Beam and User Selection Technique in Millimeter Wave Communications

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Abstract—**Millimeter wave (mmWave) communication is a promising technology to fulfill the requirements of future wireless networks. It provides very large spectrum and a large number of antennas can be practicable due to the small wavelength to exploit the array gain. However, there are several challenges, restricting the utilization of mmWave, such as hardware complexity and power consumption. To overcome these challenges, hybrid analog/digital architecture providing lower dimensional beamspace multiple input multiple output (MIMO) system is used. For the hybrid architecture, beam selection techniques exploiting the sparse nature of the mmWave channel become significant. In this paper, we consider a downlink mmWave communication when the large number of antenna is utilized at the base station. For that system, we propose a beam selection and a correlation based user selection algorithms to maximize the sum data rate.**

Index Terms—**millimeter wave, beam selection, beamspace MIMO, user selection**

I. INTRODUCTION

For the next generation wireless networks, various emerging technologies have been defined to respond the need of ever-increasing data traffic. From that viewpoint, millimeter wave (mmWave) communication has drawn great interest over the past few years thanks to its favourable opportunities. The mmWave spectrum from 30 GHz to 300 GHz has large available bandwidth providing a great enhancement in data rates. Moreover, mmWave has small wavelength enabling to fit hundreds of antennas into a small area. Especially for the massive number of antennas, mmWave communication can be the solution of space limitation issue.

On the other hand, there are various challenges about the mmWave communication and hardware implementation. The main issue is the high path loss to which the mmWave propagation is exposed. This issue restricts the range of communication to a few hundreds of meters. Another challenge about the propagation is the penetration loss related to the non-line of sight (NLoS) communication [1]. The challenges associated with the implementation can be enumerated as high power consumption and hardware complexity. Power consumption is an essential criterion in practice. Due to the high frequency and usage of large number of antennas to compensate the high path loss, the power consuming on the hardware components can be excessive. Furthermore, the complexity of circuitry increases when the large number of antennas are utilized. Therefore, a different approach to beamforming is required to reduce the hardware complexity and power consumption [2]. At that

point, the hybrid analog/digital beamforming technique can be qualified as a key solution. With this technique, analog and digital beamformers are employed jointly to exploit the benefits of both when the number of antennas are high. Although the spectral efficiency obtained by using digital beamforming cannot be achieved by hybrid beamforming, it offers the suboptimal solution with less power requirement. In a hybrid transmitter, all beams can not be transmitted simultaneously due to the limited number of radio frequency (RF) chains [3]. Hence, the beams to be transmitted are selected depending on a specific criterion, and the selected beams are digitally precoded before the transmission. Herein, the beam selection method plays a decisive role in the system performance.

For the mmWave MIMO system discussed in this study, several beam selection methods are available in the literature and different selection criterion are handled such as the magnitude maximization (MM) [4], the signal to interference plus noise ratio (SINR) maximization [5], the interference aware (IA) beam selection [6], the iterative beam selection [7], the ant colony optimization (ACO) based beam selection algorithms [8]. The MM algorithm in [4] is the simplest way to realize the beam selection in beamspace MIMO systems. Although it provides a low complexity approach, it neglects the multi-user interference which considerably restricts the system performance. When the users are nearly located, the channel will be highly correlated in the mmWave propagation. Therefore, the sparsity mask assigns the same beam for the users whose channels are similar. It is presented in [6] that the probability of assigning the same dominant beam to more than one user is extremely high especially when the large number of antennas exist at the BS. Hence, the users are exposed to a serious multi-user interference. Additionally, assigning the same beam results in a mis-use of RF chains. According to the channel condition, the number of active RF chains in the system is altered, which is undesirable. On the other hand, the IA beam selection in [6] purposes to solve the multi-user interference problem that comes to exist in the MM algorithm and [5] presents two algorithms which are iteratively eliminates the beams by maximizing the capacity and the SINR. However, the algorithms in [4]–[8] address the beam selection issue in a sparse system having equal number of users and RF chains. Therefore, the system can serve all users by selecting the beams in an appropriate way. On the other hand,

a dense system having higher number of users than RF chains makes the user selection inevitable in addition to the beam selection. Hence, the system performance can be maximized by utilizing the beam and the user selection together.

