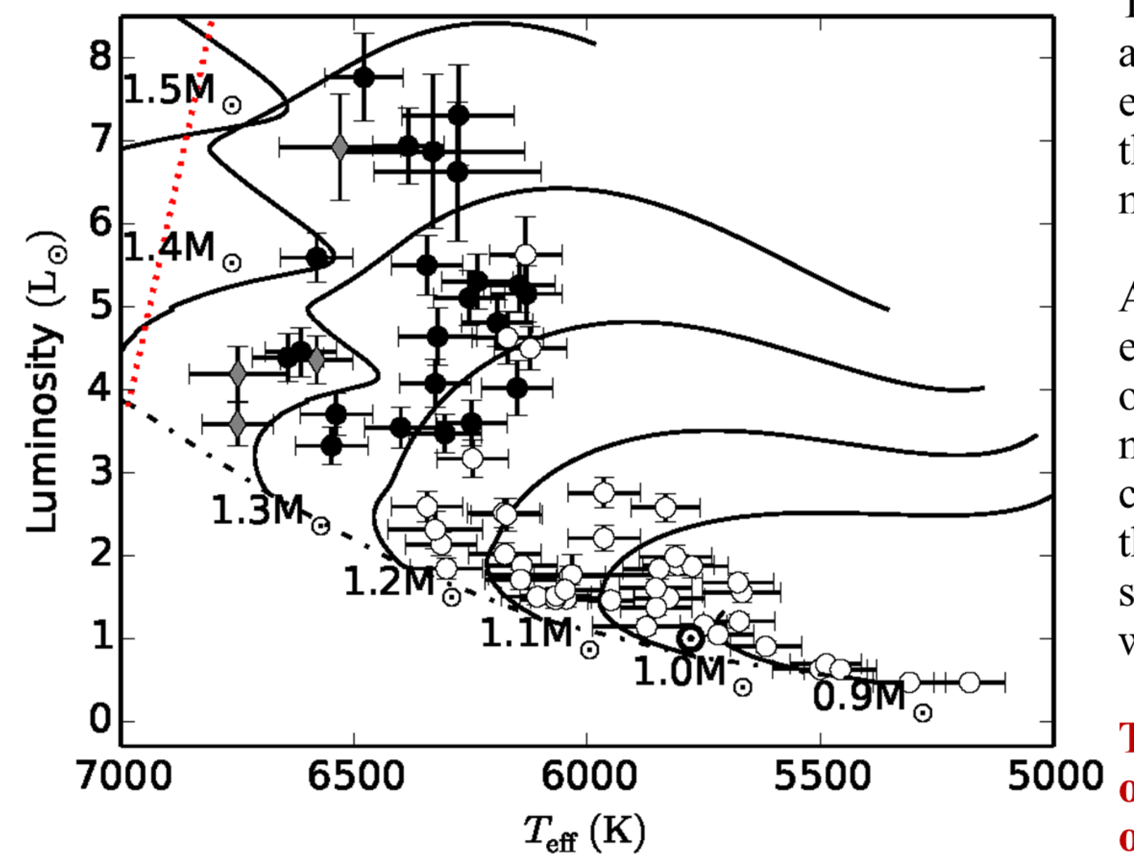
Probing Stellar Cores by Asteroseismic Inversions

Alexander Kosovichev (NJIT) and Irina Kitiashvili (NASA/AMES)

Summary

- Precision asteroseismology data from Kepler and TESS provide a unique opportunity to investigate the interior structure of stars at various stages of stellar evolution.
- Detection of mixed acoustic-gravity oscillation modes has opened perspectives for probing properties of energy-generating cores. Most of the previous analysis was focused on fitting standard evolutionary stellar models using mode frequency splitting and scaling laws for oscillation properties.
- We present results of direct asteroseismic inversions using the method of optimally localized averages (OLA), which effectively eliminates the surface effects and attempts to resolve the stellar core structure. The inversions are presented for various structure properties, including the sound speed and density.
- The results show that the mixed modes observed in F-type stars allow us to resolve the stellar core structure, and reveal significant deviations from the evolutionary models.

Kepler Asteroseismic Legacy Targets



Compton, D.L., Bedding, T.R., Stello, D., 2019, MNRAS, 485, 560-569
Silva Aguirre, V. et al., 2017, ApJ, 835, 173

The Kepler and TESS mission provide a wealth of stellar oscillation data enabling asteroseismic investigation of the internal structure and rotation of many stars across the HR diagram.

A primary tool of asteroseismology employed for interpretation of observed oscillation frequencies used a method of grids of stellar models. In combination with spectroscopic data, this approach provides estimates of stellar radius, composition, and age with unprecedented precision.

