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THE DYNAMICS OF A RESONANCE SINGLE-MASS VIBRATORY MACHINE WITH A VIBRATION EXCITER OF TARGETED ACTION THAT OPERATES ON THE SOMMERFELD EFFECT

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This paper reports a study into the dynamics of a vibratory machine composed of a viscoelastically-fixed platform that can move vertically and two identical inertial vibration exciters. The vibration exciters' bodies rotate at the same angular velocities in opposite directions. The bodies host a single load in the form of a ball, roller, or pendulum. The loads' centers of mass can move relative to the bodies in a circle with a center on the axis of rotation. The loads' relative movements are hindered by the forces of viscous resistance.

It was established that a vibratory machine theoretically possesses the following:

– one to three oscillatory modes of movement under which loads get stuck at almost constant angular velocity and generate total unbalanced mass in the vertical direction only;

– a no-oscillation mode under which loads rotate synchronously with the bodies and generate total unbalanced mass in the horizontal direction only.

At the same time, only one oscillatory mode is resonant and exists at the above-the-resonance speeds of body rotation, lower than some characteristic speed.

At the bodies' rotation speeds:

– pre-resonant; there is a globally asymptotically stable (the only existing) mode of load jams;

– above-the-resonance, lower than the characteristic velocity; there are locally asymptotically stable regimes – both the resonance mode of movement of a vibratory machine and a no-oscillations mode;

– exceeding the characteristic velocity: there is a globally asymptotically stable no-oscillations mode.

Computational experiments have confirmed the results of theoretical research. At the same time, it was additionally established that it would suffice, to enter a resonant mode of movement, to slowly accelerate the bodies of vibration exciters to the above-the-resonance speed, less than the characteristic speed.

The results reported here could be interesting both for the theory and practice of designing new vibratory machines

Keywords: resonant vibratory machine, Sommerfeld effect, inertial vibration exciter of targeted action, single-mass vibratory machine

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1. Introduction

In resonant vibratory machines, low-mass inertial vibration exciters induce the intense vibrations of platforms [1]. This makes vibratory machines energy-efficient, increases the reliability and durability of their operation.

The simplest, purely mechanical way to excite resonance oscillations is based on the Sommerfeld effect [2]. In those vibration exciters that operate on the Sommerfeld effect, the unbalanced mass:

– gets stuck at one of the resonant frequencies of the oscillations of a vibratory machine, thereby exciting intensive resonant oscillations;

– reacts to a change in the resonant frequencies of a vibratory machine caused by a change in the platform loading.

Due to these features, resonant vibration exciters that operate on the Sommerfeld effect do not need an automatic control system and, therefore, have the simplest design. That additionally increases the operational reliability and durability of a vibration exciter and the vibratory machine in general.

To build resonant vibratory machines with the translational movement of platforms, it is important to design and investigate the performance of an inertial vibration exciter of targeted action, which operates on the Sommerfeld effect. Such a vibration exciter could induce perturbing forces only in the direction of platform movement, and would not additionally load the vibratory machine (its frame, guides, supports, etc.).

2. Literature review and problem statement

Analytically, the application of the Sommerfeld effect to build resonant vibratory machines was studied in the following works:

– [3] – for a pendulum rigidly mounted on the shaft of a low-power DC electric motor installed on one of the platforms in a two-mass system;

– [4] – for a wind wheel with an unbalanced mass installed on one of the platforms in a three-mass system;

– [5] – for a pendulum rigidly mounted on the shaft of an induction electric motor installed on a platform that fluctuates horizontally.

The studies reported in [3–5] found that unbalanced masses (pendulums, wind wheels, etc.) get stuck at one of the resonant frequencies of platform oscillations. Using an electric motor under a jammed mode overloads the electrical circuit. The use of an air wheel does not provide for a high efficiency due to the peculiarities of converting air energy into mechanical movement.

The Sommerfeld effect was discovered and investigated in rotor machines with passive auto-balancers in the following works:

– [6] – for a two-ball auto-balancer with the static balancing of the rotor, which executes spatial movement;

– [7] – for two two-pendulum auto-balancers with the dynamic balancing of the rotor, which executes spatial movement;

– [8] – for a two-ball auto-balancer within a flat rotor model on isotropic supports

The studies reported in [6–8] found that balls, pendulums, etc. get stuck at one of the resonance frequencies of rotor oscillations but the body of the auto-balancer is warranted to accelerate.

Works [6–8] considered the effect of load getting stuck to be undesirable. However, [9] proposed to use passive auto-balancers as exciters of two-frequency vibrations. At the same time, slow resonance oscillations excite loads in an auto-balancer when they get stuck at resonance speed. Rapid oscillations are induced by the unbalanced masses attached to the body of the auto-balancer.

It has been proven that a vibration exciter in the form of a passive auto-balancer is applicable to one- [10], two- [11], three-mass [12] vibratory machines with the translational movement of platforms when it is rigidly installed on one of the platforms. Paper [13] proved the feasibility of a two-ball auto-balancer elastically installed on the platform as a vibration exciter.