This paper proposes a correlation based user selection and beam selection algorithm for the downlink mmWave system containing massive number of antennas and large number of users.

II. SYSTEM MODEL

In this paper, a downlink communication system which contains a BS and multiple users is dealt with. At the BS, a discrete lens array (DLA) revealing the concept of beamspace multiple-input multiple-output (MIMO) is utilized. In order to model the DLA, it can be possible to use a uniform linear array (ULA) which is composed of identical antenna elements. The number of antenna elements in the array is denoted by N_T. The spacing between each element is described as the half of the carrier wavelength, which corresponds to critically sampled ULA. In addition, a linear precoder is employed at the digital part of the BS and there are N_{RF} RF chains.

At the receiver side, the total number of users is K in which each has a single antenna. The system meets the condition that $N_T \gg K > N_{RF}$. However, the BS can serve as many users as the number of RF chains, simultaneously. For this reason, the system requires a user selection algorithm in addition to a beam selection. The received signal for the kth user is given as:

$$r_k = \mathbf{h}_k^H \mathbf{x} + n_k \qquad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{N_T \times 1}$ is the channel vector and $n_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN) for the kth user, $\mathbf{x} \in \mathbb{C}^{N_T \times 1}$ is the transmitted signal vector defined by $\mathbf{x} = [x_1, x_2, \ldots, x_{N_T}]^T$. By gathering the received signals for all the K users, the received signal vector $\mathbf{r} = [r_1, r_2, \ldots, r_K]^T$ reveals the system equation in spatial domain as:

$$\mathbf{r} = \mathbf{H}^H \mathbf{x} + \mathbf{n} \qquad (2)$$

where $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N_T \times K}$ is the channel matrix specifying the system and $\mathbf{n} \in \mathbb{C}^{K \times 1}$ represents the AWGN vector with $\mathbf{n} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_K)$ where $\mathbf{I}_K$ is the $K \times K$ identity matrix.

After the linear precoder at the BS is provided, the system equation is described as follows:

$$\mathbf{r} = \mathbf{H}^H \mathbf{P} \mathbf{s} + \mathbf{n} \qquad (3)$$

where $\mathbf{P} \in \mathbb{C}^{N_T \times K}$ is the digital precoding matrix, $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the symbol vector and its correlation matrix satisfies that $\mathbf{\Lambda}_s = \mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$. In other words, the transmitted symbols for all the K users are independent from each other and they have unit energy. Furthermore, the constraint related to the total transmit power ρ is identified by:

$$\mathbb{E}[\|\mathbf{x}^2\|] = \text{tr}(\mathbf{P}\mathbf{\Lambda}_s\mathbf{P}^H) = \text{tr}(\mathbf{P}\mathbf{P}^H) \leq \rho \qquad (4)$$

where the transmitted signal $\mathbf{x} = \mathbf{P}\mathbf{s}$, and $\text{tr}(.)$ and $\mathbb{E}[.]$ denotes the trace and expectation operations, respectively.

A. Channel Model

When an N_T-element uniform linear array (ULA) is considered, the array steering vector, which is an N_T dimensional column vector, can be described as follows:

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{N_T}} \left[e^{-j2\pi\theta m} \right]_{m \in \mathcal{Z}(N_T)} \qquad (5)$$

where $\mathcal{Z}(N_T) = \{ n - (N_T - 1)/2 : n = 0, 1, \ldots, (N_T - 1) \}$ is a set which contains the indices of the antenna elements at the BS and they are symmetrically located around zero. The spatial angle θ is defined by:

$$\theta = \left(\frac{d}{\lambda} \right) \sin(\vartheta), \quad d = \lambda/2 \qquad (6)$$

where $\vartheta \in [-\pi/2, \pi/2]$ is the physical angle, λ is the wavelength of propagation and d is the antenna spacing which satisfies the critical spacing. Therefore, the spatial angle is $\theta = 0.5\sin(\vartheta)$, and it is an element of the range $[-0.5, 0.5]$.

