



**CLIMATE
RESEARCH
FOUNDATION**

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Detection of inhomogeneities in daily data: a test based in the Kolmogorov- Smirnov test

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(FIC, Fundación para la Investigación del Clima), Madrid
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- ◆ Temperature
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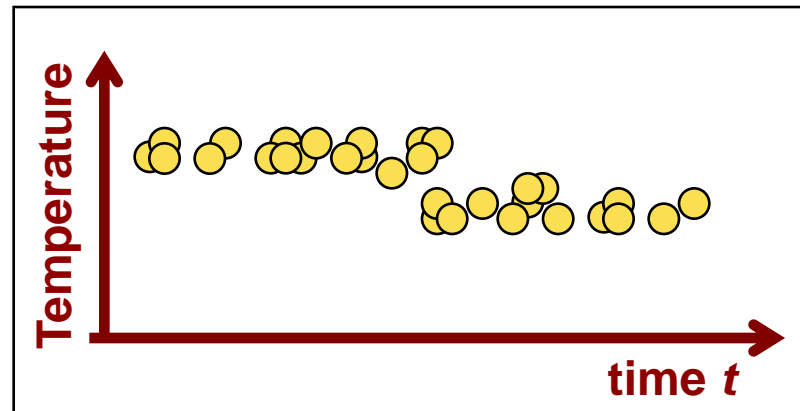
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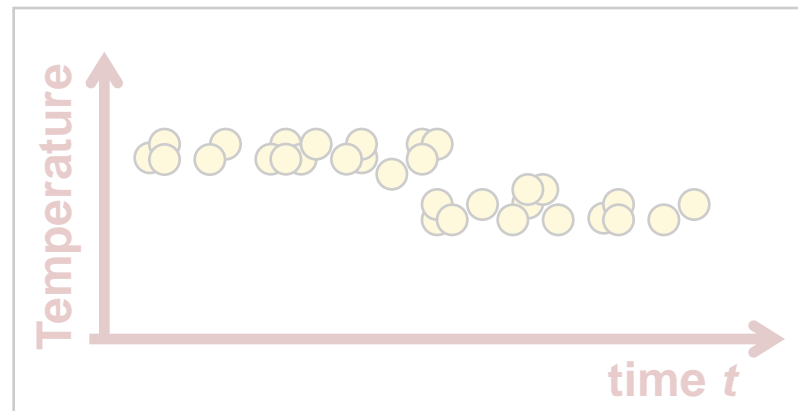


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Problem: Abrupt change in observations

- Changes in the weather shelter (best / worst ventilation, white painting, ...)
- Changes in meteorological sensors
- Changes in the location of weather stations
- Changes in the environment (vegetation, buildings, ...)



Introduction

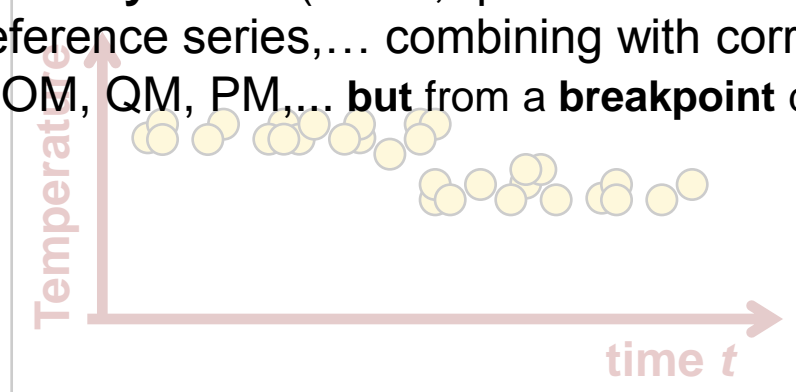
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Problem: Abrupt change in observations

- Changes in the weather shelter (best / worst ventilation, white painting, ...)
- Changes in meteorological sensors
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- Changes in the environment (vegetation, buildings, ...)

Current solutions

- SNHT: Standard Normal Homogeneity Test (by Alexandersson, 1986)
- Others methods at **monthly scale**: Buishand range test, Pettitt test, von Neumann ratio tests,...
- Some methods at **daily scale** (mean, quantiles or moments): Using parallel measurements, reference series,... combining with corrections as HOM, HOMAD, SPLIDHOM, QM, PM,... **but** from a **breakpoint** detection at **monthly** scale.



Introduction

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Problem: Abrupt change in observations

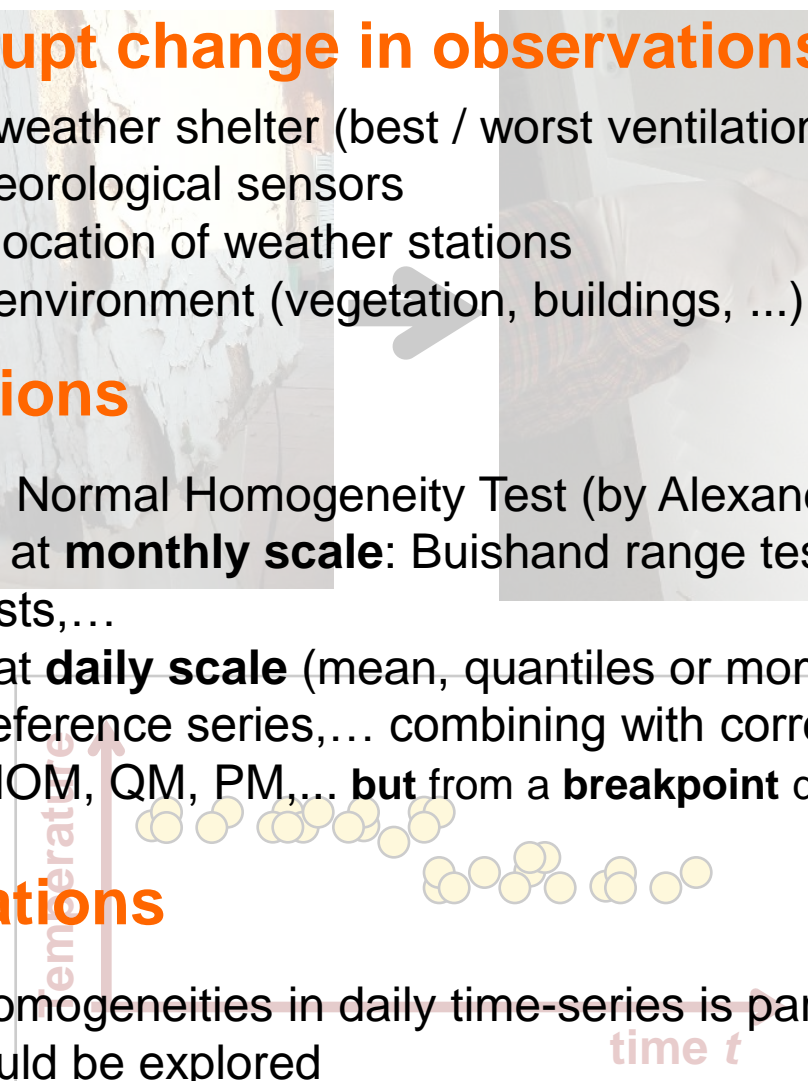
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Current limitations

- Detection of inhomogeneities in daily time-series is partially saved.
- Alternatives should be explored



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Definitions

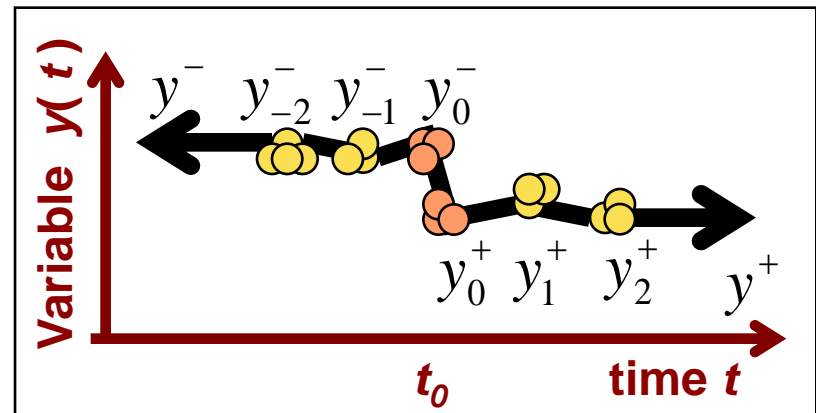
$y_i \equiv$ set of daily data

$t_0 \equiv$ inhomogeneity candidate

$t^- \equiv$ time values to the left

$t^+ \equiv$ time values to the right

PV \equiv p-value of KS test



Methodology

Definitions

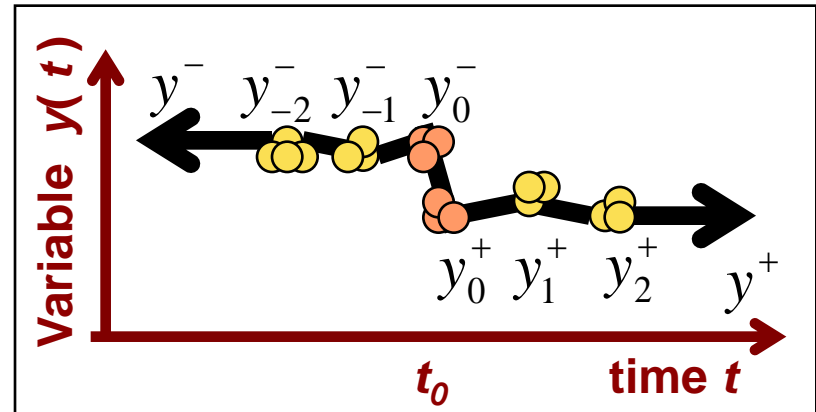
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Measure of the dissimilitude between two sets (e.g. 365 days)

$$LPV_j^i \equiv \log_{10}(PV(y_i, y_j))$$

$$PV_j^i \in (0, 1) \longrightarrow LPV_j^i \in (-\infty, 0)$$

High \leftrightarrow low dissimilitude

Methodology

Definitions

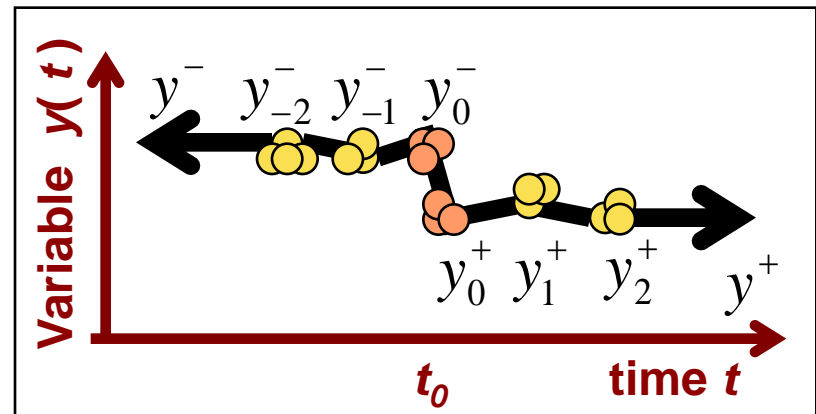
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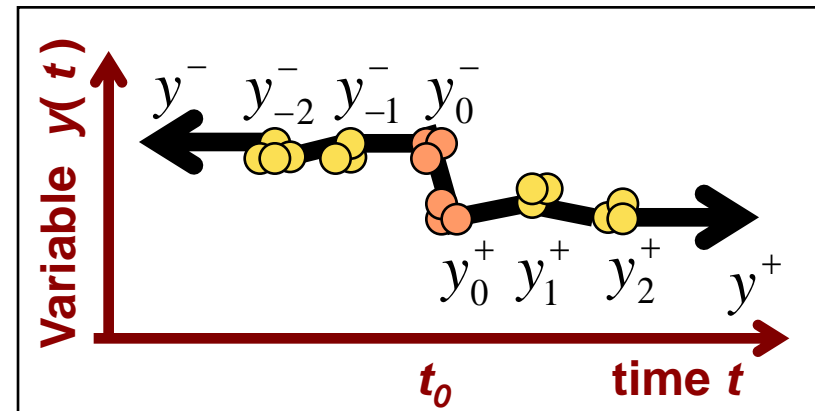
Measure of possible Inhomogeneity jumps: Dissimilitude between 2 contiguous sets

$$LPV_{i+1}^i \equiv \log_{10}(PV(y_i, y_{i+1}))$$

Methodology

Control Analysis

Introducing an artificial inhomogeneity to each set of data $y_i \rightarrow \tilde{y}_i$

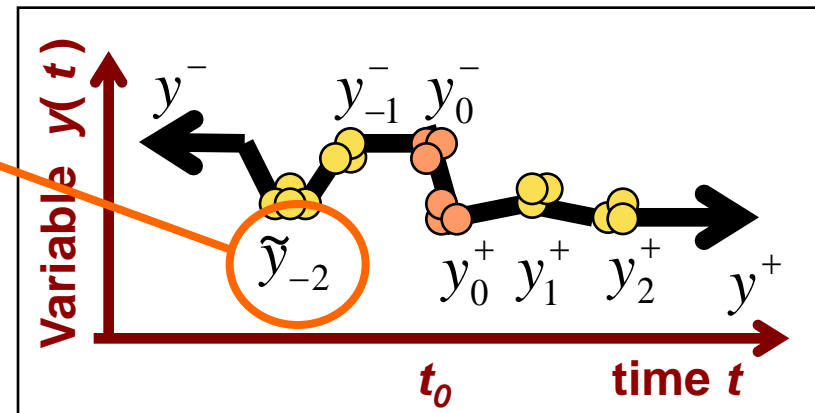


Methodology

Control Analysis

Introducing an artificial inhomogeneity to each set of data $y_i \rightarrow \tilde{y}_i$

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Methodology

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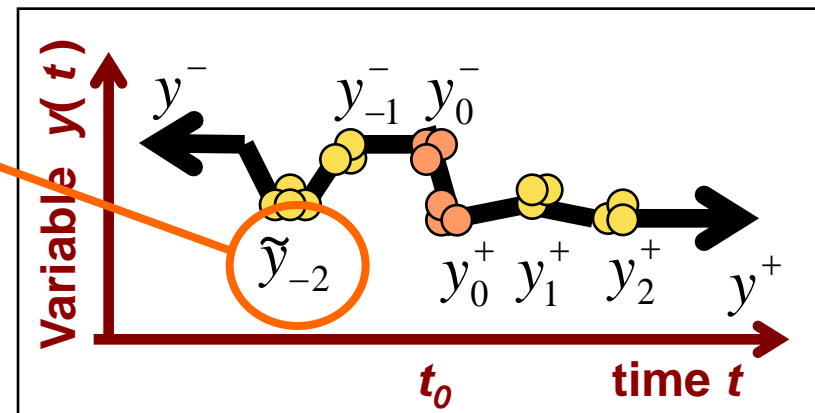
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Variable	a	b
Temperature	1	2
Precipitation	3	0

EXAMPLE

$$\tilde{y}_i \equiv a^{\pm 1} \cdot y_i \pm b$$

Artificial inhomogeneity



Methodology

Control Analysis

Introducing an artificial inhomogeneity to each set of data $y_i \rightarrow \tilde{y}_i$

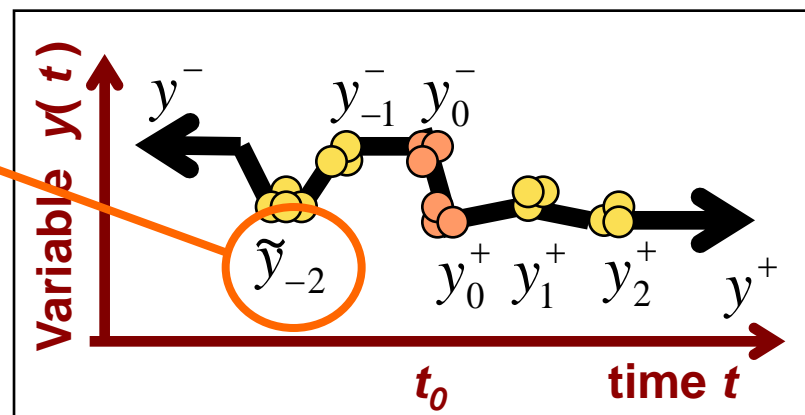
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Artificial inhomogeneity



Reference LPV is defined from the average value: inhomogeneity of control

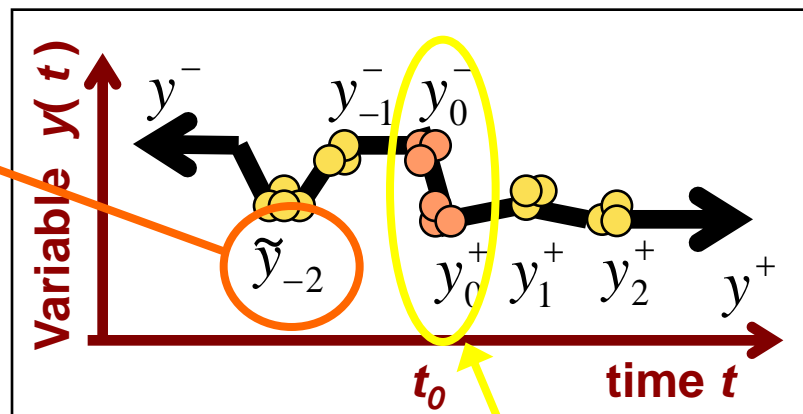
$$LPV_{Inh} \equiv \frac{1}{N-1} \sum_{i=1}^{N-1} LPV_{\{i+1\}}^i$$

Methodology

Control Analysis

Introducing an artificial inhomogeneity to each set of data $y_i \rightarrow \tilde{y}_i$

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Artificial inhomogeneity

Reference LPV is defined from the average value: inhomogeneity of control

$$LPV_{Inh} \equiv \frac{1}{N-1} \sum_{i=1}^{N-1} LPV_{\{i+1\}}^i$$

$$LPV_{i+1}^i \leq LPV_{Inh} \rightarrow \begin{cases} y_0^- = y_i \\ y_0^+ = y_{i+1} \end{cases}$$

Methodology

Inhomogeneity detection

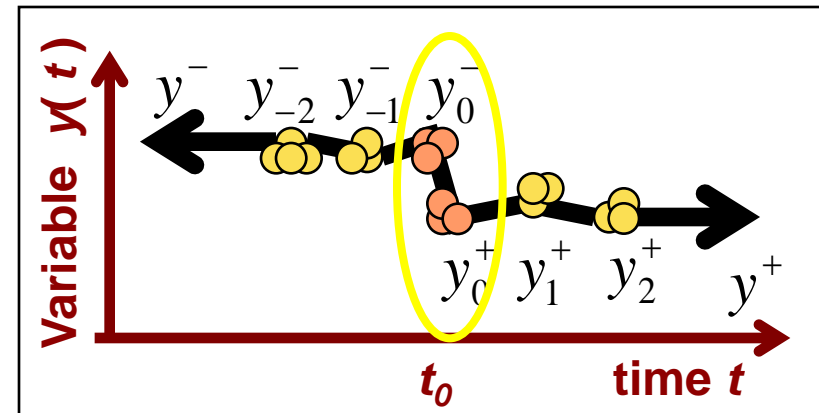
Similarity between the **candidates** y_0 and the other **populations** that are on the left (-) and right (+)

$$LPV_-^- \equiv \log_{10}(PV(y^-, y_0^-)) \quad \text{Similar}$$

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$$LPV_+^- \equiv \log_{10}(PV(y^-, y_0^+)) \quad \text{different}$$

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Methodology

Inhomogeneity detection

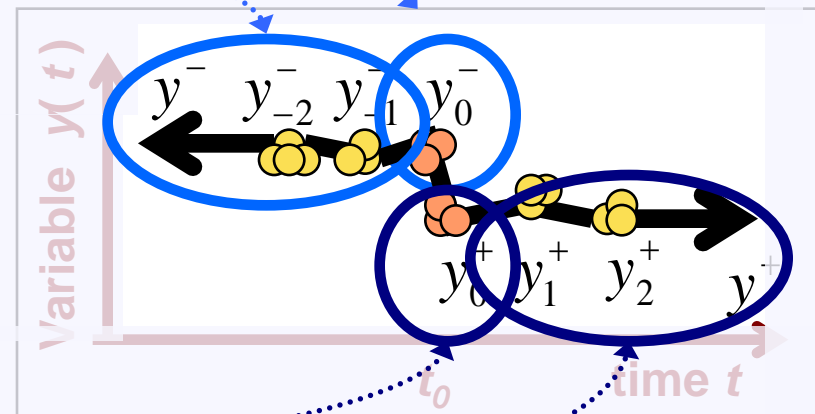
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Methodology

Inhomogeneity detection

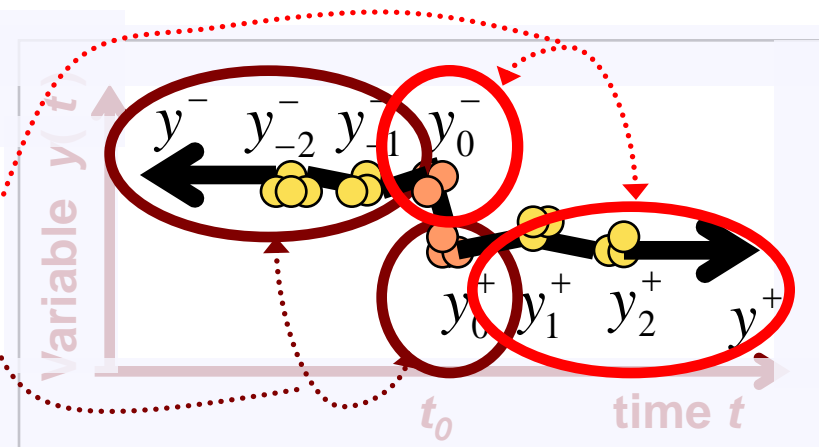
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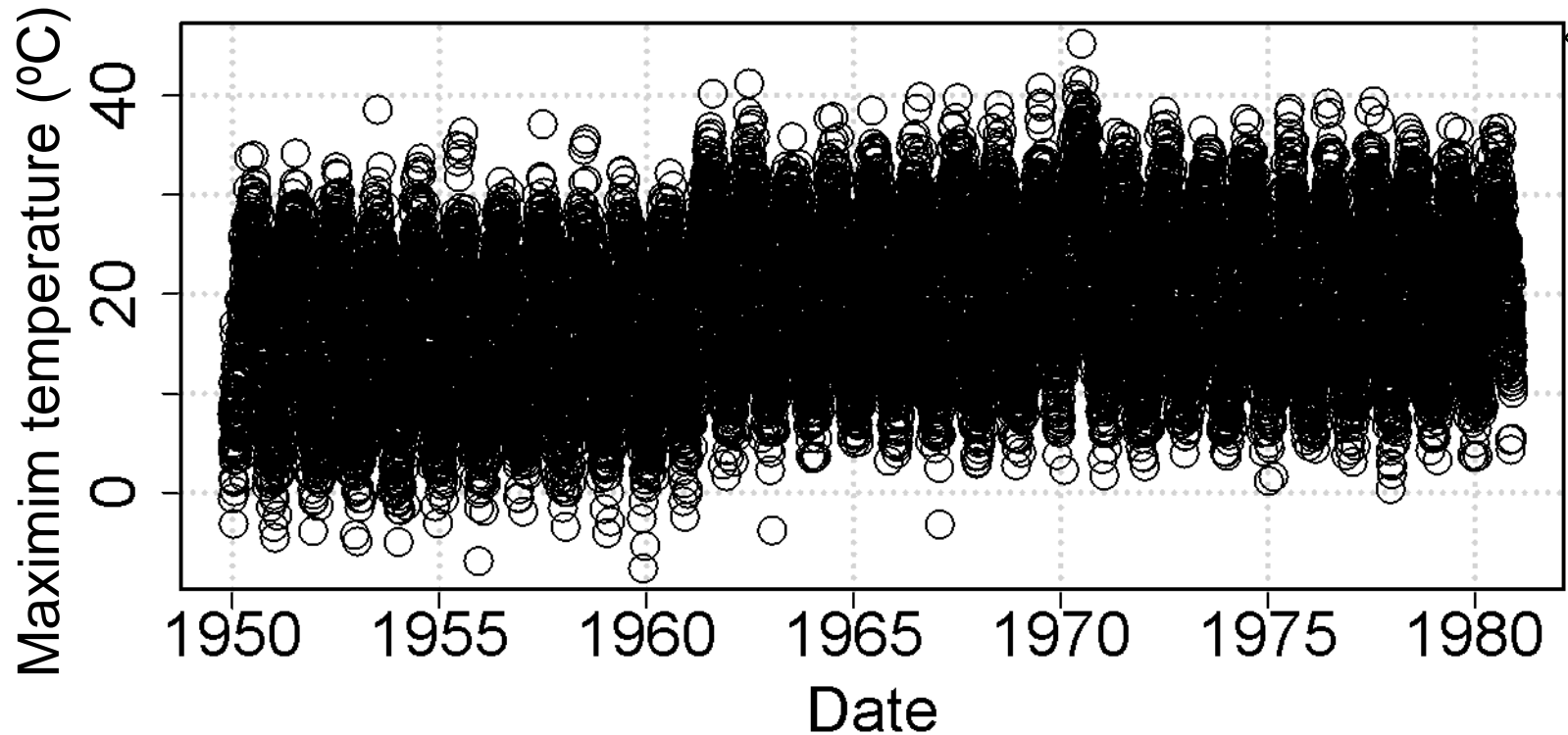


