Efficient transit light curves for oblate and rapidly rotating stars

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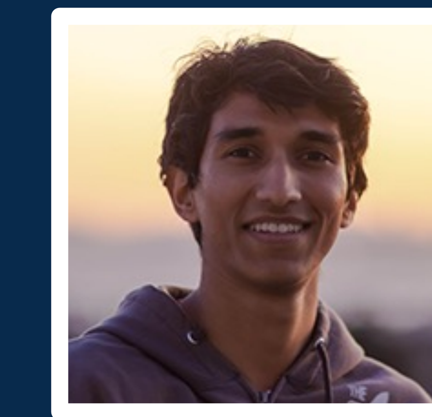
https://github.com/rodluger/starry



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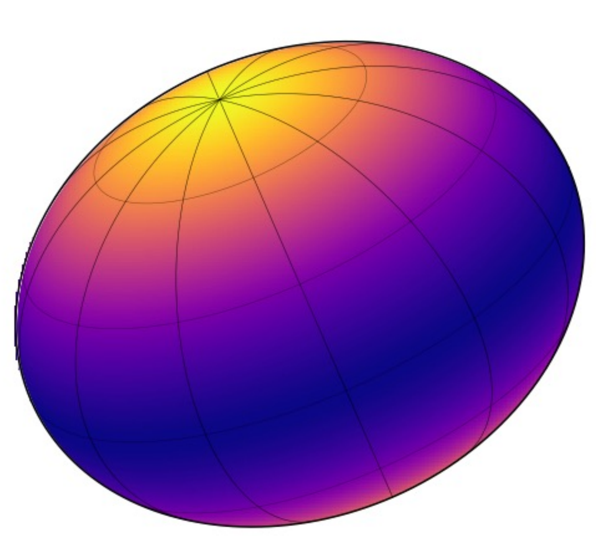
INTRODUCTION AND MOTIVATION

Observing certain effects of stellar rotation can serve as a protractor to measure the angle between the planet's orbit and the star's rotation axis. This **spin-orbit alignment** measurement is important to learn about the dynamical history of exoplanet systems. Spectroscopic methods are frequently used to measure spin-orbits but require the use of large ground-based telescopes and only provide the projected spin-orbit angle.

Rapidly rotating stars exhibit certain effects, including oblateness and gravity darkening, that make it possible to constrain the true spin-orbit angle using transit photometry alone. This makes spin-orbit measurements feasible with large photometric datasets such as TESS.



Left: A rapidly rotating star demonstrating oblateness and gravity darkening. Centrifugal force causes the equator to bulge outwards, distorting the star into an oblate spheroid. The density of material at the poles is greater than at the equator, causing a latitude-dependent temperature and luminosity variation called gravity darkening.

CHALLENGES

Using the rapid rotation of exoplanet hosts to measure spin-orbit angles is not a new technique but is difficult to model. The technique is usually **computationally intensive**, involving 2D surface integrals at every flux computation in the model. As a result, the technique has in the past been used mostly with least squares fitting on a handful of more obviously asymmetric transits.

Making the technique applicable to large datasets such as TESS <u>requires the use of a model that is both accurate and enables fast posterior inference</u>

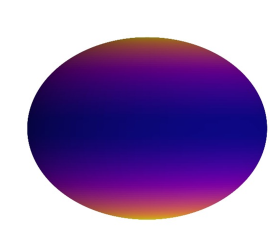
MODELING: GRAVITY DARKENING

Von Zeipel Law:

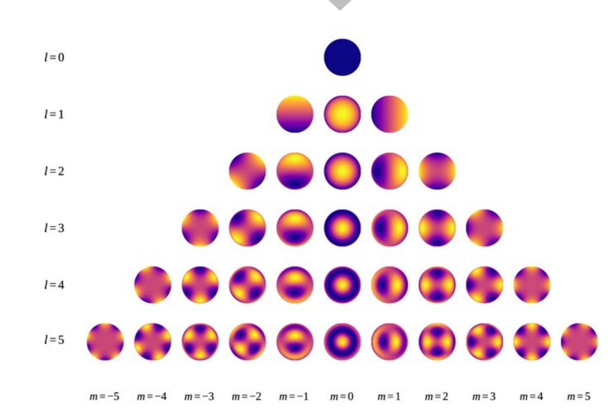
$$T = T_{pole}\frac{g^\beta}{g^\beta_{pole}}$$

Planck's Law:

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}.$$



Stellar surface map



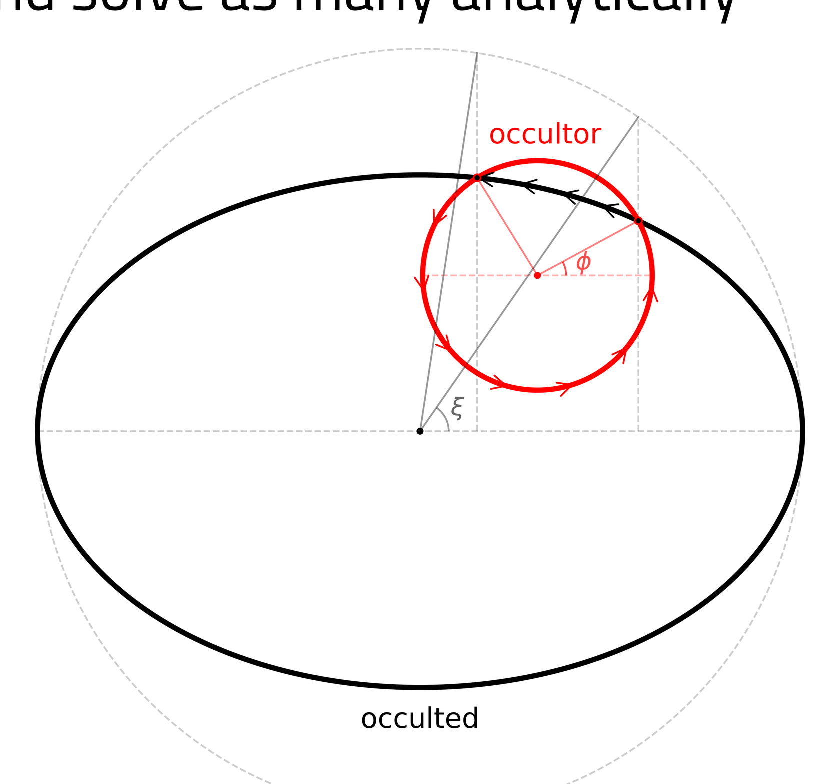
Spherical harmonic transform

We model gravity darkening starting with the Von Zeipel Law, which relates the local surface gravity of a star to the temperature. We plug this into the Planck function and expand it out in terms of **spherical harmonics** to represent the surface map of the star. This allows fast computation of gravity darkened transit light curves.

MODELING: OBLATENESS

To model oblateness, we must sum the flux blocked by the planet in transit. This is usually done by performing a 2D surface integral of the flux under the planet at every point in the transit. Using Green's theorem, we write this 2D surface integral as a **1D line integral** around a closed region. We classify the forms of this integral for different spherical harmonic terms and solve as many analytically as possible.

Right: The geometry of the oblateness problem showing the star in black and the planet in red. The boundary of the integral, shown with arrows, is parametrized with two angles ξ and ϕ and is always performed counterclockwise around a closed region.