Considering a mmWave system with multiple transmit antennas, the channel vector [4] associated with the kth user is described as:

$$\mathbf{h}_k = \beta_k^{(0)} \mathbf{a}_T \left(\theta_{T,k}^{(0)} \right) + \sum_{p=1}^{N_p} \beta_k^{(p)} \mathbf{a}_T \left(\theta_{T,k}^{(p)} \right) \qquad (7)$$

where $\beta_k^{(0)}$ and $\theta_{T,k}^{(0)}$ represents the channel gain and the angle of departure of the LoS path for the kth user, respectively. It is assumed that each user receives a LoS path $|\beta_k^{(0)}| \neq 0, \forall k$.

B. Beamspace System Representation

Beamspace system is a virtual representation of the traditional MIMO channel. In the MIMO architecture, DLA at the transmitter realizes this transformation from spatial domain to beamspace domain via the beamforming matrix.

The beamforming matrix $\mathbf{U} \in \mathbb{C}^{N_T \times N_T}$ whose columns are formed by the array steering vectors is determined as:

$$\mathbf{U} = \left[\mathbf{a} \left(\theta_m = \frac{m}{N_T} \right) \right]_{m \in \mathcal{Z}(N_T)} \qquad (8)$$

where the specified directions θ_m are generated by dividing the whole space into N_T, evenly. Thus, the beamforming matrix provides N_T orthogonal beams. Furthermore, it is a unitary Discrete Fourier Transform (DFT) matrix satisfying $\mathbf{U}\mathbf{U}^H = \mathbf{U}^H\mathbf{U} = \mathbf{I}$. The beamspace channel vector for the kth user is identified as follows:

$$\mathbf{h}_{b,k} = \mathbf{U}^H \mathbf{h}_k \qquad (9)$$

where $\mathbf{h}_{b,k} \in \mathbb{C}^{N_T \times 1}, \forall k = 1, 2, \ldots, K$. By extension, the beamspace channel matrix can be written as:

$$\mathbf{H}_b = \mathbf{U}^H \mathbf{H} \qquad (10)$$

where $\mathbf{H}_b = [\mathbf{h}_{b,1}, \mathbf{h}_{b,2}, \ldots, \mathbf{h}_{b,K}] \in \mathbb{C}^{N_T \times K}$.

Hence, the beamspace system which is an equivalent representation of (3) due to the unitary nature of $\mathbf{U}$, is defined as:

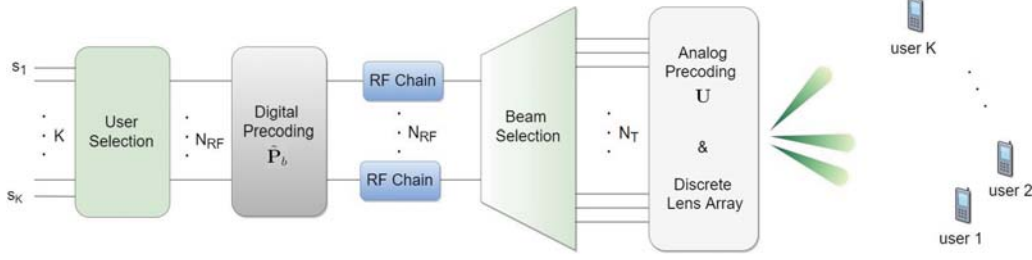$$\mathbf{r} = \mathbf{H}_b^H \mathbf{P}_b \mathbf{s} + \mathbf{n} \qquad (11)$$

Fig. 1. Transceiver architecture of the considered mmWave system

where $\mathbf{P}_b = \mathbf{U}^H\mathbf{P}$ is the beamspace precoder. It is worth mentioning that the beamspace channel has a sparse nature. Sparsity states that the channel has only a few number of non-zero coefficients. In other words, there are slight number of multipath components which are weaker than the LoS component by the nature of mmWave communication.

