks that require a lot of time and effort (de Graaff & Kolmos, 2003; Ronis, 2008). Besides, students need to know what to do instead of teachers assigning tasks. Furthermore, students interested in learning from books may not be comfortable with PBL's rules that require teamwork and the need to seek and process information gathered from outside the book (Ronis et al., 2008). In addition, de Graaff and Kolmos (2003) argue that while deeper levels of understanding can be achieved in learning, students may fall short in the breadth of vision and knowledge (de Graaff & Kolmos, 2003).

Assessment in PBL

According to Lu, Bridges and Hmelo-Silver (2014), assessment in PBL can focus on mastery of knowledge and skills or proficiency in problem-solving processes. Also, through synthesis from other studies, the authors' Lu, Bridges and Hmelo-Silver (2014) argue that systematic assessment can go beyond knowledge structures or the effectiveness of PBL in training skills such as reasoning, problem-solving, and decision making well as soft skills of self-directed and collaborative learning. More emphasis may be placed on "process" assessments such as group participation rather than "product" assessments such as writing or exams (Lu, Bridges & Hmelo-Silver, 2014).

Theoretical framework

Relationship within a triangle in Mathematics program of Vietnam

The General Education Program in Mathematics (2018) shows that the goals to be achieved in teaching the topic of Relationships within a triangle in the Geometry 10 program are as follows: (1) Provide knowledge about relations quantities in triangles, relationships and formulas for calculating the area of a triangle; (2) Add calculation skills, convert formulas; (3) Instructions for applying the knowledge of the lesson to solving triangles and solving some problems in practice (Ministry of Education and Training [MoET], 2018).

At the same time, the program requires students to (1) explain the basic quantification systems in triangles: theorem of cosine, theorem of sine, the formula for calculating the area of a triangle; (2) describe how to solve a triangle and apply it to address some problems with real-world content (for example, determining the distance between two places when encountering obstacles, determining the height of objects when cannot be measured directly) (MoET, 2018).

PBL model

Based on the characteristics and classroom conditions of the experimental class, the study proposes a 6-step learning model based on the model proposed by Busfield and Peijs (2003) as follows:

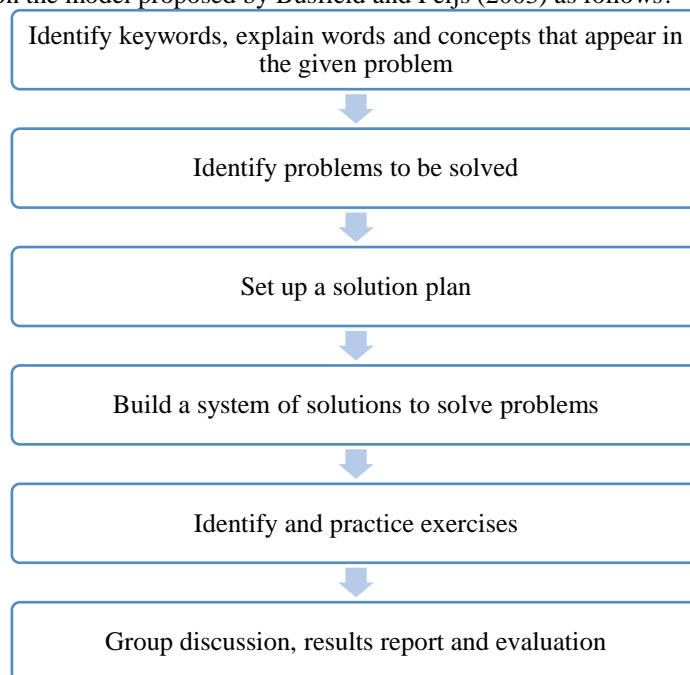


Figure 1. The 6-step learning model

From Figure 1, in step 1, after the problem is posed, each student learns by himself, identifies the keywords, words, and concepts that appear in the problem and explains them; if unclear, he can discuss with his classmates. in groups or consult with the teacher. In step 2, students explore the problem and determine the tasks

that need to be solved; in this step, the teacher can guide them by asking questions related to the problem, stimulating students' curiosity. In step 3, the students in each group discuss and plan to deal with the problem; the teacher can give suggestions if the groups tend to stray from the problem, out of focus. After that, the groups themselves discuss, analyze and find solutions to cope with the problem and decide the possible solution to address the problem in step 4. In step 5, each student of each group will be assigned specific tasks and perform those tasks. Finally, after performing the individual tasks, the groups discuss and report the results, the teacher will evaluate and conclude.

Class organization

It is necessary that teachers are well prepared for the following aspects of teaching using the PBL model in order to have an effective lesson:

- (1) Developing lesson plans, designing teaching situations according to the problem-based teaching method for the lesson "The relationships within a triangle and solution of triangles," encouraging the use of electronic lesson plans;
- (2) Prepare projectors, extra boards, pictures (if necessary), teaching tools such as rulers, chalks, pens;
- (3) Design study sheets, student self-assessment cards, post-teaching tests;
- (4) Planning to divide study groups;
- (5) Remind students to prepare before class.

At that time, the teacher in the role of a guide should pay attention to remind students of the things that need to be prepared in advance as follows: (1) Review the formulas about the quantification system in the right triangle that has been learned in classes before; (2) Review lessons and knowledge learned in previous lessons, especially the dot product of two vectors; (3) Prepare all necessary learning tools; (4) Convenient seating arrangement for study groups.

For group study, teachers can do the following: the class is divided into four groups, each group has about ten people. In each group, select a group leader responsible for leading the group during the discussion, leading the group to solve problems and a secretary to record all information during discussion and participation. Students' activities in the group are subject to individual self-assessment after the lesson.

Regarding organizing learning, (1) students in each group will exchange, discuss issues and give opinions. All opinions are respected, and the team will analyze to select the optimal ideas, then the team leader will assign each task and report the results; (2) the groups will report the results of the study; (3) the teacher evaluates the activity and after each activity, the students self-assess their performance.

The study's purpose, as well as its research questions

This study was conducted to determine the effectiveness and feasibility of project-based learning (PBL) in teaching the relationships within a triangle and triangles' solutions. The following questions must be answered in order to achieve this goal. Research is required to accomplish this.