Methodology

Inhomogeneity detection: EXAMPLE 1

Ideal case with a jump and one unusual year

Theoretical case built from normal distributions

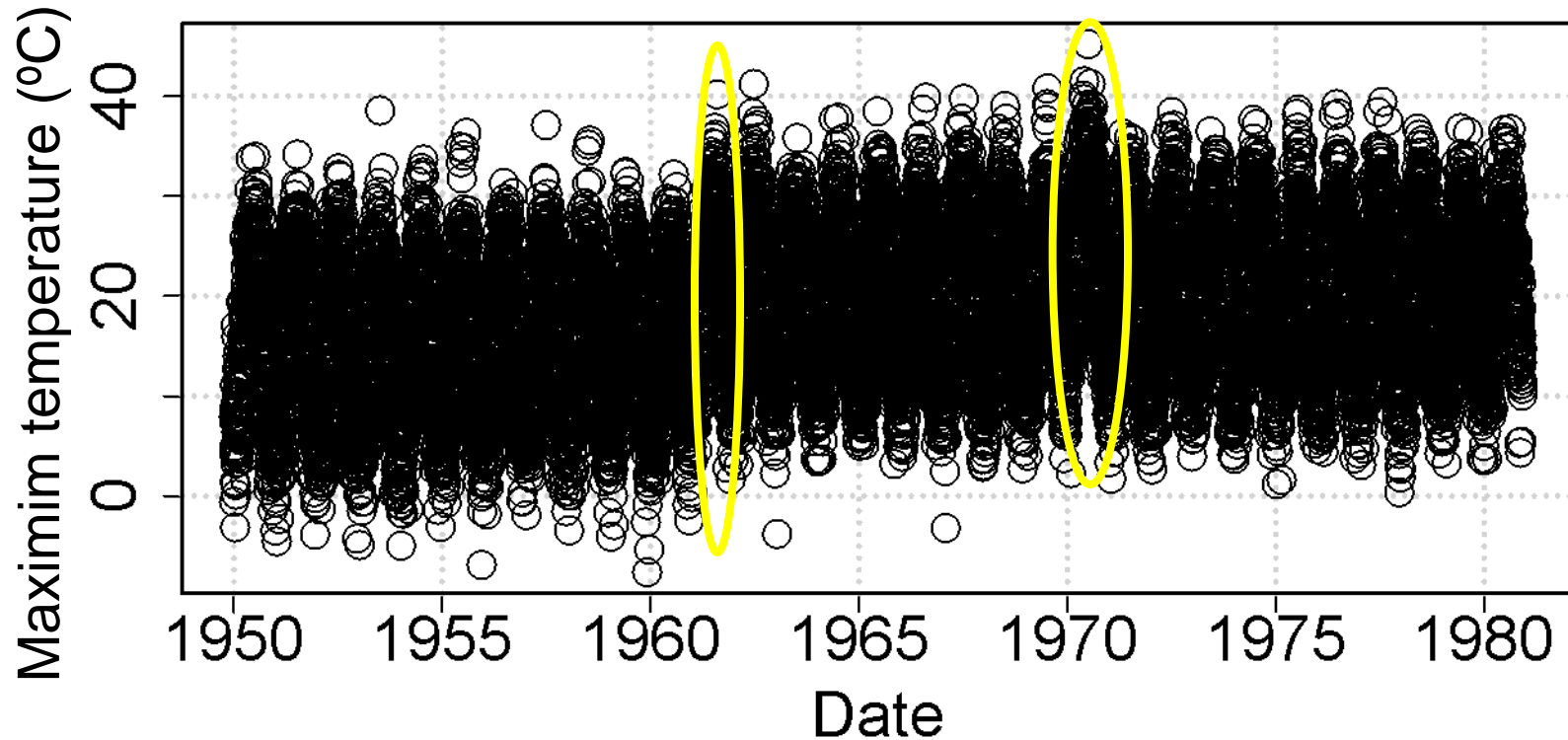


Methodology

Inhomogeneity detection: EXAMPLE 1

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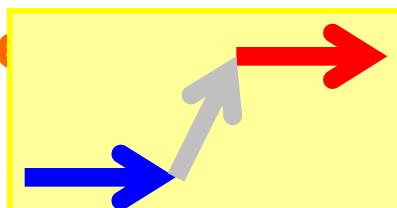
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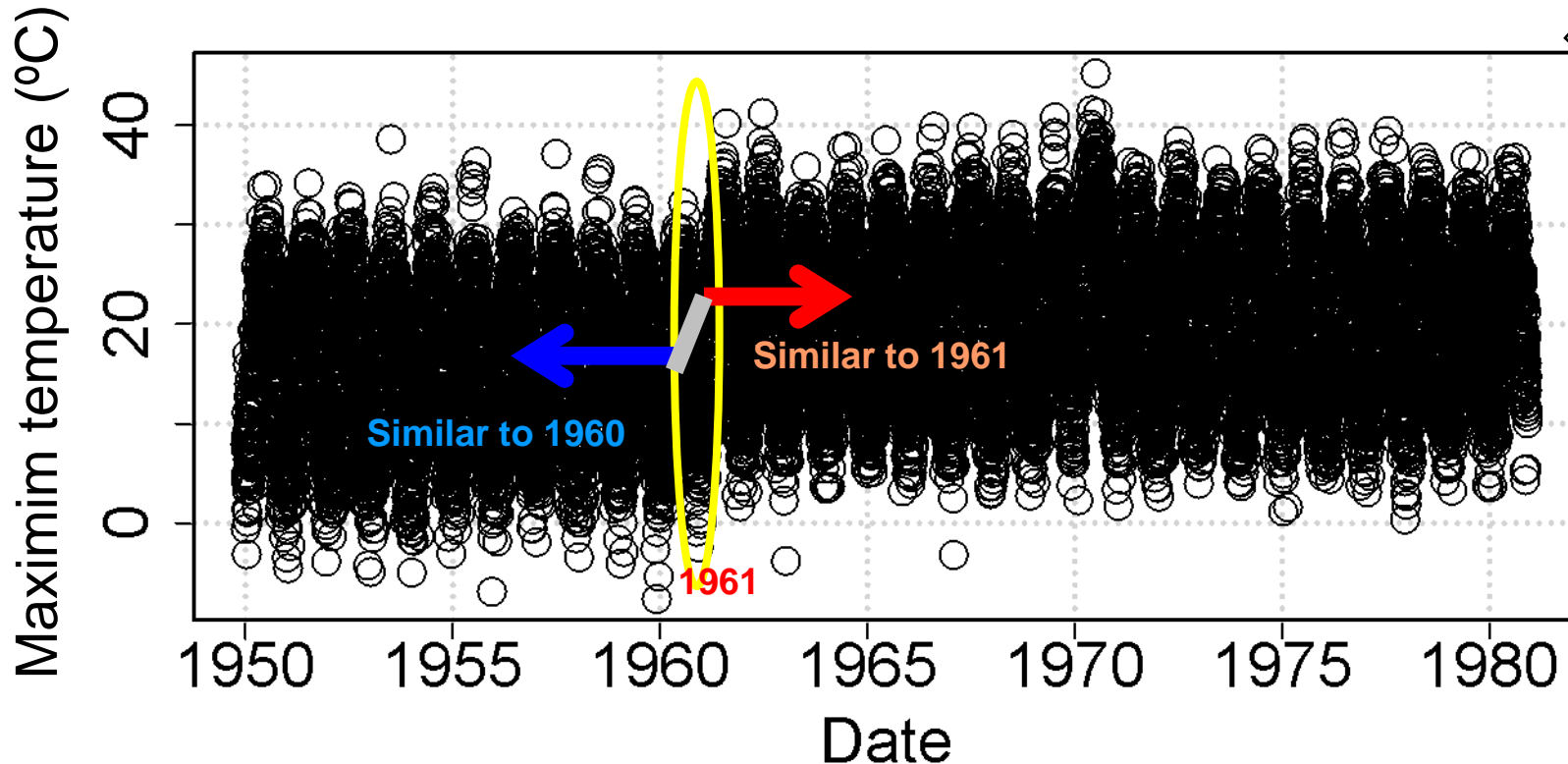
Methodology

Inhomogeneity detection: **MPLE 1**

Ideal case of the jump



Theoretical case built from normal distributions



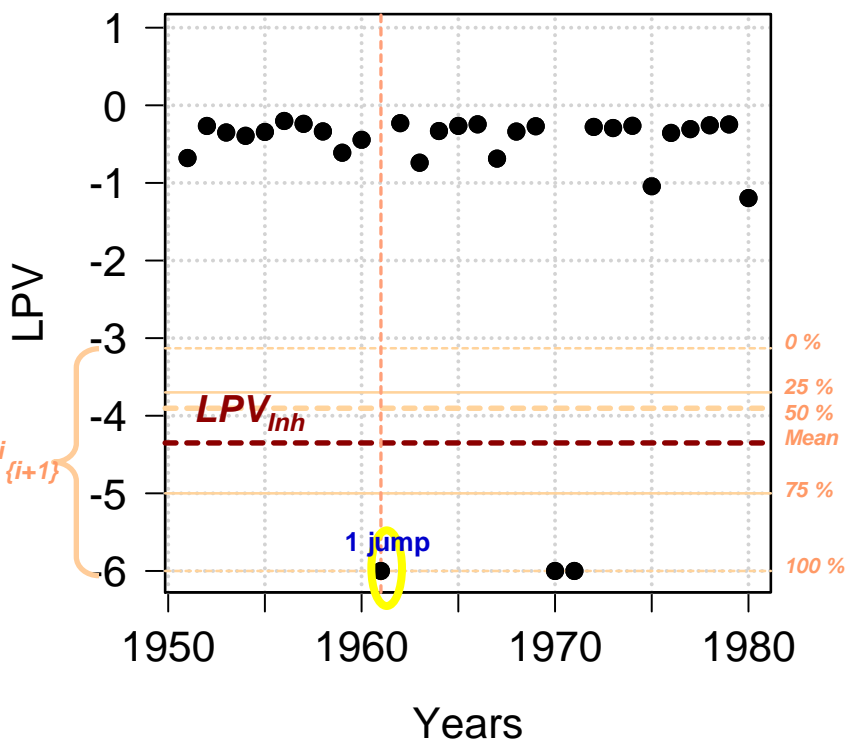
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Inhomogeneity detection: EXAMPLE 1

LPV diagrams for the ideal case of jump

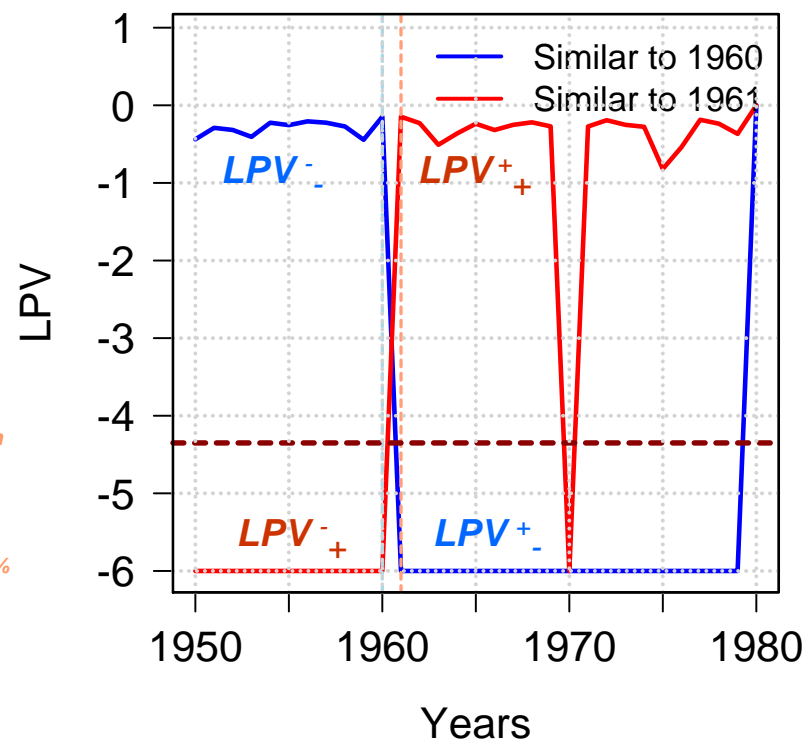
LPV_{i+1}^i

Control Analysis



LPV_{\pm}^{\pm}

Similarity between years



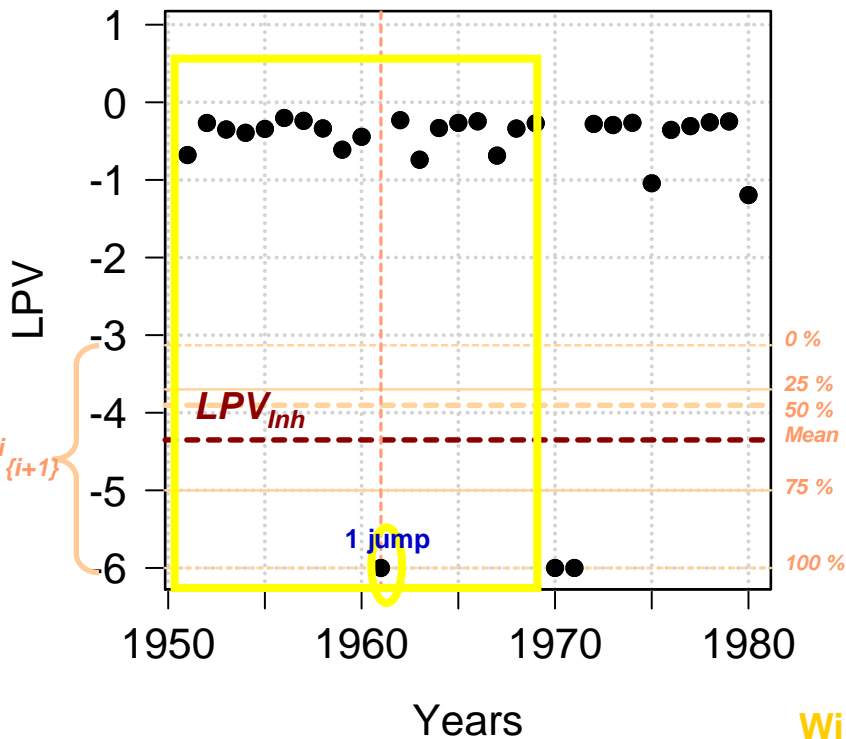
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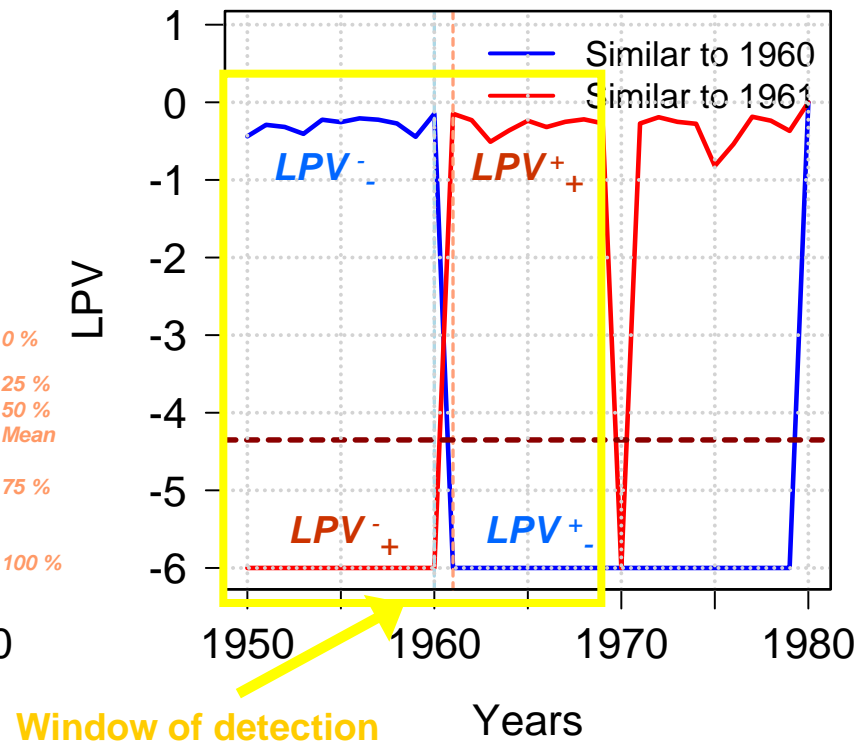
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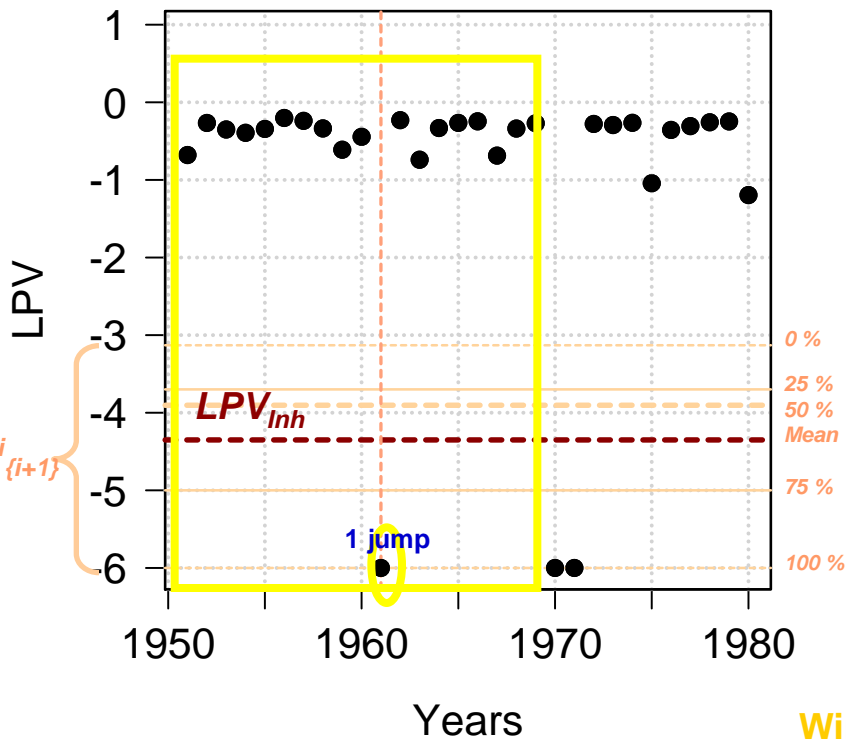
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Inhomogeneity detection: EXAMPLE 1

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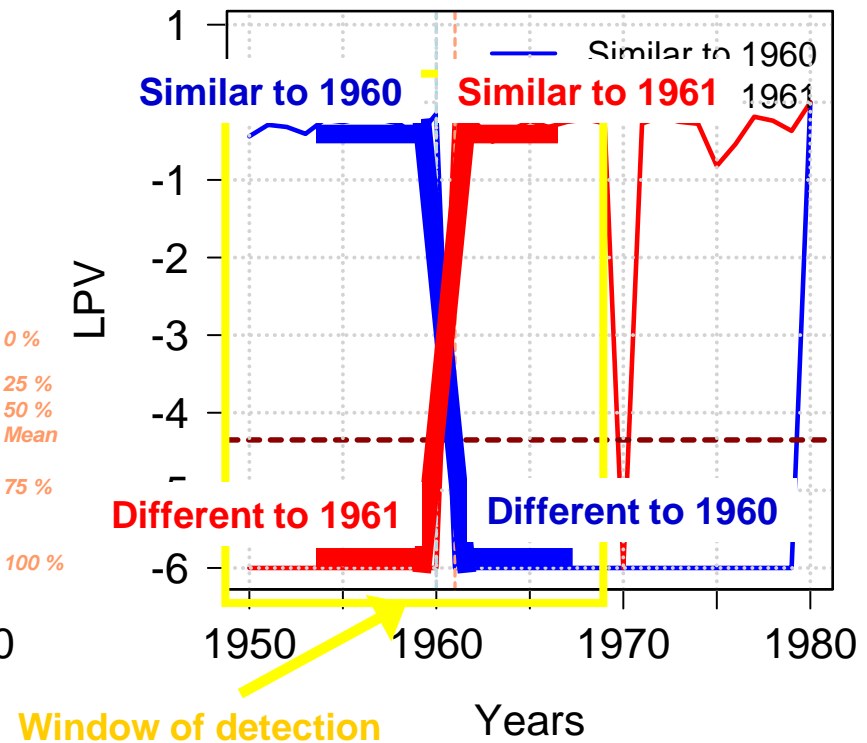
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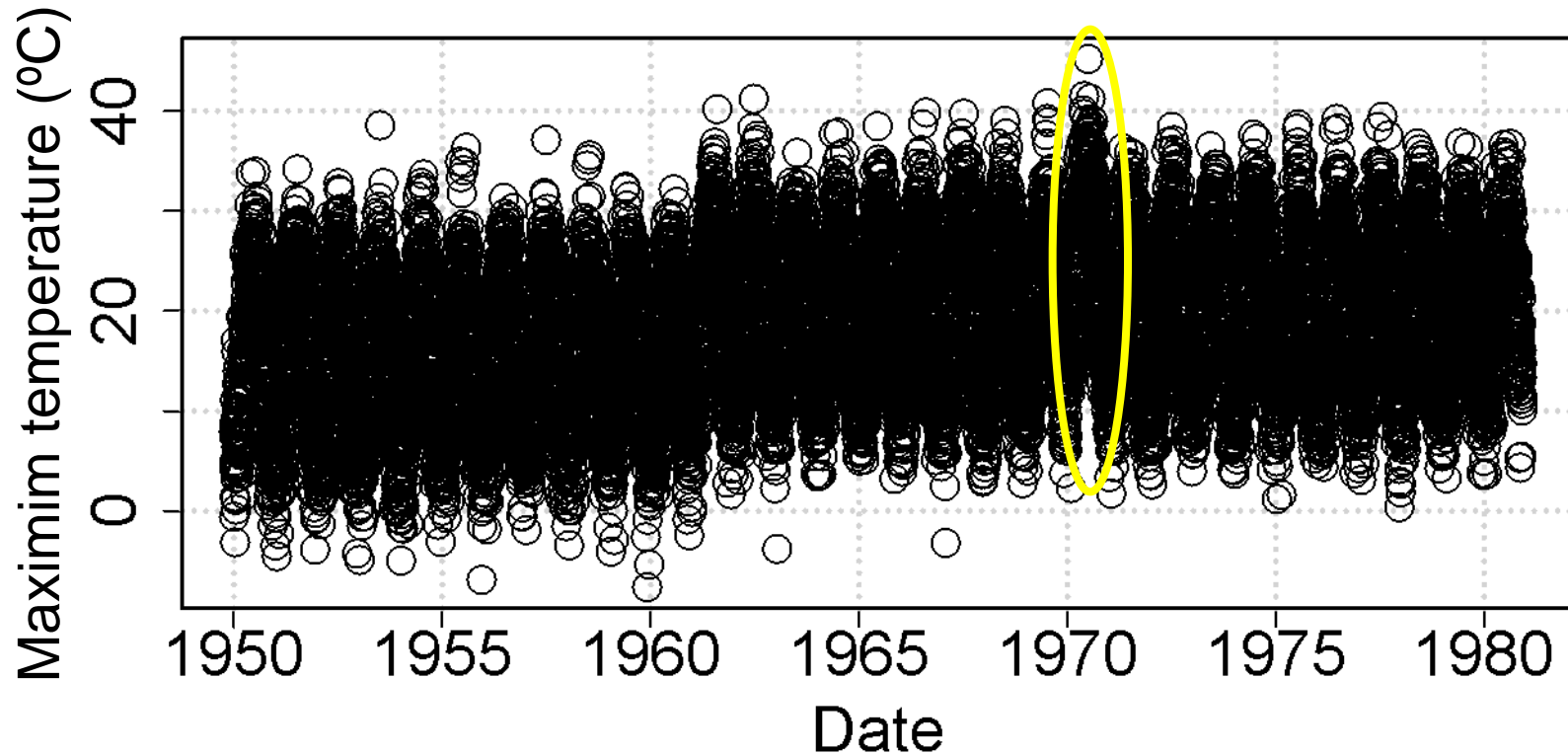