C. Digital Precoding

In this section, zero forcing (ZF), matched filter (MF) and QR decomposition based digital precoders are given. For the system equation in (3), the precoder matrix $\mathbf{P}$ can be described by:

$$\mathbf{P} = \alpha\mathbf{W} \tag{12}$$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_K] \in \mathbb{C}^{N_T \times K}$ is the unscaled precoder matrix and α is the power scaling factor guaranteeing the condition given in (4) is satisfied. The scaling factor is then identified as [4]:

$$\alpha = \sqrt{\frac{\rho}{tr(\mathbf{W}\mathbf{\Lambda}_s\mathbf{W}^H)}} \tag{13}$$

For the general expressions given in (12) and (13), we can describe the unscaled precoder matrices for ZF, MF and QR precoders. When ZF precoder which ensures that all users have null interference including inter-user interferences is utilized, the Moore-Penrose inverse of the channel is realized to construct the precoder matrix. Therefore, the unscaled precoder matrix is indicated by the following:

$$\mathbf{W}_{ZF} = \mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1} \tag{14}$$

When MF precoder is provided, the unscaled precoder matrix which is matched with the channel matrix is defined as:

$$\mathbf{W}_{MF} = \mathbf{H} \tag{15}$$

Apart from those precoders, QR precoder presented in [9] exploits QR decomposition of the channel such that $\mathbf{H}^H = \mathbf{R}^H\mathbf{Q}^H$ where $\mathbf{Q}$ and $\mathbf{R}$ are unitary and upper triangular matrices, respectively. Then, the unscaled precoder matrix is identified by:

$$\mathbf{W}_{QR} = \mathbf{Q}\mathbf{L}^{-1}\mathbf{L}_D \tag{16}$$

where $\mathbf{L} = \mathbf{R}^H$ and diagonal matrix $\mathbf{L}_D$ is generated with the diagonal elements of $\mathbf{L}$. The precoder provides an interference free system since $\mathbf{L}_D$ is a diagonal matrix. However, it can be utilized only for square channel matrices due to the diagonalization and inverse operations.

III. PROPOSED ALGORITHM

For the proposed system given in Fig. 1, it is required to apply both user and beam selection. Firstly, a correlation based user selection algorithm is performed. Most correlated users are eliminated to mitigate the inter-user interference. Therefore, the correlation between i^{th} and j^{th} user is calculated by:

$$c(i,j) = \frac{|\mathbf{h}_{b,i}^H\mathbf{h}_{b,j}|}{\|\mathbf{h}_{b,i}\|\|\mathbf{h}_{b,j}\|} \tag{17}$$

where $i = 1, \ldots, K$ and $j = 1, \ldots, K$ while $i \neq j$.

The user pairs that has the correlation greater than the specified threshold c_{th} are determined, namely the users i and j are determined such that:

$$c(i,j) > c_{th} \tag{18}$$

Among the user pairs, the user that has the lower channel gain is eliminated, namely $\mathbf{h}_{b,i}$ is removed from $\mathbf{H}_b$ matrix such that:

$$\|\mathbf{h}_{b,i}\| < \|\mathbf{h}_{b,j}\| \tag{19}$$

In this manner, the algorithm can select different number of users depending on the threshold and the channel condition. If the set of selected users are denoted by $\mathcal{U}$, the resulting beamspace channel matrix $\hat{\mathbf{H}}_b$, which is less correlated, can be expressed by selecting the jth column of $\mathbf{H}_b$:

$$\hat{\mathbf{H}}_b = \left[\mathbf{H}_b(:,j)\right]_{j\in\mathcal{U}} \tag{20}$$

where $\hat{\mathbf{H}}_b \in \mathbb{C}^{N_T \times p}$ and $p = |\mathcal{U}|$ is the total number of selected users where $p > N_{RF}$.