1. How do students learn about triangular relationships in the 10th-grade math textbook?
2. In the PBL model, is it possible for students to make significant progress in their mathematical knowledge if they learn about relationships within triangles and triangle solutions while still in the stages of the learning process?
3. What strategies did students employ to improve their problem-solving skills after gaining knowledge from the stages outlined previously?

Method

The research resulted in teaching lessons on triangle relationships and solutions developed specifically for the experimental class. At the same time, the study conducted class observation and post-test for the experimental class and the control class. In addition, a survey of students' opinions about the lessons was carried out to assess the students' attitudes in the experimental class. As previously stated, a study design that included a pre-test, an intervention phase, and a post-test phase was implemented to achieve the research's overall objective. Research designs like these, which allow for the evaluation of the effectiveness of educational innovations, are extremely common in the field of educational research (Dugard & Todman, 1995; Tesch, 2016; Papadakis et al., 2016).

Participants

The pedagogical experiment was conducted in January 2021 for students in grades 10A4 (experimental class) and 10A5 (control class) of Doan Van To High School, Cu Lao Dung District, Soc Trang Province, Vietnam.

Instruments

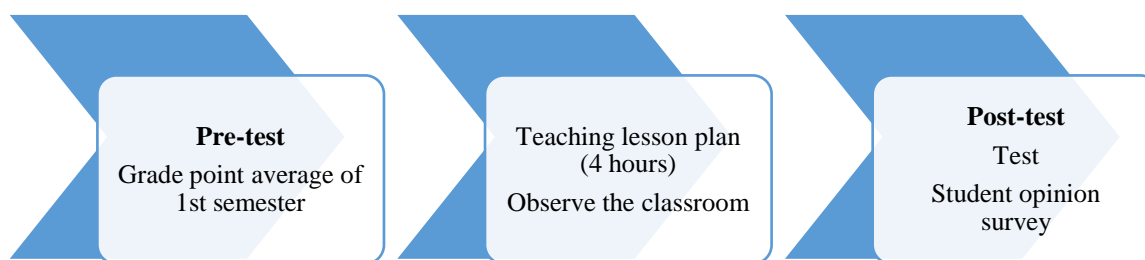


Figure 2. Experimental progress

According to Figure 2, the students who took part in the post-test for the two groups were divided into two groups in the previous experiments. Following the post-test and pre-test, the preliminary research was carried out to determine whether or not the hypothesis was correct. Validation and testing were required in order to determine whether or not the experiment would be successful. By developing appropriate and high-quality instruments, researchers contributed to the overall effort. According to two experts in mathematics education, the tests were valid and reliable, who agreed. It was determined that the instruments and research were of high quality, and several changes were made. A panel of experts confirmed that the instrument had not been revised further, and they unanimously agreed that the instrument was appropriate. They ultimately agreed to review the tests because they believed the results were relevant to the research question. Aside from content coverage, researchers can assess academic skills and academic content coverage. Table 1 shows the results of students' participation in group activities.

Table 1. Scale for rating the level of group activity

Point range	Classification
0.0 – 2.0	Weak
2.1 – 4.0	Medium
4.1 – 6.0	Good
6.1 – 8.0	Very good

The results of the students' post-test were graded using a scale shown in Table 2 to determine the effectiveness of the experiment in terms of knowledge gained by the students.

Table 2. The scale of grading assignments after the experiment

Classification	Poor	Weak	Medium	Good	Very good	Excellent
Point range	0.0 – 1.9	2.0 – 3.4	3.5 – 4.9	5.0 – 6.4	6.5 – 7.9	8.0 – 10.0

The results of the students' work were also classified according to the following levels for each question in the post-test in Table 3:

Table 3. The scale of grading the level of completion in each question of the post-test

Classification	Weak	Medium	Good	Very good
Item 1	0.0 – 1.0	1.1 – 2.0	2.1 – 3.0	3.1 – 4.0
Item 2	0.0 – 1.0	1.1 – 2.0	2.1 – 3.0	3.1 – 4.0
Item 3	0.0 – 0.5	0.6 – 1.0	1.1 – 1.5	1.6 – 2.0

The students' opinion survey questions were designed based on a Likert scale with five levels including Totally agree, Agree, Neutral, Disagree and Completely disagree.

Data collection and analysis

Data were collected based on the average score of 1st-semester Mathematics, the results of class observations, the post-experiment test of both classes, and students' survey result in the experimental class. The data were analyzed quantitatively (via SPSS 20 software) and analyzed qualitatively.

Results and Discussion

Results involving pre-test

This study used the average score of Mathematics in the first semester of students in both control and experimental classes to verify the equivalence of academic performance. Tables 4 and 5 below presented descriptive statistics of pre-test and independent t-test results of control and experimental classes.

Table 4. Descriptive statistics of pre-test results

	N	Mean	Std Dev	StdErr	Minimum	Maximum
Experimental class	43	5.7209	1.255	0.1914	3.5	8.0
Control class	38	5.8368	1.256	0.2037	3.3	8.3

Table 5. Independent t-test of pre-test results

df	t Stat	Mean difference	p-Value – 2 tailed
79	-0.4147	-0.11591	0.680

An independent t-test was used to test the significance of the mean difference between the experimental class and the control class. With significance level $\alpha=0.05$ and degrees of freedom $df=79$, the critical value $p=0.680$ is greater than 0.05. Thus, the difference in mean scores between the experimental class and the control class is not significant. Based on the above results, it can be concluded that the levels of the experimental and control classes are equivalent in importance.

Results of experimental lessons

In the practical lessons in class 10A4 on the relationships within triangles and the solution of triangles, the study gave a total of 5 large activities, including warm-up activities and activities from 1 to 4, respectively. Students from each group would exchange ideas, perform individual exercises, and the group secretary would record these activities for exchange and self-assessment in class. After five activities, the total score was recorded in Table 6 by different levels.

