

# Some General Solutions for Linear Bragg-Hawthorne Equation

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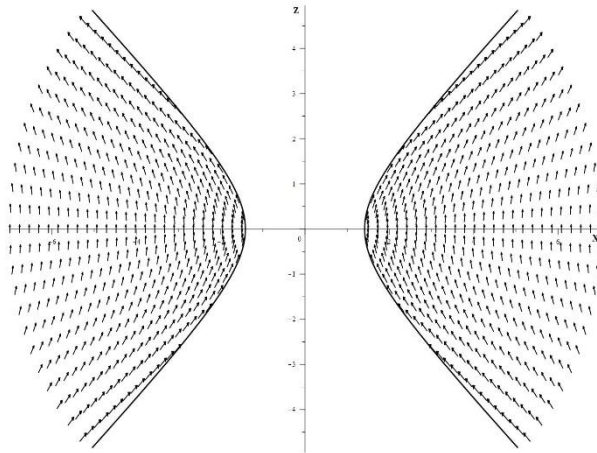
**Abstract:** Linear cases of Bragg-Hawthorne equation for steady axisymmetric incompressible ideal flows are systematically discussed. The equation is converted to a more convenient form in spherical coordinate system. A new vorticity decomposition is derived. General solutions for 16 linear cases of the equation are obtained. These solutions can be specified to gain new analytical vortex flows, as examples in the paper demonstrate. A lot of well-known solutions like potential flow past a sphere, Hill's vortex with and without swirl, are included and extended in these solutions. Special relations between some vortex flows are also revealed when exploring/comparing related solutions.

**Keywords:** axisymmetric flow; Bragg-Hawthorne equation; Grad–Shafranov equation; Euler equations; Beltrami flow; spherical vortex

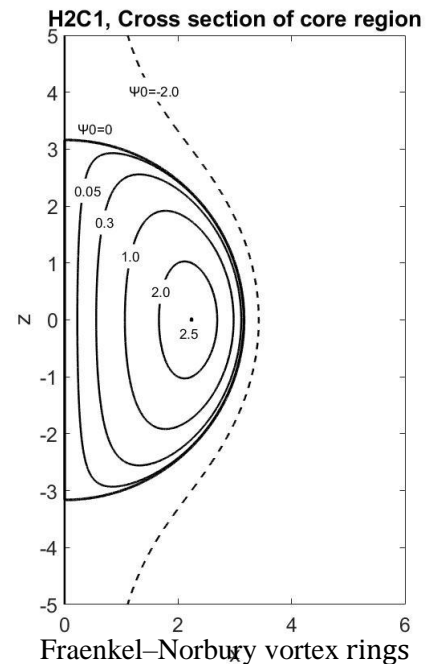
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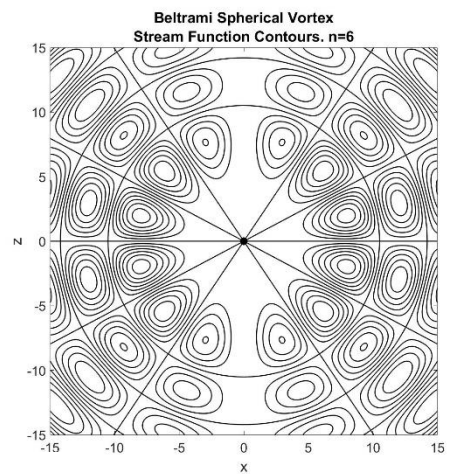
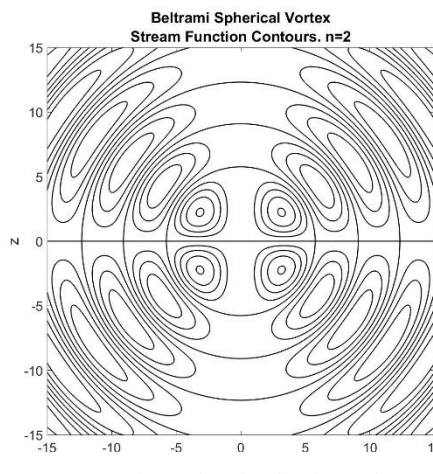
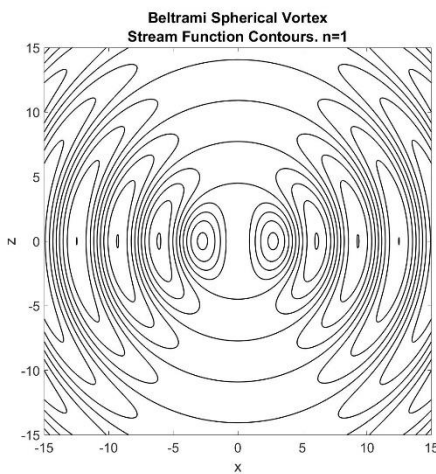
# Examples of vortex solutions



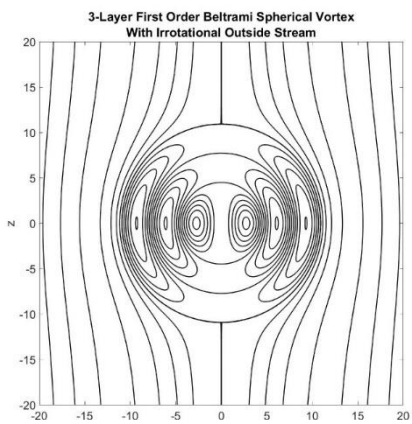
Non-linear Beltrami flow outside a hyperboloid



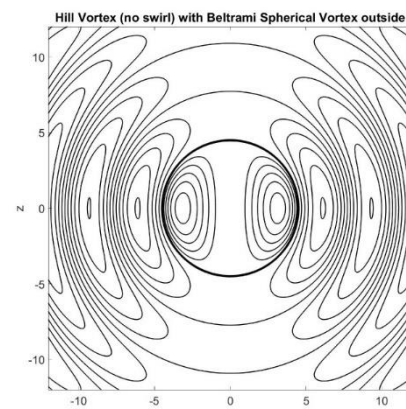
Fraenkel–Norbury vortex rings



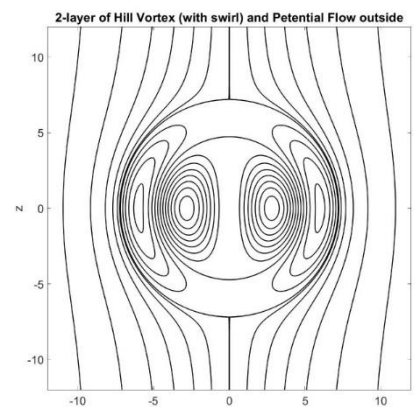
Beltrami spherical vortices



3-layer Beltrami spherical vortex



Hill's vortex (without swirl) with hollow Beltrami spherical vortex



2-Layer Hill's Vortex with swirl