After the user selection is performed as shown in Fig. 2, a beam selection is applied to serve the selected users with their strongest beams. Beam selection makes a low complexity system available by utilizing the sparse nature of the beamspace channel. Thus, not only the hardware complexity and the dimension of the system are reduced but also no significant performance loss occurs. So, the reduced dimensional beamspace channel matrix $\tilde{\mathbf{H}}_b$, by selecting the ith row of $\hat{\mathbf{H}}_b$, is represented by:

$$\tilde{\mathbf{H}}_b = \left[\hat{\mathbf{H}}_b(i,:)\right]_{i\in\mathcal{S}} \tag{21}$$

where $\mathcal{S}$ is a set involving the indices of beams which are chosen to be transmitted. Therefore, the received signal with lower dimension is given as follows:

$$\mathbf{r} = \tilde{\mathbf{H}}_b^H\tilde{\mathbf{P}}_b\mathbf{s} + \mathbf{n} \tag{22}$$

where $\tilde{\mathbf{P}}_b \in \mathbb{C}^{\ell \times N_{\mathrm{RF}}}$ is the reduced dimensional precoder matrix which corresponds to $\tilde{\mathbf{H}}_b \in \mathbb{C}^{\ell \times N_{\mathrm{RF}}}$ and $\ell = |\mathcal{S}| = N_{\mathrm{RF}}$ where $\ell \leq K$. The precoder matrix $\tilde{\mathbf{P}}_b$ for ZF, MF and QR precoders is obtained by applying (14), (15) and the QR decomposition to $\tilde{\mathbf{H}}_b$ which is described in (21).

For the beam selection, the most dominant beam (or the 1st strongest beam) is determined for each selected users initially. This corresponds to finding 1-beam sparsity masks $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_p$ where $\mathcal{M}_p$ denotes the set containing the strongest beam for the pth user. Then, the 2nd strongest beams are also specified for each selected users. At that point, the algorithm controls whether the most dominant beams of two or more users coincide or not. If they do not, the algorithm assigns their most dominant beams for each selected user. In this case, there will be no problem associated with the multi-user interference. But if they coincide, which is likely to occur in the proposed system, the algorithm has to take the interference into consideration. Hence, the users are classified as interference users (IUs) and non-interference users (NIUs), and the set of user index for NIUs and IUs are denoted by $\mathcal{S}_{NIU}$ and $\mathcal{S}_{IU}$, respectively.
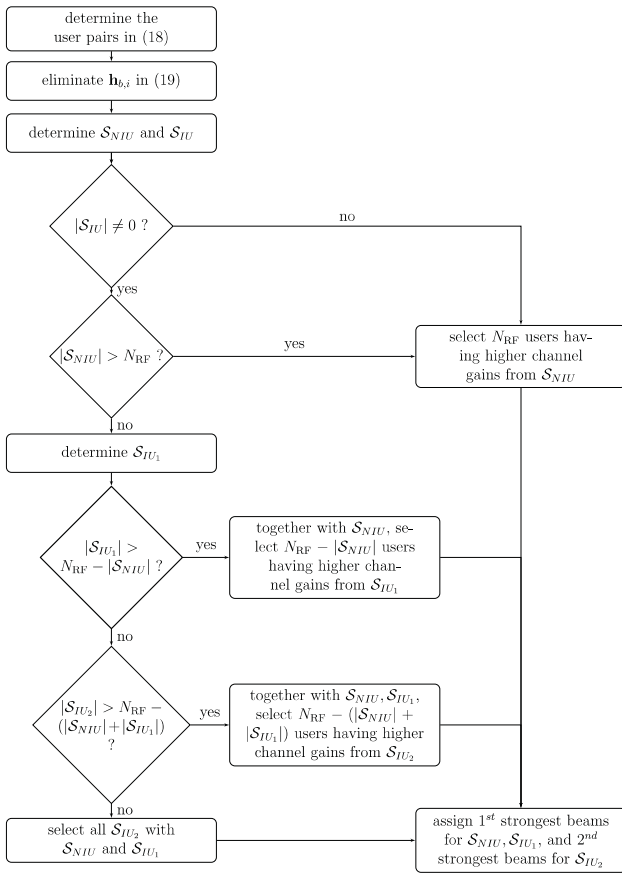