Table 6. Results of self-assessment of students in the experimental class

Point range	Classification	Frequency	%
0.0 – 2.0	Weak	1	2.33%
2.1 – 4.0	Medium	5	11.63%
4.1 – 6.0	Good	20	46.51%
6.1 – 8.0	Very good	12	27.90%
8.1 – 10.0	Excellent	5	11.63%
	Sum	43	100%

Specifically, most of the students in the class participating in the exchange activities were at a good level, 20 students out of the total 43 students in the class (accounting for 46.51%). Twelve students participated in activities at a good level (accounting for 27.90%), and five students participated at an excellent level (accounting for 11.63%). In addition, there were still some students who had not participated much in group activities and the process of lesson development; specifically, there were five students who only participate at an average level (11.63%), and one student performance was at a weak level (accounting for 2.33%).

In general, when teaching by the PBL method for the relationships within triangles and the solution of triangles, the students of the class were active and enthusiastic in participating in activities and building lessons. This outcome shows an effective part of teaching associated with using a new teaching method instead of the usual methods used in the classroom.

Observation results of control and experimental classes

As shown in Table 7, the experimental class outperformed the control class in observing relationships within triangles and solving them. The findings were analyzed and compared in teaching methods, learning methods, skills, and attitudes.

Table 7. Observational results of the experimental and control classes

Contents	Experimental class	Control class
Teaching methods	<ul style="list-style-type: none"> - Take students as the center. - Teachers use slide show combined with a whiteboard in the teaching process. - Teachers lead students to form new knowledge through problem-solving. - The process of teaching new content is carried out in 6 steps of the PBL method. 	<ul style="list-style-type: none"> - Take the teacher as the center. - Teacher teaches by blackboard. - The teacher presents new knowledge, likes it and gives illustrative examples. - The process of teaching new knowledge content is asking questions, giving formulas, solving problems, and applying examples.
Learning methods	<ul style="list-style-type: none"> - Working in groups, discussing together, combining many skills such as analysis, prediction, presentation, criticism. - Learn new knowledge through problem-solving. - Combination of individual work and group work. 	<ul style="list-style-type: none"> - Work individually, listen to the teacher's questions and express their own opinions. - Acquiring new knowledge through the teacher's communication process. - Listen to lectures and take notes of information and lesson content conveyed by the teacher.
Skills	Skills in teamwork, communication, analysis, prediction, presentation, criticism.	Analytical and predictive skills.
Student's attitude	<ul style="list-style-type: none"> - Actively participate in group activities and exchange ideas. - Eager to speak, comment and absorb ideas. 	<ul style="list-style-type: none"> - Being passive, a few students enthusiastically expressed their opinions.

Results involving post-test

After the teaching process in the experimental class, the researchers conducted a post-experiment test (30-minute test) in the experimental class 10A4 (January 19, 2021) and the control class 10A5 (January 20). January 2021. The test score results were statistically and calculated in Table 8 and Table 9 presented below.

Table 8. Descriptive statistics of post-test results

	N	Mean	Std Dev	StdErr	Minimum	Maximum
Experimental class	43	5.9186	1.48135	0.2259	1.3	8.0
Control class	38	3.8605	1.71062	0.2775	1.0	7.8

Before and after the experiment, the classes were stable in the initial number. The average score of the experimental class increased slightly compared to the results of the pre-experiment statistics; for the control class, it decreased sharply and based on this table shows the meaningful difference in the mean scores of the two classes. The standard deviation and standard error factors were not much different; for the smallest and largest values, the experimental class was higher than the control class, but not too much difference. In terms of the specific score distribution, it would be presented in the tables that follow this paragraph.

Table 9. Independent t-test of post-test results

df	t Stat	Mean different	p-Value – 2 tailed
79	5.803	2.05808	0.000

The researchers used an independent t-test to test the significant difference between the experimental and control classes with a hypothesis like pre-experiment. With significance level $\alpha=0.05$ and degrees of freedom $df=79$, the critical value $p=0.000$ is less than 0.05. It can be noticed that the difference in mean scores between the experimental class and the control class is significant. From the above results, it is shown that the level of the experimental class and the control class is not equivalent after the experiment. The value of the calculated research influence is $1.39 > 1.00$, so it can be concluded that the experimental effects have a great influence. The researchers detailed the score distribution of the control and experimental classes and the level of achievement in each post-test question in Table 10.

Table 10. Post-test results

Point range	Intermediate value	Frequency	
		Experimental class	Control class
0.0 – 1.9	0.95	1	2
2.0 – 3.4	2.7	1	15
3.5 – 4.9	4.2	10	9
5.0 – 6.4	5.7	13	9
6.5 – 7.9	7.2	15	3
8.0 – 10.0	9.0	3	0
Sum		43	38

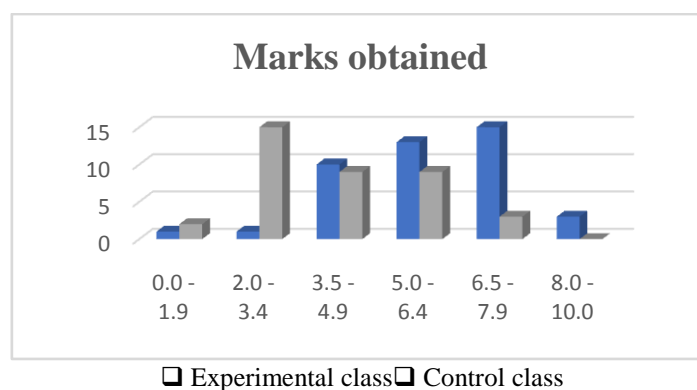


Figure 3. Point distribution chart after the experiment

Data from Figure 3 revealed that in the control class, the post-test scores had the highest frequency in the range of 2.0 - 3.4 (15 students) and the lowest frequency in the range of 8.0 - 10.0 (0 students), the test scores for the control group focused on the range from 2.0 to 6.4 (33 students out of 38 students). In the experimental class, the test scores with the highest frequency ranged from 6.5 to 7.9 (15 students), and the lowest frequency ranged from 0.0 to 1.9 (1 student); the test scores focused on a range from 3.5 to 7.9. There was a moderate difference in the range of scores from 3.5 to 6.4 between the two classes. In the range of scores from 2.0 to 3.4, there was a big difference, the control class had 14 students more than the experimental class, but in the range of 8.0 - 10.0, there were no students in the control class. In general, the experimental class scores were average and good; the control class scores were below average.

