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**THE USE OF GAMIFICATION IN MATHEMATICS:**

**CHESS AS A MANIPULATIVE MATERIAL**

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## **ABSTRACT**

Nowadays mathematics is very tedious for students, specifically in problem solving. For this reason, in this work, chess has been used as manipulative material to improve the result among Primary Education students. This research has been carried out during the 5th year of Primary School at the Juan Pablo II School in Alcorcón, for two weeks. During this time, a comparison has been made between the traditional methodology and the gamified one, in order to know which of them increases the learning achievement of the students. The results obtained have been favorable to the gamified methodology, thus proving that the use of chess as a manipulative material increases the learning achievement of the students significantly as well as their motivation. In turn, the use of chess as a manipulative material has been a help for those students with special educational needs.

## **KEY WORDS**

Chess, manipulative material, gamification, learning achievement, fraction

## **RESUMEN**

Las matemáticas resultan muy tediosas para los alumnos, en concreto la resolución de problemas. Por ello, en este trabajo se ha utilizado el ajedrez como material manipulativo para mejorar el rendimiento entre los alumnos de Educación Primaria. Esta investigación se ha llevado a cabo en el curso de 5º de Primaria en el colegio Juan Pablo II de Alcorcón, durante dos semanas. Durante este tiempo se ha realizado una comparación entre la metodología tradicional y la gamificada, para conocer cuál mejora el rendimiento de los alumnos. Los resultados obtenidos han sido favorables a la metodología gamificada, probando así, que el uso del ajedrez como material manipulativo aumenta el rendimiento de los alumnos significativamente así como su motivación y sus ganas de aprender. A su vez, el uso del ajedrez como material manipulativo ha supuesto una ayuda para aquellos alumnos con necesidades educativas especiales.

## **PALABRAS CLAVE**

Ajedrez, material manipulativo, gamificación, rendimiento, fracción.

## **THE USE OF ACTIVE METHODOLOGIES AND GAME IN THE LEARNING PROCESS**

In Spain for a long time until now, mathematics has been considered to be the most difficult subject for the students. For the majority, it becomes an impossible subject to understand, memorize and unworkable for the application of rules and procedures, leading to a lack of motivation for the subject. Unfortunately, the statistics show us that the academic failure in general, and particularly in mathematics, is growing over time

Nowadays the mathematics that are imparted in the primary school classes is not related with the previous experiences of the students, with their daily life. Many students have great difficulty to understand the mathematical concepts and also to memorize the rules and/or the procedures. But, thanks to the educational innovation, different methodologies are being discovered, which are helping to enhance the learning achievement in classrooms. One of this, is the gamification, now what is the gamification? The term itself was coined by Pelling in 2002, although it did not gain popularity until 2010. We understand gamification as the application of game mechanics in non-recreational environments with the aim of influencing motivation and improving learning.

Currently, the gamification's implementation is widely extended among: business environments related to marketing campaigns, to involve their client and users; in finance, to increase customer loyalty and promote the use of online banking services; in social networks, which incorporate rankings and levels in the profiles; in sports as a tool and social network that promotes sport actions; or in teaching, to motivate students. The implementation of this methodology in the classroom is given to strengthen a commitment on the teacher as well as the students.

According to Herranz, to be able to apply gamification in classrooms we have to consider its four fundamental elements: dynamics, mechanics, components of the game and type of players. First of all, dynamics are global aspects to which a gamified system must orient itself, it is related to the effects, motivation and desires that are intended to generate in the participant.

Secondly, mechanics are a series of rules that try to generate games that can be enjoyed, that cause a certain "addiction" and commitment on the part of the users, by providing them with challenges and a path through which to travel. These rules are: establish challenges, taking users out of their comfort environment to introduce them into the game's mechanic (Werbach, 2013); offer opportunities, competition and collaboration, presenting the best way for the participants to behave themselves; the existence of a feedback will indicate the fact of obtaining prizes for well performed or completed actions; and the established rewards, which can be staggered according to effort, level, risk, among others (Herranz, 2013).

Regarding the components of gamification, these are elements associated to the two previous. They can change depending on the type and the quantity, everything depends on the creativity in which the game is developed. These are: achievements, gifts, conquests and/or advances, it is important that one or more needs of the participants are met; the avatars; the badges, that the student obtains as he advances in the subject; the levels are previous stages that the students overcome before reaching the final objective; the formation of teams, motivating the socialization and the sensitization of the people in joining to compete; achieve common goals and obtain a final reward; points and leaderboards or rankings.

Finally, according to Bartle we can find four different profiles of players in gamification: the killers, seek to compete with other players; the socializers, they are attracted by social aspects over the same strategy of the game; the achievers, aim to solve challenges successfully and get a reward for it; and the explorers, want to discover and learn anything new or unknown from the system.

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## I. DIFFERENCES BETWEEN GAME-BASED LEARNING AND GAMIFICATION

The term gamification is becoming increasingly popular in the educational environment, but many people confuse or do not really understand the difference between gamification and game based-learning.

The Game Based-Learning (GBL) or Serious Game, consists in the use of games as a support tool of learning, assimilation or assessment of the knowledge. For its part, gamification is based on incorporating dynamics or game mechanisms (points, rankings, badges, game rules, etc.) to entities outside the game to encourage specific behavior. In short, the game based-learning, as the name suggests, learning while playing, and gamification is not playing, however it uses different elements and techniques of the game to achieve greater motivation of students, and capture their attention helping them to improve their learning achievement. According to Piaget, the use of these mechanics helps in learning and improves both oral and written skills. Even more children have fun, create, imagine and develop both socially and personally.

*Table 1: Comparison GBL system and gamification.*

<b>GBL or SERIOUS GAME</b>	<b>GAMIFICATION</b>
Use of games for educational purposes	Use of mechanics of the game
Ludic environments	Non-ludic environments
The content is transformed to suit the story and the scenes of the game.	Game characteristics are added for the learning management system.
Easy to understand and entertained to solve. Playful experience.	To motivate and attract the attention.
Has rules and objectives well defined	Mechanics and dynamics: rankings, avatars, badges...