Methodology

Inhomogeneity detection: EXAMPLE 1

Ideal case of unusual year (i.e., 2 jumps)

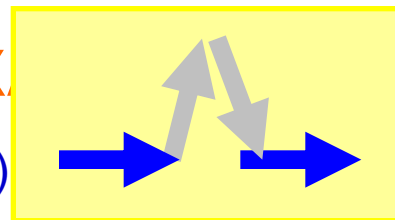
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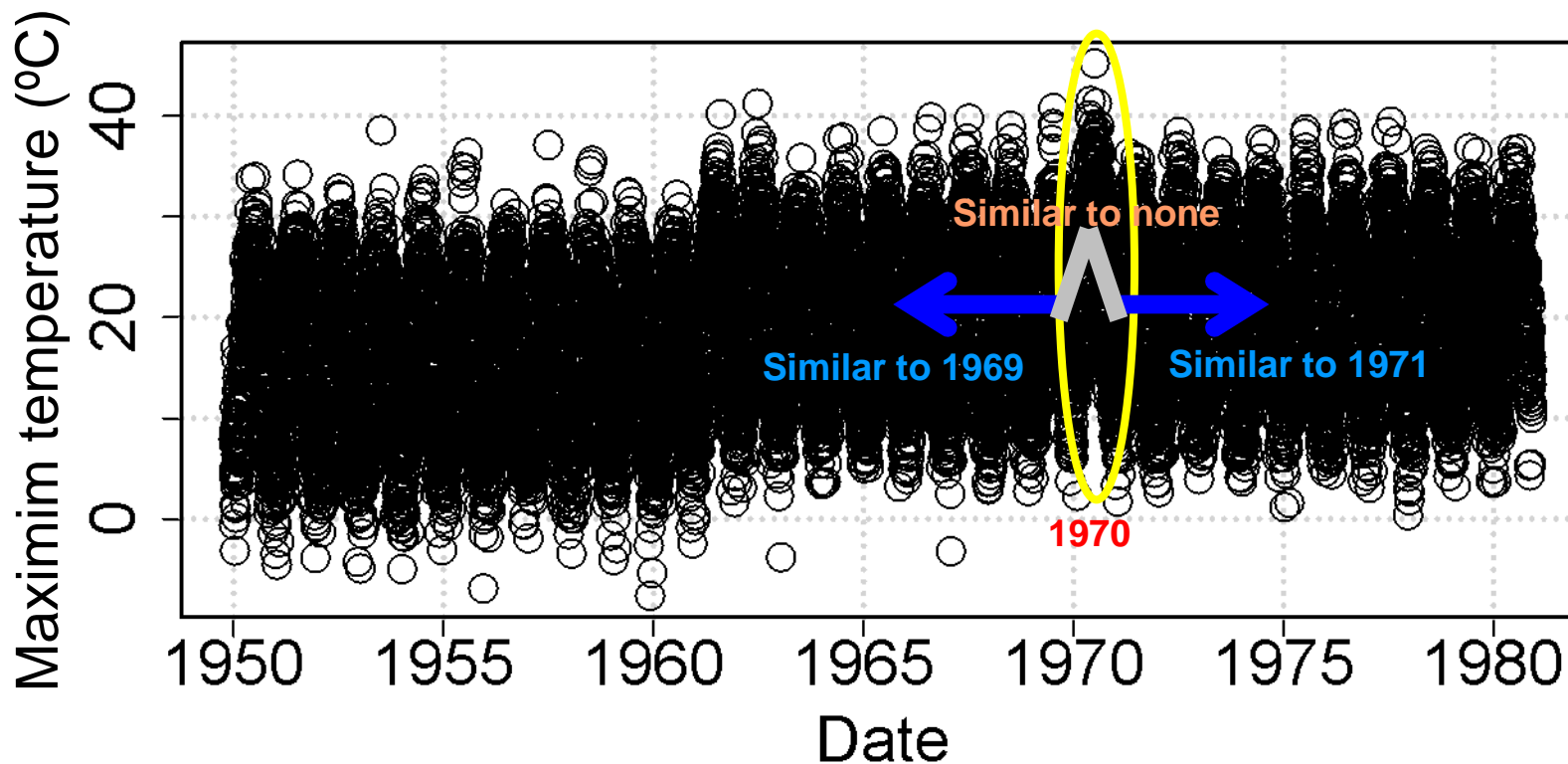
Methodology

Inhomogeneity detection: EX

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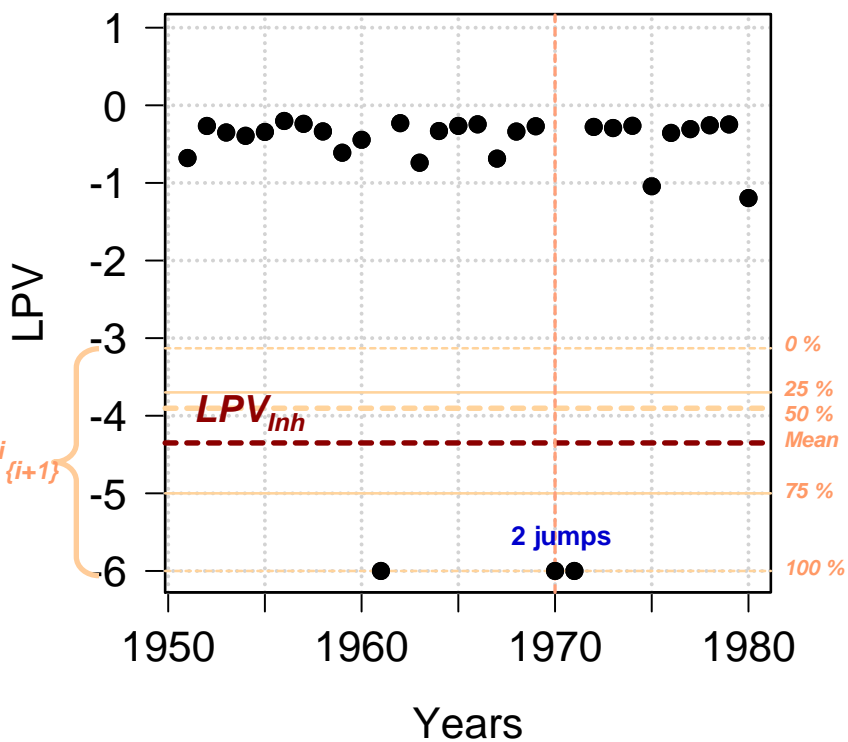
Methodology

Inhomogeneity detection: EXAMPLE 1

LPV diagrams for the ideal case of unusual year (2 jumps)

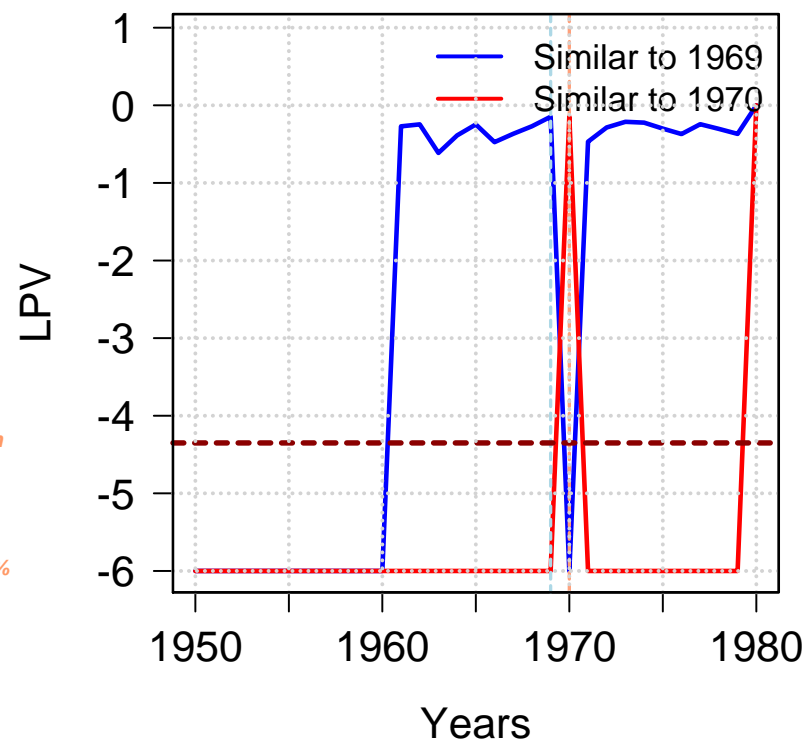
LPV_{i+1}^i

Control Analysis



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Similarity between years



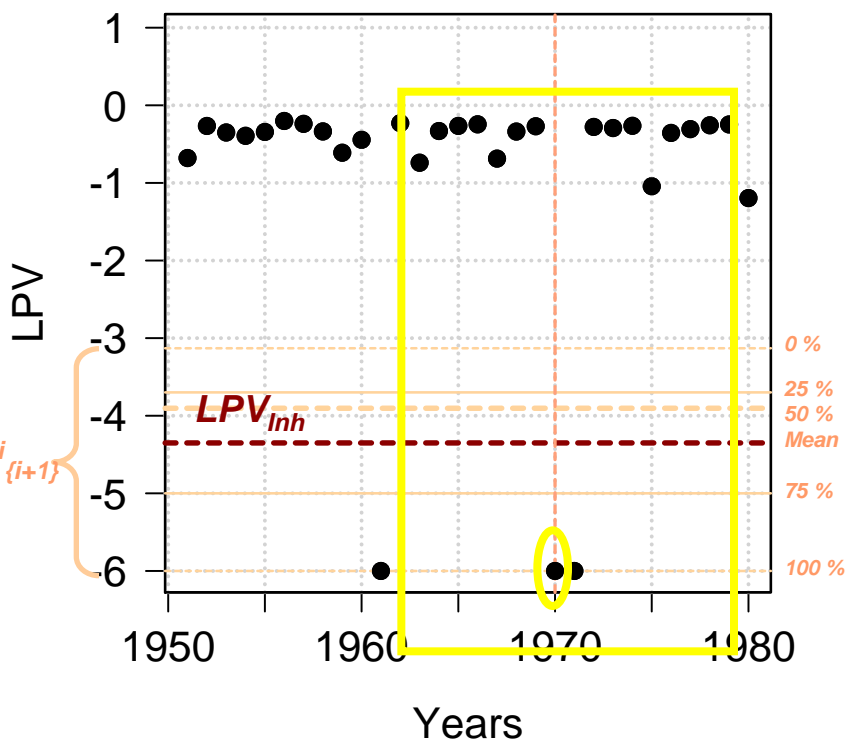
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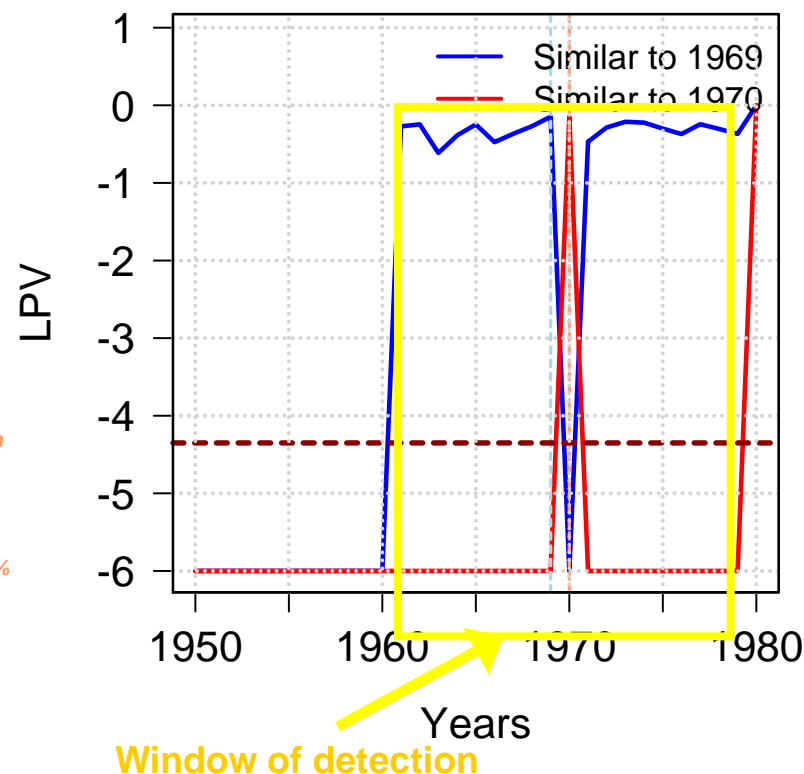
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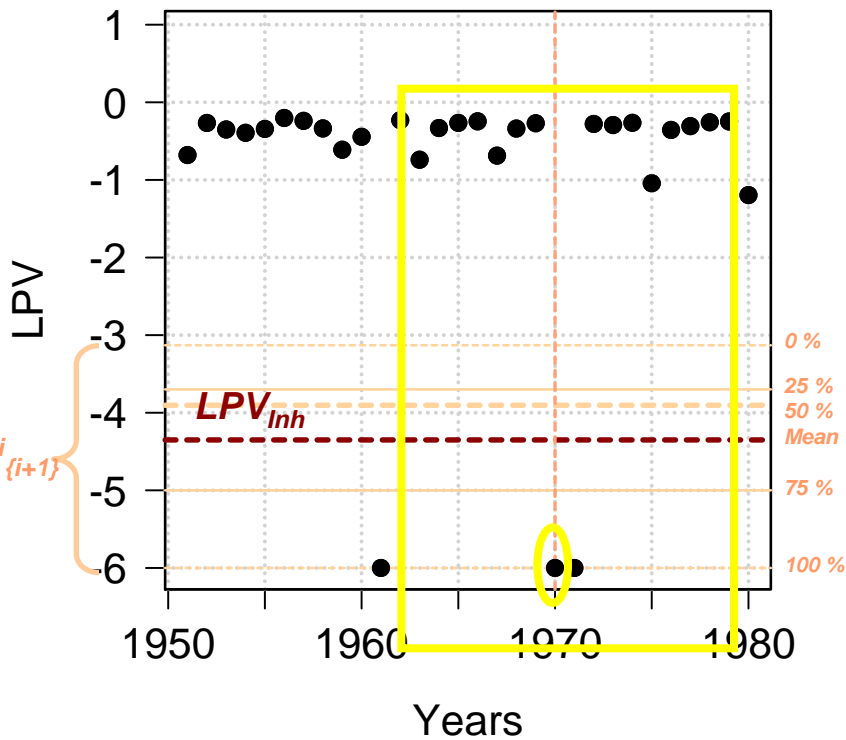
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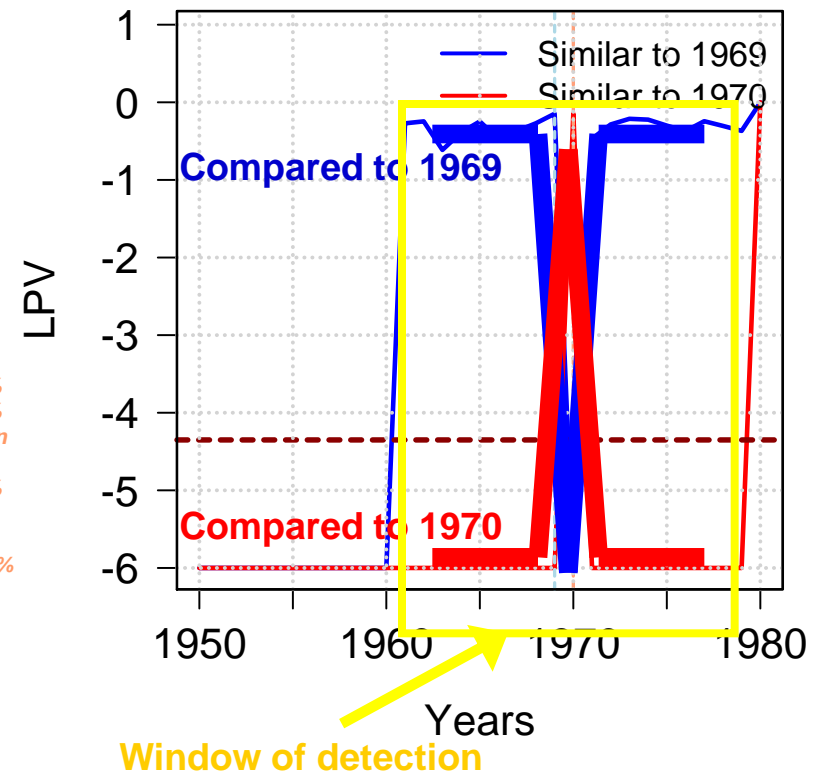
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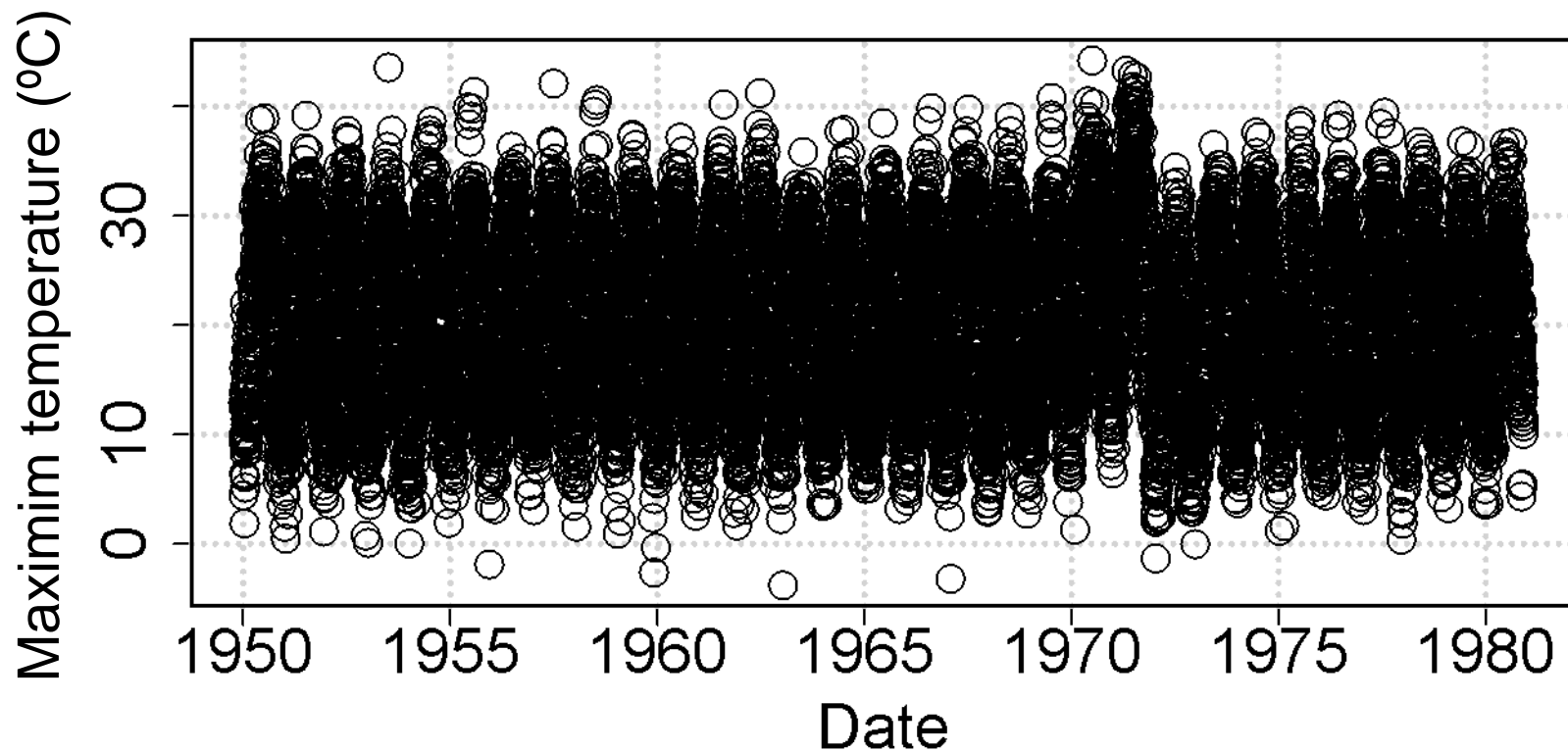


Methodology

Inhomogeneity detection: EXAMPLE 2

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Ideal case of inhomogeneity of second order (i.e., more of 2 jumps)