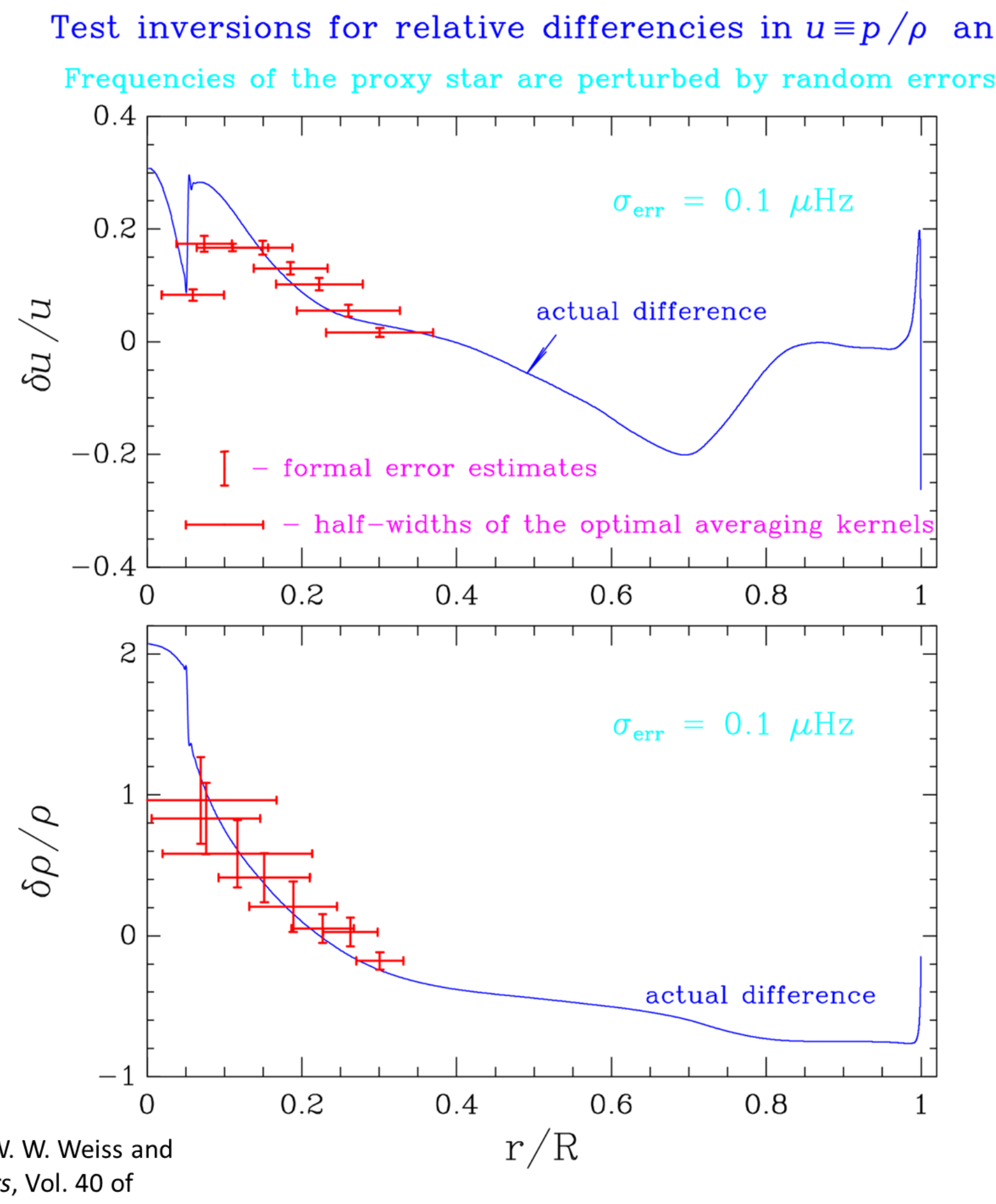
The high accuracy measurements of oscillation frequencies open a new opportunity for asteroseismic inversions which allow us to reconstruct the internal structure and test the evolutionary stellar models.

Test inversion

A specific feature of asteroseismology data is that only low-degree oscillations can be observed. An additional difficulty is caused by uncertainties in the mass and radius of stars.

Gough and Kosovichev (1993) showed that this difficulty can be overcome by an additional condition in the inversion procedure, which constraints a frequency scaling factor.

Test inversions showed that if the low-degree p-mode frequencies are measured with precision of about 1 μHz then the structure of the stellar core can reconstructed even when the stellar mass and radius are not known.