*Source: Prepared by the author*



Therefore, gamification is never going to be a game, it will be a methodology in which the mechanics and dynamics of the games are used to achieve whatever we want from the point of view of learning.

## **II. CHESS AS A GAME AGAINST THE MANIPULATIVE MATERIAL**

The use of manipulative materials in classrooms, provides a significant learning. It helps us to present contents in different subjects. At the present time, has been an extraordinary growth regarding the use of manipulative materials in the mathematical classroom, such as chess, abacus, blocks, tangram and geoboard. All of them help us to work and consolidate the contents presented, motivate students, awaken interest in mathematics, develop creativity and apply strategies to solve problems. Among them, we can highlight chess, is a game supported almost entirely by logic and mathematics, in addition to possessing a level of imagination and creativity. It favors the becoming of logical reasoning as a characteristic element to play correctly.

For some time now, chess has been implemented in the school environments. In 1995, the Spanish Senate suggested as an optional subject the implementation of chess in the Spanish classrooms. This was the starting point that set an important advance in the environment the field of chess in the classrooms. However, one of the most significant advances was on March 13, 2012, the date on which the European Parliament approves the introduction of the program “Chess in the school” in the education systems in the European Union, in the declaration 50/2011.

Later, on February 11, 2015, the Socialist Group presented to the Congress “*la propuesta no de ley sobre la implantación y fomento de la práctica del ajedrez en escuelas y espacios públicos y su promoción como deporte*” (file number 161/002598). This proposal is of great importance, since it was one of the few in which all the political parties were in favour. All of them fought for the implementation of chess in schools and public spaces, promoting it in turn as a sport.

Chess could be applied in a classroom in two different ways, as a game with its rules, corresponding to the Serious Game; or on the other hand, applying only the mechanics and techniques of it (in a non-ludic environment), in this case, chess becomes a manipulative material, to implement gamification.

Now, having clear the difference between Serious Game and gamification; knowing what the manipulative materials are and how can they help us in classrooms; and knowing how can we use the chess in the classroom as manipulative material and not just as a game, how can we apply all of this in our classrooms in the mathematics subject, so that they stop being boring?

### **III. MATHEMATICS WITH GAMES OR MANIPULATIVE MATERIALS**

Nowadays there are several studies that affirm the weakness in training mathematics, despite being an essential part in the education of people and in the development of their daily life. An objective example can be taken from the results of PISA 2015. This shows the score in mathematics obtained by Spain is 486 points, slightly below the OECD's averaging with 490 points and the EU's averaging with 493 points. Of the 35 member countries of the OECD Spain is ranked 25th.

One way to solve this problem is to improve those factors that directly concern the teaching of mathematics and to seek significant learning, in which the new knowledges is based on the concepts that the person has already learnt before. For this we can use gamification in the teaching-learning process, improving the motivation and the attitude of the students during the lessons, facilitating the significant learning. The goal is to lead students to learn for themselves, engage them in learning as they do for games and do it for the satisfaction of doing it, and not for the reward they will receive if they get good grades. All this can be achieved and it brings many benefits for students.

The use of chess, as a manipulative material to teach mathematics, can improve the learning achievement in this subject and achieve a high level of interest and great motivation in students. The similitudes between the practice of chess and the exercising of mathematics are present since, through chess students put into practice the numerical calculation, the planning and evaluation of each position on the board, the logical reasoning, the resolution of problems and intuition, which are strategies used by the student in the study of mathematics.

In this research, chess is used as a manipulative material in the classroom, more specifically in the Mathematics subject. A gamified methodology is followed, with a reward system (the sum of certain point leads to a prize), levels, challenges, feedback and a group work, for the correct learning achievement of the activity, thus increasing the motivation and involvement of the students.

The objectives of this work are: O1. To demonstrate that chess can be used as manipulative material. O2. To present gamification and its use of the game's own mechanics (in our case of chess) to achieve a meaningful learning of mathematics. Both in order to increase the learning achievement of the students.

## **GAMIFICATION AND CHESS IN THE MATHEMATICS CLASSROOM**

The research has been carried out in a privately-owned but state-funded bilingual center, Juan Pablo II School of Alcorcón, in fifth year of Primary, due to that is the course according to the *Curriculum del Boletín Oficial de la Comunidad de Madrid* (see Annex 1), in which the students begin to see the addition and subtractions of fractions with the same denominator and with a different denominator.

This school works with classrooms differentiated by sex, so we have had four different classrooms, two of them female and two male. The research has counted on a sample of 93 students, of whom 55 were boys and 38 were girls. Among them it should be noted 1% of the students with SEN (Especial Educational Needs), between them we find: a student with autism, who evades himself and also he gets frustrated if he is not able to do something; a student with ACNEAE (*Alumnos con Necesidades Específicas de Apoyo Educativo*), who does not follow any class because she is not able to understand it; two students with ADD (Attention Deficit Disorder), who are not concentrated due to their disorder; a student with ADHD (Attention Deficit Hyperactivity Disorder) and dyslexia, she does not understand most of the things because she is not capable of understanding what she reads; three dyslexic students and two of them also with dyscalculia, are absent from class because they do not understand most things; and finally a student with ADHD and Tourette Syndrome<sup>1</sup>, this student is not able to sit still for a long period of time, which is why most of the time he is not paying attention. All these students have in common a lack of attention in class, which means that there is no learning, and over time this increases their failure (see Table 1)

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<sup>1</sup> Tourette Syndrome is a neurological disorder characterized by repetitive, stereotyped, involuntary movements and vocalizations called tics.

*Table 1: Summary of special educational needs present in the classroom.*

<b>SEN</b>	<b>CHARACTERISTICS</b>
<b>AUTISM</b>	Evasion during the classes
<b>ADD/ADHD</b>	Continuous movement and evasion
<b>ACNEE</b>	Lack of knowledge
<b>ADHD and TOURETTE</b>	Continuous movements and noises
<b>DYSLEXIA/ DYSCALCULIA</b>	Lack of understanding, due to the change of letters and / or numbers.