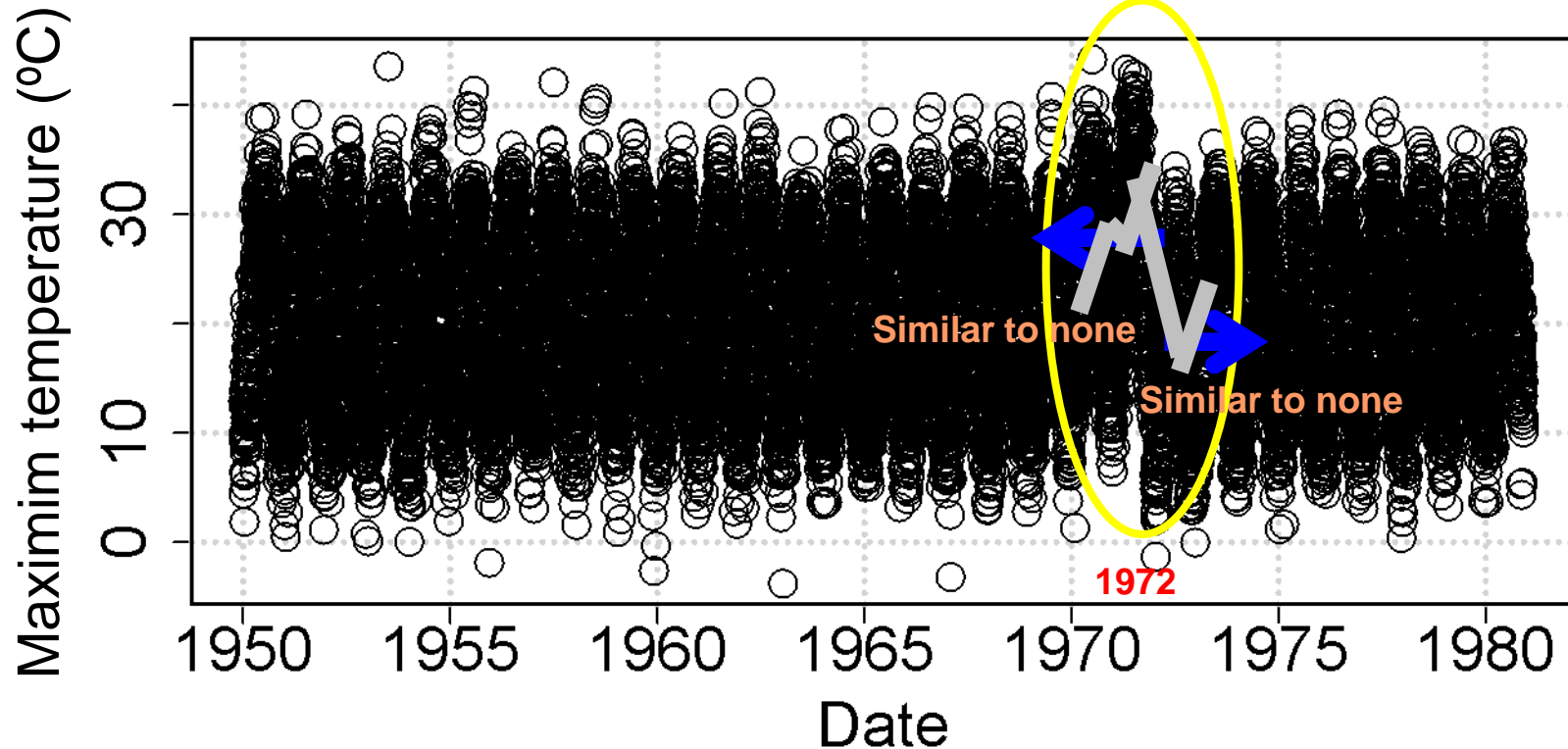


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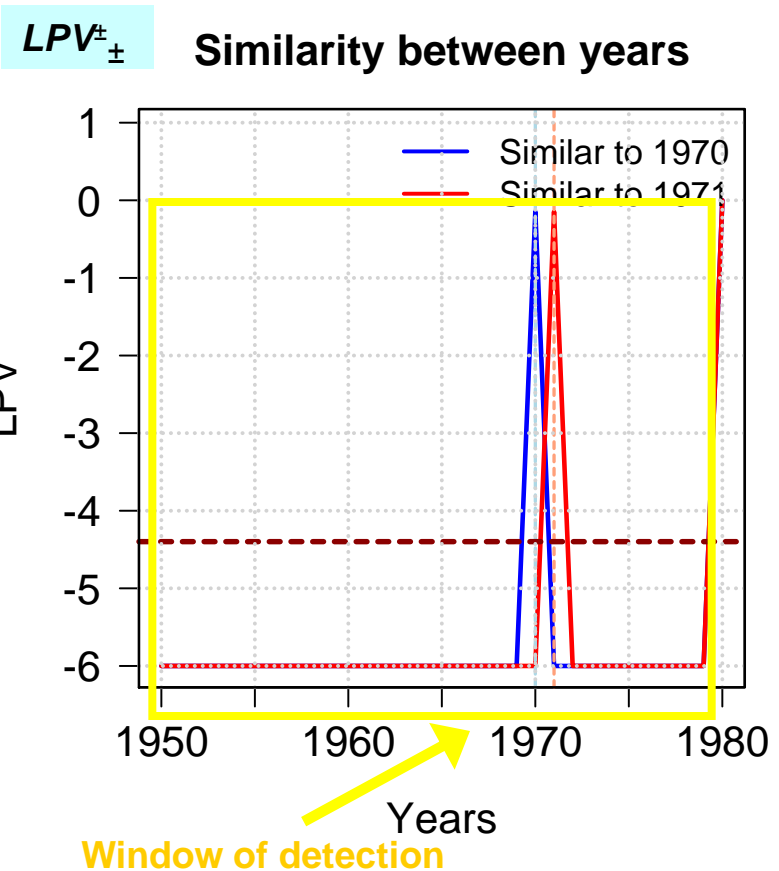
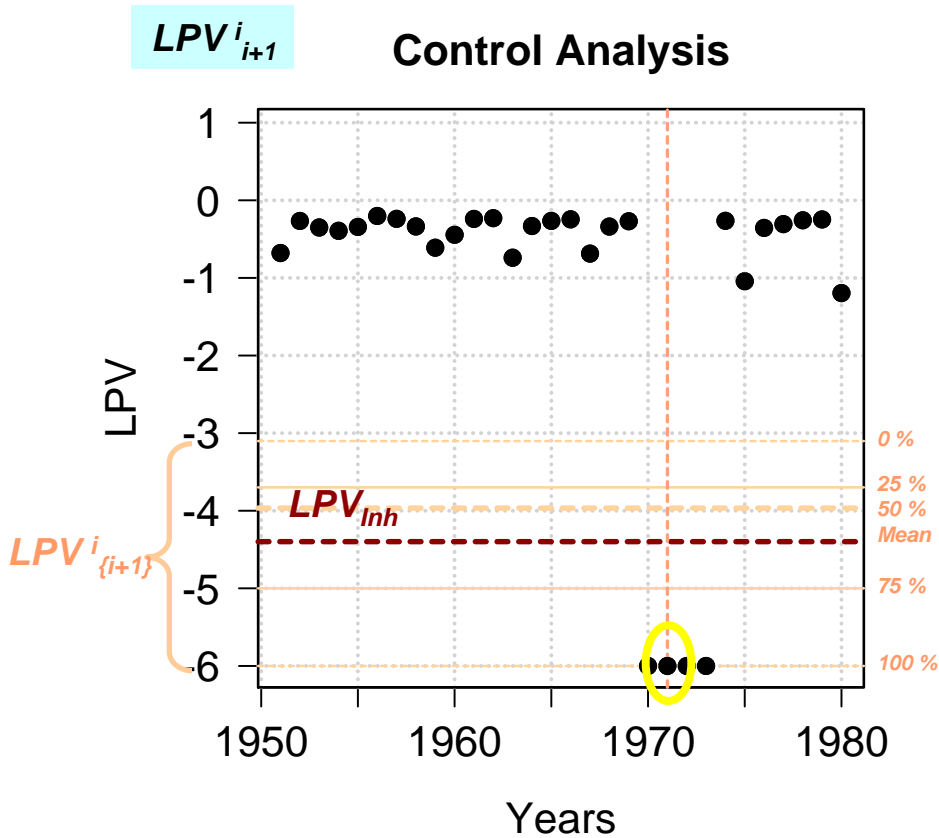
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Inhomogeneity detection: EXAMPLE 2

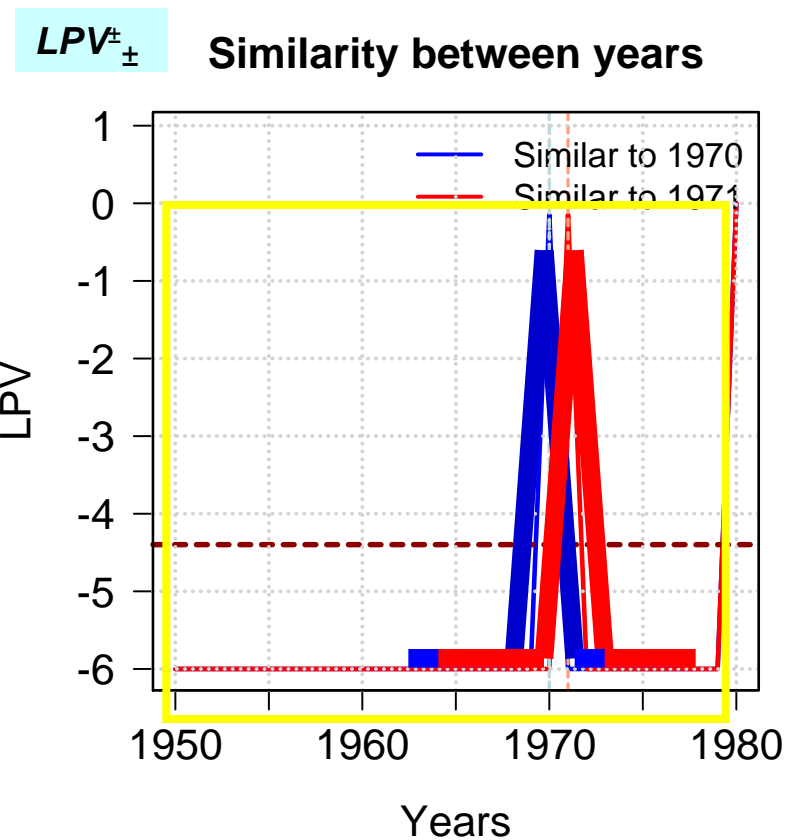
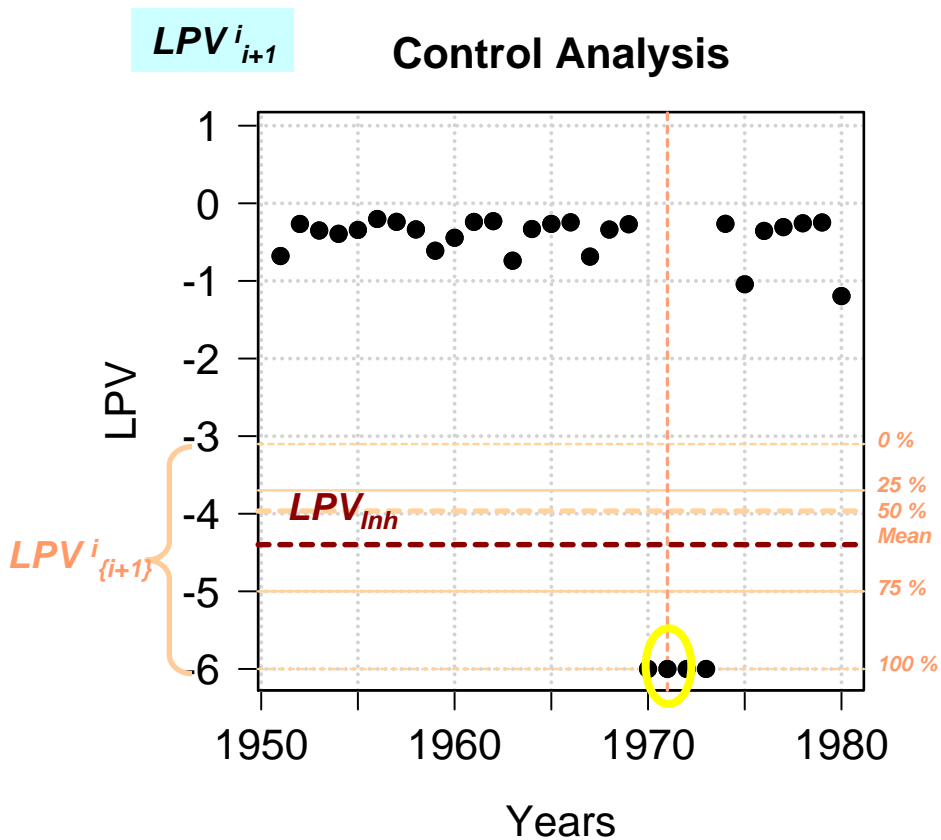
LPV diagrams for the ideal case of inhomogeneity of second order



Methodology

Inhomogeneity detection: EXAMPLE 2

LPV diagrams for the ideal case of inhomogeneity of second order



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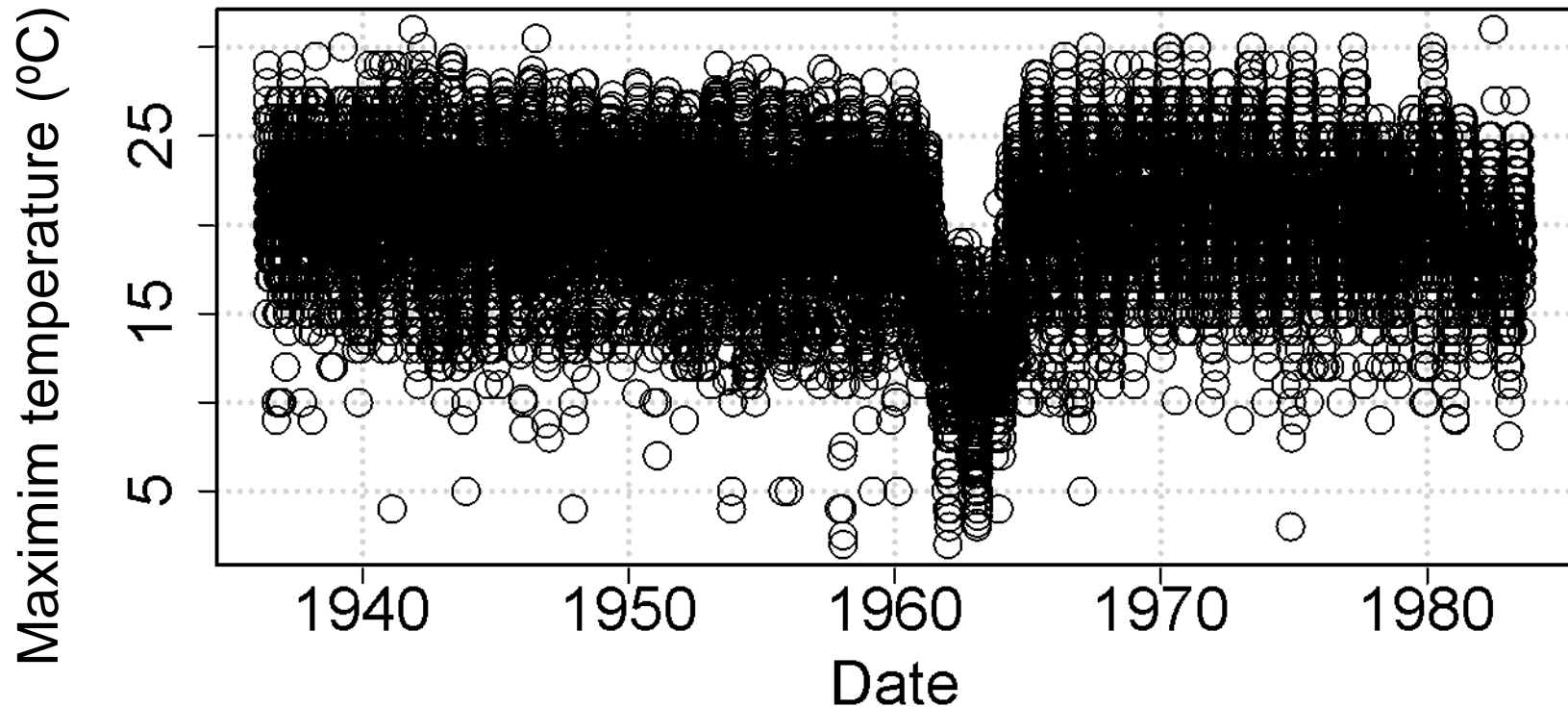
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Real cases

Temperature: CASE 1

Inhomogeneities in temperature

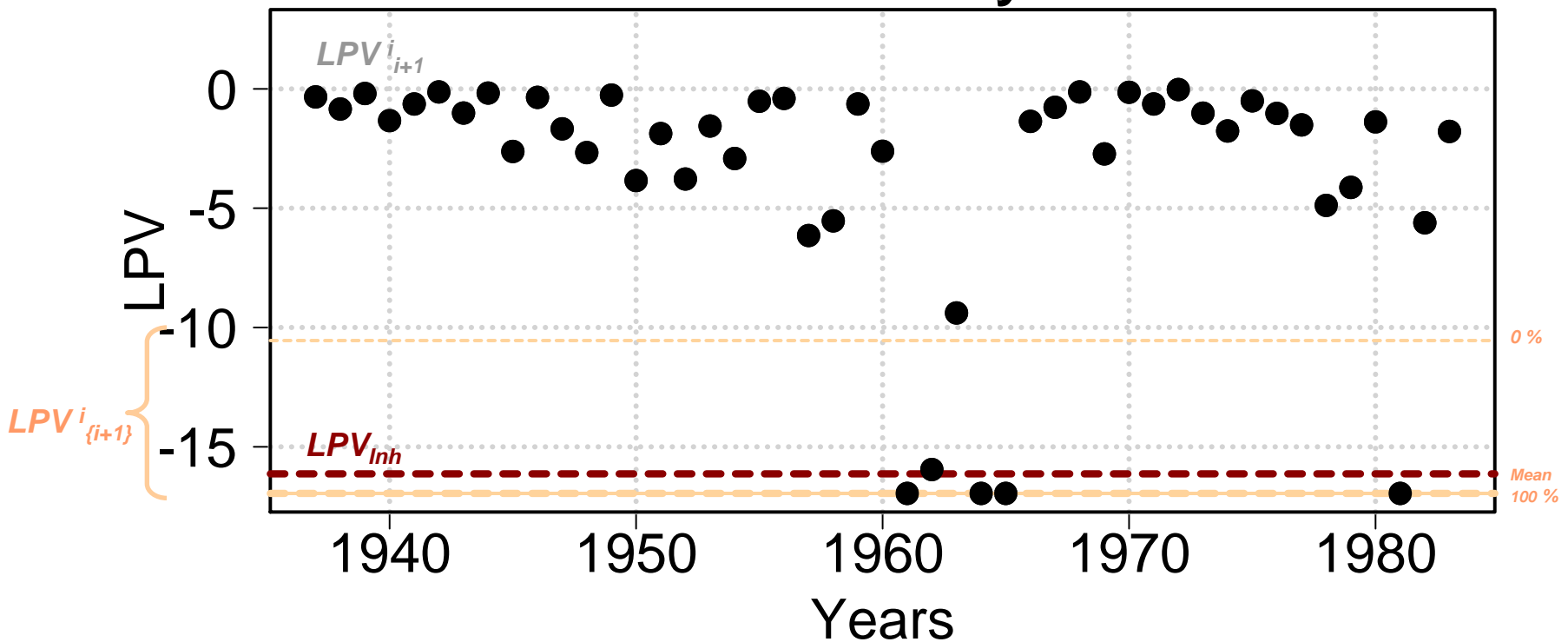


Real cases

Temperature: CASE 1

Inhomogeneities in temperature: LPV diagrams

Control Analysis

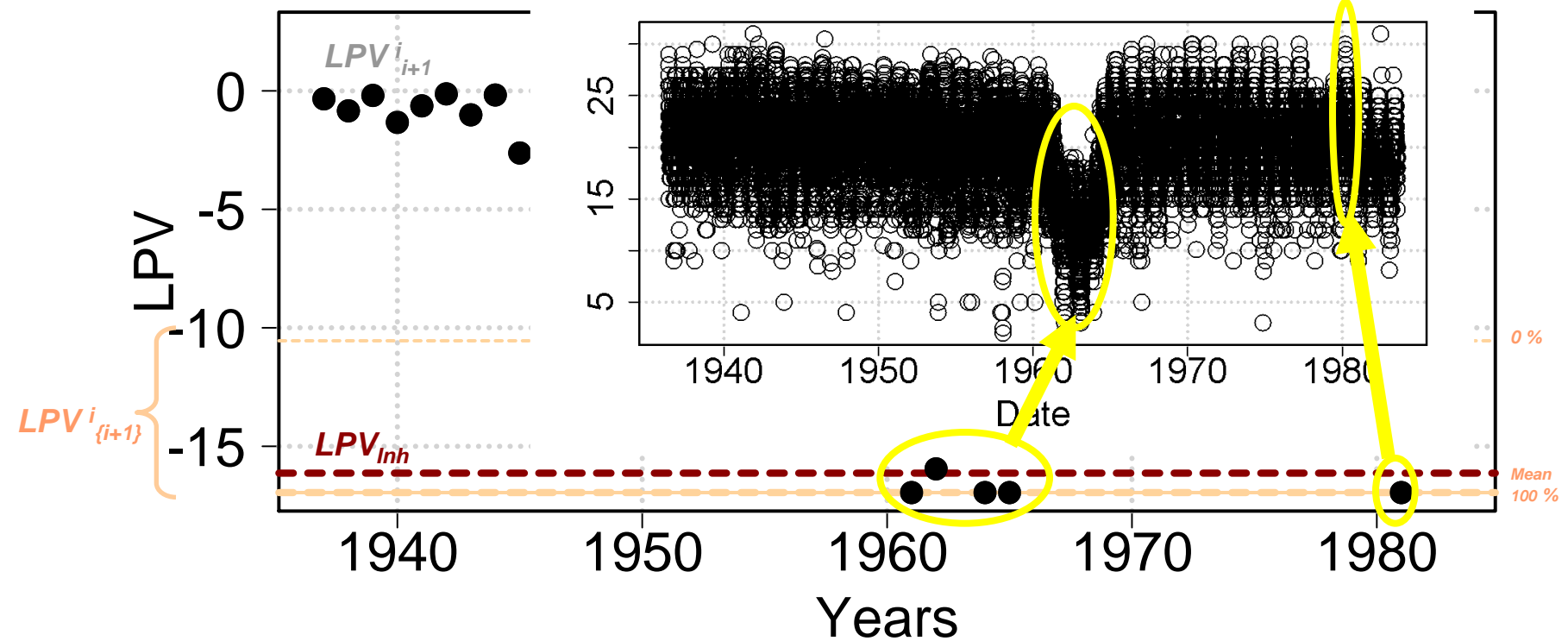


Real cases

Temperature: CASE 1

Inhomogeneities in temperature: LPV diagrams

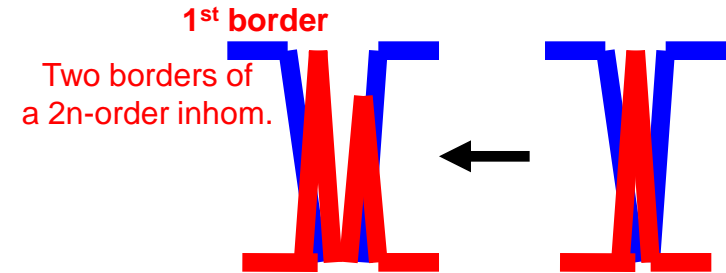
Control Analysis



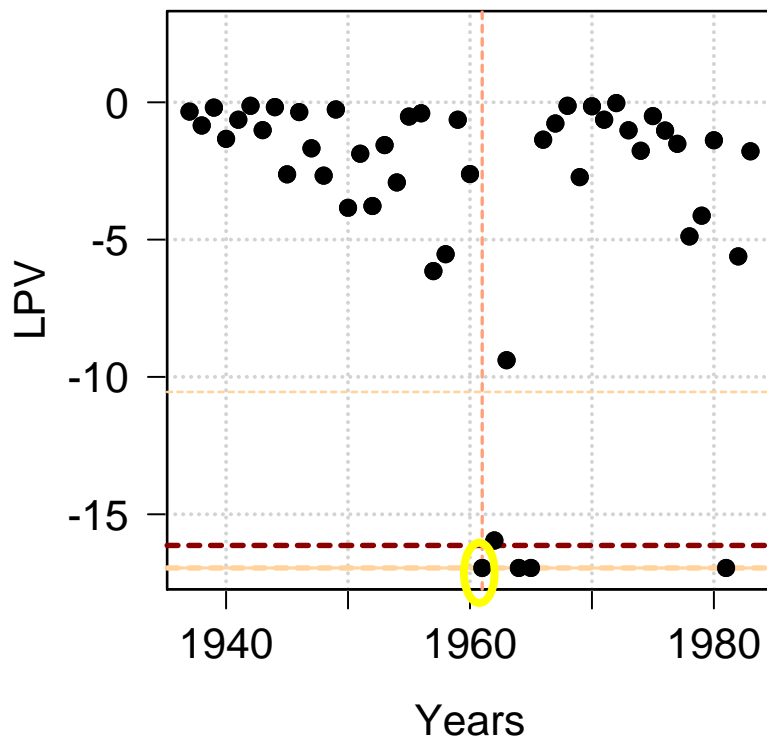
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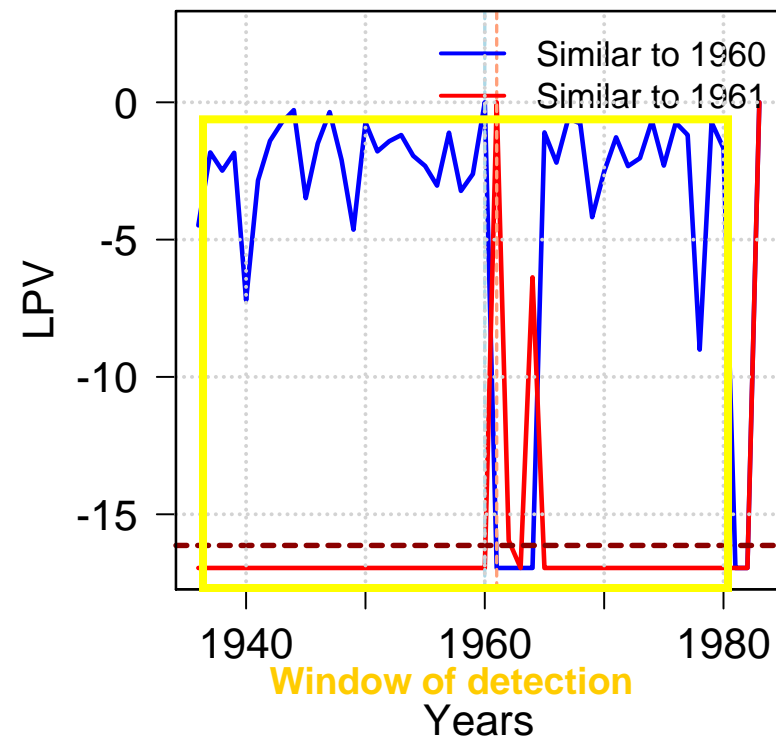
Inhomogeneities in temperature: LPV diagrams