Gough, D. O. and Kosovichev, A. G.: 1993a, in W. W. Weiss and A. Baglin (eds.), *IAU Colloq. 137: Inside the Stars*, Vol. 40 of *Astronomical Society of the Pacific Conference Series*, p. 541

Asteroseismic Inversions

Detection of mixed modes in the oscillation spectra of F-type stars allows us to resolve the structure of the inner stellar cores. The mixed modes have properties of internal gravity waves (g-modes) in the convectively stable helium core and properties of acoustic modes outside the core. The oscillation frequencies of these modes may be quite sensitive to the properties of the core. Including them in the inversion procedure allows us to localize the averaging kernels in the core region.

Using explicit formulations for the variational principle, frequency perturbations can be reduced to a system of integral equations for a chosen pair of independent variables, e.g. for (ρ,γ)

$$\frac{\delta\omega^{(n,l)}}{\omega^{(n,l)}} = \int_0^R K_{\rho,\gamma}^{(n,l)} \frac{\delta\rho}{\rho} dr + \int_0^R K_{\gamma,\rho}^{(n,l)} \frac{\delta\gamma}{\gamma} dr,$$

where $K_{\rho,\gamma}^{(n,l)}(r)$ and $K_{\gamma,\rho}^{(n,l)}(r)$ are sensitivity (or 'seismic') kernels. These are calculated using the initial solar model parameters, ρ_0 , P_0 , γ , and the oscillation eigenfunctions for these model, $\vec{\xi}$.

Kosovichev, A.G. 1999, J. Comput. Appl. Math., Vol. 109, No. 1 - 2, p. 1 - 39

Taking into account uncertainties in mass and radius

The sensitivity for various pairs of solar parameters, such the sound speed, Brunt-Väisälä frequency, temperature, and chemical abundances, can be obtained by using the relations among these parameters, which follow from the equations of solar structure ('stellar evolution theory'). These 'secondary' kernels are then used for direct inversion of the various parameters, e.g. f=P/ρ and helium abundance Y.

It is important to take into account potential systematic uncertainties in the stellar mass and radius. Because the oscillation frequencies are scaled linearly with the factor $q = M/R^3$, then, following, the mode frequencies ω_i can be expressed in terms of their relative small difference $\delta\omega_i^2/\omega_i^2$ from those of a standard reference model of similar mass and radius according to the linearized expression

$$\delta\omega_i^2/\omega_i^2 = \int_0^1 \left(K_{f,Y}^i \frac{\delta f}{f} + K_{Y,f}^i \delta Y \right) dx - I_q^i \delta q,$$

where $x = r/R$, $q = M/R^3$, Y is the helium abundance, and f can be any function of p and ρ; M and R are stellar mass and radius, in solar units, $K_{f,Y}$ and $K_{Y,f}$ are appropriate kernels, and I_q is an integral over the reference model.

Optimally Localized Averaging Inversion

These constraints can provide localized averages of $\delta \ln f$ and estimates of Y and δq of the kind