Table 11. Grading results of test scores after the experiment

Classification	Poor 0.0 – 1.9	Weak 2.0 – 3.4	Medium 3.5 – 4.9	Good 5.0 – 6.4	Very good 6.5 – 7.9	Excellent 8.0 – 10.0
Experimental class	1 2.3%	1 2.3%	10 23.3%	13 30.2%	15 34.9%	3 7%
Control class	2 5.3%	15 39.5%	9 23.7%	9 23.7%	3 7.8%	0 0%

According to Table 11, in the experimental class, good ratings accounted for the highest percentage (34.9%), poor and weak ratings accounted for the lowest percentage (total 4.6%). Scores of good and average also accounted for a high percentage. As a whole, the grade rating for the class was quite satisfactory. In the control class, the weak rating accounted for the highest percentage (39.5%), and the excellent rating accounted for the lowest percentage (0%). The percentages of good and average scores were calculated as relative percentages. Regarding distribution, the control group scores were distributed at an approximately average level compared to the scores of the other groups.

Between the two classes, there was a difference in the percentage of ratings as well as the distribution, specifically in the lower level of the control class, accounting for more than 3% compared to the experimental class; the control class was weaker than the experimental class by 37.2%, this difference was quite large; medium grade, both classes had roughly equal percentages (23.3% and 23.7%). For the good level, the experimental class accounted for 6.5% more than the control class; good grade in experimental class accounted for more percentage than control class 27.11%; excellent grade in the experimental class accounted for 7% of the total, only the control class did not have excellent grades.

Item 1. (4 points) Let ABC be a triangle, as shown in the figure. Calculate \hat{A} , S_{ABC} , m_c , r .

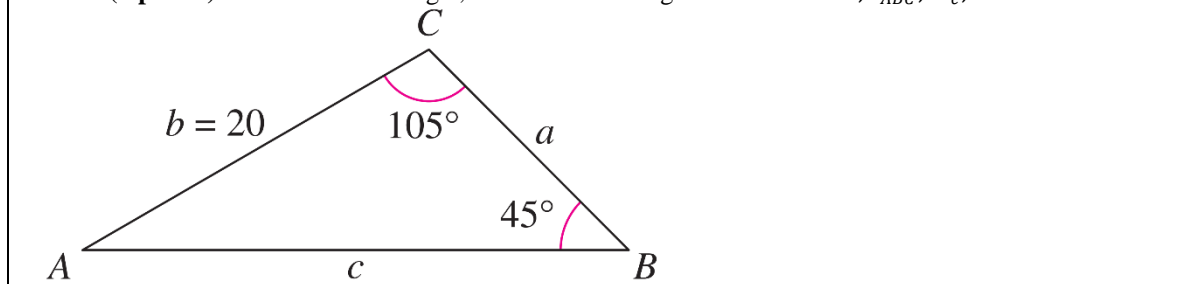


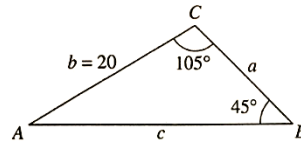
Table 12. Analysis of test results of Item 1

Classification	Weak 0.0 – 1.0	Medium 1.1 – 2.0	Good 2.1 – 3.0	Very good 3.1 – 4.0
Experimental class	1 2.3%	5 11.6%	17 39.5%	20 46.5%
Control class	5 13.2%	15 39.5%	11 28.9%	7 18.4%

Based on data from Table 12, in Item 1 (the question at the level of understanding and application), the majority of students in the experimental class achieved good and good scores (accounting for 46.5% and 39.5%); this shows that students had a good grasp of the formulas given to learn and know how to apply these formulas correctly in calculations. Nonetheless, many students in the class did not do well (accounting for 2.3% and 11.6%); these students could only calculate small ideas such as finding the measure of (A) when knowing the number measure of the other two angles.

For the control class, the distribution of scores of the question focused on the average and good level (accounting for 39.5% and 28.9%), the number of students with good scores accounted for 18.4%, and the number of students with weak points accounted for the least percentage is 13.2%; however, this figure was still quite high compared to the experimental class. In general, in Item 1, most of the students of the two classes met the requirement of remembering formulas and applying formulas to calculate specific problems, inferring to find the elements of a triangle sense.

Câu 1. (4 điểm) Cho tam giác ABC như hình vẽ. Tính \hat{A} , S_{ABC} , m_c , r của tam giác.



$$\begin{aligned} \text{Câu 1} \\ \hat{A} = 180^\circ - 105^\circ - 45^\circ = 30^\circ \\ c = \frac{\sin(105^\circ) \cdot 20}{\sin(45^\circ)} = 27,32 \\ a = \frac{b \cdot \sin A}{\sin B} = \frac{20 \cdot \sin(30^\circ)}{\sin(45^\circ)} = 10\sqrt{2} \\ S_{ABC} = \frac{1}{2} c \cdot b \cdot \sin A = \frac{1}{2} \cdot 27,32 \cdot 20 \cdot \sin(30^\circ) = 136,6 \\ m_c^2 = \frac{2(a^2 + b^2) - c^2}{4} = \frac{2(20^2 + 10\sqrt{2})^2 - 27,32^2}{4} = 113,40 \\ \Rightarrow m_c = \sqrt{113,40} = 10,64 \\ S = pr \Rightarrow r = \frac{S}{p} = \frac{136,6}{\frac{(a+b+c)}{2}} = \frac{136,6}{\frac{(10\sqrt{2} + 20 + 27,32)}{2}} = 4,4 \end{aligned}$$

Figure 4. Student's work TN08 – Good work

For example, in Figure 4, student TN08 already knew how to apply the sine and cosine theorems, the area of a triangle formula, the length of the triangle median, and the calculation of the problem's elements, while also satisfying the criterion that the calculation was.