Control Analysis



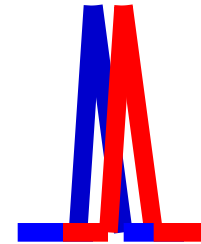
Similarity between years



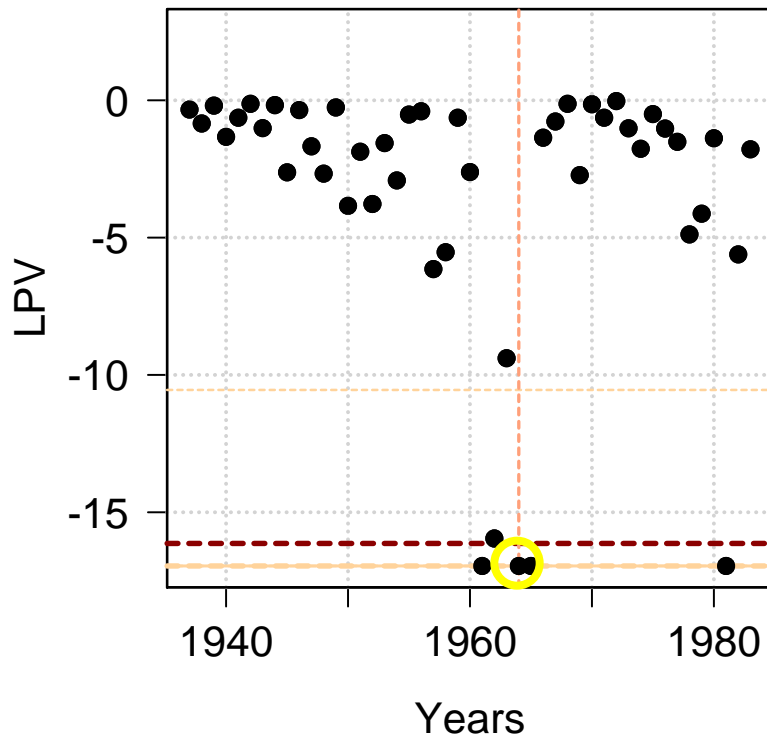
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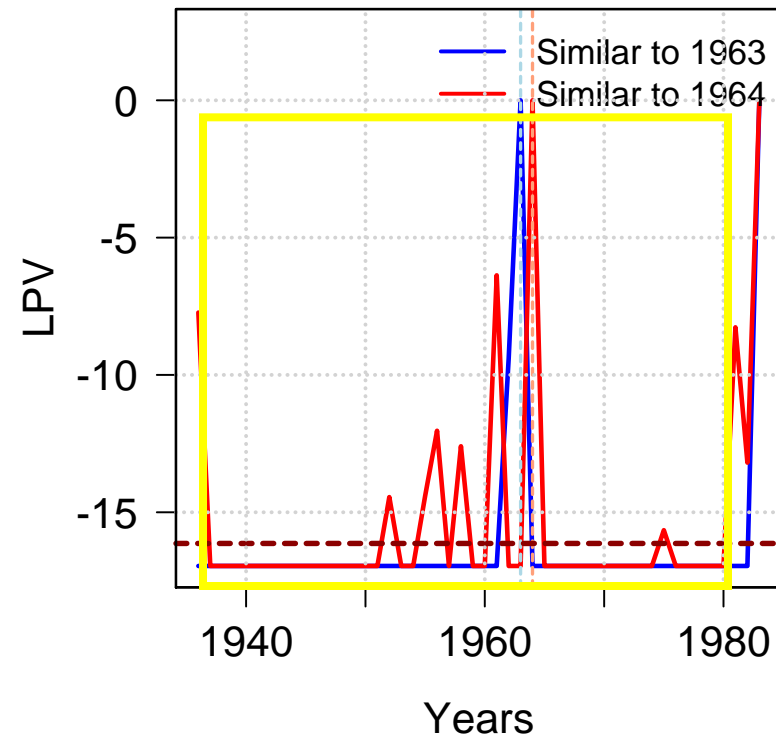
Inhomogeneities in temperature: LPV diagrams



Control Analysis



Similarity between years



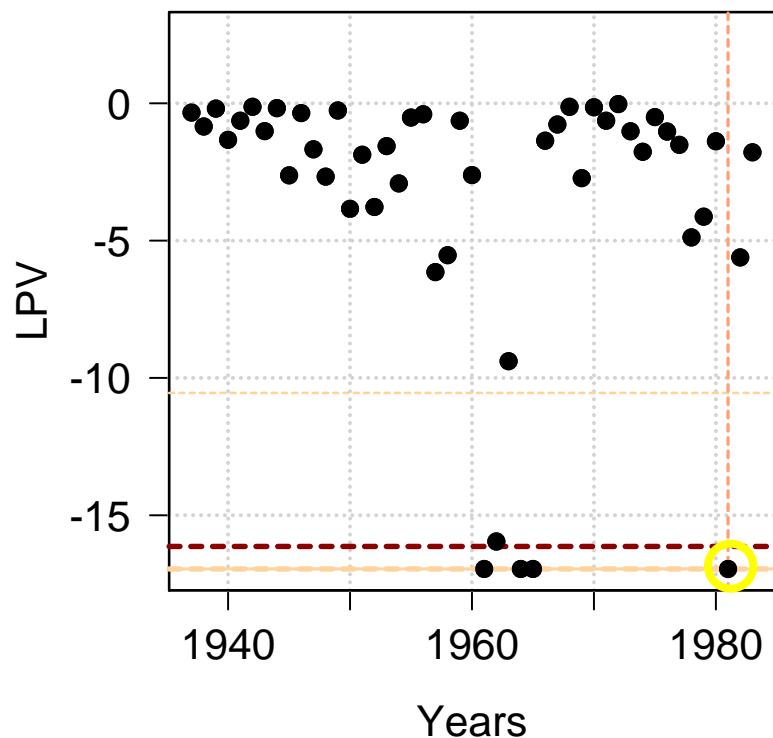
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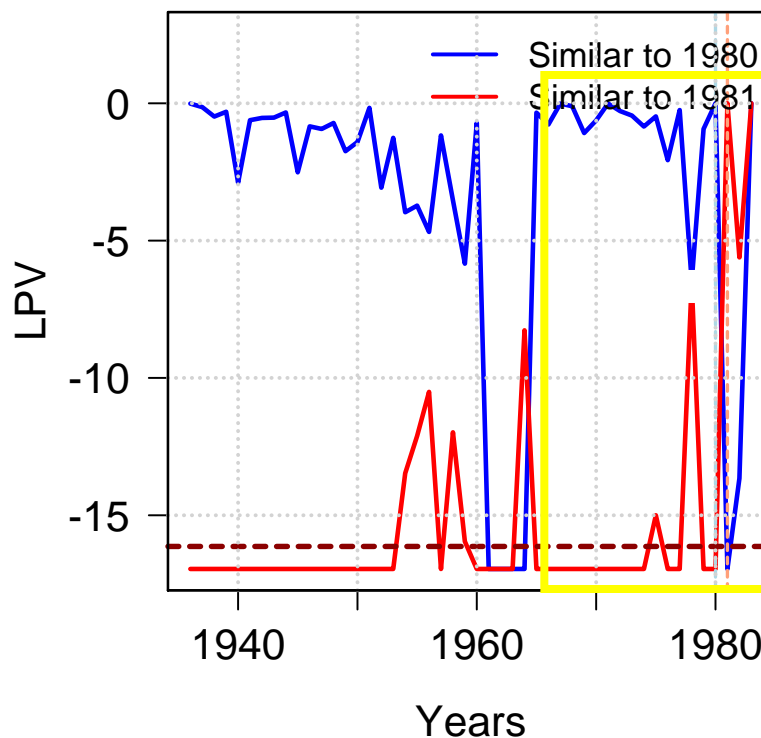
Inhomogeneities in temperature: LPV diagrams



Control Analysis



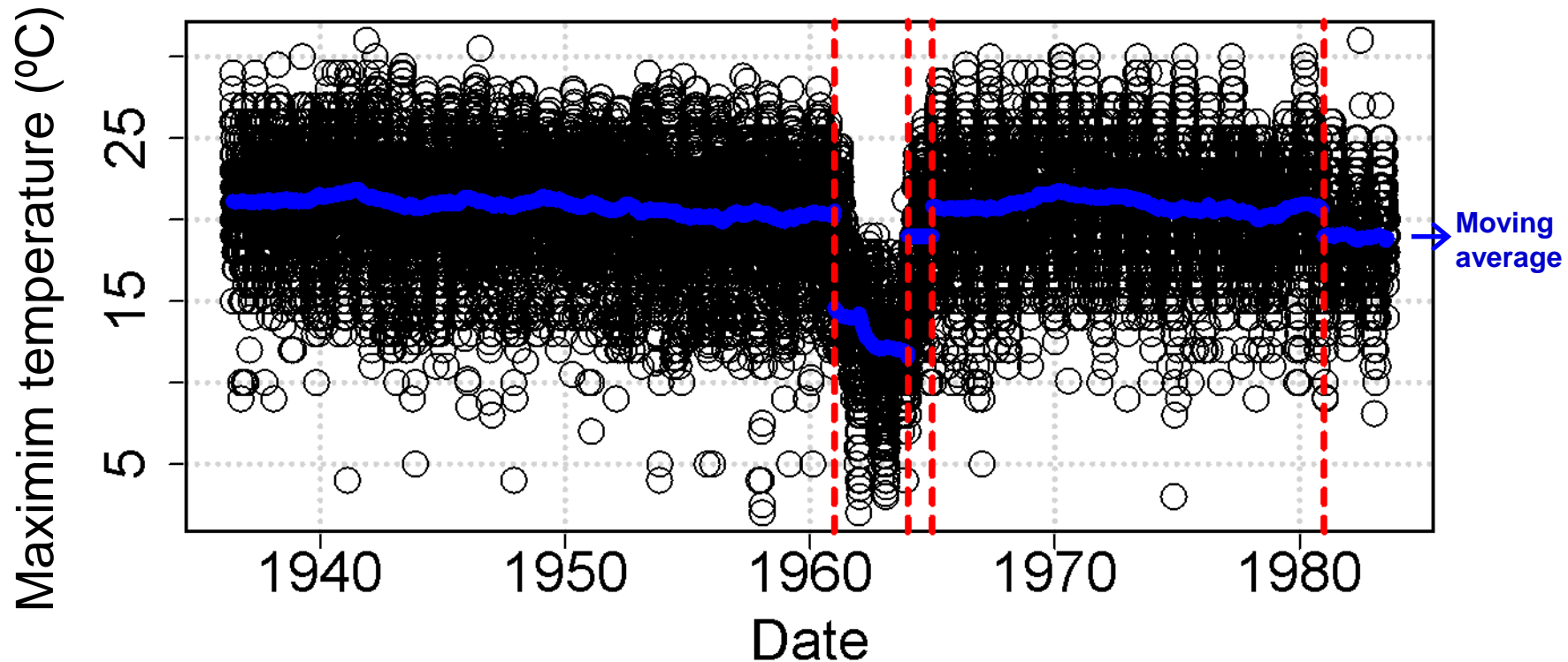
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Real cases

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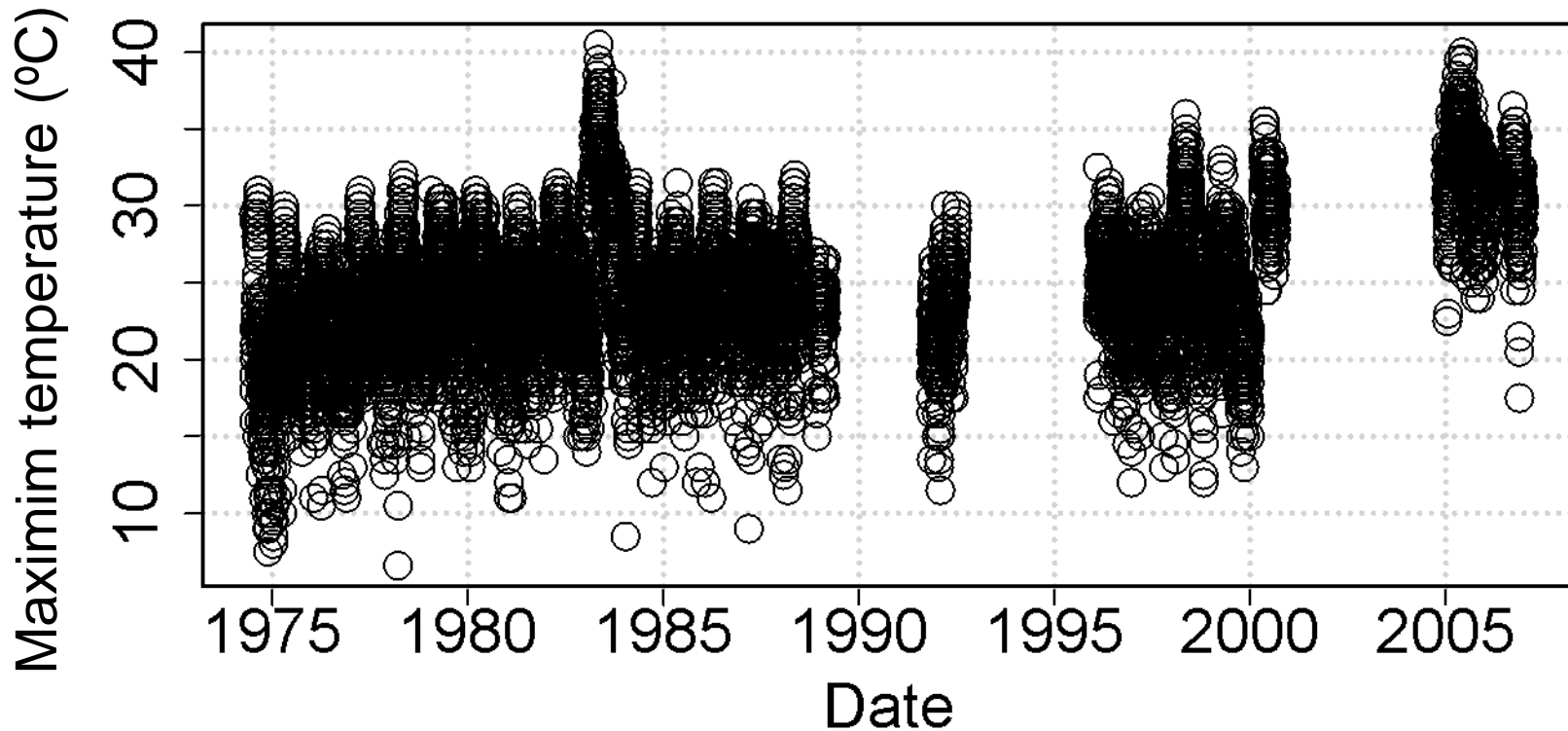
Inhomogeneities in **temperature**: final analysis



Real cases

Temperature: CASE 2

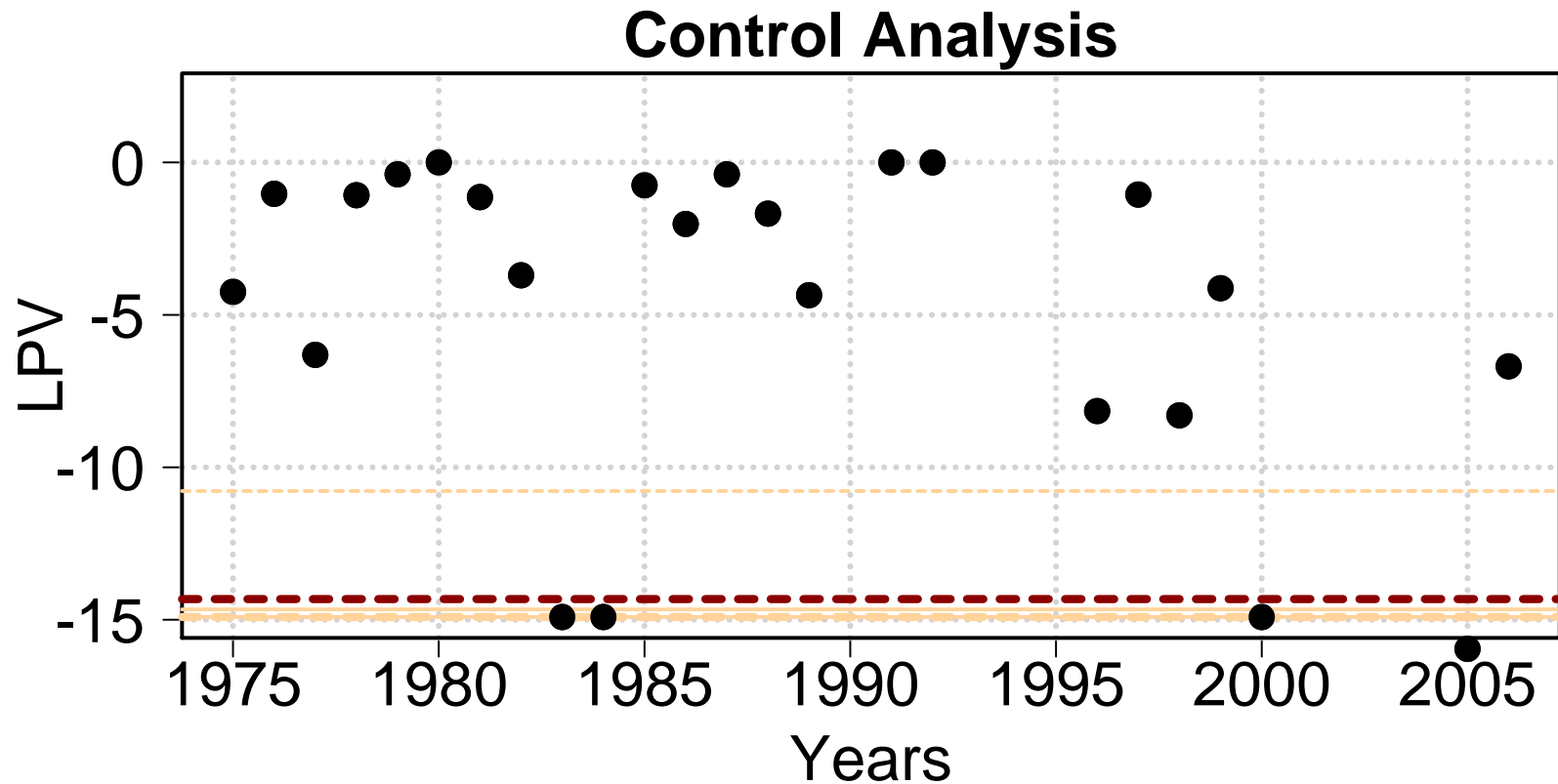
Inhomogeneities in temperature



Real cases

Temperature: CASE 2

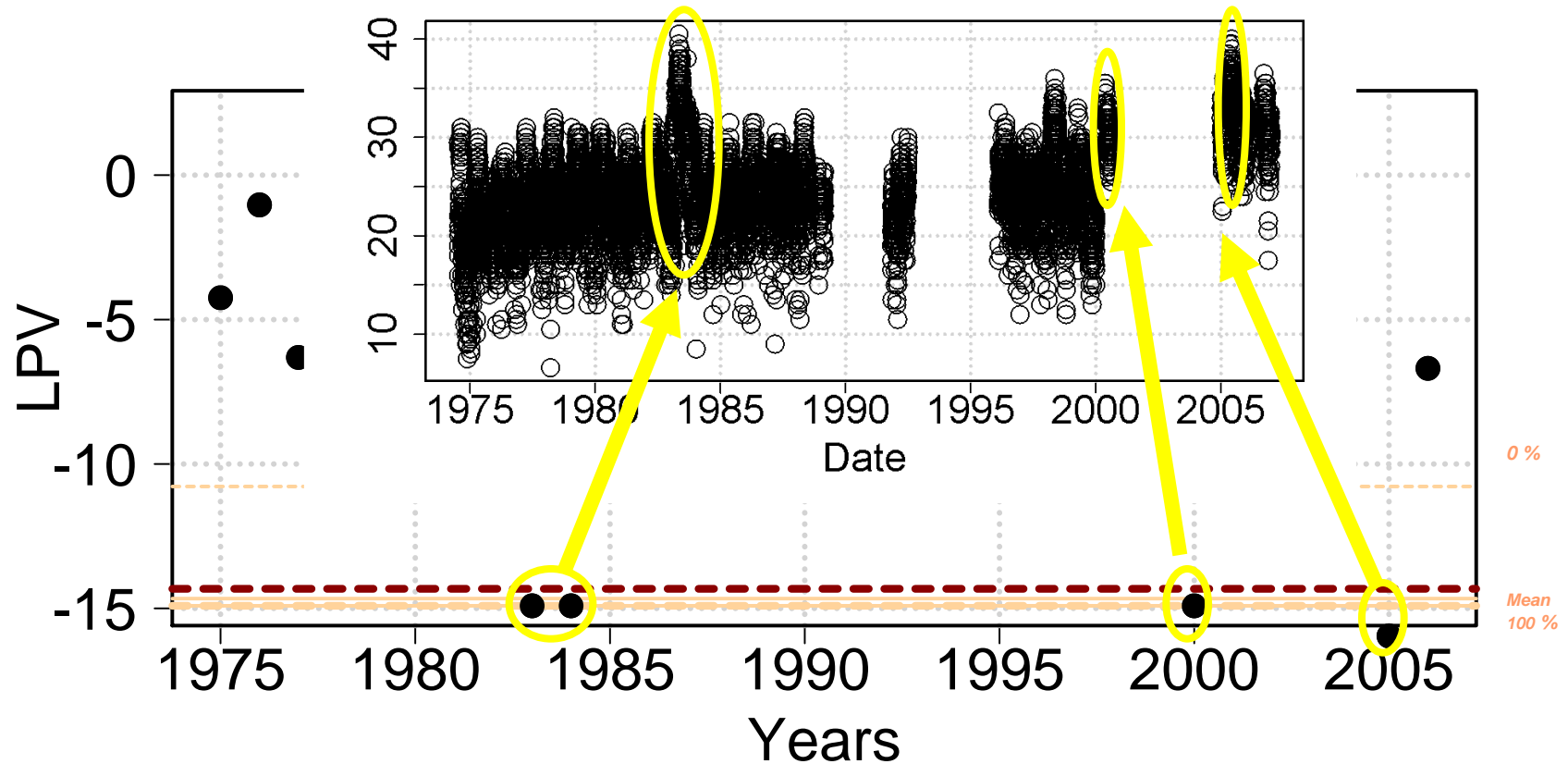
Inhomogeneities in temperature: LPV diagrams



Real cases

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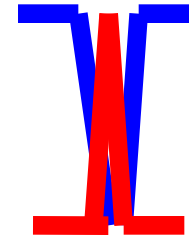
Inhomogeneities in ~~temperature~~ LPV diagrams