Fig. 2. Flowchart of the proposed user and beam selection algorithm

If the number of NIUs, $|\mathcal{S}_{NIU}|$ is greater than N_{RF}, the algorithm has to select N_{RF} users and the corresponding most dominant beams. In order to do that, channel gains of NIUs are considered, namely the users that have higher channel gains are selected until the total number of users to be served reaches N_{RF}. The selected users in NIUs are served by their most dominant beams. Therefore, that beams are selected and added to the set $\mathcal{S}$. On the other hand, if $|\mathcal{S}_{NIU}|$ is less than N_{RF}, it is required to add $N_{\mathrm{RF}} - |\mathcal{S}_{NIU}|$ users among IUs. Due to the fact that an IU shares the same strongest beams with another IUs, the algorithm must search the primarily selectable users among IUs. For that purpose, the users having the same 1st strongest beams, called as *beam partners* are found out. In other words, the set $\mathcal{S}_{IU}$ is separated to its subsets and each of the subsets is formed by beam partners. For each subset, the algorithm chooses one user whose 1st beamspace channel gain is the greatest one among its 1st beam partners. These users construct the set $\mathcal{S}_{IU_1}$ and these users will be served by their 1st strongest beams. Then, the algorithm controls whether $|\mathcal{S}_{IU_1}|$ is sufficient or not. If it is higher than $N_{\mathrm{RF}} - |\mathcal{S}_{NIU}|$, the algorithm chooses $N_{\mathrm{RF}} - |\mathcal{S}_{NIU}|$ users having higher channel gains among $\mathcal{S}_{IU_1}$. Also, their most dominant beams are selected and added to the set $\mathcal{S}$. If it is not sufficient, adding $N_{\mathrm{RF}} - (|\mathcal{S}_{NIU}| + |\mathcal{S}_{IU_1}|)$ users from the set of remaining users $\mathcal{S}_{IU_2}$ satisfying $\mathcal{S}_{IU_2} = \mathcal{S}_{IU} \setminus \mathcal{S}_{IU_1}$ is needed. If $|\mathcal{S}_{IU_2}|$ is higher than the required number of users, the users having higher channel gains among $\mathcal{S}_{IU_2}$ is selected and their 2nd strongest beams are assigned to these users. Totally, the algorithm selects N_{RF} beams out of N_{T} beams in order to serve N_{RF} users while $K - N_{\mathrm{RF}}$ users are out of service because of the high density of the environment.

IV. SIMULATION RESULTS

In this section, for different number of antennas and users, the sum data rate comparisons of three beamspace MIMO precoders are provided through the proposed and the MM algorithm. Considering the mmWave massive MIMO system having a carrier frequency $f_c = 28$ GHz, we set the number of transmit antennas $N_{\mathrm{T}} = 128$, firstly. Moreover, we consider a large number of users $K = 64$ which is higher than the number of RF chains $N_{\mathrm{RF}} = 32$. For the kth user, channel vector $\mathbf{h}_k$ which is specified in (7) has one LoS path with $\beta_k^{(0)} \sim \mathcal{CN}(0,1)$, two NLoS path with $\beta_k^{(p)} \sim \mathcal{CN}(0,0.1)$ and the spatial angles for LoS and NLoS paths with $\theta_{\mathrm{T},k}^{(0)}, \theta_{\mathrm{T},k}^{(p)} \sim \mathcal{U}(-0.5, 0.5)$ when $p = 1, 2$.