The inertial vibration exciters considered in [3–5, 9–13] are not vibration exciters of targeted action. This additionally loads vibratory machines with the translational movement of the platforms, induces undesirable vibrations of a vibratory machine's frame, foundation, etc.

Two electric motors are used as vibration exciters of targeted action, whose shafts host rigidly mounted pendulums while rotating in opposite directions. Of interest are the simplest structures whose shafts are not connected at all [14–17]. In this case, over time, the pendulums, due to the phenomenon of self-synchronization [14], begin to rotate synchronously in opposite directions. Self-synchronization occurs at the rectilinear [15] and flat-parallel [16] platform movements. Rotors can be accelerated to the rated rotational speed [15, 16], and rotors may also get stuck at the resonance oscillation frequency of the platform [17]. The rotor jamming mode is caused by the Sommerfeld effect. In this case, intense resonance oscillations of the platform are excited but, at the same time, the electric circuit of electric motors is overloaded.

There is an issue whether two auto-balancers rotating in opposite directions would work as a vibration exciter of targeted action. Once feasible, such a structure would not overload electric motors. To address this issue, the performance of a vibration exciter for the case of a single-mass vibratory machine is investigated below.

3. The aim and objectives of the study

The purpose of this work is to study the dynamics of a resonant single-mass vibratory machine with a vibration exciter of targeted action that operates on the Sommerfeld effect. This is necessary both for the construction of a general theory and the design of such vibratory machines.

To accomplish the aim, the following tasks have been set:

– to build a mechanical-mathematical model of a vibratory machine and find the steady modes of its movement;

– to investigate the stability of steady motion modes;

– to test the results of the theoretical study using a computational experiment.

4. The study materials and methods

To build a mechanical-mathematical model of the vibratory machine, we used the results reported in [10]. To search for the steady modes of movement of the vibratory machine, a small parameter is introduced, and elements of perturbation theory, the theory of nonlinear oscillations [18] were applied.

The stability of steady motion modes was investigated by the first Lyapunov method using elements of perturbation theory and the theory of nonlinear oscillations [18].

The results of the theoretical research were tested using a computational experiment. To this end, the differential equations of vibratory machine movement were integrated over a long period of time, sufficient to set a certain mode of movement.

5. Results of studying the dynamics of a resonant single-mass vibratory machine with a vibration exciter of targeted action

5. 1. Constructing a mechanical-mathematical model of the vibratory machine and searching for the steady modes of its movement

5. 1. 1. Description of the mechanical-mathematical model of the vibratory machine, differential equations of its movement

A vibratory machine (Fig. 1) is composed of a platform, mass M, and two inertial vibration exciters whose unbalanced mass is a ball, a roller (Fig. 1, *b*), or a pendulum (Fig. 1, *c*). The platform can only move vertically along the guides. The platform rests on a viscoelastic support with a stiffness coefficient

k and a viscosity coefficient b. The position of the platform is determined by the coordinate y, equal to zero in the position of the static equilibrium of the platform.

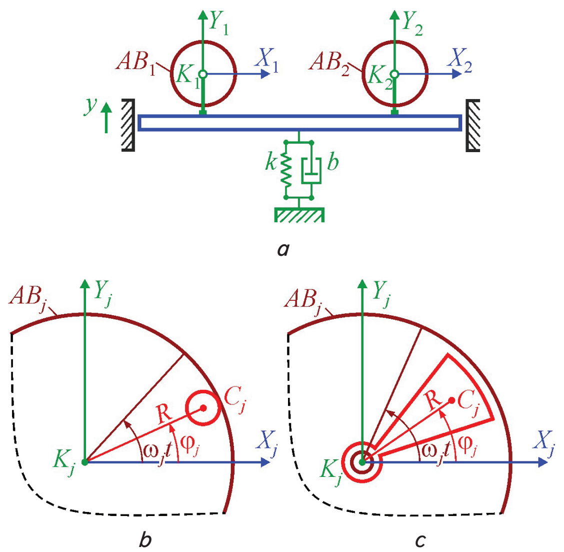

Fig. 1. A vibratory machine model; the motion kinematics of: *a* — a platform; *b* — a ball or a roller; *c* — a pendulum

The body of vibration exciter number j revolves around the point K_j at a constant angular velocity ω_j. The position of the j-th body is determined by angle $\omega_j t$, where t is the time.

The mass of one load is m. The center of load mass can move in a circle of radius R with the center at point K_j (Fig. 1, *b*, *c*). The position of load number j with respect to the $K_jX_jY_j$ coordinate system is determined by the angle φ_j, $/j=1,2/$. The movement of the load relative to the body of the vibration exciter is prevented by the force of viscous resistance whose module is $F_j = b_W R\,|\,\varphi_j' - \omega_j\,|$, $/\,j=1,2/$, where b_W is the coefficient of the viscous resistance force, and the bar in magnitude denotes the time derivative t.