$$\begin{aligned} \Delta ABC \text{ có góc } \hat{A} &= 180^\circ - \hat{B} + \hat{C} \\ &= 180^\circ - 150^\circ \\ &= 30^\circ \\ \Rightarrow \hat{A} &= 30^\circ \\ \text{Diện tích của tam giác } ABC \\ \text{ta có: } S_{ABC} &= \frac{1}{2} bc \sin A = \frac{1}{2} \cdot 45 \cdot 105 \cdot \sin 30^\circ = 1181,25 \\ \text{Đường trung tuyến } C \text{ của tam giác } ABC \\ \text{ta có: } m_c^2 &= \frac{2(a^2 + b^2) - c^2}{4} = \frac{2(30^2 + 20^2) - 185^2}{4} = -2106,25 = \sqrt{-2106,25} \approx 46 \\ \text{Đường tròn ngoại tiếp tam giác } ABC \text{ có:} \\ \frac{b}{\sin B} &= \frac{20}{\sin 45^\circ} = 20\sqrt{2} \end{aligned}$$

Figure 5. Student's homework DC04 – Bad homework

In addition to the good and good work, some students' work was still not good. The image above was the work of student DC04 (see Figure 5). The above student knew how to calculate the measure of angle A, and in general, this student knew how to use the correct formula to calculate it. However, in calculating the triangle area, DC04 had replaced the error in the formula; namely, it could not distinguish the measure of the side and the measure of the angle, leading to incorrect calculation. Next was calculating the length of the median line, although using the correct formula, with the same mistake as above, DC04 calculated the wrong result and did not realize the serious error that was the square of the line length median output negative numbers. In general, in the work of student DC04, it can be reported that this student knew how to apply previously learned formulas to the calculation according to the requirements of the problem but did not understand the formulas and symbols clearly, leading to misuse and poor performance. The mistake of student DC04 was also the mistake of some students in both the experimental and control classes, which raises the problem that teachers need to take appropriate measures so that students can understand the formula and apply the correct formula and knowledge has been learned.

Item 2. (4 points) Cu Lao Dung is a district in the easternmost part of Soc Trang province, positioned at the end of the poetic Hau River with Dinh An and Tran De estuaries flowing into the East Sea. Cu Lao Dung has many different names, such as Huynh Dung Chau, Ho Chau, Cu Lao Vuong, Cu Lao Chang Be. To get to Cu Lao Dung, we need to go through ferry routes, including Dai Ngai ferry, going from point A (bank B) to point C (right bank), but cannot go directly because in the middle of the river there is another island, so Dai Ngai ferry will get from point A to point B, and then go to point C (as shown). According to you, the Dai Ngai ferry has to travel many more kilometers than going directly from A to C if you know $\widehat{ABC} = 123^\circ$?

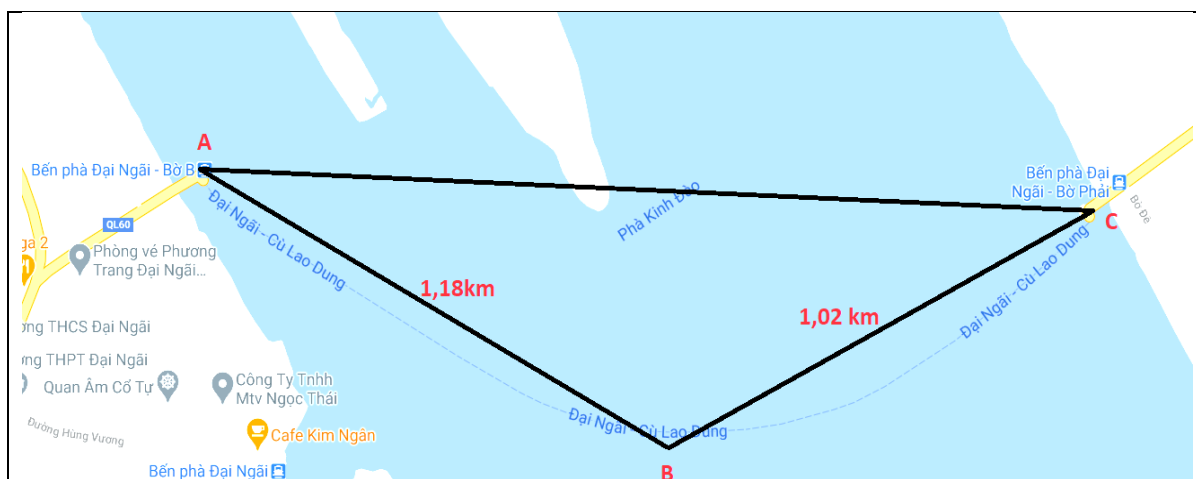


Table 13. Analysis of test results in Item 2

Classification	Weak	Medium	Good	Very good
	0.0 – 1.0	1.1 – 2.0	2.1 – 3.0	3.1 – 4.0
Experimental class	5 11.6%	9 20.9%	20 46.5%	9 20.9%
Control class	18 47.4%	13 34.3%	6 15.8%	1 2.6%

According to Table 13, in Item 2 (a question at the application level), students of the control class achieved the weakest score (accounting for 47.4%), and the lowest percentage was at a good level (accounting for 2.6%). Besides, the group of average scores also accounted for a relatively high percentage (34.3%). The lowest percentage was in the weak level (11.6%) for the experimental class, and the highest percentage was in the good group (46.5%). In general, there was a clear difference in the distribution of scores in the experimental and control classes in the second question, which shows that students had not been able to apply the formulas to calculate the math problem in the control class. Because of the valuable content, most of the students in the class did not get good marks, some students in the class did not recognize the normal triangle, so they used the formula of the Pythagorean theorem to calculate. In contrast, in the real class, the test results were better; many students still got bad grades (14 students), and many students made mistakes in the presentation and use of mathematical symbols.