In Table I, the average number of selected users, K_u and the sum data rates for ZF, MF and QR precoders based on the correlation threshold, c_{th} are demonstrated. The performance evaluations show that the average number of selected users increases and the sum data rates for all precoders decreases when c_{th} is increased. In other words, as the number of eliminated users increases, the performance of the proposed algorithm is improved. For the comparison of the precoders, we can initially state that the MF precoder can not manage multi-user interference since it is interference limited. Therefore, it does not provide good sum data rate meaning that the residual interference restricts its performance. For ZF precoder, the table shows that it achieves better performance than MF precoder

$N_\text{T} = 128$, $K = 64$, $N_\text{RF} = 32$				
	Sum data rates (bps/Hz)			
Threshold, c_{th}	QR	ZF	MF	K_u
0.2	58.57	53.89	13.68	46.98
0.3	56.97	51.78	13.50	51.88
0.4	55.60	49.37	13.40	55.42
0.5	53.81	46.30	13.19	58.30

since ZF completely eliminates the interference. However, it degrades the performance while eliminating the interference. Furthermore, QR precoder provides the best sum data rate performance for all threshold values. Because it can cancel the interference without degrading the performance unlike ZF.

In Fig. 3, the sum data rate results are provided for different number of antennas while the number of users and RF chains are fixed and QR precoder is used. The proposed algorithm with 128 antennas gives almost the same performance of the MM algorithm with 256 antennas although it has less number of antennas. Because, the MM algorithm is more sensitive to the highly correlated user channels. Also, decreasing the number of antennas degrades the performance of the MM algorithm much more than the proposed algorithm because of the correlation issue. Therefore, the sum data rate of MM algorithm with 128 antennas is quite low.
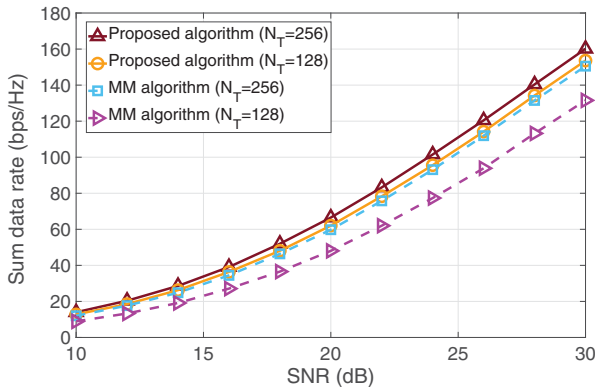

Fig. 3. Sum data rate results of the MM and the proposed algorithms for QR precoder with 64 users, 32 RF chains and $c_{th} = 0.2$.

In Fig. 4, the performance results are demonstrated for different number of users while the number of antennas and RF chains are fixed. The sum data rate is proportional to the number of users for both algorithms although the total number of users to be served is 32. Because, as the number of users is increased, the probability to select well-conditioned users increases. Therefore, the inter-user interference is decreased namely the sum data rate increases.

V. CONCLUSION

In this paper, we have proposed the algorithm which performs the beam and the user selection by taking into account
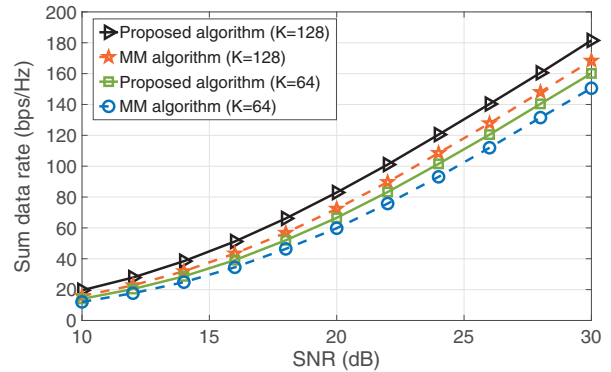

Fig. 4. Sum data rate results of the MM and the proposed algorithms for QR precoder with 256 antennas, 32 RF chains and $c_{th} = 0.2$.

highly correlated user channels, that is inherent in mmWave communication. The performance evaluation of the proposed algorithm for different types of precoder and threshold values has been given. Moreover, the sum data rate performance comparisons of the proposed algorithm against the MM algorithm for different number of antennas and users have been provided. The simulation results indicate that the proposed algorithm achieves better sum data rate performance.

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