The differential equations of vibratory machine movement take the following form

$$M_\Sigma y'' + by' + ky + S_y'' = 0,$$
$$\kappa m R^2 \varphi_j'' + b_W R^2\left(\varphi_j' - \omega_j\right) +$$
$$+ mgR\cos\varphi_j + mRy''\cos\varphi_j = 0, \quad /\,j=1,2\,/. \qquad (1)$$

In (1), $M_\Sigma = M + 2m$ is the mass of the entire system (the massed of vibration exciters are attributed to the mass of the platform),

$$S_x = mR\left(\cos\varphi_1 + \cos\varphi_2\right), \quad S_y = mR\left(\sin\varphi_1 + \sin\varphi_2\right), \qquad (2)$$

g is the free-fall acceleration module; for a ball, a roller, and a pendulum, respectively, $\kappa = \{7/5,\ 3/2,\ 1 + J_C/(mR^2)\}$.

Note that the form of the differential equations of movement of system (1) does not depend on the type of load.

In further analytical studies, the effect of gravity on a load is neglected.

5. 1. 2. Reducing motion equations to a dimensionless form

Introduce the dimensionless variables and time

$$v = y/\tilde{y}, \quad s_x = S_x/\tilde{s}, \quad s_y = S_y/\tilde{s}, \quad \tau = \tilde{\omega} t, \qquad (3)$$

where $\tilde{y}$, $\tilde{s}$, $\tilde{\omega}$ is the characteristic scale to be selected later.

Then the differential equations of motion (1) are reduced to the following form

$$\ddot{v} + \frac{b}{M_\Sigma \tilde{\omega}}\dot{v} + \frac{k}{M_\Sigma \tilde{\omega}^2} v + \frac{\tilde{s}}{M_\Sigma \tilde{y}}\ddot{s}_y = 0,$$
$$\ddot{\varphi}_j + \frac{b_W}{\kappa m \tilde{\omega}}\left(\dot{\varphi}_j - \frac{\omega_j}{\tilde{\omega}}\right) + \frac{\tilde{y}}{\kappa R}\ddot{v}\cos\varphi_j = 0, \qquad (4)$$

where a dot above a value denotes a derivative for $\mathbf{t}$.

Introduce the characteristic scale and dimensionless parameters:

$$\tilde{\omega} = \sqrt{k/M_\Sigma}, \quad \tilde{s} = 2mR, \quad \tilde{y} = 2mR/M_\Sigma;$$
$$\varepsilon = \frac{2m}{\kappa M_\Sigma}, \quad \beta = \frac{b_W M_\Sigma}{2m^2 \tilde{\omega}}, \quad h = \frac{b}{2M_\Sigma \tilde{\omega}}, \quad n_j = \frac{\omega_j}{\tilde{\omega}}. \qquad (5)$$

Then equations (4) take the following form:

$$\ddot{v} + 2h\dot{v} + v + \ddot{s}_y = 0,$$
$$\ddot{\varphi}_j + \varepsilon\beta\left(\dot{\varphi}_j - n_j\right) + \varepsilon\ddot{v}\cos\varphi_j = 0, \quad /\,j=1,2\,/. \qquad (6)$$

To excite the vibrations of targeted action, the bodies of vibration exciters must rotate at the same speeds n in opposite directions:

$$n_1 = n, \quad n_2 = -n. \qquad (7)$$

Move to the new coordinates that determine the movement of loads

$$\varphi_1 = \psi_1, \quad \varphi_2 = \pi - \psi_2. \qquad (8)$$

Then, taking into consideration (7), (8), the dimensionless differential equations of vibratory machine movement (5) take the following form

$$\ddot{v} + 2h\dot{v} + v + \ddot{s}_y = 0,$$
$$\ddot{\psi}_j + \varepsilon\beta\left(\dot{\psi}_j - n\right) + \varepsilon\ddot{v}\cos\psi_j = 0, \quad /\,j=1,2\,/. \qquad (9)$$

In (9)

$$s_x = \left(\cos\psi_1 - \cos\psi_2\right)/2, \quad s_y = \left(\sin\psi_1 + \sin\psi_2\right)/2. \qquad (10)$$

The derived differential equations (9), with accuracy to designations, coincided with the differential equations of motion of a single-mass vibratory machine [10] for the case of two loads in a (single) vibration exciter.

5. 1. 3. Steady motion modes under which loads rotate in opposite directions, at zero approximation (e=0)

At $\varepsilon=0$, the system of differential equations (9) takes the following form

$$\ddot{v} + 2h\dot{v} + v + \ddot{s}_y = 0, \quad \ddot{\psi}_1 = 0, \quad \ddot{\psi}_2 = 0. \qquad (11)$$

Note that the system of differential equations (11) does not include implicit dimensionless time $\mathbf{t}$. Therefore, the last two equations allow such a solution in which loads rotate at the same angular speeds Ω in opposite directions

$$\tilde{\psi}_1 = \Omega\tau + \psi_0, \;\; \tilde{\psi}_2 = \Omega\tau - \psi_0. \tag{12}$$

In (12), the ψ_0 parameter determines the angle of rotation at which one load is ahead and the other lags behind the average angle of rotation $\Omega\tau$ of two loads.