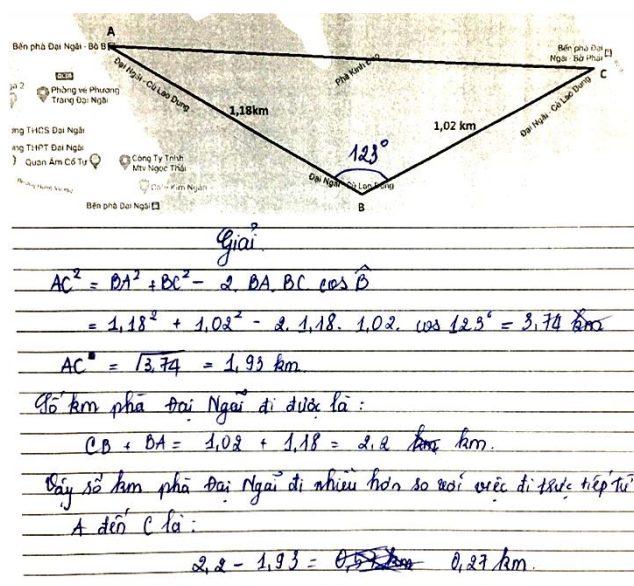


Figure 6. Student's work TN40 – Good work

According to Figure 6, the student TN40 had realized the problem requires finding the length of one side when knowing the lengths of the other two sides and the angle between those two sides of the triangle, then compared

the length of the side with the sum of the lengths of the two sides. In this solution, TN40 had well applied the cosine formula and calculated it accurately to satisfy the requirements of the problem.

Phụ Đại Ngã đã phải đi nhiều hơn số km so với việc đi trực
 tiếp từ A đến C là:
 Ta có ΔABC có:
 $AB = 1,8 \text{ km}$
 $BC = 1,02 \text{ km}$
 Theo định lý Py-ta-go
 $AC^2 = AB^2 + BC^2$
 $AC^2 = 1,8^2 + 1,02^2$
 $AC = 2,4 \text{ km}$

Figure 7. Student's homework DC09 – Bad work

Figure 7 indicates that the student DC09's work made the mistake of using the Pythagorean theorem in a non-square triangle, leading to incorrect results. In the control class, many students made mistakes similar to DC09, through which it can be seen that many students did not understand the requirements of the given question, nor could they apply the formulas they had learned to some mathematical problems in real life. Thus, in teaching, teachers need to help students understand the rational use of formulas in different problems, especially real-world problems, to meet the needs of students and modern education and see the role of mathematics in some real-life situations.

Item 3. (2 points) Calculate the slope of the hill in the figure given below.

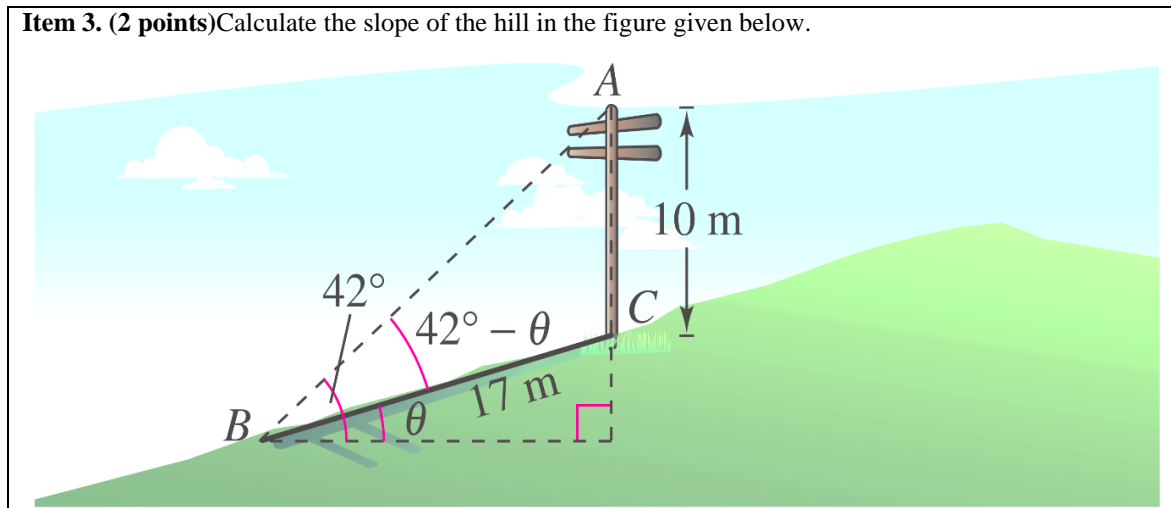


Table 14. Analysis of test results in Item 3

Classification	Weak 0.0 – 0.5	Medium 0.6 – 1.0	Good 1.1 – 1.5	Very good 1.6 – 2.0
Experimental class	32 74.4%	8 18.6%	3 7%	0 0%
Control class	30 78.9%	8 21.1%	0 0%	0 0%

Table 14 indicates that in Item 3 (a question at a high level of application), the problem asked to find the measure of an angle, and students needed to infer and apply formulas to the calculation accordingly. Specifically, the experimental and control class students did not achieve the maximum score; only a few students in the experimental class achieved a good score, accounting for 7%. Most of the students in the two classes had weak points, and the average score (accounting for more than 70%) shows that the difference between the two groups in this question was not much. Nevertheless, all students scored at weak and average levels; there were no students with good and good scores; in the experimental class, only three students got good scores (accounting for 7%), and no students got good grades which gets good grades. Although there was a difference between the two classes, in general, most students of both classes still did not know the math form, and the application of formulas to the problem still has many errors.