*Source: Prepared by the author*

The intervention in the center has been developed with two different didactic sequences corresponding to the traditional methodology and gamified, for this, two classrooms of different sex have been chosen for the traditional methodology, and the same for the gamified methodology.

For the traditional methodology the resources present in a common classroom have been required: a blackboard, papers, pens and the textbook of the students. However, for the gamified methodology, chess has been used as a manipulative material, along with the blackboard and school material. This material was given to the students at the beginning of the first session to promote the teaching-learning process, this material consists of a board with its pieces in two dimensions for each student.

## **I. TRADITIONAL METHODOLOGY**

This was carried out in two different classes during 8 sessions of 20 minutes. The students did a test in the first lesson and in the last lesson (both equal), to check and analyze their evolution. During the other 6 sessions the fractions, the types of fractions, equivalences and the addition and subtraction of them were explained, using the textbook and the blackboard.

In the first session, the pretest was given to the two classes with the aim of knowing the base level of each student and the concepts they knew. The pretest consists of seven questions ordered from least to greatest complexity. The questions include: name and classification of fractions; definition of equivalence and equality, as well as knowing how to find them; and addition and subtraction of fractions with different denominators, knowing how to use the steps correctly (see Annex 2).

During the second session the concept of fraction was explained, as well as the different types of fractions: equal-one, proper and improper. The class started with a brainstorming, to achieve a complete definition of fraction. Once this concept was understood, the parts of a fraction were explained as well as the types of fractions. As materials were used the use of the blackboard and the book for those students who had not understood it could reinforce the contents. The objective of this session has been to be familiar with the terms of: fraction, numerator, denominator, proper, improper and equal-one.

The next session referred to the practice of the contents taught during the previous session. The objectives have been to put into practice the concepts seen in the previous day, and to clarify any doubts that may exist among the students.

Once the basic concepts and practices were assimilated, the fourth session was opened, in which the difference between equality and equivalence was explained. To this end, examples of daily life were used and, as a material, the blackboard. The objectives of this session have been to differentiate between the terms of equivalence and equality; as well as to learn how to obtain them from a fraction.

The fifth session was an implementation of the concepts seen in the fourth session. For this, the textbook was used, and several exercises were carried out that were corrected in a loud voice explaining the failures. The objectives have been to know how to find new equivalent fractions and equalities from a fraction; and to be able to take these terms to the daily life.

Regarding the sixth session, the addition and subtraction of fractions with common denominator (which had been seen in the previous course) was quickly reviewed and the addition and subtraction of fractions with a different denominator was explained. The objective of the session has been to understand the difference between the addition and subtraction with different or common denominator, as well as to be able to understand the steps to add and subtract with different denominator.

During the seventh session, the textbook was used as a resource to perform addition and subtraction exercises for fractions with different denominators, in order to have a better understanding of the concepts and the steps to follow. The objective has been to achieve the addition and subtraction with different denominator, comparing them to know what formula to apply.

In the last session they were given the posttest, which, as already mentioned, is the same as the pretest delivered in the first session (see Annex 2). The objectives of this session have been: to distinguish the types of fractions as well as to recognize them; to know the difference between equality and equivalence and put it into practice; and to know the steps of adding and subtracting fractions with different denominators. This posttest has been compared with the pretest to know the evolution of each student (see Table 2).

Table 2: Summary traditional methodology sessions.

<b>SESSION</b>	<b>DEVELOPING</b>	<b>OBJECTIVES</b>
1 <sup>st</sup>	Pretest.	To know the base with which the students leave.
2 <sup>nd</sup>	Explanation of the different types of fractions (equal-one proper and improper).	To be familiar with the terms of: fraction, numerator, denominator, own, improper and equal-one.
3 <sup>rd</sup>	Exercises of types of fractions (apparent, proper and improper).	To carry out the concepts seen in the previous day.
4 <sup>th</sup>	Differentiation between equality and equivalent fractions.	To be able to differentiate the terms and learn to obtain equalities and equivalences.
5 <sup>th</sup>	Exercises of equality and equivalent fractions	To find equivalent fractions and equalities.
6 <sup>th</sup>	Explanation of addition and subtraction of fractions.	To know how to differentiate between addition and subtraction with different or common denominator
7 <sup>th</sup>	Exercises of addition and subtraction of fractions to assimilate concepts.	To implement of the addition and subtraction with the common and different denominator.
8 <sup>th</sup>	Post-test	To differentiate fraction types, equality and equivalence, and the addition and subtraction of fractions.

Source: Prepared by the author

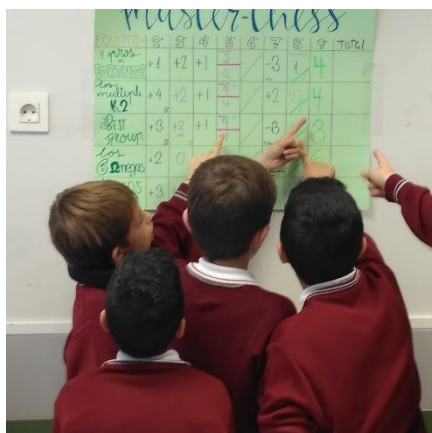


## II. GAMIFIED METHODOLOGY

The gamified methodology, like the traditional one, was developed in two different classrooms. This methodology consisted of 10 sessions of 20 minutes each, except the ninth session, which was 30 minutes. During these sessions, we explained: the types of fractions; equivalence and equality; and the addition and subtraction of the same; through the blackboard and chess as manipulative material.

Throughout the sessions, the students were getting points for the team that was reflected in the ranking of the class (see Illustration 1). These points could be achieved by: speed in preparing and removing materials; silence once the teacher has indicated it; know how to behave during the explanations; respect for colleagues; and fellowship, helping everyone in the team that needs it.

*Illustration 1: Ranking of the gamified male classroom.*



*Source: Prepared by the author*

At the same time, students were able to obtain badges as a reward for personal improvement at each level. In the last session a contest was held, which served to review all the concepts and contents seen during the previous sessions.