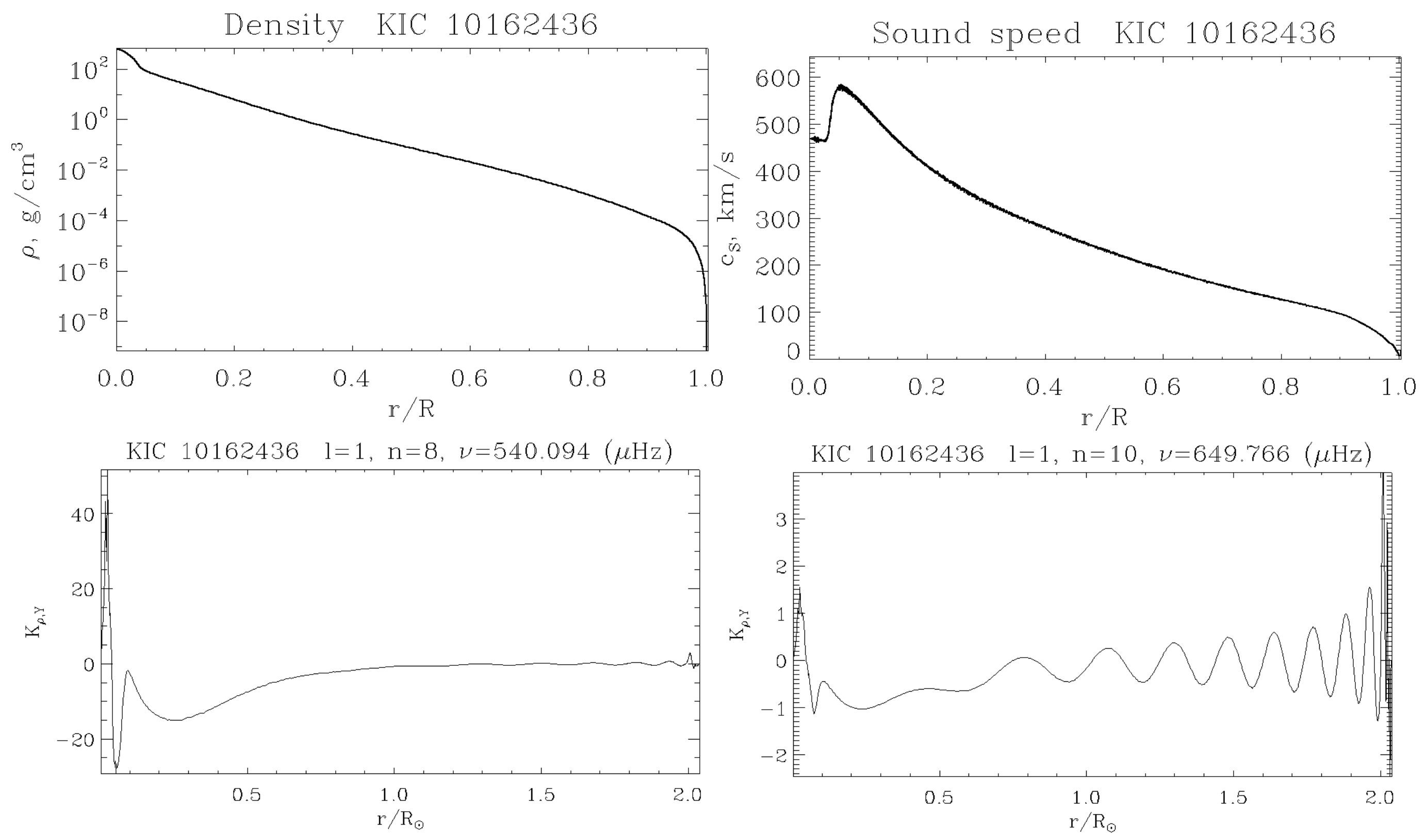
$$\overline{\delta \ln f} \equiv \int_0^1 \sum_i a_i(x_0) K_{f,Y}^i \delta \ln f \, dx \equiv \int_0^1 A_{f,Y}(x,x_0) \delta \ln f \, dx = \sum_i a_i(x_0) \frac{\delta\omega_i^2}{\omega_i^2}$$

by minimizing the functional amongst coefficients $a_i(x_0)$:

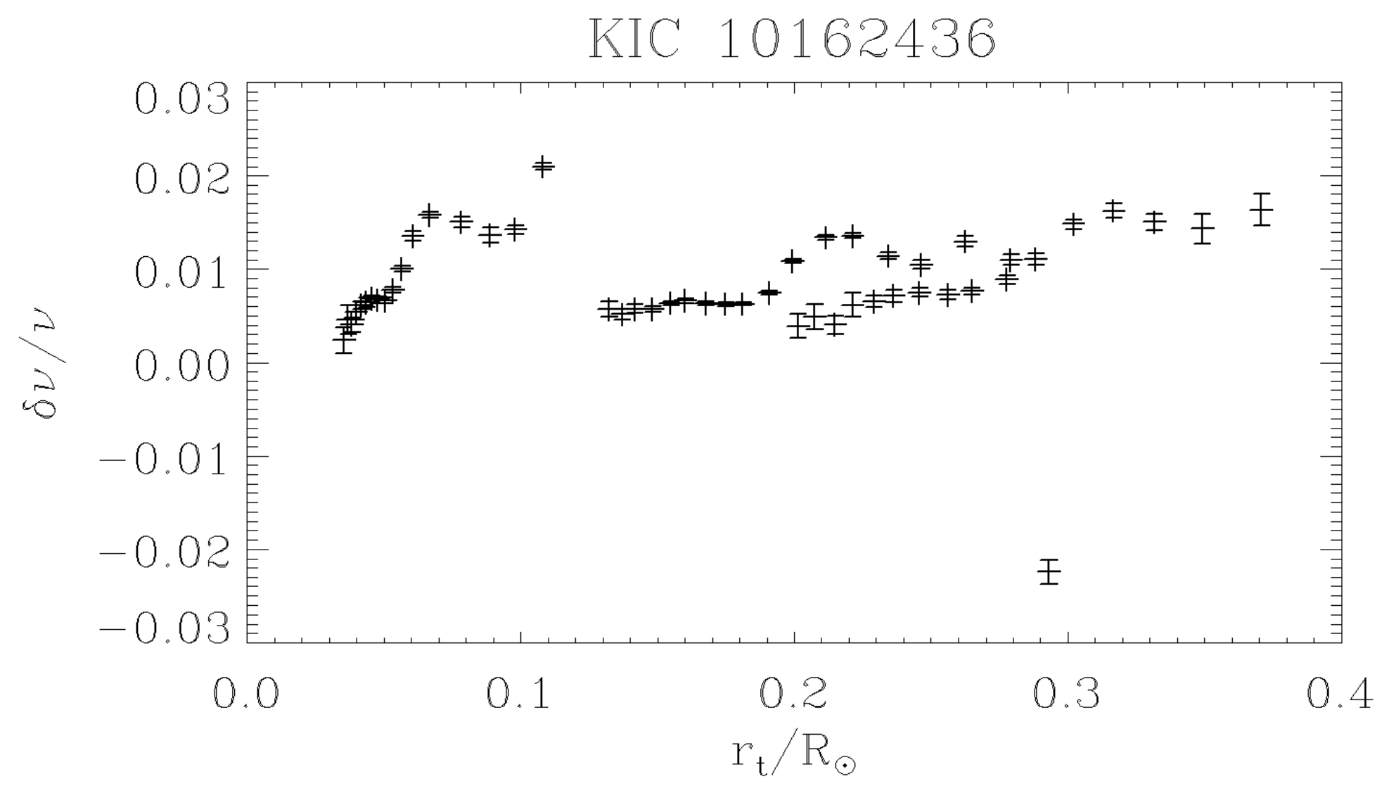
$$\int_0^1 A_{f,Y}^2(x,x_0) J_f dx + \lambda_1 \int_0^1 \left(\sum_i a_i K_{Y,f}^i \right)^2 J_Y dx + \lambda_2 \left(\sum_i a_i I_q^i \right)^2 + \alpha \sum_i a_i^2 \epsilon_i^2$$