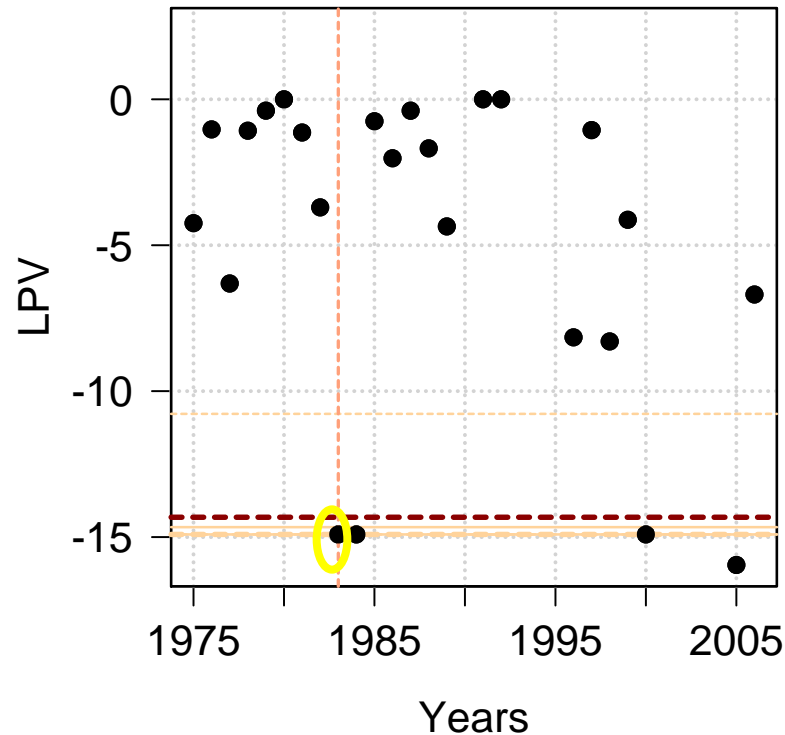
Real cases

Temperature: CASE 2

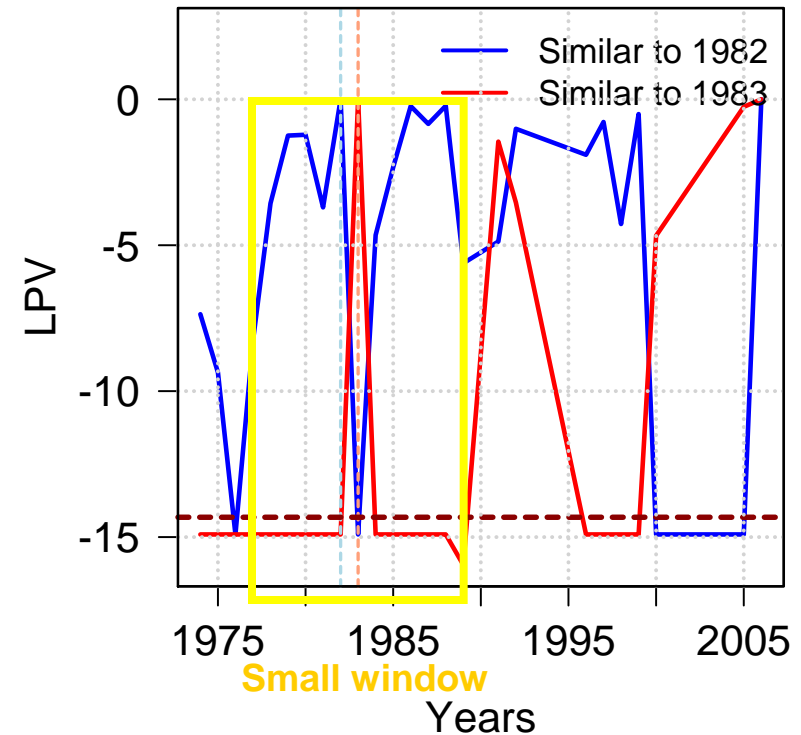
Inhomogeneities in temperature: LPV diagrams



Control Analysis



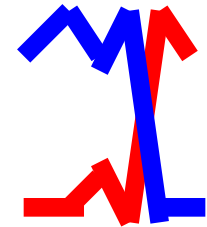
Similarity between years



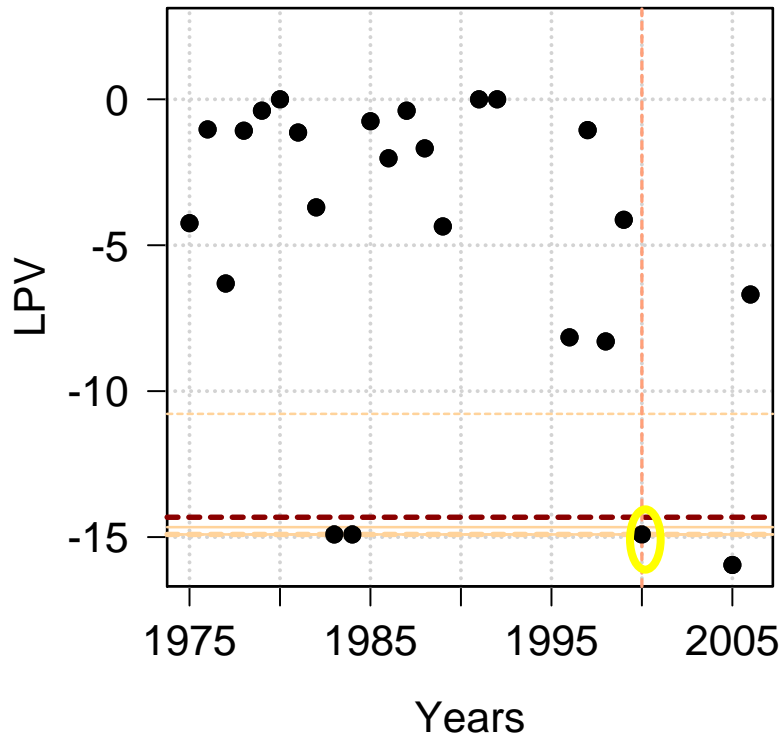
Real cases

Temperature: CASE 2

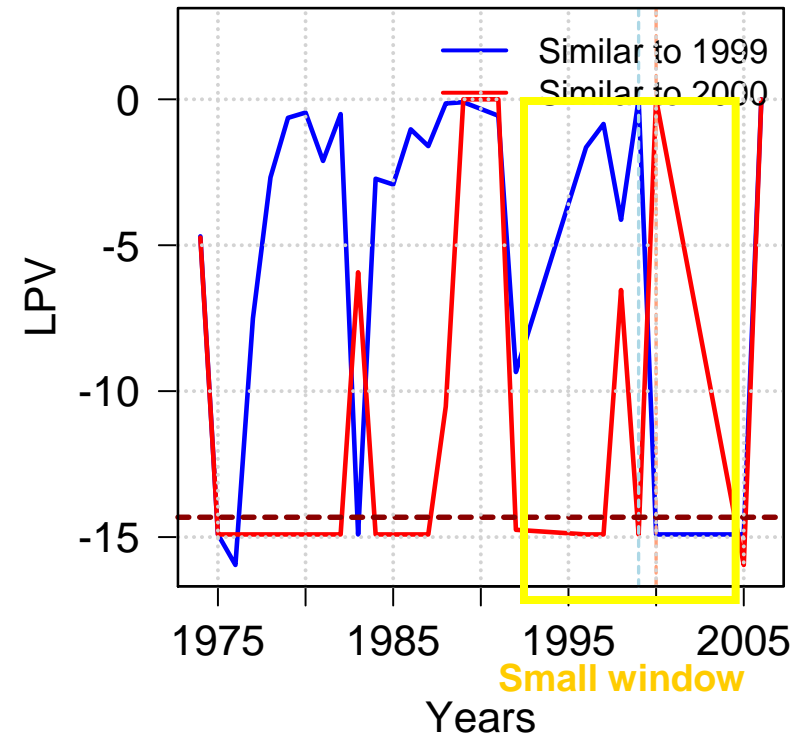
Inhomogeneities in temperature: LPV diagrams



Control Analysis



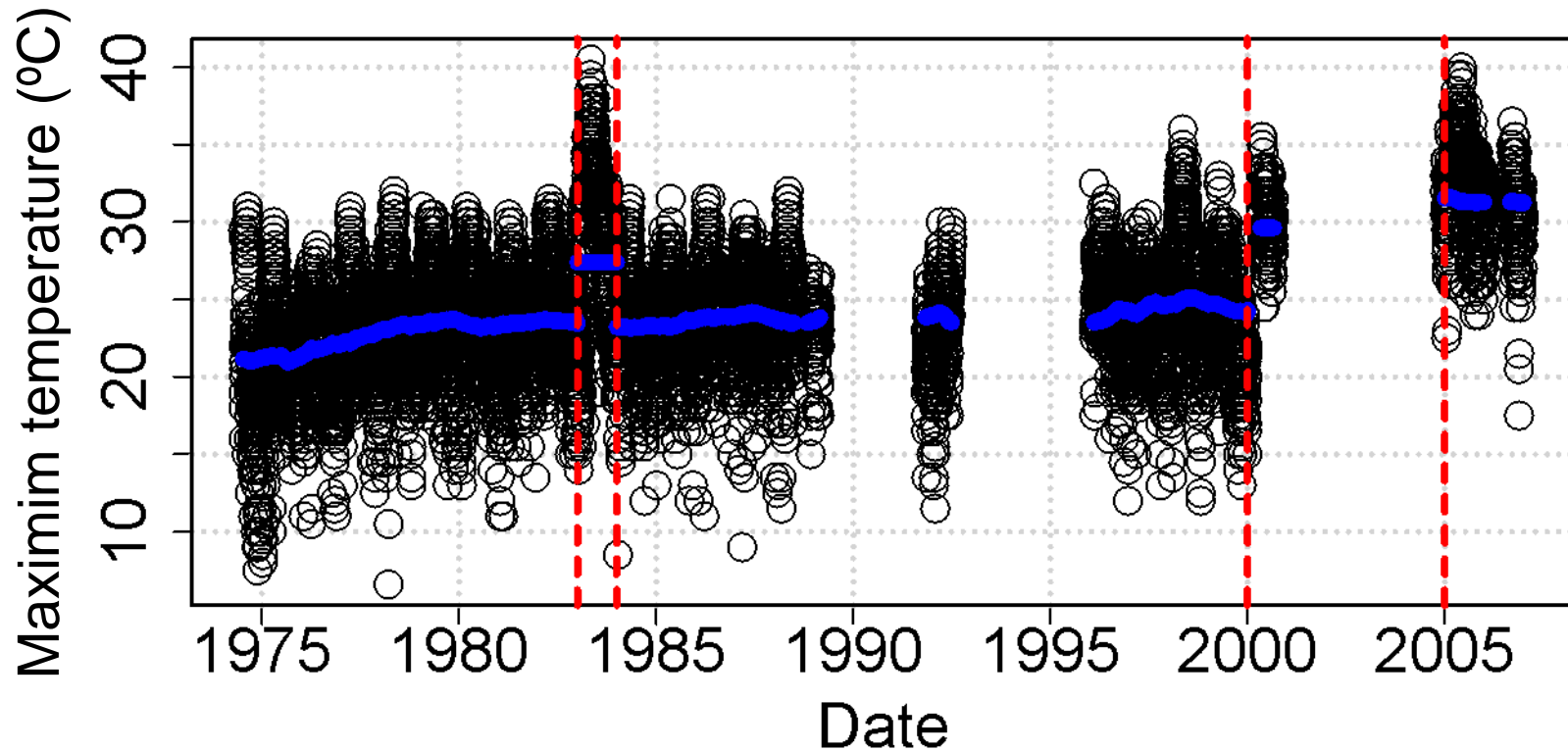
Similarity between years



Real cases

Temperature: CASE 2

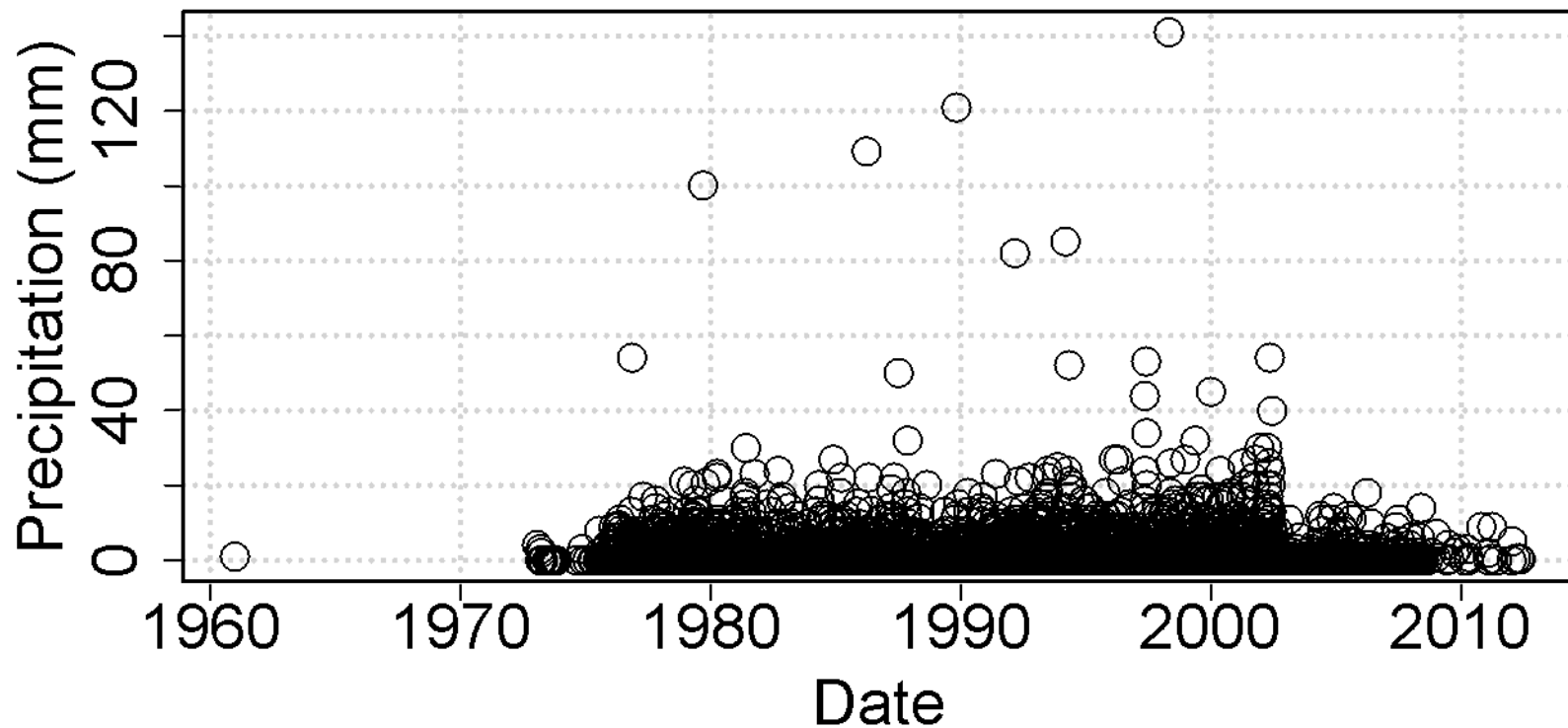
Inhomogeneities in **temperature**: final analysis