The first session was divided into two parts. At the beginning of the class they were given a pretest (see Annex 2), which has been purchased at the end of the development of the methodology with the posttest. While the students were taking the test, they were given: a board and a set of cards (see Annex 3), as manipulative material; and a chess player's card (see Annex 4), in which they were going to be able to stick the badges that they achieved in each level.

At the end of the test, the class was divided into 4 or 5 groups depending on the classroom. Once they were grouped, each one chose: a name for the group and a captain. In turn, they had to design their own avatar for the chess card and assign a name for it. The objectives of this session have been: to know with what knowledge the students started and to divide the class into groups.

During the second session, the students were shown the placement of the pieces on the board as well as the movements of each of them. To consolidate this knowledge, games such as "Mouse that catches you the cat" or "Escape from the labyrinth" were made. Once the movements of the pieces were assimilated, the students were asked to transform their traditional chess boards into numerical boards (see Annex 5), where the fractions would later work.

At the end of the class, those students who assimilated these concepts, obtained their badges corresponding to the movements and the recognition of each pieces, thus passing level (see Annex 6). The objective of this session has been: to familiarize the students with the manipulative material they were going to use, as an aid to assimilate the concepts of fractions during the rest of the sessions.

In the third session the values that each piece of chess was going to have as manipulative material were explained (see Table 3). Then, we began to see the concept of fraction as well as the types of fractions, as they appear in the *Boletín Oficial de la Comunidad de Madrid* (see Annex 1), for 5th year of Primary.

Table 3 : Values of the pieces

<b>King</b>	1	<b>Knight</b>	4
<b>Pawn</b>	2	<b>Rook</b>	5
<b>Bishop</b>	3	<b>Queen</b>	6

Source: Prepared by the author

To explain the fractions, chess was used. The students created fractions using the number-board and the pieces with their different values. For this they had to place the pieces in different cells, the piece would correspond to the numerator and the cell to the denominator, in such a way that the position of the piece was equal to a fraction (see Illustration 2).

Illustration 2: Creating fractions

The illustration shows an 8x8 chessboard grid with columns numbered 1 to 8 and rows labeled A to H. A Rook piece is placed on square G2, and a Knight piece is placed on square C6. To the right of the board, handwritten notes show the calculation of fractions based on the piece's value (N) and the cell's value (D):

$$\frac{N \text{ (Piece's value)}}{D \text{ (cell's value)}}$$

$$\text{Knight C6} = \frac{4}{3}$$

$$\text{Rook G2} = \frac{5}{2}$$

Source: Prepared by the author

The objectives of this session have been: to understand the concept of fraction and the differences between the three types of fractions (proper, improper and equal-one); and to understand how fractions are formed using the pieces and the chessboard.

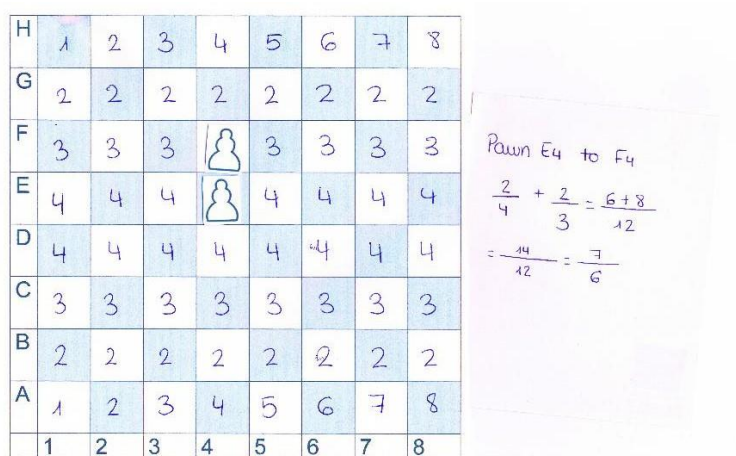
In the fourth session the students completed their first challenge. This consisted of a series of activities related to the proper, improper and apparent fractions, seen during the previous session (see Annex 7). After twenty minutes have passed, the challenges were picked up and corrected. Once the results were obtained, the corresponding badges were given to the students who had attained the level, and the points as a group were added to the "Master Chess" ranking. The objectives of this session have been: to put into practice the knowledge learned the previous day; to learn how to place and create fractions with the board; and to differentiate the three types of fractions.

During the fifth session, the difference between equality and equivalence was explained. For this, examples of daily life were used, which the students solved with the help of chess, as manipulative material. The objective of this session has been to differentiate the terms and learn to obtain equalities and equivalences.

In the next session the second challenge was presented. This included four activities, all of them on the equivalence and equality of fractions, seen in the previous session (see Annex 8). As in the first challenge, at 20 minutes were collected and corrected, to be able to deliver the corresponding badges to each student, depending on their successes (see Annex 6). The objective of this session has been to demonstrate that they understood the differences between equal and equivalent fractions.

Regarding the seventh session, there was a quick review of the addition and subtraction of fractions with common denominator (viewed in the previous course). After this, the addition and subtraction of fractions with a different denominator was explained, through the movement of pieces in the chessboard. If a piece moved forward or sideways, the initial and final positions will be added. On the contrary, if a piece receded, the subtraction of positions will be performed (see Illustration 3).

Illustration 3: Example of addition



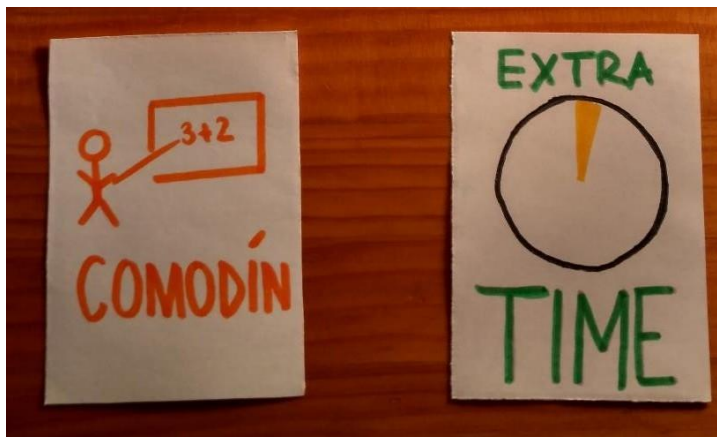
Source: Prepared by the author

The objectives of the session have been: to differentiate between both addition and subtractions, as well as to understand the steps to add and subtract with different denominator, with the help of chess.