Then

$$s_x = -\sin\psi_0 \sin(\Omega\tau), \;\; s_y = \cos\psi_0 \sin(\Omega\tau). \tag{13}$$

Taking into consideration (13), the first equation in (11) takes the following form

$$\ddot{v} + 2h\dot{v} + v = \Omega^2 \cos\psi_0 \sin(\Omega\tau). \tag{14}$$

A partial solution to differential equation (14) takes the following form

$$\tilde{v} = \frac{\Omega^2 \cos\psi_0}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2}\begin{bmatrix}\left(1-\Omega^2\right)\sin(\Omega\tau) - \\ -2h\Omega\cos(\Omega\tau)\end{bmatrix}. \tag{15}$$

Note that one cannot find parameters Ω and ψ_0 at zero approximation.

5. 1. 4. Refining the steady motion modes applying the first approximation

By substituting (12), (15) in the second and third equations in (9), we obtain the following two equations

$$\varepsilon\beta(\Omega - n) - \varepsilon\frac{\Omega^4 \cos\psi_0 \cos(\Omega\tau \pm \psi_0)}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2}\times$$
$$\times\left[\left(1-\Omega^2\right)\sin(\Omega\tau) - 2h\Omega\cos(\Omega\tau)\right] = 0, \tag{16}$$

where the upper character in «±» corresponds to the first equation and the lower character – to the second.

Leave in (16) the non-periodic components (interfering with the frequency of movement of loads), and we obtain

$$\varepsilon\beta(\Omega - n) + \frac{\varepsilon}{2}\frac{\Omega^4 \cos\psi_0}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2}\times$$
$$\times\left[\pm\left(1-\Omega^2\right)\sin\psi_0 + 2h\Omega\cos\psi_0\right] = 0. \tag{17}$$

Subtract the second equation from the first equation in (17), we obtain

$$\varepsilon\frac{\Omega^4\left(1-\Omega^2\right)\sin\psi_0\cos\psi_0}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2} = 0. \tag{18}$$

Condition (18) is met if $\sin\psi_0\cos\psi_0 = \sin(2\psi_0)/2 = 0.$ Hence, we find

$$2\psi_0 = 0, \pm\pi, \ldots \tag{19}$$

Add the second equation to the first equation in (17), and we obtain

$$\varepsilon\beta(\Omega - n) + \varepsilon\frac{h\Omega^5\cos^2\psi_0}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2} = 0. \tag{20}$$

Consider the following possibilities.

1. At $\psi_0 = \pm\pi/2$, $\pm 3\pi/2, \ldots$ equation (20) takes the form $\varepsilon\delta(\Omega - n) = 0$. Hence, we find the angular velocity of load rotation

$$\Omega = n. \tag{21}$$

At this movement, the loads rotate synchronously with the bodies of vibration exciters. At the same time

$$s_x = \pm\sin(\Omega\tau), \;\; s_y = 0, \tag{22}$$

and there are no platform oscillations.

2. At $\psi_0 = 0$, $\pm\pi, \ldots$ equation (20) takes the following form

$$\varepsilon\beta(\Omega - n) + \varepsilon\frac{h\Omega^5}{\left(1-\Omega^2\right)^2 + 4h^2\Omega^2} = 0. \tag{23}$$

From (23), we find the angular velocity at which the loads get stuck.

Note that equation (23) was studied in work [10]. The main results are as follows.

In the cases where the forces of external and internal resistance are small, the weight of loads is much less than the weight of the platform, etc. there are three characteristic speeds of rotor rotation $\tilde{n}_1$, $\tilde{n}_2$, $\tilde{n}_3$. At the same time, $1 < \tilde{n}_1 << \tilde{n}_2 < \tilde{n}_3 << n$ and if:

– $0 < n < \tilde{n}_1$ then there is a single frequency at which loads get stuck Ω_1, and $0<\Omega_1<1$;

– $\tilde{n}_1 < n < \tilde{n}_2$ then there are three frequencies at which loads get stuck $\Omega_{1,2,3}$, such that $0<\Omega_1<1<\Omega_2<\Omega_3<n$;

– $\tilde{n}_2 < n < \tilde{n}_3$ then there are three frequencies at which loads get stuck $\Omega_{1,2,3}$, such that $1<\Omega_1<\Omega_2<<\Omega_3<n$;

– $n > \tilde{n}_3$ then there is a single frequency at which loads get stuck Ω_1, such that $1<<\Omega_1<n$.

5. 2. Investigating the stability of steady movement modes

5. 2. 1. A mode of the synchronous rotation of loads with the bodies of vibration exciters

Introduce the unperturbed motion under a synchronous rotation mode

$$\tilde{\psi}_1 = n\tau - \pi/2, \;\; \tilde{\psi}_2 = n\tau + \pi/2, \;\; \tilde{v} = 0. \tag{24}$$

Introduce perturbed motion

$$\tilde{\psi}_1 = n\tau - \pi/2 + x_1, \;\; \tilde{\psi}_2 = n\tau + \pi/2 + x_2, \;\; \tilde{v} = x_0. \tag{25}$$