for tradeoff parameters λ_1, λ_2 and α, where ϵ_i are standard relative errors in the data.
Appropriate weight functions J depend on the averages sought.
For determining δq, we set $J_f = 1, J_Y = 1, \lambda_1 \neq 0$ and $\lambda_2 = 0$;
for $\overline{\delta \ln f}(x_0)$ we set $J_f = (x-x_0)^2, J_Y = 1, \lambda_1, \lambda_2 \neq 0$;
for $\overline{\delta Y}$ we set $J_f = 1, J_Y = 1, \lambda_1, \lambda_2 \neq 0$.

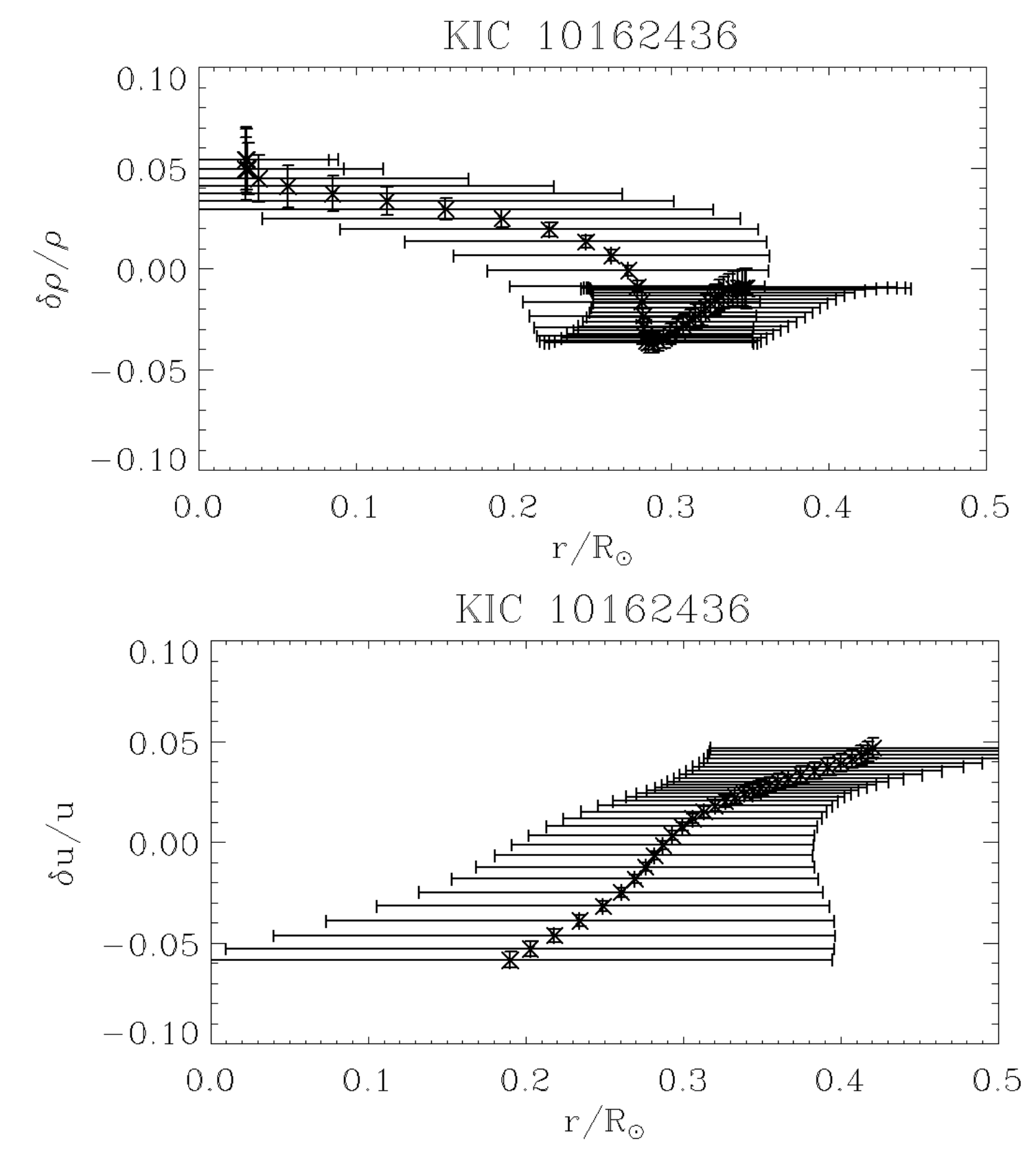
KIC 10162436: Evolutionary model and sensitivity kernels