During the eighth session the students faced their last challenge. In this, they could find four activities related to the subtraction and the addition of fractions (see Annex 9). At the end of this, they were given the badges in correspondence with the successes of each student and the points were added to the ranking of the class. The objective of this challenge has been to solve problems through the movement of the pieces.

The ninth session, in contrast to the other sessions, lasted 30 min. In this, two activities were carried out, with which the concepts seen in the previous eight sessions were reviewed. At the beginning of the class, the groups exchanged the points earned during the rest of the sessions for: "extra time", 1 point was required and "teacher's wild card", 5 points were required (see Illustration 4).

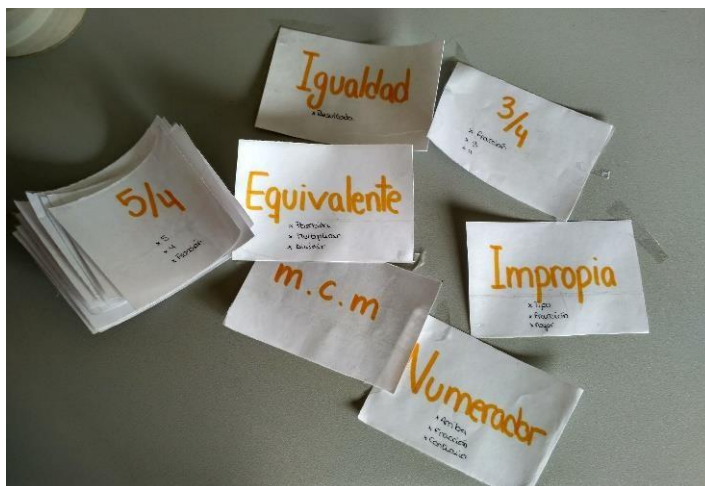
Illustration 4: Wildcards for contests



Source: Prepared by the author

The first of the activities was called "Password". For this one student from each team was needed. Each child took a flashcard (in turns) and tried to describe what was written in it, with only one word in each turn. Once the group found out the word, the chosen student could pick up another flashcard to solve. Each student had 1 minute to find out the most flashcards they could (see Illustration 5).

Illustration 5: Password's flashcards



Source: Prepared by the author

For example, a student picked up the flashcard in which "Numerator" was written, the student had to try to describe it without saying "above, fraction, opposite", and only with one word for each shift. The student said "Denominator", his group "down", "the other"... so on until the group found out the word.

Regarding the second one, it was a "*Cifras y letras*". For this, all the groups took a paper. In each game the teacher dictated an addition or subtraction with same or different denominator. The students had to do the necessary operations to find the correct result. When a group finished shouting "stop", at the same time, the whole class rose their hands. The professor checked that the answer was correct. If it was correct they obtained a bonus, in case of being incorrect the whole class could keep trying.

The objectives of this ninth session have been to review all the concepts and strengthen those that failed to understand in previous sessions, all through a playful activity.

Finally, in the last session, the post-test was delivered, which, as already mentioned, is the same as the initial pretest delivered in the first session (see Annex 2). The objectives of this session have been: to distinguish the three types of fractions as well as to recognize them; to recognize the difference between equality and equivalence and put it into practice; and to know how to add and subtract fractions of different denominator (see Table 5)

*Table 4: Summary gamified methodology sessions.*

<b>SESSION</b>	<b>DEVELOPING</b>	<b>OBJECTIVES</b>
<b>1°</b>	Pretest and delivery of materials	To know with what knowledge the students started and to divide the class into groups.
<b>2<sup>a</sup></b>	Presentation of chess and its use	To familiarize the students with the manipulative material they were going to use as an aid to assimilate the concepts of fractions during the rest of the sessions.
<b>3<sup>a</sup></b>	Type of fractions (proper, improper and equal-one)	To understand the concept of fraction and the differences between the three types of fractions (proper, improper and equal-one); and to understand how fractions are formed using the pieces and the chessboard.
<b>4<sup>a</sup></b>	First challenge	To put into practice the knowledge learned the previous day; to learn to place and create fractions with the board; and to differentiate the three types of fractions.
<b>5<sup>a</sup></b>	Explanation about equal and equivalent fractions	To differentiate the terms and learn to obtain equalities and equivalences.



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<b>SESSION</b>	<b>DEVELOPING</b>	<b>OBJECTIVES</b>
<b>6<sup>a</sup></b>	Second challenge	To demonstrate that they understood the differences between equal and equivalent fractions.
<b>7<sup>a</sup></b>	Explanation of addition and subtraction of fractions	To differentiate between both addition and both subtractions, as well as to understand the steps to add and subtract with different denominator, with the help of chess.
<b>8<sup>a</sup></b>	Third challenge	To solve problems through the movement of the pieces.
<b>9<sup>a</sup></b>	Activities	To review all the concepts, and strengthen those that failed to understand in previous sessions, all through a playful activity.
<b>10<sup>a</sup></b>	Post-test	To distinguish the three types of fractions as well as to recognize them; to recognize the difference between equality and equivalence and put it into practice; and to know how to add and subtract fractions of different denominator

*Source: Prepared by the author*

---

## RESULTS

Once the results of all the pretests and post-tests made by the 5th grade students have been collected, they have been analyzed and compared in different ways. In first place, classrooms of the same sex but with different methodologies, traditional and gamified, have been compared. Once the four classrooms have been analyzed, the methodologies have been compared separately, first the traditional female classroom with the traditional male classroom; and secondly, the gamified male classroom with the gamified female classroom. Finally, students with SEN (Special Educational Needs) have been analyzed separately.