Linearize differential equations of motion (9) to obtain

$$\ddot{x}_0 + 2h\dot{x}_0 + x_0 + \frac{d^2}{d\tau^2}\left[\frac{x_1 - x_2}{2}\sin(n\tau)\right] = 0,$$

$$\ddot{x}_1 + \varepsilon\beta\dot{x}_1 + \varepsilon\ddot{x}_0\sin(n\tau) = 0, \;\; \ddot{x}_2 + \varepsilon\beta\dot{x}_2 - \varepsilon\ddot{x}_0\sin(n\tau) = 0. \tag{26}$$

Introduce new variables

$$w = (x_1 + x_2)/2, \;\; z = (x_1 - x_2)/2. \tag{27}$$

Then the system of differential equations (26) is transformed to the following form

$$\ddot{x}_0 + 2h\dot{x}_0 + x_0 + \frac{d^2}{d\tau^2}\left[z\sin(n\tau)\right] = 0,$$

$$\ddot{z}+\varepsilon\beta\dot{z}+\varepsilon\ddot{x}_0\sin(n\tau)=0,\quad \ddot{w}+\varepsilon\beta\dot{w}=0. \tag{28}$$

(28) demonstrates that z is a slow-changing function. Then the first equation in (28), with an accuracy to the values of the zero order of smallness (for ε), takes the following form

$$\ddot{x}_0+2h\dot{x}_0+x_0=zn^2\sin(n\tau). \tag{29}$$

A partial solution to equation (29) takes the following form

$$\tilde{x}_0=\frac{zn^2}{(1-n^2)^2+4h^2n^2}\left[(1-n^2)\sin(n\tau)-2hn\cos(n\tau)\right]. \tag{30}$$

Then the second equation in (28) takes the following form

$$\ddot{z}+\varepsilon\beta\dot{z}-\varepsilon\frac{zn^4}{(1-n^2)^2+4h^2n^2}\times$$
$$\times\left[(1-n^2)\sin(n\tau)-2hn\cos(n\tau)\right]\sin(n\tau)=0. \tag{31}$$

Its time-averaging in the interval [0, $2\pi/n$] produces

$$\ddot{z}+\varepsilon\beta\dot{z}+\frac{\varepsilon n^4(n^2-1)}{2\left[(1-n^2)^2+4h^2n^2\right]}z=0. \tag{32}$$

Consequently, the mode of the synchronous rotation of loads is stable at the above-the-resonance speeds of rotation of the bodies of vibration exciters (n>1).

5. 2. 2. Modes of load jamming

Introduce the unperturbed motion under the mode when loads get stuck

$$\tilde{v}=\frac{\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau)-2h\Omega\cos(\Omega\tau)\right],\ \tilde{\psi}_1=\tilde{\psi}_2=\Omega\tau. \tag{33}$$

Introduce the perturbed motion

$$v=\frac{\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau)-2h\Omega\cos(\Omega\tau)\right]+x_0,$$

$$\psi_1=\Omega\tau+x_1,\ \psi_2=\Omega\tau+x_2. \tag{34}$$

Then

$$s_y\approx\sin(\Omega\tau)+\frac{x_1+x_2}{2}\cos(\Omega\tau). \tag{35}$$

After linearization, the system of differential equations (9) takes the following form

$$\ddot{x}_0+2h\dot{x}_0+x_0+\frac{d^2}{dt^2}\left(\frac{x_1+x_2}{2}\cos(\Omega\tau)\right)=0,$$

$$\ddot{x}_j+\varepsilon\beta\dot{x}_j+\varepsilon\begin{bmatrix}-\ddot{\tilde{v}}x_j\sin(\Omega\tau)+\\+\ddot{x}_0\cos(\Omega\tau)\end{bmatrix}=0,\quad /j=1,2/. \tag{36}$$

In the new variables (27), system (36) takes the following form

$$\ddot{x}_0+2h\dot{x}_0+x_0+\frac{d^2}{dt^2}\left[w\cos(\Omega\tau)\right]=0,$$

$$\ddot{w}+\varepsilon\beta\dot{w}+\varepsilon\left[-\ddot{\tilde{v}}w\sin(\Omega\tau)+\ddot{x}_0\cos(\Omega\tau)\right]=0,$$

$$\ddot{z}+\varepsilon\beta\dot{z}+\varepsilon\left[-\ddot{\tilde{v}}z\sin(\Omega\tau)\right]=0. \tag{37}$$

Averaging the third equation in (37) for time in the interval [0, $2\pi/\Omega$] produces

$$\ddot{z}+\varepsilon\beta\dot{z}+\frac{\varepsilon\Omega^4(1-\Omega^2)}{2\left[(1-\Omega^2)^2+4h^2\Omega^2\right]}z=0. \tag{38}$$

(38) demonstrates that only such an oscillatory mode could be stable under which loads get stuck at the pre-resonant speed (Ω<1).