Real cases

Precipitation: CASE 1

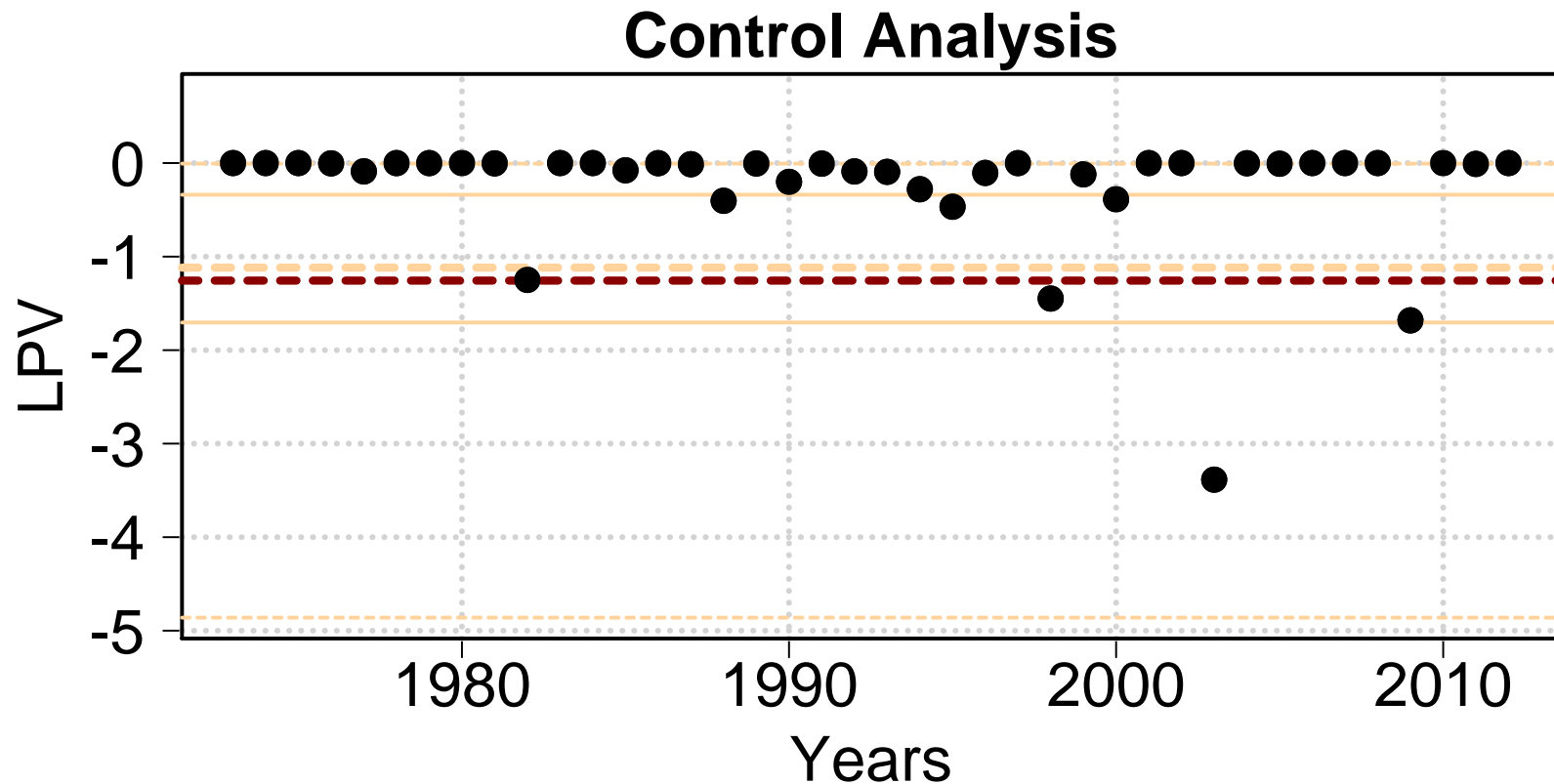
Inhomogeneities in precipitation



Real cases

Precipitation: CASE 1

Inhomogeneities in precipitation: LPV diagrams

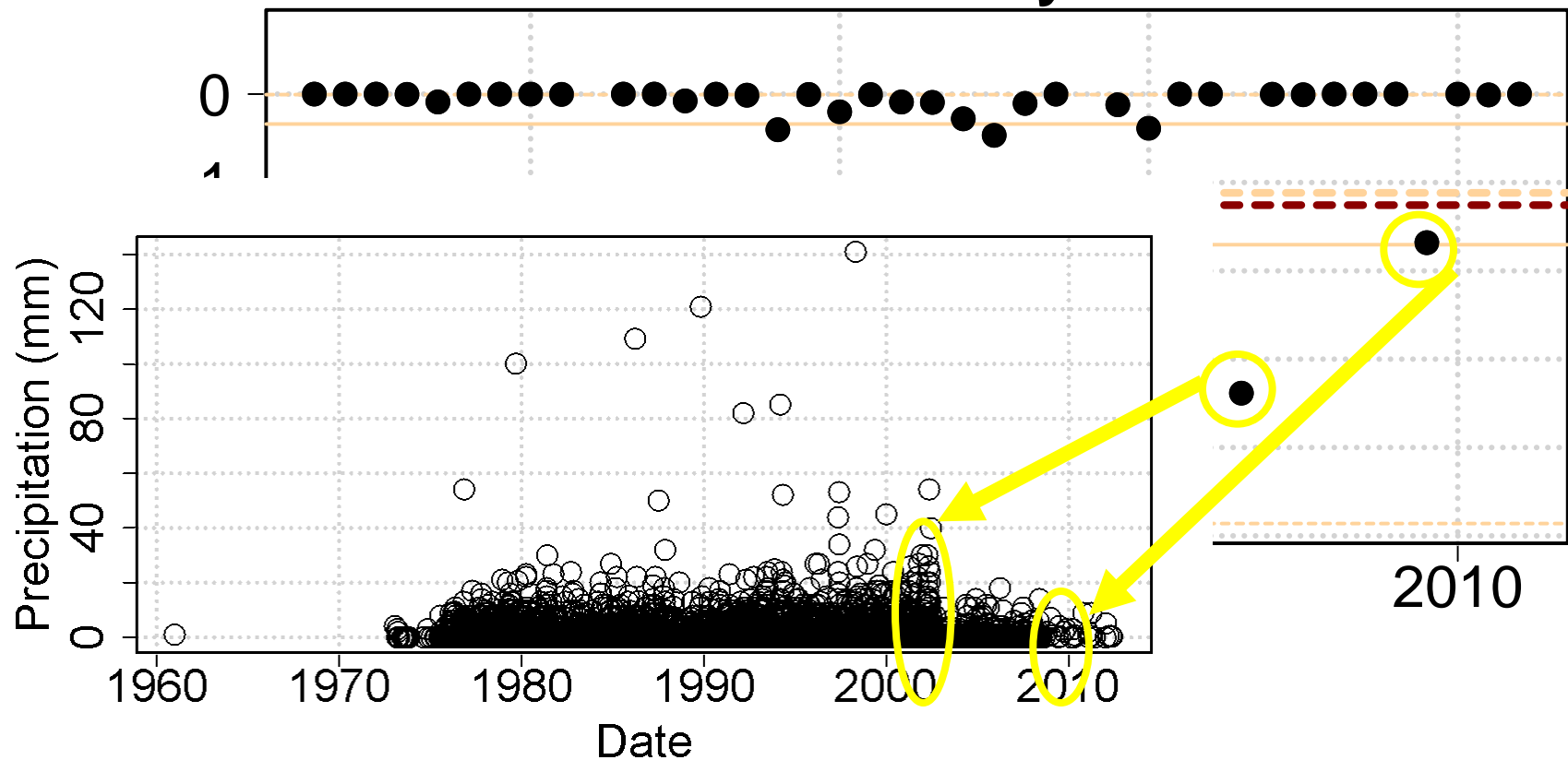


Real cases

Precipitation: CASE 1

Inhomogeneities in precipitation: LPV diagrams

Control Analysis

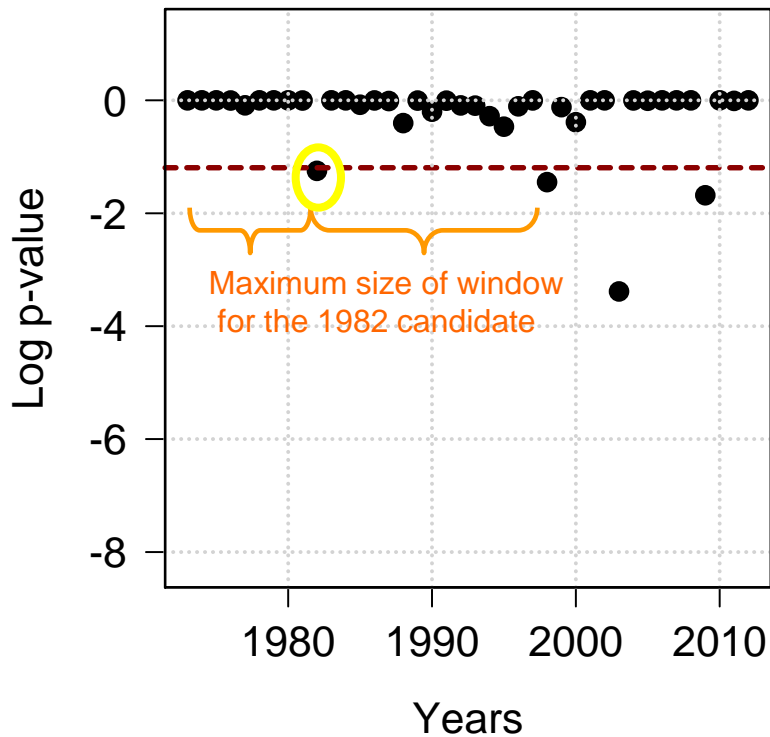


Real cases

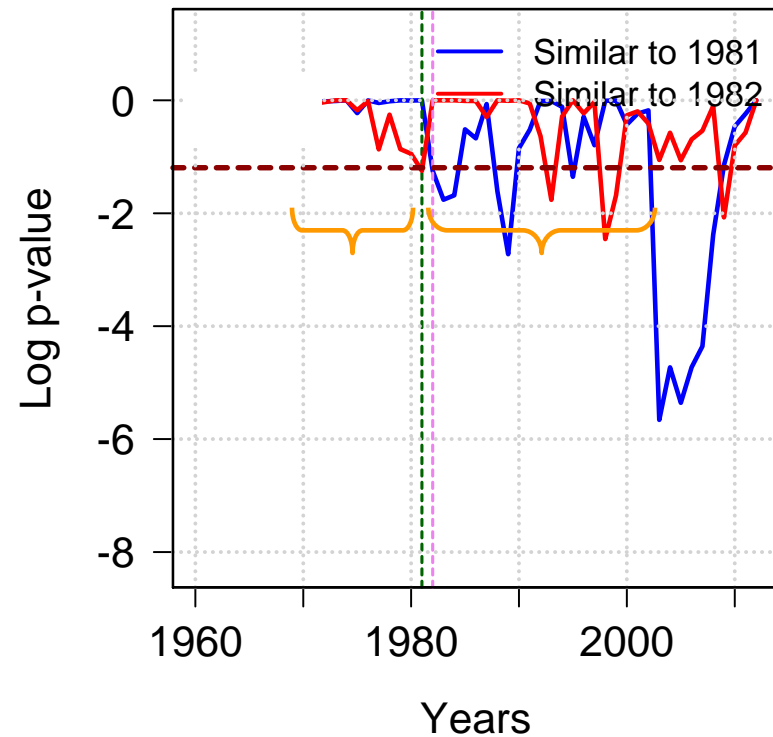
Precipitation: CASE 1

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



Similarity between years

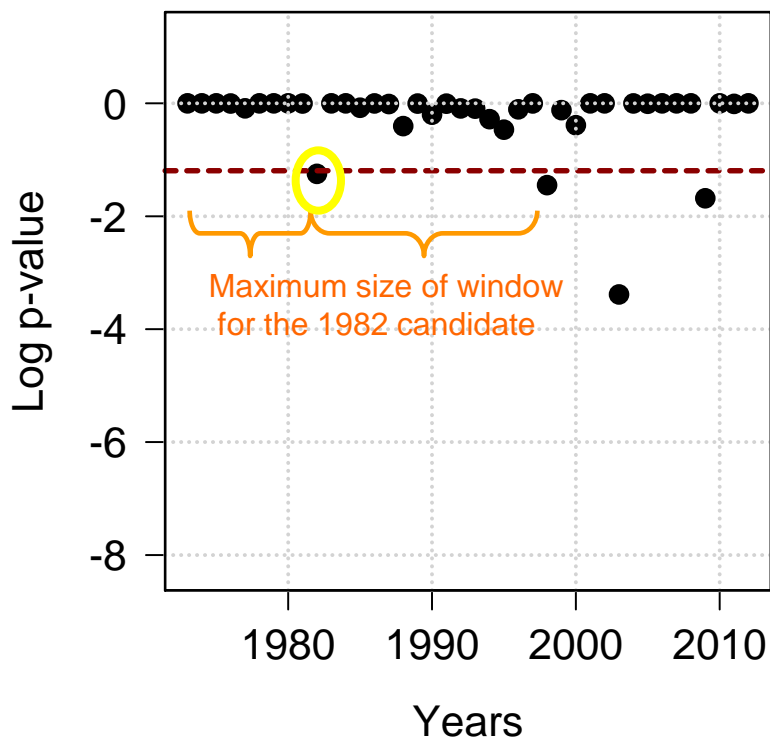


Real cases

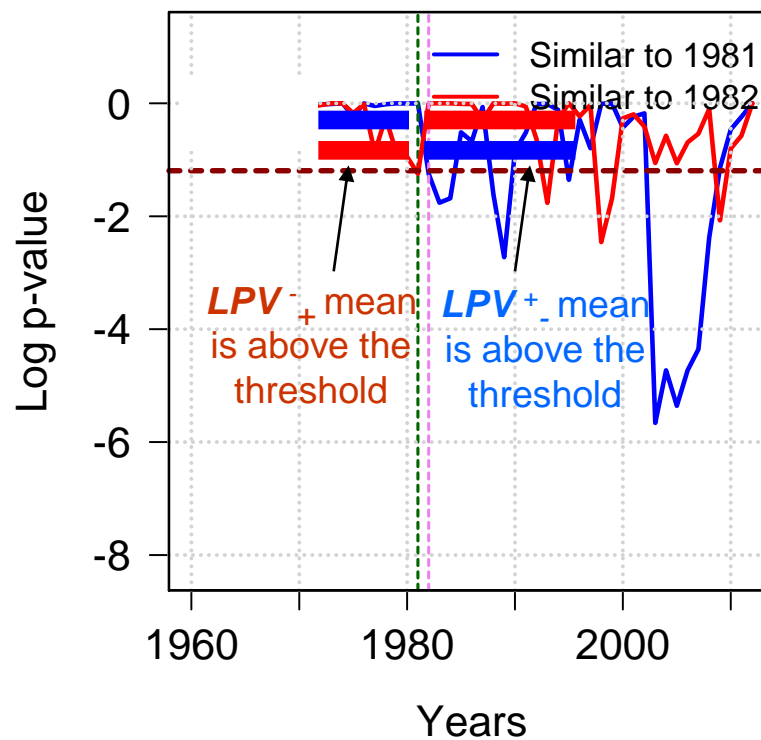
Precipitation: CASE 1

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



Similarity between years



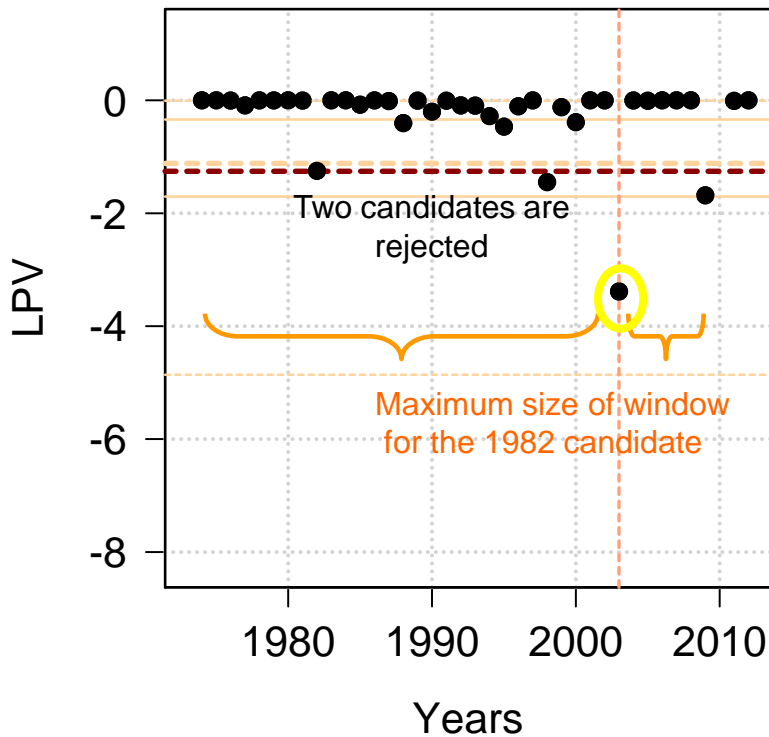
Real cases

Precipitation: CASE 1

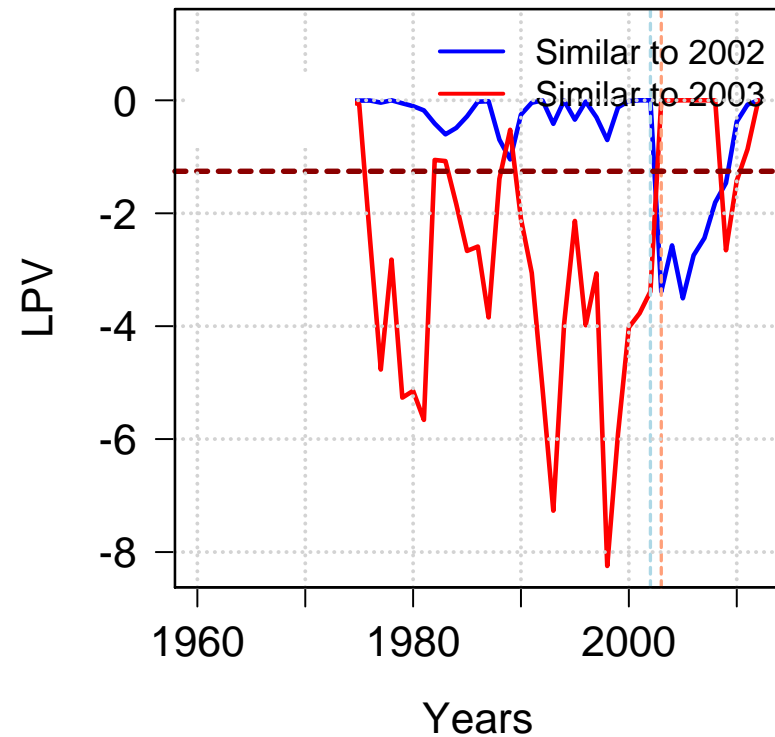
Inhomogeneities in precipitation: LPV diagrams



Control Analysis



Similarity between years



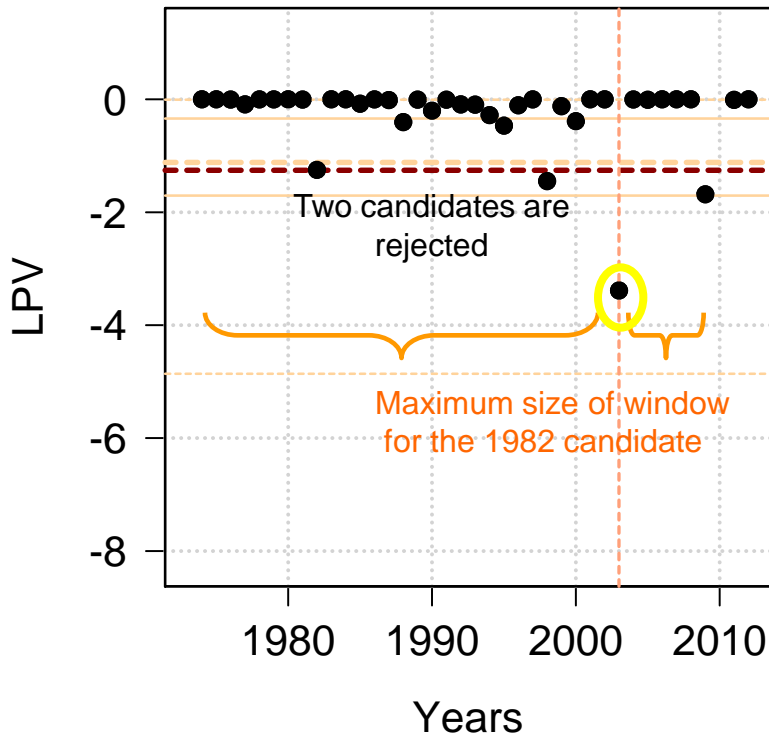
Real cases

Precipitation: CASE 1

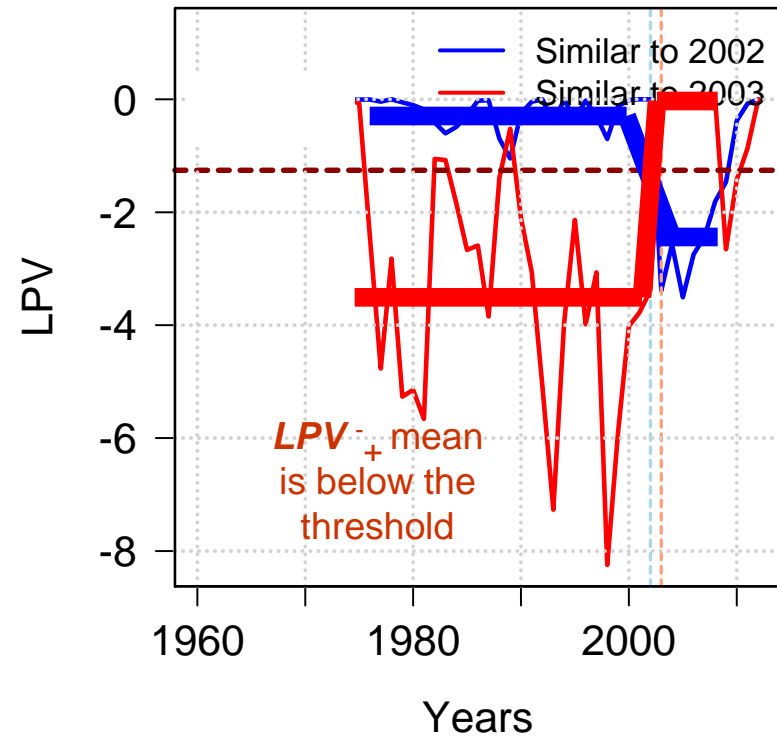
Inhomogeneities in precipitation: LPV diagrams



Control Analysis



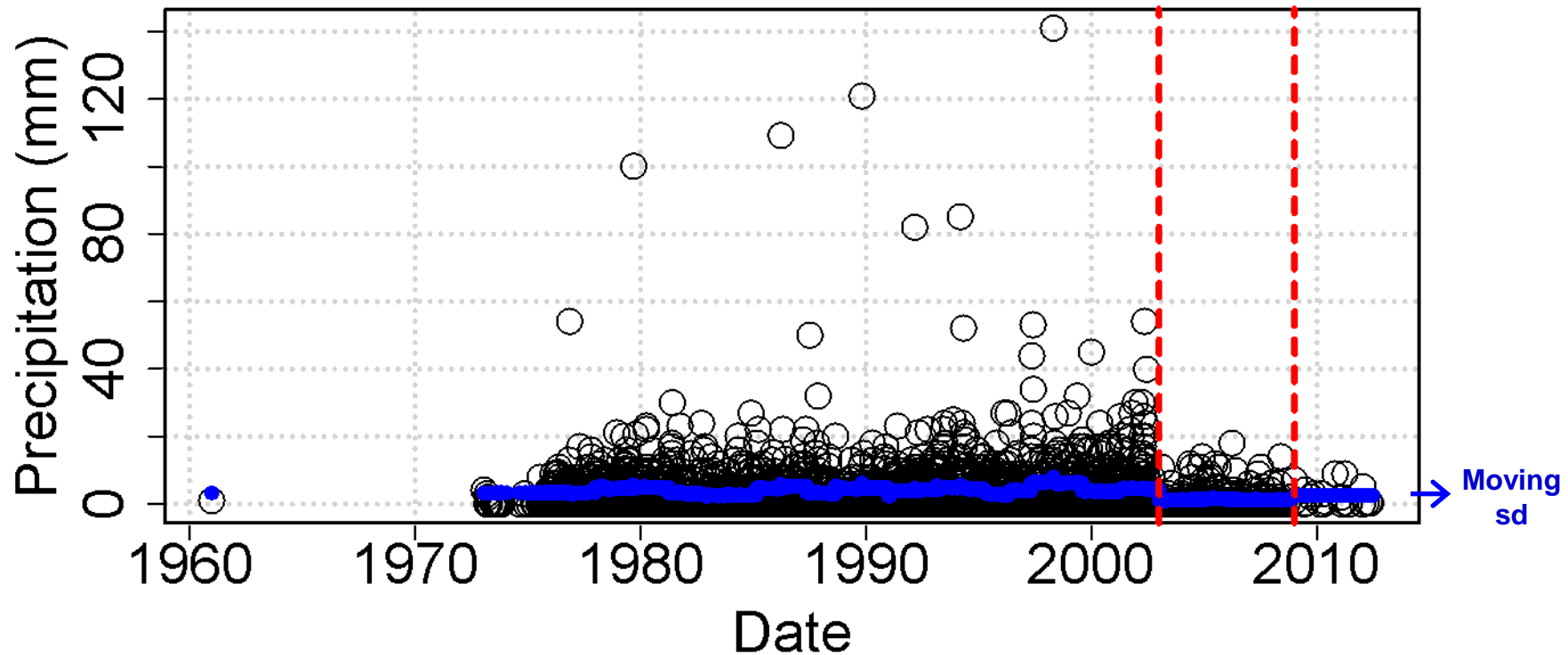
Similarity between years



Real cases

Precipitation: CASE 1

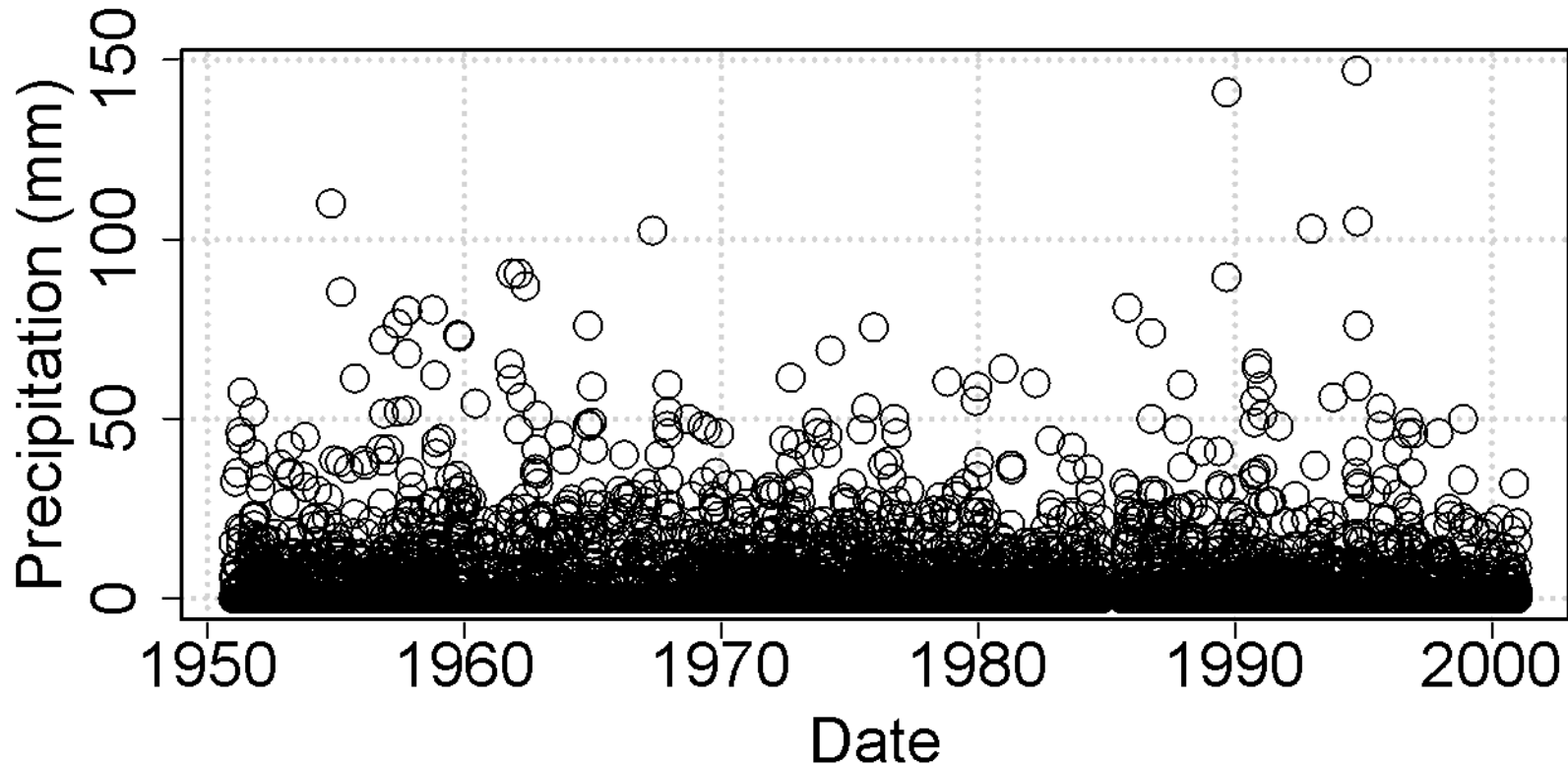
Inhomogeneities in **precipitation**: final analysis