Firstly, when comparing male classrooms, several students were observed to have increase their learning achievement from pre-test to post-test. In the gamified classroom 86% of students improved it, compared to 63% of students in the traditional classroom (see Table 5 and Table 6). Focusing on the students who have progressed during the investigation, it could be observed that the average of points obtained over 10, in the post-test has been of 5.35 in the gamified classroom slightly higher compared to the traditional classroom with an average of 3.24. The average of points of the students has been calculated in base 10, since it is the decimal numbering system, most common par excellence (see Table 5 and Table 6).

*Table 5: Percentage of learning achievement in the male gamified classroom.*

<b>GAMIFICATION</b>	<b>% STUDENTS</b>	<b>AVERAGE</b>
<b>INCREASE</b>	86%	5,35
<b>EQUAL</b>	14%	0
<b>DECREASE</b>	0%	0

*Source: Prepared by the author*

Table 6: Percentage of learning achievement in the male traditional classroom.

<b>TRADITIONAL</b>	<b>% STUDENTS</b>	<b>AVERAGE</b>
<b>INCREASE</b>	63%	2,56
<b>EQUAL</b>	37%	0
<b>DECREASE</b>	0%	0

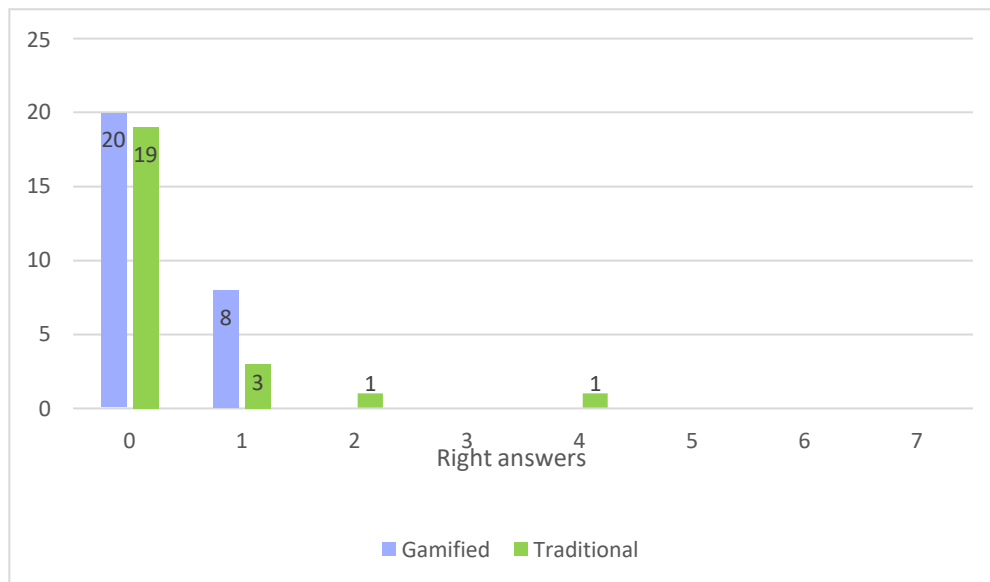
*Source: Prepared by the author*

Now if you study both classes, you can see an increase in the average of correct answers between the pretest and the 60% post-test in the gamified classroom, which compared to 23% in the traditional classroom, is more than double.

In the following graphics we can see how many students answered correctly both in the initial test, pretest (see Graphic 1) and in the last one, post-test (see Graphic two). Likewise, the blue colours correspond to the gamified classroom and the green colours correspond to the traditional classroom.

In Graph 1 it can be seen how almost 100% of the students in both classrooms start in this research without any basis. Only one student in the traditional classroom starts with a higher base, due to external factors of the school.

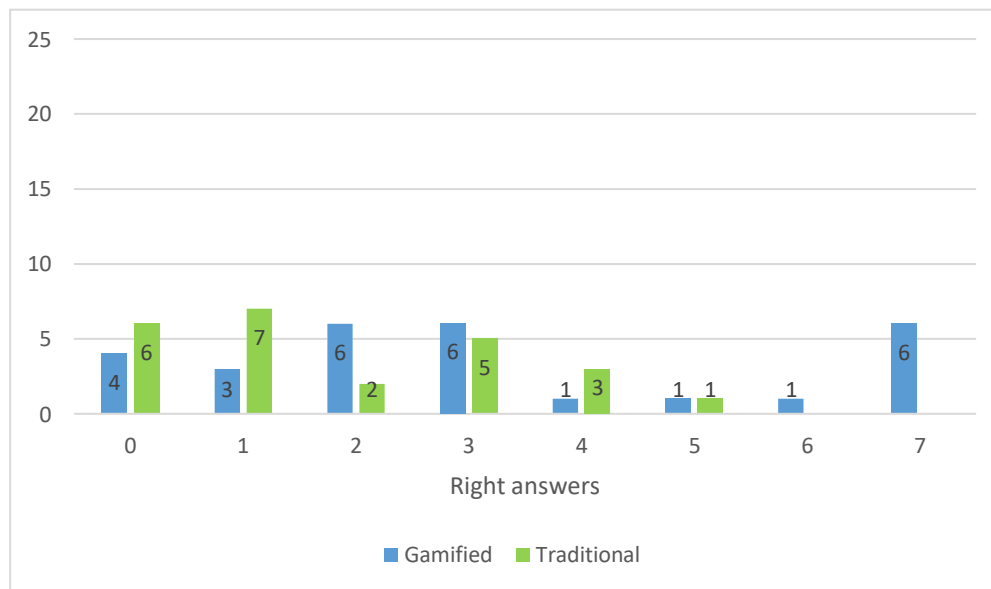
Graphic 1: Comparison of right answers in the pretest between the male gamified and traditional classrooms.



Source: Prepared by the author

In the analysis of the post-tests of male classrooms, it is possible to highlight the failures and the excellences in both classrooms. In traditional classrooms there have been 8 absolute failures, none of them have met the expectations of the research, doubling that in the gamified classroom. In terms of excellence during the tests, the gamified classroom has had 6 students, who have obtained the maximum number of correct answers, which indicates that they have internalized the new contents (see Graphic 2).

Graphic 2: Comparison of right answers in the post-test between the male gamified and traditional classrooms.