The second equation in (37) demonstrates that w is a slow-changing function. Then the first equation in (37), with an accuracy to the values of the zero order of smallness (for ε), takes the following form

$$\ddot{x}_0+2h\dot{x}_0+x_0=w\Omega^2\sin(\Omega\tau). \tag{39}$$

A partial solution to equation (39) takes the following form

$$\tilde{x}_0=\frac{w\Omega^2}{(1-\Omega^2)^2+4h^2\Omega^2}\begin{bmatrix}(1-\Omega^2)\sin(\Omega\tau)-\\-2h\Omega\cos(\Omega\tau)\end{bmatrix}. \tag{40}$$

Then the second equation in (37) takes the form

$$\ddot{w}+\varepsilon\beta\dot{w}+\varepsilon\frac{w\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau)-2h\Omega\cos(\Omega\tau)\right]\sin(\Omega\tau)-$$
$$-\varepsilon\frac{w\Omega^4}{(1-\Omega^2)^2+4h^2\Omega^2}\times$$
$$\times\left[(1-\Omega^2)\sin(\Omega\tau)-2h\Omega\cos(\Omega\tau)\right]\cos(\Omega\tau)=0.$$

Averaging it for time in the interval [0, $2\pi/\Omega$] produces

$$\ddot{w}+\varepsilon\beta\dot{w}+\frac{\varepsilon\Omega^4(\Omega^2-1+2h\Omega)}{2\left[(1-\Omega^2)^2+4h^2\Omega^2\right]}w=0. \tag{41}$$

(39), (41) demonstrate that at the pre-resonant speeds of load jamming (Ω<1) the perturbation is $w, x_0 \xrightarrow[\tau\to+\infty]{} 0$, which corresponds to the asymptotic stability of movement.

Thus, among all possible modes under which loads get stuck, the steady one is the mode under which loads get stuck at pre-resonance speed (0<Ω<1).

Based on the results reported in [10], the second characteristic speed of rotor rotation is

$$\tilde{n}_2=1+\frac{1}{4h\beta}=1+\frac{m^2\omega_0^2}{b_Wb}. \tag{42}$$

Moreover, there is only one pre-resonance frequency of load jamming – Ω_1 ($0<\Omega_1<1$), and only at speeds lower than $\tilde{n}_2$ but at any values of other parameters.

5. 3. Verifying the results of theoretical research using a computational experiment

Below are the results of integrating the system of differential equations (6) under the initial conditions

$$y = \dot{y} = 0; \quad \varphi_j = -\pi / 2,$$

$$\dot{\varphi}_j = 0, \quad / j = 1,2 /. \qquad (43)$$

Estimated data:

$$h = 0.1, \quad \beta = 1, \quad \varepsilon = 0.05. \qquad (44)$$

The speed of rotation of the shafts varies by the following law

$$n_1(\tau) = -n_2(\tau) = \begin{cases} 2n\tau / T, \text{ if } \tau < T/2; \\ n, \text{ otherwise.} \end{cases} \qquad (45)$$

where T=2,000 and [0, T] is the dimensionless time interval in which the differential equations of motion are integrated.

Taking into consideration (44), formula (42) produces $\tilde{n}_2 = 3.5$. The results of the computational experiment are as follows.

At the pre-resonance speeds of body rotation, the only stable mode of movement is the mode when loads get stuck. This movement is globally asymptotically stable and occurs under any initial conditions. Fig. 2 shows the result of the integration of the system of differential equations (6) at n=0.9. On the left are the charts of magnitude changes throughout the integration interval. On the right are the charts of magnitude changes after setting the movement – in the interval [T–Δτ, T], where Δτ=0.98T.

The jamming mode begins to appear even during the acceleration of the vibration exciter bodies.

Fig. 3 shows the results of integrating differential motion equations at the above-the-resonance speeds of rotation of vibration exciter bodies not exceeding $\tilde{n}_2 = 3.5$ (n=3.5).

Under the slow acceleration of the bodies of vibration exciters, the mode under which loads get stuck appears first. Next, the jamming mode maintains stability with an increase in the speed of rotation of bodies to a maximum value not exceeding $\tilde{n}_2 = 3.5$. However, for any $n \in (1, \tilde{n}_2]$, a stable mode can be a mode of the synchronous rotation of loads with bodies. Typically, this mode occurs at the rapid acceleration of the bodies. Thus, at $n \in (1, \tilde{n}_2]$, the locally stable are the two modes of movement of a vibratory machine. Of course, each mode has its own pull zone.

Fig. 4 shows the results of integrating differential motion equations at the above-the-resonance speeds of rotation of the bodies of vibration exciters exceeding $\tilde{n}_2 = 3.5$ (n=3.6).

The only stable steady mode of movement is the mode of the synchronous rotation of loads together with the bodies of vibration exciters. Moreover, with slow acceleration, the mode at which loads get stuck appears first. However, when the speed of rotation n exceeds the characteristic speed $\tilde{n}_2 = 3.5$, the jam mode loses stability.