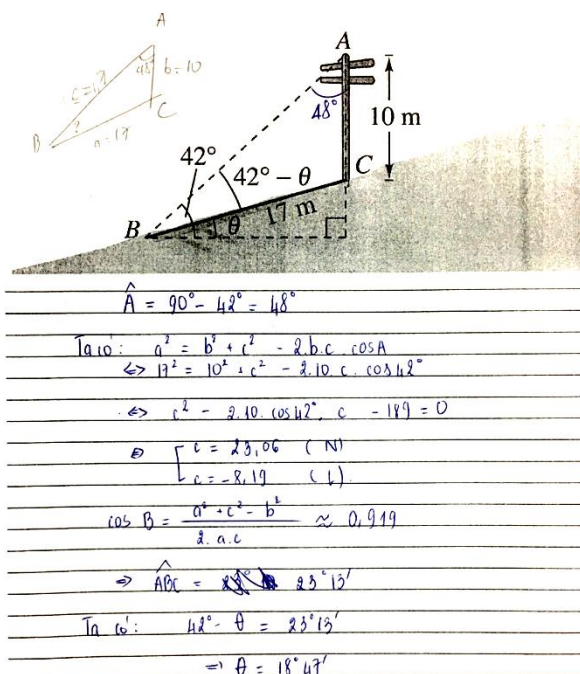


Figure 8. Student's homework of TN43 – The work was not good

The above worksheet from Figure 8 clearly shows that TN43 students had formed a good idea to resolve the requirements of the problem posed. TN43 found the measure of angle A and applied the cosine theorem to find the length of side AB, but this student made a mistake in changing numbers and calculations, leading to the results of the remaining parts being wrong even though the all steps were reasonable. Through the work of TN43 students, teachers need to emphasize that the calculation must be careful and accurate in the process of doing the test to get the best results. Besides, in this question, a few students of the experimental and control classes had the idea of finding the length of side AB by applying the sine theorem to triangle ABC, but the calculation was not good. Most students of both classes could calculate angle A; specifically, the work of TN19 students was presented in Figure 9.

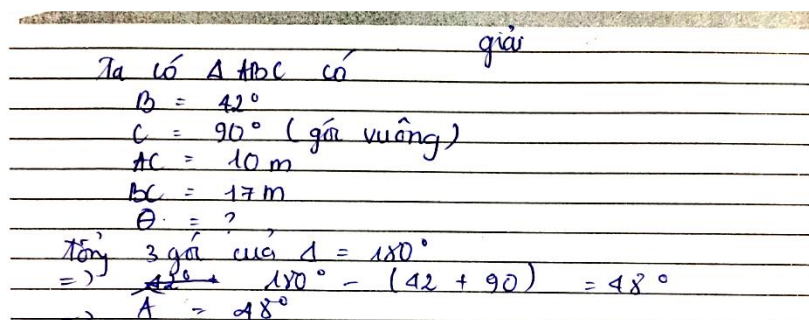


Figure 9. Student's homework of TN19 – Bad work

Students of TN19 and most of the rest of the two classes could calculate this step and stop here; only a few students continued to apply formulas such as the sine theorem and the cosine theorem to answer the question posed. This question shows that there was not much difference in the experimental and control classes when the question at a high level of application required students to reason to do the test.

Survey results of students' opinions after the experiment

The Cronbach's Alpha test in SPSS 20 was used to determine the reliability of the student attitude scale on practical lessons in this research. The obtained results are that the observed variables (7 questions) all had correlation coefficients – the appropriate sum (not less than 0.3) and Cronbach's Alpha coefficient is equal to 0.834 (greater than 0.7), so the scale met the requirements of reliability. After the experiment in class 10A4 and giving students a test, the researchers surveyed to get students' opinions to assess students' attitudes towards practical lessons. The tables and graphs below present all the data we obtained after surveying 43 experimental class 10A4.

Table 15. Student survey results of Item 1

Item 1. I like the experimental lessons on the relationships within a triangle and the solution of triangles.					
Level	Completely	Disagree	Neutral	Agree	Totally agree

	disagree				
f	0	0	4	11	28
%	0%	0%	9.3%	25.6%	65.1%

According to Table 15, in Item 1, 100% of the class's students answered fully; specifically, 28 students (accounting for 65.1%) completely agreed that they liked learning experimental lessons on the relationships within a triangle and the solution of triangle problems. Eleven students agreed (25.6%) in the triangle, and four students (9.3%) chose the normal answer; no student disagreed or completely disagreed with the above question. From the data presented above, when studying the lesson on triangle relationships and the solution of triangles utilizing the PBL method in the class, the class students were particularly intrigued by the experimental lessons in class.

Table 16. Student survey results of Item 2

Item 2. I find that the process of organizing activities in these lessons helps me study more effectively.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	0	2	18	23
%	0%	0%	4.7%	41.9%	53.5%

Based on Table 16, in Item 2, most students of the class completely agreed and agreed that the process of organizing the activities of the experimental class helped them learn more effectively (accounting for 53.5% and 41.9%) and besides two students chose the normal answer to the question (accounting for 4.7%). None of the students chose the answer that they disagreed or completely disagreed with. Based on the above data, it can be demonstrated that students can adapt to a new teaching process according to the PBL method compared to the normal teaching process learned in class; this also contributes to motivating teachers when applying other teaching methods in instructing.

Table 17. Student survey results of Item 3

Item 3. I find that approaching new knowledge based on the problem to be solved helps me to absorb knowledge faster and easier.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	0	5	12	26
%	0%	0%	11.6%	27.9%	60.5%

According to Table 17, it is observed that the methodological approach by problem-based teaching helped students acquire knowledge faster and easier and obtain the desired results. Specifically, 26 students (accounting for 60.5%) completely agree with the statement given, 12 students (accounting for 27.9%) agreed, and five students (accounting for 11.6%) gave the opinion that it was neutral. In this question, no student disagreed or completely disagreed with the given statement. Thus, through collecting opinions from students, research shows that problem-based teaching was one of the methods that could help students acquire knowledge quickly and easily.

Table 18. Student survey results of Item 4

Item 4. I find that problem-based learning helps me relate what I learn in class to real-life problems.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	1	4	11	27
%	0%	2.3%	9.3%	25.6%	62.8%

It can be observed from Table 18; there were 27 students (accounting for 62.8%) who completely agreed that problem-based learning helped students relate the knowledge learned in class with real problems, and 11 students (accounting for 25.6%) agreed with the recognition specified above. Besides, in this question, four students (9.3%) chose the normal level, and one student (2.3%) disagreed with the statement that problem-based learning helped to relate knowledge awareness of real-world problems. Thus, in instructing by this method, teachers need to pay attention to the appropriate use of real problems to see the role of mathematics in solving some problems of life, the relationship between theory and practice.