Source: Prepared by the author

When comparing female classrooms, you can see a number of students that have increased their learning achievement from pretest to post-test. In the gamified classroom 94% of students improved it, compared to 63% of students in the traditional classroom (see Table 7 and Table 8). Focusing on the students who have progressed during the research, it can be observed that the average of points obtained over 10, in the post-test was 3.73 in the gamified classroom more than double compared to the traditional classroom with an average of 1.28. The average points of the students have been calculated in base 10, since it is the decimal numbering system (see Table 7 and Table 8).

*Table 7: Percentage of learning achievement in the female gamified classroom.*

<b>TRADITIONAL</b>	<b>% STUDENTS</b>	<b>AVERAGE</b>
<b>INCREASE</b>	63%	3,73
<b>EQUAL</b>	37%	0
<b>DECREASE</b>	0%	0

*Source: Prepared by the author*

*Table 8: Percentage of learning achievement in the female traditional classroom.*

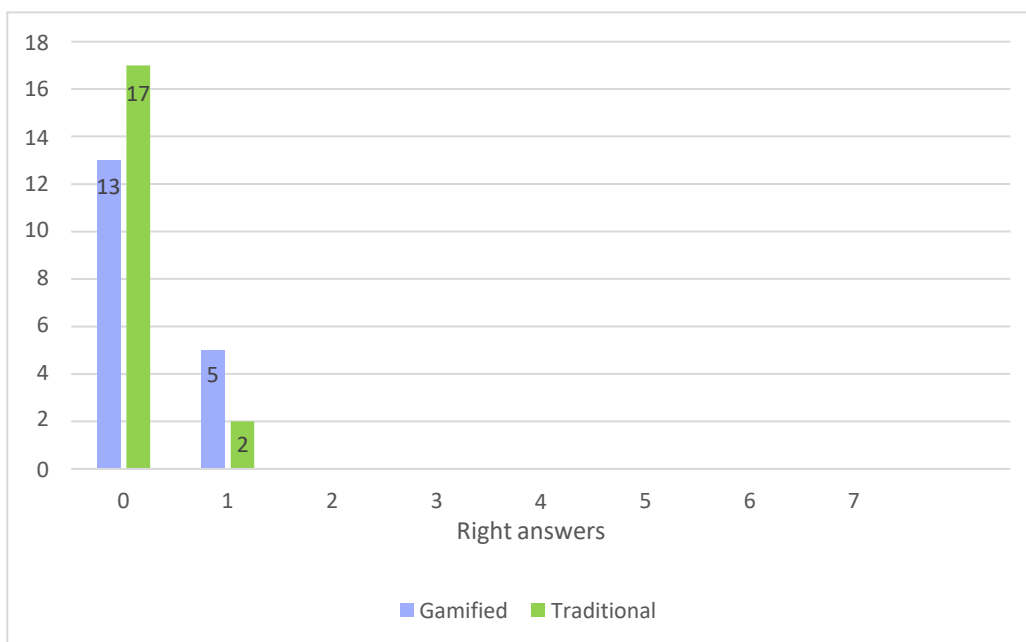
<b>GAMIFICATION</b>	<b>% STUDENTS</b>	<b>AVERAGE</b>
<b>INCREASE</b>	94%	1,28
<b>EQUAL</b>	6%	0
<b>DECREASE</b>	0%	0

*Source: Prepared by the author*

Now if you look at both classes, you can see an increase in the average of hits between the pretest and the post-test of 45% in the gamified classroom, compared to 17% in the traditional classroom. As in male classrooms, learning achievement has been higher in the gamified classroom than in the traditional classroom.

Now comparing the female classrooms with different methodologies, it can be observed that none of the students at the beginning of the investigation had a previous basis, since 100% of the students are between 0 and 1 right answers (see Graphic 3)

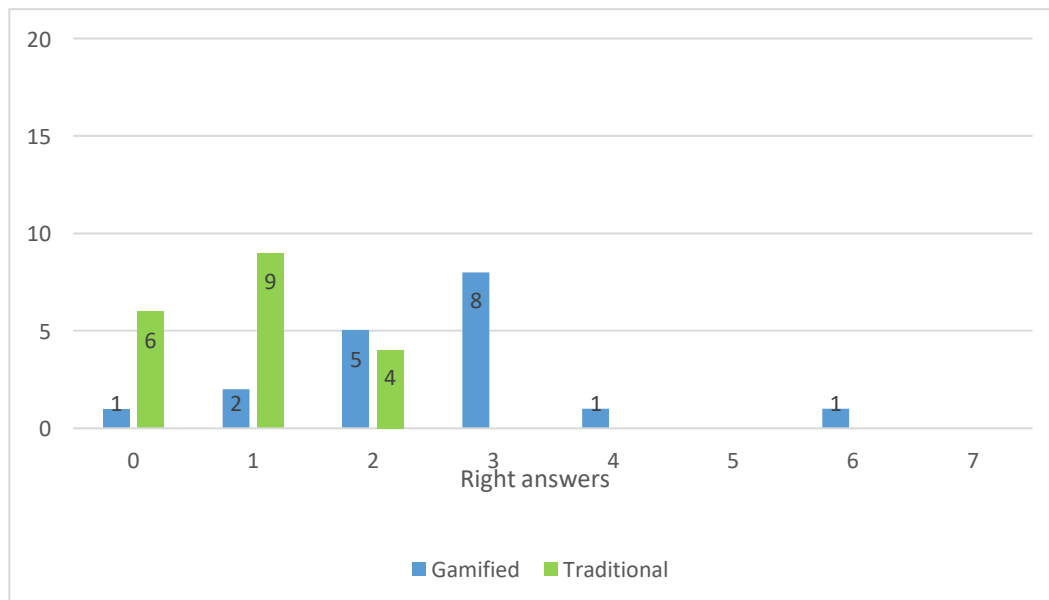
*Graphic 3: Comparison of right answers in the pretest between female gamified and traditional classrooms.*



*Source: Prepared by the author*

In these classrooms, unlike those of the students, there has not been any case of excellence. Regarding failures, the traditional classroom has had 8 failures, doubling that in the gamified classroom where there have been only 4 failures (see Graphic 4).