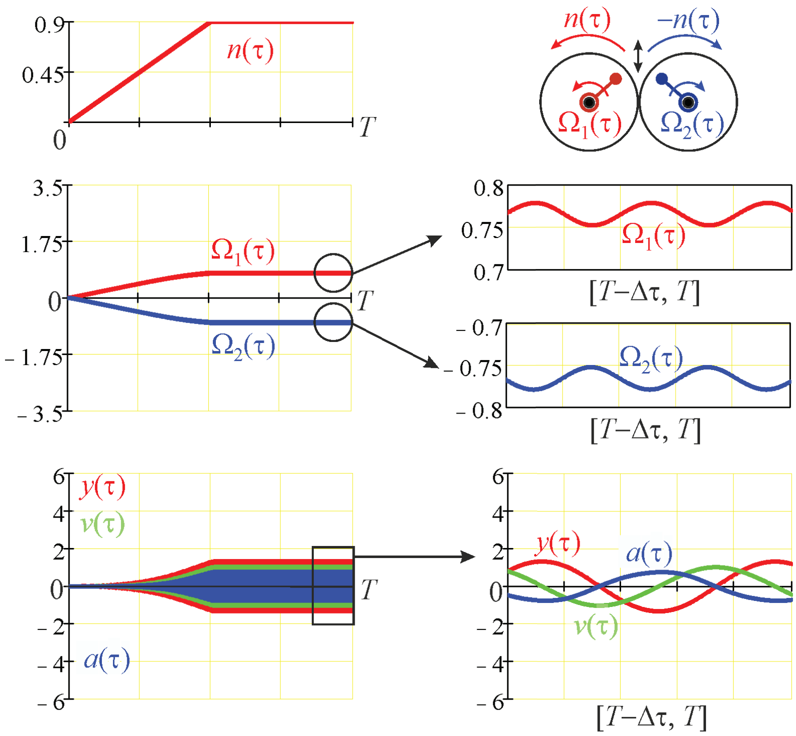

Fig. 2. Results of the integration of differential equations of movement at the pre-resonance speeds of rotation of vibration exciter bodies (n=0.9, Δτ=0.98 T)

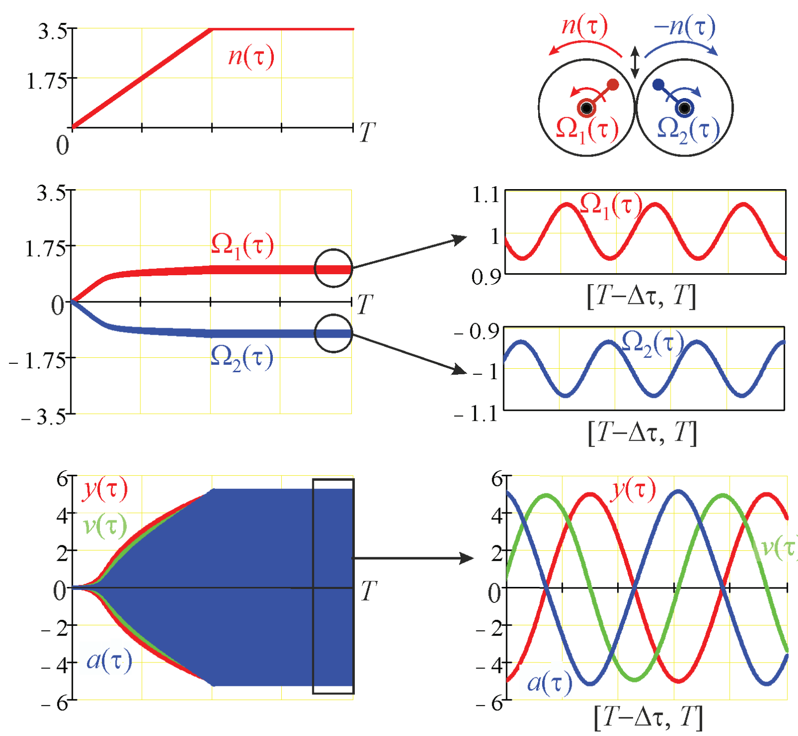

Fig. 3. Results of integrating differential motion equations at the above-the-resonance speeds of rotation of vibration exciter bodies not exceeding $\tilde{n}_2 = 3.5$ (n=3.5, Δτ=0.99 T)