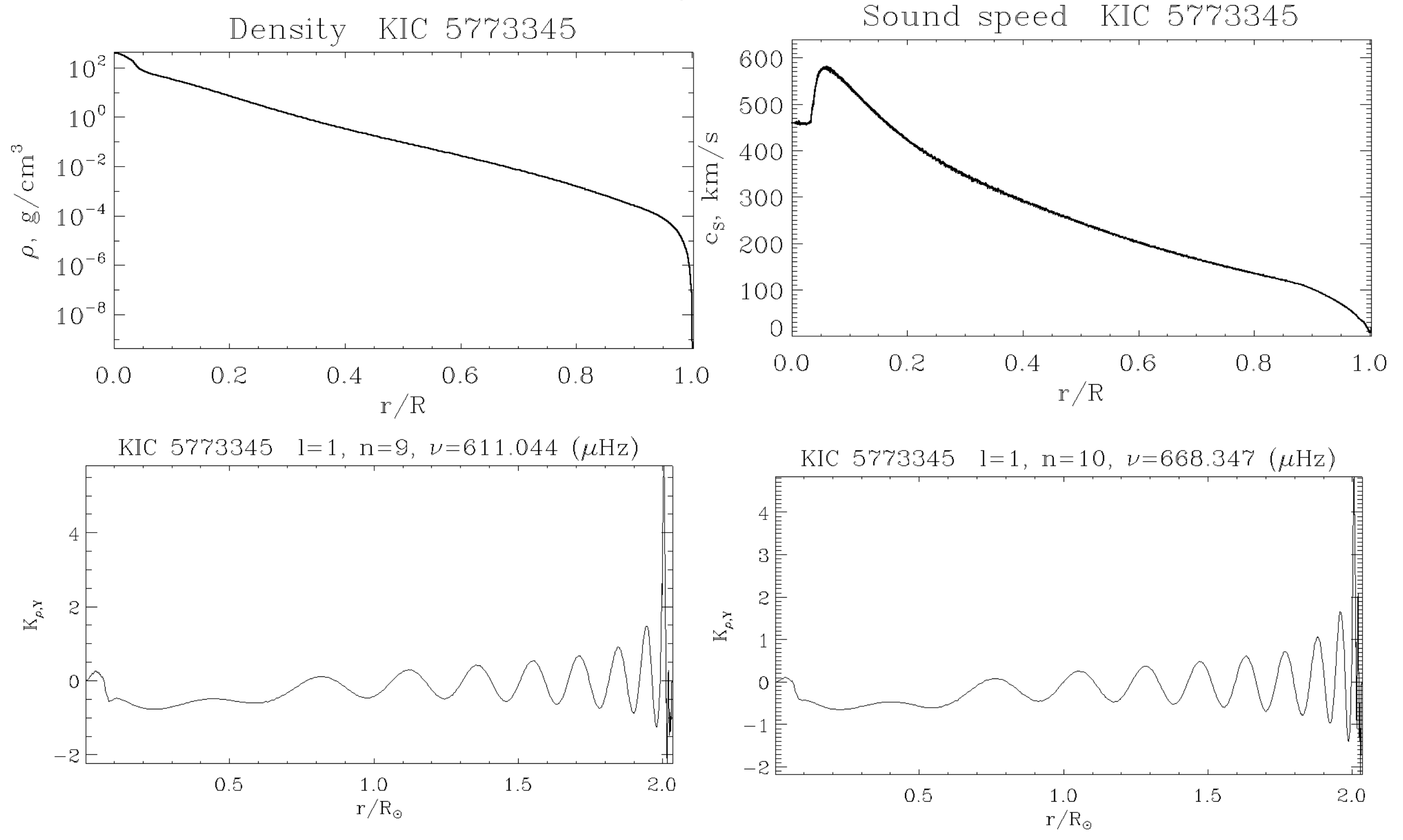
Frequency difference between the observations and model as a function of the inner turning point radius