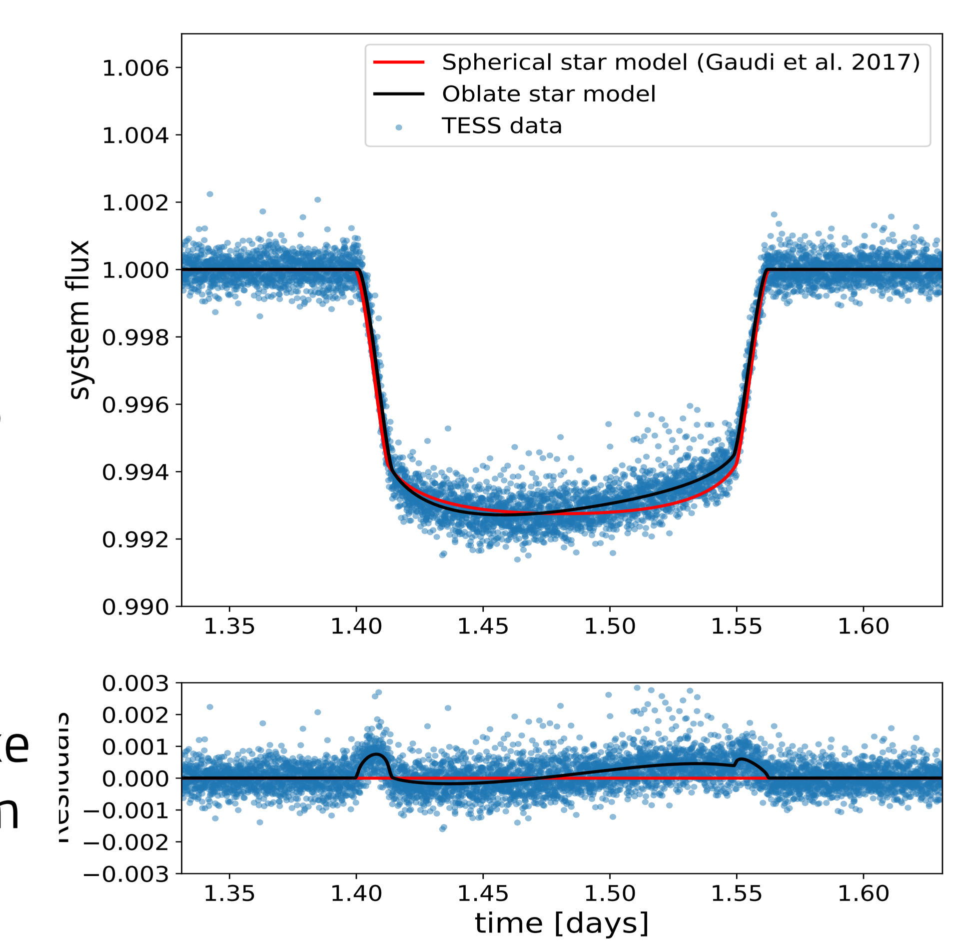
PERFORMANCE

We implement the models for gravity darkening and oblateness in C++ the package `starry`. Since `starry` is designed for surface maps with spherical harmonics and allows integration with pymc3, the model is ideal for posterior inference. We find the implementation to be **4 orders of magnitude faster** than an equivalent numerical method and much more precise, especially at higher rotation rates, than the Barnes (2009) transit model.

EARLY RESULTS AND NEXT STEPS

We test the new `starry` algorithm on an unbinned TESS light curve of KELT-9b, a system that shows visible asymmetry due to gravity darkening. While we are still working on reparametrizing the model for better speed, we find that a laptop computer is sufficient to run the posterior inference. We are currently working on **using the model on more such systems**, including those without perceptible asymmetry. The method is best used in tandem with Doppler tomography as a way of identifying the true spin orbit inclination and making stronger constraints.

Right: TESS light curve of KELT-9b with both the best fitting spherical and oblate, gravity darkened model. Notably, there are significant departures from the spherical model at ingress and egress, implying that oblateness alone can be used to make constraints on the system geometry.



<u>2000 exoplanets in TESS will orbit A/F stars, out of which a significant proportion will be rapidly rotating and amenable to constraints from this method.</u>