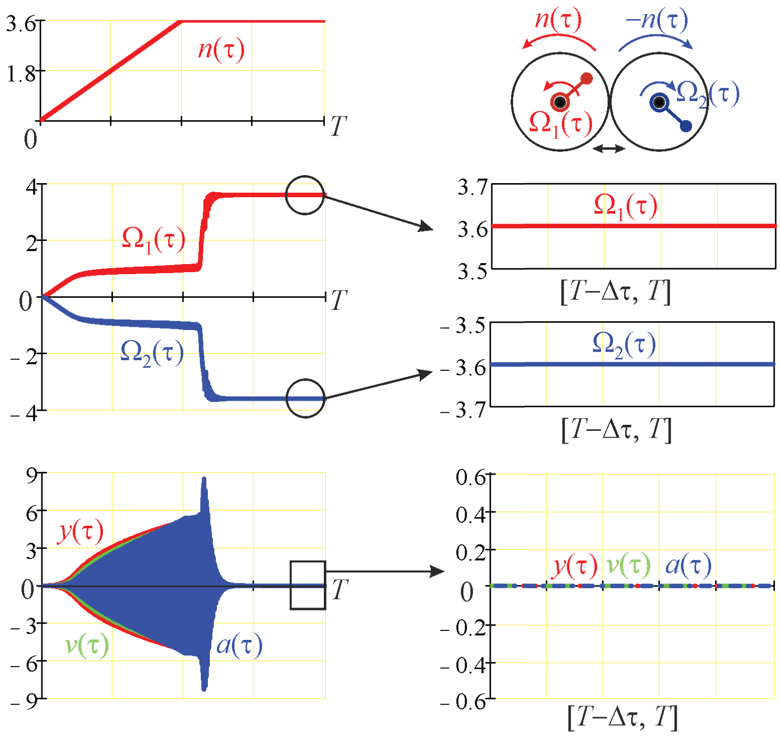

Fig. 4. Results of integrating differential motion equations at the speeds of rotation of vibration exciter bodies greater than $\tilde{n}_2 = 3.5$. (n=3.6, $\Delta\tau$=0.99 T)

6. Discussion of results of studying the dynamics of a resonant single-mass vibratory machine with a vibration exciter of targeted action

Our study has shown that a vibratory machine possesses the following:

– one to three oscillatory modes of movement under which loads get stuck at an almost constant angular velocity whose value is determined from equation (23);

– a no-oscillations mode under which loads rotate synchronously with the bodies, and their total unbalanced mass in the vertical direction is zero.

Only under a single oscillation mode do the loads get stuck at an almost constant angular rotational velocity Ω, less than the resonance oscillation frequency of the platform n=1. With an increase in the speed of rotation of the bodies of vibration exciters, the load rotation frequency approaches a resonance frequency, which excites intense resonance oscillations. At the same time, the platform oscillation amplitude increases monotonously.

At the pre-resonance speeds of the rotation of vibration exciter bodies, there is a globally asymptotically stable mode of load jamming at the pre-resonance rotation speeds. At the above-the-resonance speeds of rotation of the bodies of vibration exciters, smaller than the second characteristic speed, the locally asymptotically stable are both the mode of load jamming at the pre-resonant speeds of rotation and the mode of the synchronous rotation of loads. At the speeds of rotation of the bodies of vibration exciters greater than the second characteristic speed, the globally asymptotically stable is the mode of the synchronous rotation of loads.

Computational experiments confirm the results of our theoretical studies on the existence and stability of steady motion modes. In addition, the computational experiments establish that when rotating the bodies of vibration exciters at above-the-resonance speeds less than $\tilde{n}_2$, the proper choice of initial conditions or the acceleration rate of the vibration exciter bodies could ensure the onset of any mode out of two possible stable modes.

To drive a vibratory machine to the resonance mode of movement, it would suffice to slowly accelerate the bodies of vibration exciters to a speed less than $\tilde{n}_2$.

It should be noted that the same unbalanced masses can be attached to the bodies of vibration exciters. Then the combined vibration exciter would work as two inertial vibration exciters of targeted action. The first would be formed by loads and would excite slow fluctuations at a resonance frequency. The second would be formed by the unbalanced masses on the bodies of vibration exciters and would excite rapid vibrations at the rotation frequency of bodies. Since the differential equations of platform movement are linear in relation to the coordinates of the platform and the total unbalanced mass, we can assume that the conditions of performance of a vibration exciter would not change [10].

This work does not investigate the effect exerted on the performance of the vibration exciter by gravity forces. Also unaddressed are the regions of attraction of the locally stable steady modes of movement of vibratory machines. However, this does not significantly affect the results obtained.

In the future, it is planned to investigate the steady modes of movement of two-mass and three-mass resonant vibratory machines with the translational movement of platforms and a vibration exciter of targeted action.

7. Conclusions

1. Vibratory machines theoretically possess the following:

– one to three oscillatory modes of movement under which loads get stuck at almost constant angular velocity and form total unbalanced mass only in the vertical direction;

– a no-oscillations mode under which loads rotate synchronously with the bodies and from total unbalanced mass only in the horizontal direction (there is no vertical component).

It has been established that only one oscillatory mode is resonant. Under it, loads get stuck at an almost constant angular rotational speed, close to the resonance frequency, thereby exciting the intense resonance oscillations of the platform. The mode exists when rotating the bodies at the above-the-resonance speeds less than some characteristic speed.

2. At the pre-resonance speeds of the rotation of vibration exciter bodies, a globally asymptotically stable is the only existing mode of load jamming (at pre-resonance speeds). At the above-the-resonance speeds of the rotation of the bodies of vibration exciters, smaller than the characteristic speed, locally asymptotically stable are both the resonant mode of movement of a vibratory machine and a no-oscillations mode. At the speeds of rotation of the bodies of vibration

exciters greater than the characteristic speed, the globally asymptotically stable is the no-oscillations mode.

3. Computational experiments confirm the results of theoretical research and allow us to establish the following:

– when rotating the bodies of vibration exciters at above-the-resonance speeds lower than the characteristic speed, the proper choice of initial conditions, or the speed of the acceleration of the bodies of vibration exciters could ensure the onset of both a resonant oscillation mode and a no-oscillations mode;

– to set a vibratory machine to a resonance mode of movement, it would suffice to slowly accelerate the bodies of vibration exciters to a speed less than the characteristic speed.

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