Real cases

Precipitation: CASE 2

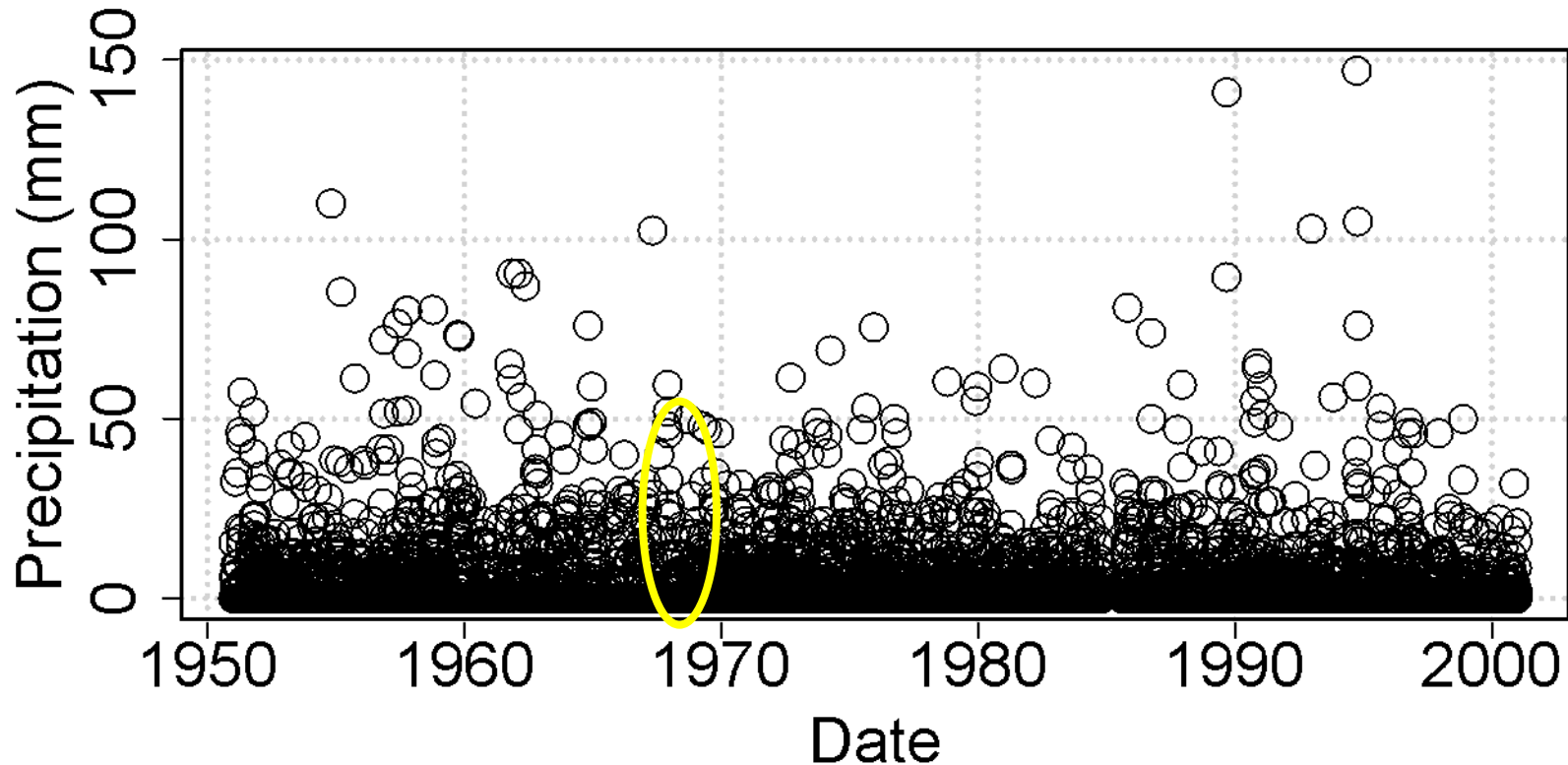
Inhomogeneities in precipitation



Real cases

Precipitation: CASE 2

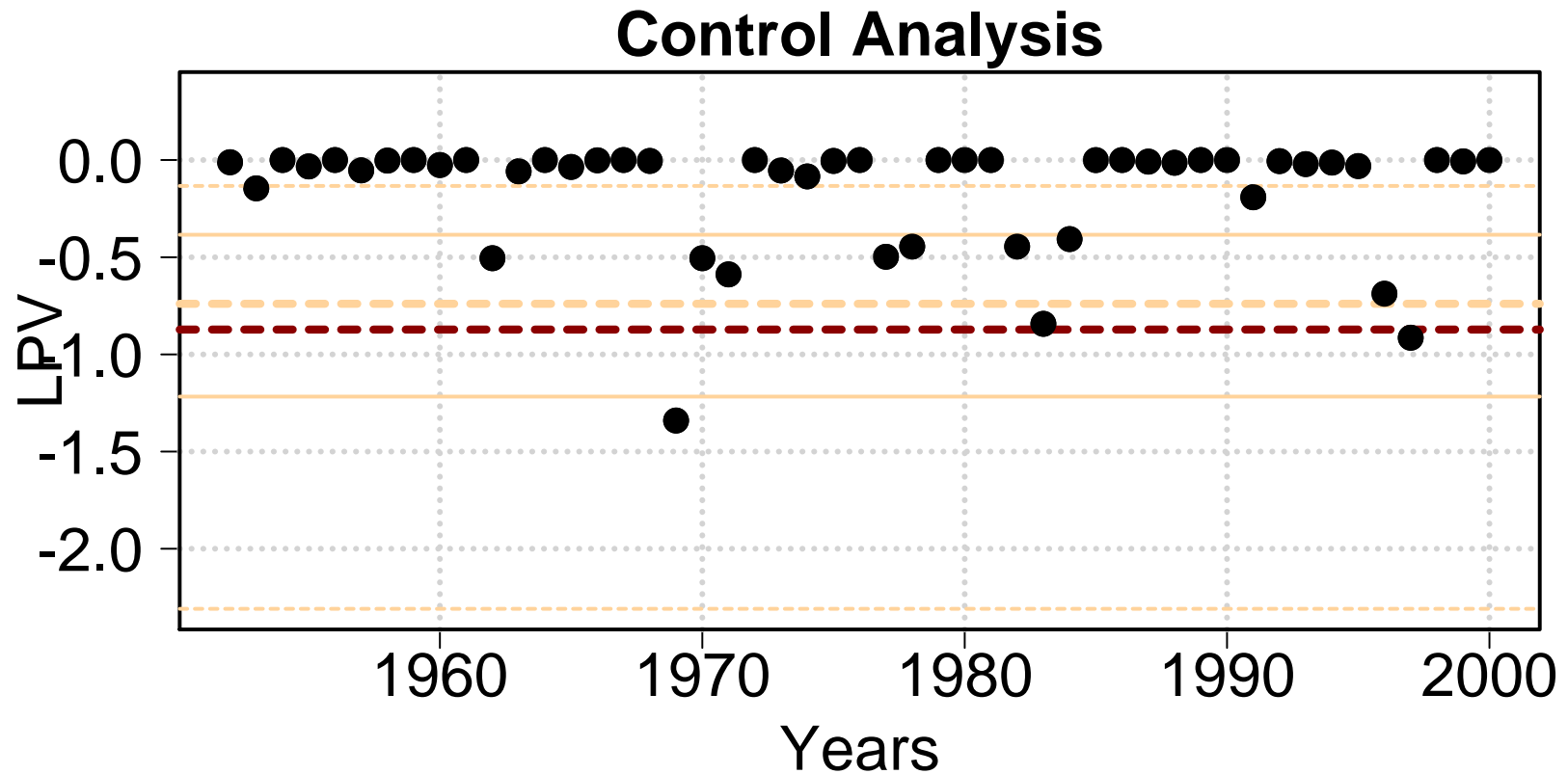
Inhomogeneities in precipitation



Real cases

Precipitation: CASE 2

Inhomogeneities in precipitation: LPV diagrams

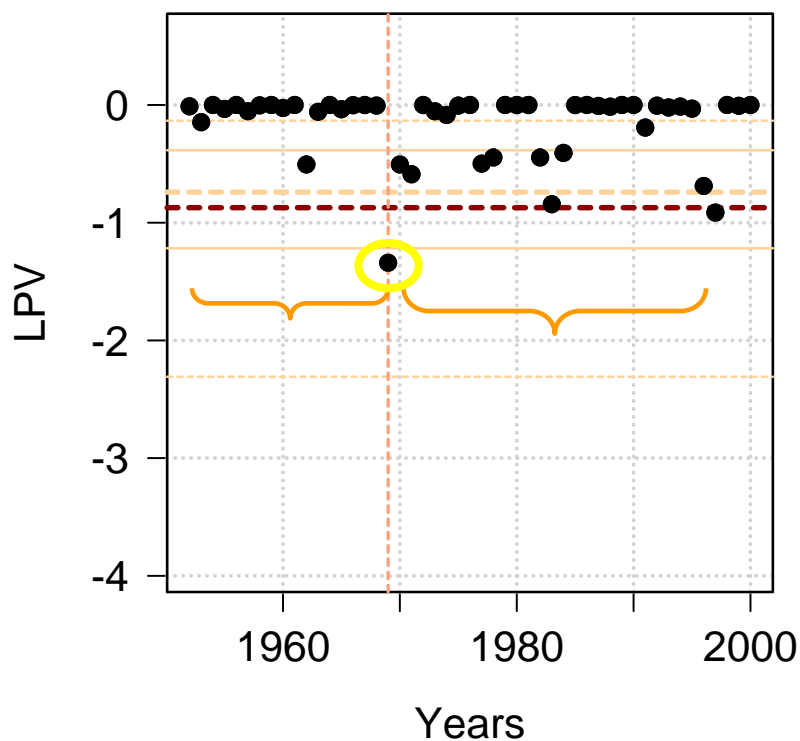


Real cases

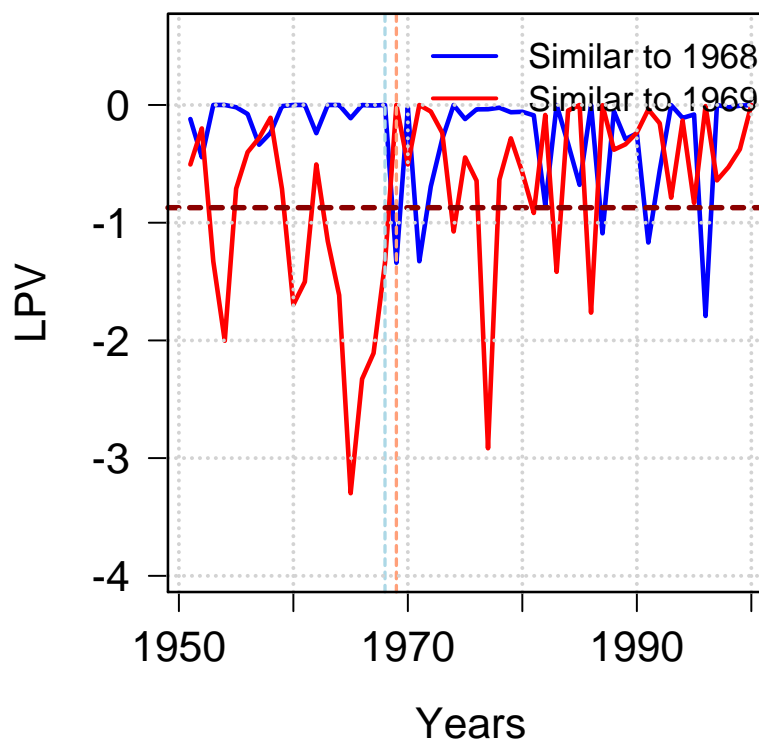
Precipitation: CASE 2

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



Similarity between years

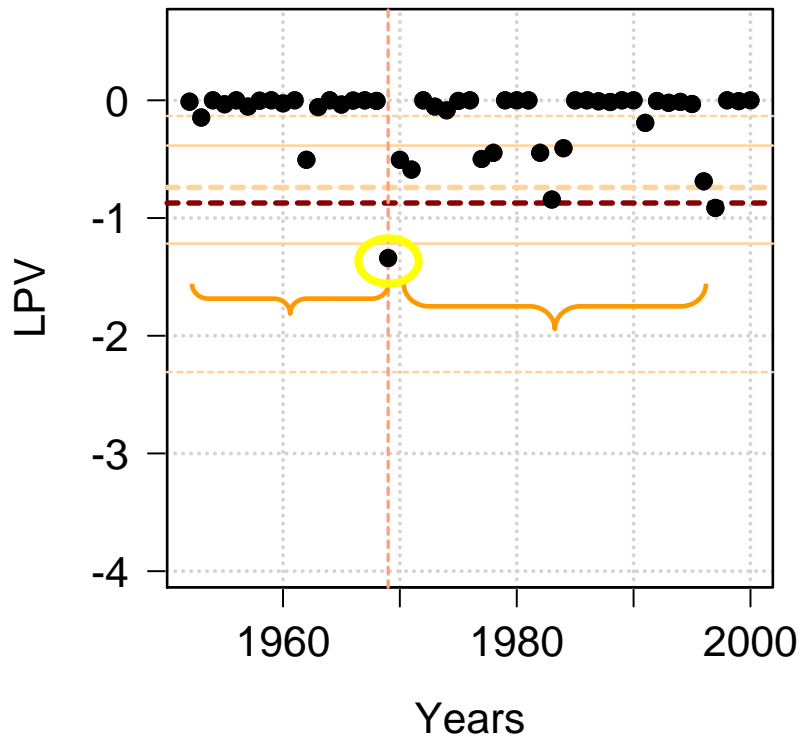


Real cases

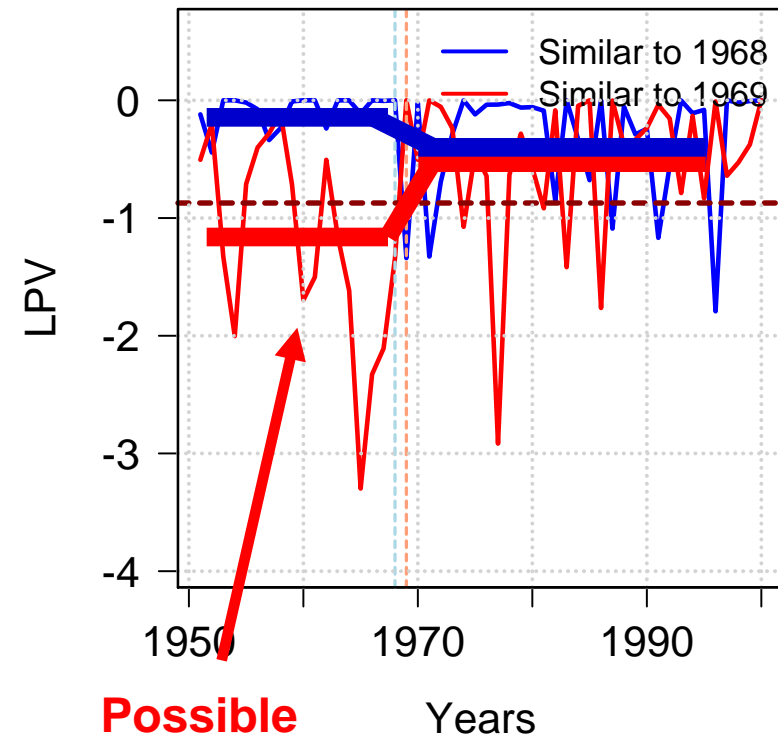
Precipitation: CASE 2

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



Similarity between years



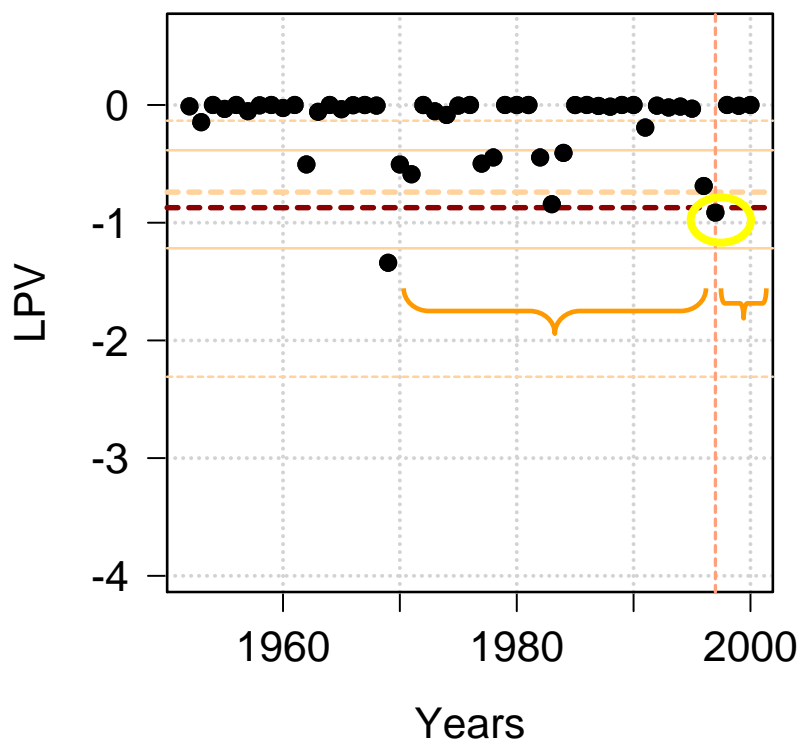
Possible inhomogeneity

Real cases

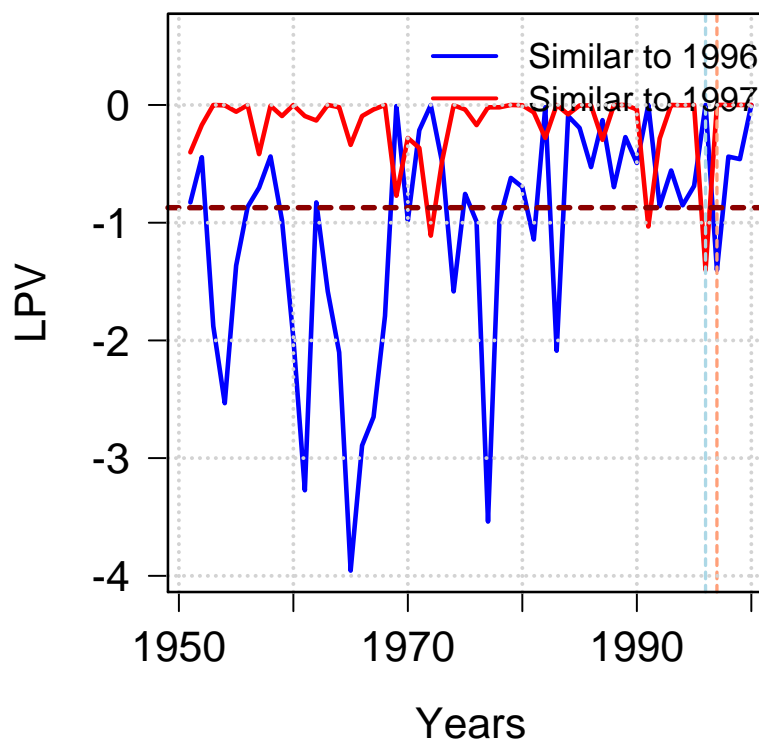
Precipitation: CASE 2

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



Similarity between years

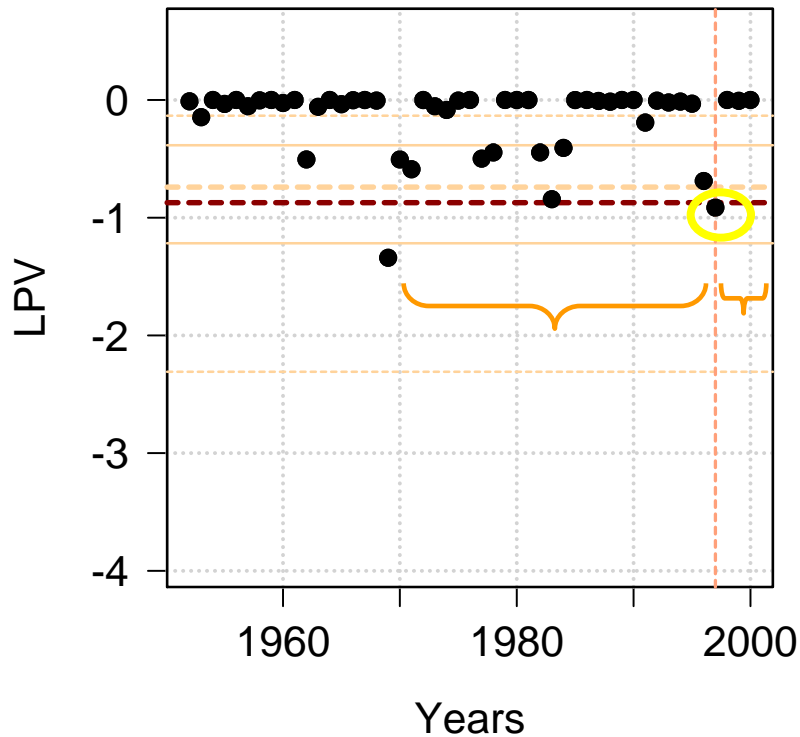


Real cases

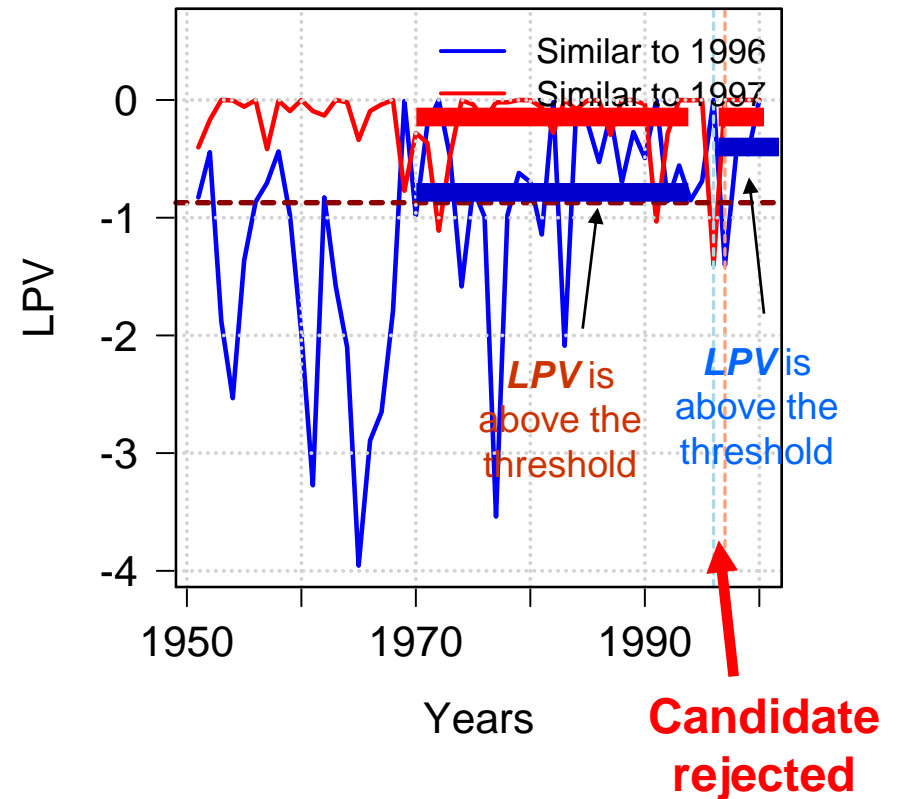
Precipitation: CASE 2

Inhomogeneities in precipitation: LPV diagrams

Control Analysis



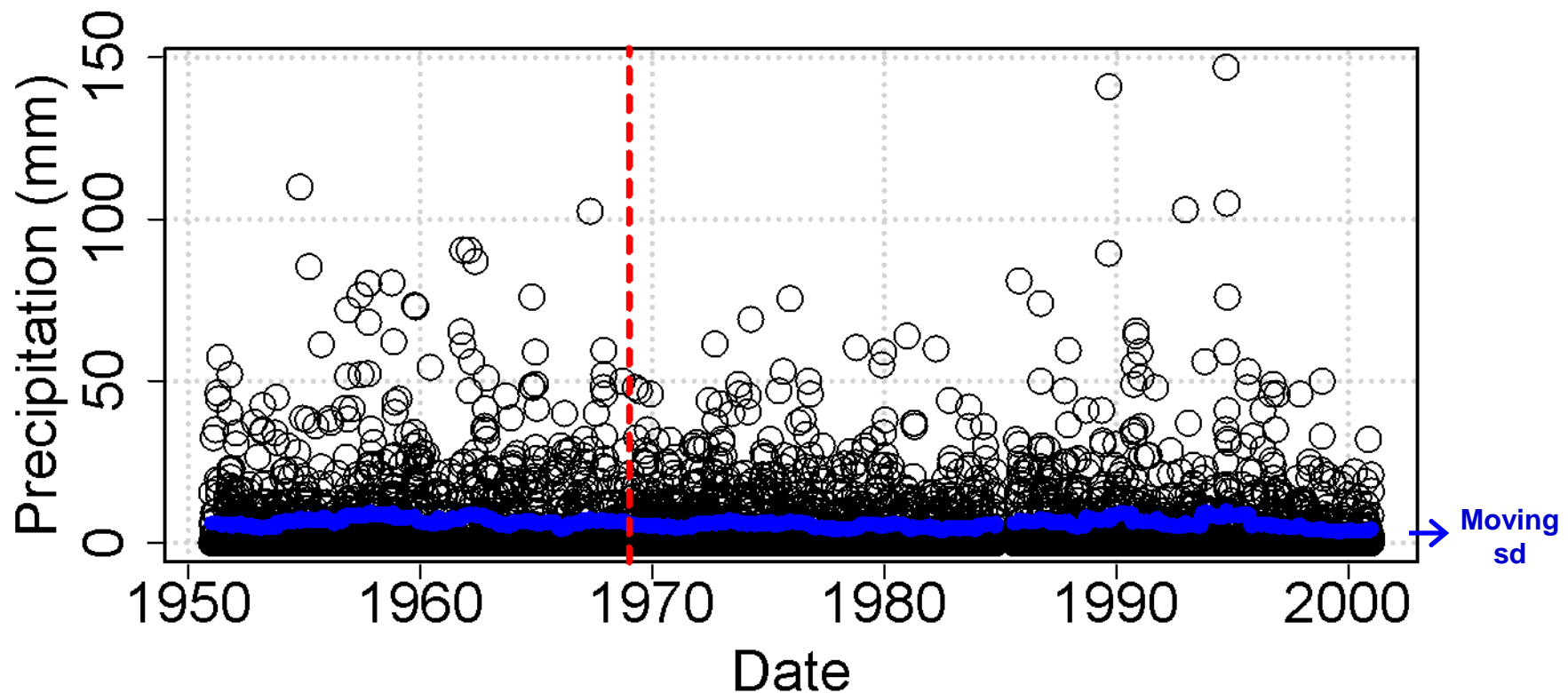
Similarity between years



Real cases

Precipitation: CASE 2

Inhomogeneities in **precipitation**: final analysis



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Discussion

- Selection of non-parametric test:

- Kolmogorov-Smirnov: It is adequate for temperature and precipitation.
- Other? Anderson-Darling: For precipitation is too much sensitive due to the high natural variability (takes a lot of extremes as “unusual year”)

Discussion

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- Introduction of the artificial inhomogeneity: Choice of parameters of the control ($a=?$, $b=?$). Our case, $a = 3$ for precipitation, $b = 2$ for temperature
- Percentile of the LPV of reference. Level of confidence for a candidate? For precipitation can be important due to the high uncertainty.

Discussion

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- Inhomogeneity detection:

- Inhomogeneity (on the left/right) must be greater than the reference jump
- Size of the detection window is limited by the candidate neighbors.
- Time-series with trends are a priori considered homogenous because increase is soft (no jumps), except if there is a long gap.

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About the methodology

- Non-parametric test of Kolmogorov-Smirnov can be used to detect inhomogeneity for both temperature and precipitation at daily scale.
- A control analysis is required introducing an artificial inhomogeneity (“reference jump”) for comparing with the original time-series: LPV diagrams.
- For temperature, the recommended “reference jump” is 2°C and for precipitation is a multiplication factor of 3.

Conclusions

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About the methodology

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About its application

- Temperature shows a reference LPV with less uncertainty than precipitation.
- Test can detect (systematic) changes in ECDF, not only in the averages
- Some candidates of precipitation can be due to natural variability.
- A more extensive study of real cases could refine the methodology.

Thanks

Thank you for your attention

and
for getting up so early...
... as me.