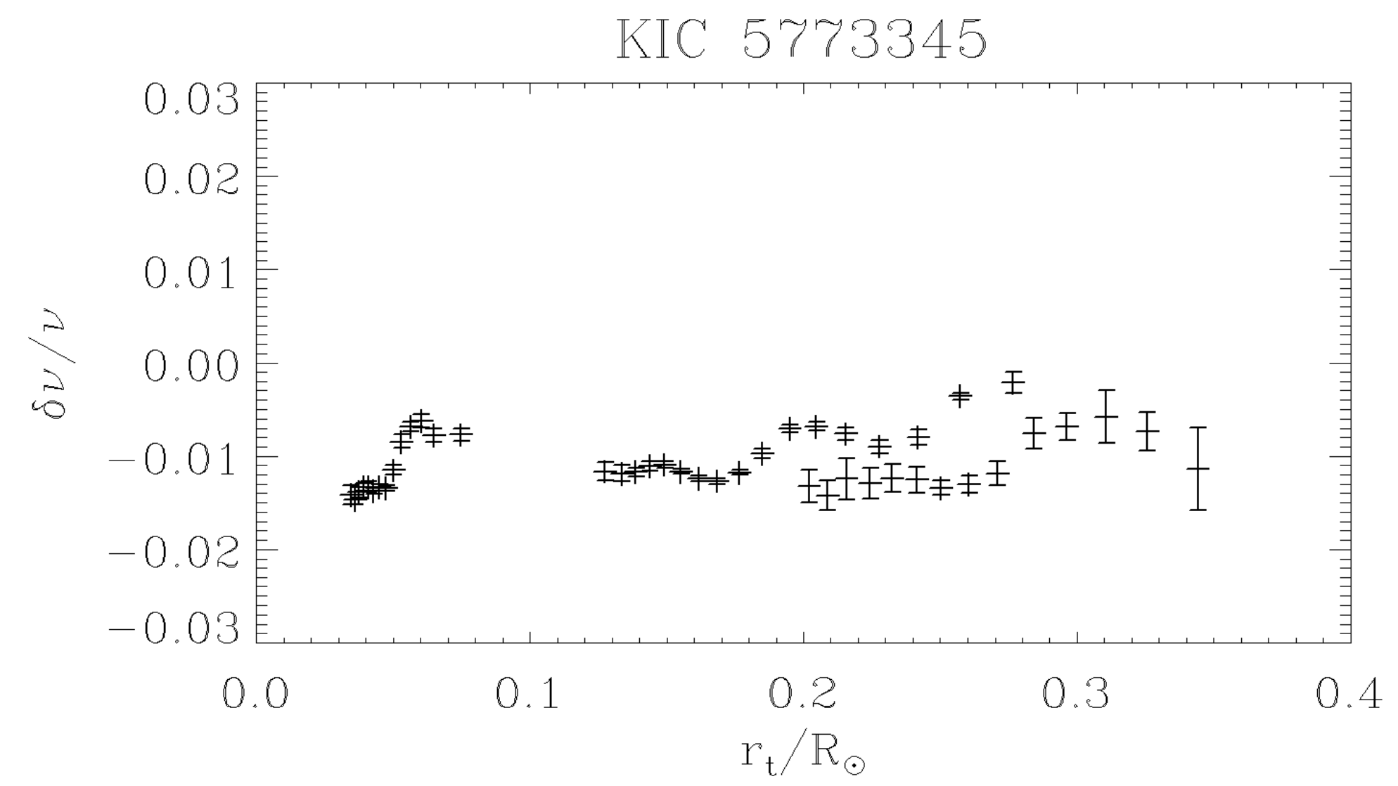
Inversion results for KIC 10162436



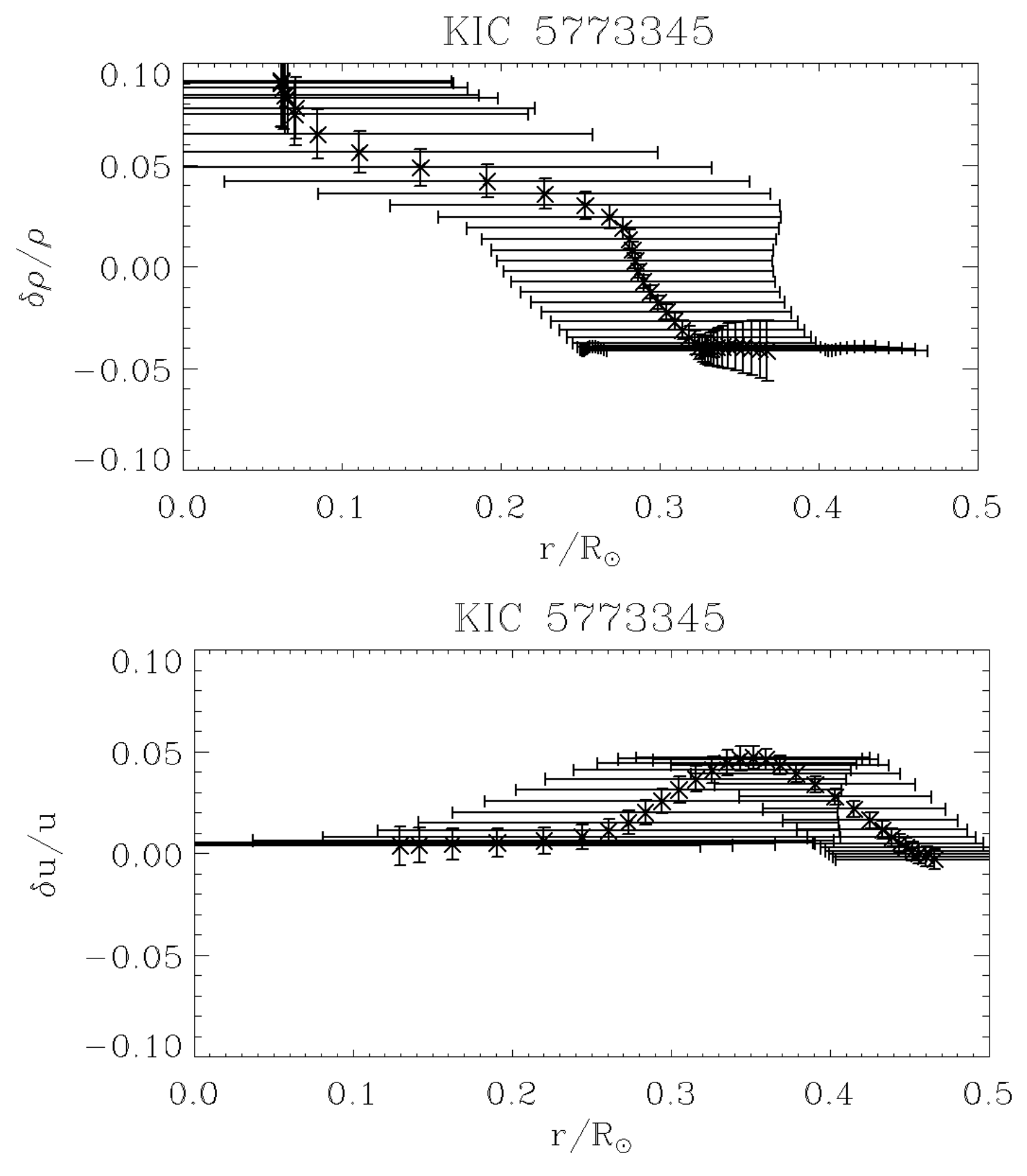
KIC 5773345 : Evolutionary model and sensitivity kernels



Frequency difference between the observations and model as a function of the inner turning point radius



Inversion results for KIC 5773345



For both stars, the inversion results show that the density of the core and the surrounding shell are about 5% higher than in the stellar models, but lower outside the energy-release shell. The boundary of the helium core is located at 0.05R$_\odot$ in KIC 10162436, and at 0.07R$_\odot$ in KIC 5773345. Outside the helium cores, the nuclear energy production shells extend to 0.3R$_\odot$, with the peak rate at ~0.08R$_\odot$ in both models. Perhaps these stellar regions involve physical processes that are not described by the evolutionary models. For understanding these deviations it will be beneficial to perform more detailed structure inversion studies for a large sample of post-main sequence stars with hydrogen-burning shells.