Table 19. Student survey results of Item 5

Item 5. I find myself making progress in the calculation, solving problems similar to those approached before.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	1	7	16	19
%	0%	2.3%	16.3%	37.2%	44.2%

Table 19 revealed that item 5 was interested in whether the students themselves could see the improvement in the calculation, handle problems similar to those approached before, and get the results. There were 19 students (accounting for 44.2%) out of 43 students who completely agreed with this, 16 students (accounting for 37.2%)

agreed, seven students (16.3%) had no opinion, and one student (accounting for 2.3%) disagreed with this comment. Thus, most students in the class noticed the progress when learning with the problem-based teaching method, and besides, there were still some students who did not notice the progress when learning with this method.

Table 20. Student survey results of Item 6

Item 6. I find that exchanging, working in groups, learning together and solving problems help me take the initiative and stimulate my interest in learning.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	0	3	11	29
%	0%	0%	7%	25.6%	67.4%

It is reported from Table 20 that in this item, 29 students (accounting for 67.4%) completely agreed that exchanging, working in groups, learning together, and solving problems help them take the initiative and stimulate their interest in learning practice. Besides, 11 students (accounting for 25.6%) agreed with the above statement; however, there were still three students (accounting for 7%) giving normal opinions. In general, most students in the class felt more interested in learning when they were active and working in groups instead of working individually. Through the results of this question, teachers need to strengthen the organization of exchange and group work activities to be proactive and stimulate interest in learning.

Table 21. Student survey results of Item 7

Item 7. I look forward to taking analogous classes in other lessons.					
Level	Completely disagree	Disagree	Neutral	Agree	Totally agree
f	0	1	2	7	33
%	0%	2.3%	4.7%	16.3%	76.7%

Data in Table 21 show that in this last item, the students were asked whether they would like to take similar lessons in other lessons instead of just learning the relationships within a triangle and solving triangle problems and getting the results. The results are as follows: There were 33 students (accounting for 76.7%) who completely agreed, seven students (accounting for 16.3%) agreed, two students (accounting for 4.7%) were neutral, and one student (accounting for 2.3%) disagree. As a result, most students expressed a desire to learn additional lessons using problem-based teaching methods applied to other subjects.

Conclusion

After going through the experimental process, the researchers obtained results that were nearly identical to those predicted. In terms of knowledge, the two classes before the experiment had the same level of proficiency, and after the experiment, there was a difference; the mean score of the experimental class was higher than that of the control class, which partly shows the effectiveness. The results of the PBL method in knowledge formation and skill training of students in applying knowledge when teaching the relationships within a triangle and the solution of triangles. Research by the authors Roh, 2003; Eric 2010; Dolmans et al., 2016; Surya and Syaputra, 2017; Hendriana, Johanto and Sumarmo, 2018; Thao, 2019; also confirms related results. In addition, the study obtained students' opinions on practical lessons; most of the questions received positive feedback from students. The findings reveal that most students believe that group activities help them progress in their calculation and problem-solving skills. Besides, group activities and approaches to real-life problems help students be more excited and active in learning and look forward to taking similar lessons in the future. This statement is also the opinion of the authors Westwood, 2011; Hendriana, Johanto and Sumarmo, 2018; Jatisunda et al., 2020; about the effectiveness of PBL on students' learning attitudes.

Thus, the results obtained after the experimental process show the feasibility of applying the problem-based teaching method to teaching the interactions that exist within a triangle and the solution of triangles. The results also show students' active learning when participating in group work and learning new knowledge when studying according to the proposed PBL. Thus, the study has verified the correctness of the original research hypothesis that "If teachers apply the problem-based teaching method well in teaching the relationships within a triangle and the solution of triangles, – Geometry 10 can help students be active in learning, self-forming knowledge and applying knowledge to resolve practical situations related to this topic.

During the experiment, the research also recognized the advantages and disadvantages of applying the PBL method in teaching. Advantages include, first and foremost, that students have a fundamental understanding of the subject matter being taught. In addition, students learn new knowledge with a step-by-step process of PBL that helps students retain knowledge more deeply (Dolmans et al., 2016; Jatisunda et al., 2020; Kardoyo et al., 2019; Lu, Bridges & Hmelo-Silver, 2014; Roh, 2003; Surya & Syaputra, 2017). This effect is because the students' knowledge is formed with the teacher's support; this is evident when the teacher asks the students to repeat the previously learned knowledge and the pos-test. In addition, when studying in a PBL environment, students have a positive attitude, interest in learning, enthusiastically speak and exchange ideas, and work together in groups (Hendriana, Johanto & Sumarmo, 2018; Jatisunda et al., 2020; Westwood, 2011). Students

can develop various skills due to this, including communication skills, teamwork skills, and problem-solving skills. (Dolmans et al., 2016; Eric 2010; Hendriana, Johanto & Sumarmo, 2018; Roh, 2003; Surya & Syaputra, 2017; Thao, 2011). In terms of difficulty, it can be observed that students only grasp basic knowledge; most of them still cannot apply knowledge to problems with a high level of application. Nonetheless, because the teaching process is different from the norm, there are still some students who have not adjusted in time (de Graaff & Kolmos, 2003; Ronis, 2008), thus making it time-consuming and difficult for teachers to manage the class because most activities are based on group work (Ingram, 2018; MacMath, Wallace & Chi, 2009), and leading to disorder and affecting neighboring classes (de Graaff & Kolmos, 2003; Ronis, 2008) (de Graaff & Kolmos, 2003; Ronis, 2008). In terms of knowledge and skills, students still make many mistakes in solving problems with practical content (clearly shown in sentences 2 and 3 of the post-test), which is the knowledge of breadth (de Graaff & Kolmos, 2003).

In addition to the obtained results, the research still has certain limitations, such as the short experiment taking place within four hours and requiring students to be exposed to a new teaching method different from other teaching methods. Since methods are frequently taught in class, some students cannot adapt to the activities taking place in the classroom. Research has also revealed that the degree of impact is extremely significant, but the results of the experiments have not been conclusive; there are still some students who have not achieved good results as well as this method has not yet affected stimulating students' active learning.

The study results have revealed that using the problem-based teaching method to teach the relationships within a triangle and the solution of triangles is both effective and efficient, despite the limitations and difficulties associated with using the method. Based on the results obtained, teachers can apply the PBL method to teaching students' knowledge.

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