*Graphic 4: Comparison of right answers in the post-test between the female traditional and gamified classrooms.*



*Source: Prepared by the author*

Secondly, the classrooms of different sex and equal methodologies have been compared. In the gamified methodology in the female classroom 94% of the students have increased their learning achievement, however in the male classroom the increase has been 86%. Although the percentage of comparison between the pretest and the post-test, is higher in the female classroom. The average of successes of the final increase is greater in the students with a 5.35 compared to a 3.73 of the students.

When comparing the failures of both classrooms, it can be observed that in male classrooms there have been twice as many failures as in the female classroom. However, among the students there have been 6 of them who have achieved excellence at the end of the research, something that has not happened in female classrooms.



On the other hand, when analyzing and counting the traditional methodology classrooms, the similarity between the percentages of both classrooms could be observed. More than half of the students have increased their learning achievement during the research, specifically 63% of the students. However, the average number of correct answers from the pretest to the post-test in the male classrooms has doubled compared to the female classrooms. Even so, the average did not surpass the minimum knowledge that was expected in this investigation.

Finally, it is worth mentioning the influence of both methodologies on the learning achievement of students with special educational needs during the investigation. Of the nine students with SEN, five of them were boys and four girls. Among the students, four of them have experienced gamification. The rest have worked with the traditional methodology.

All the students with SEN began the research without the basic knowledge required, even something lower than the basis that the rest of their classmates had. At the end of the sessions, all the students present in the gamified methodology increased their learning achievement, with an average of 5 points in the post-test. Among these students, two of them stand out, reaching excellence (the maximum number of successes in the final test). Regarding the students with traditional methodology, none of them met the expectations of the intervention.

## CONCLUSIONS

From the results obtained during this research, it has been proven that the objectives set have been achieved: demonstrate that chess can be used as manipulative material; and present the gamification and its use of the mechanics of the game (in our case of chess) to achieve a significant learning of mathematics. Both in order to increase the learning achievement of the students.

With the development of this research it has been fulfilled that the use of chess as a manipulative material to teach mathematics, specifically the addition and subtraction of fractions, improves the learning achievement of the student in this subject. In turn, thanks to the use of chess as a board game in an educational chess version, it has been observed that, in addition to helping to achieve the learning of mathematical objectives, the motivation of the students increases. In addition, the children said they felt more important because they considered chess to be a game of elders.

During the sessions, a greater involvement of the students was appreciated, since they were more stimulated and this was transmitted at their homes. This motivation in them, translates into a favorable learning and a better acquisition of concepts by children.

Regarding students with special needs, the use of chess as a manipulative material has been an aid to their motivation, thus increasing their capacity for attention in class and increasing their learning achievement, avoiding failure.

Any research work developed with a minimum of enthusiasm helps to clear up some unknowns about the topic but, simultaneously, generates new questions, new ideas and / or opens new ways of working. In relation to the use of chess as manipulative material, a future interesting line of work would be the multiplication and division of fractions by means of chess. Regarding gamification as an educational methodology, it could be studied the application of the same method in another different subject, in order to achieve the proposed objectives and motivate students to continue learning.

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## ANNEXS

### ANNEX 1. BOLETÍN OFICIAL DE LA COMUNIDAD DE MADRID

#### Quinto curso

##### *Números y operaciones*

Números naturales. Nombre y grafía de los números menores que un millón. Ordenación. Descomposición según el valor posicional de las cifras.

1. Lee, escribe al dictado con cifras y letras, descompone en forma aditiva y aditivo-multiplicativa, atendiendo al valor posicional de sus cifras números naturales menores que un millón.

2. Construye reglas graduadas a partir de otras sin graduar, hechas de materiales asequibles, en las que se hayan señalado previamente el 0 y el 1.

3. Redondea a los millares, centenas y decenas números menores que un millón.

Divisibilidad. Múltiplos y divisores. Números primos.

4. Define las relaciones "divisor de" y "múltiplo de" entre dos números y determina si un número es múltiplo o divisor de otro.

5. Calcula los primeros múltiplos de un número dado.

6. Halla todos los divisores de cualquier número menor que 50.

7. Define número primo y número compuesto y memoriza la lista ordenada de los números primos menores que 30.

8. Conoce las reglas de divisibilidad por 2, 5 y 10.

Numeración romana.

9. Conoce la numeración romana y las equivalencias con la numeración decimal.

10. Utiliza el sistema de numeración romana para datar hechos históricos.

Fracciones. Fracciones y decimales. Equivalencia de fracciones. Ordenación de fracciones de igual denominador. Simplificación de fracciones.

11. Es capaz de dar automáticamente la expresión fraccionaria de decimales sencillos.

12. Encuadra el valor numérico de una fracción entre dos naturales consecutivos.

13. Sitúa o intercala fracciones en una recta graduada entre dos naturales consecutivos.



5. Rodea la opción correcta:

- $5/3 + 8/5 =$   
b)  $13/15$     b)  $15/11$     c)  $49/15$     d)  $13/5$
  
- $1/4 + 3/2 =$   
b)  $7/4$     b)  $4/6$     c)  $3/7$     d)  $2/4$
  
- $3/5 - 2/4 =$   
a)  $1/1$     b)  $1/4$     c)  $1/10$     d)  $5/9$

6. ¿Qué pasos hay que seguir para sumar y restar fracciones?

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7. Realiza las siguientes operaciones:

- $15/3 + 1/4 + 1/2 =$  \_\_\_\_\_
- $10/5 + 3/2 =$  \_\_\_\_\_
- $8/4 - 1/2 =$  \_\_\_\_\_
- $22/3 - 12/6 =$  \_\_\_\_\_

ANWERS

CURSO \_\_\_\_\_

1

Nº LISTA \_\_\_\_\_

SEXO: H / M

1. ¿Cuántos tipos de fracciones hay? Cítalas

Hay 3. Propias, impropias y aparentes.

2. Clasifica según la tabla:

2/3    7/7

5/6    1/2

10/13    4/3

8/6    15/5

1

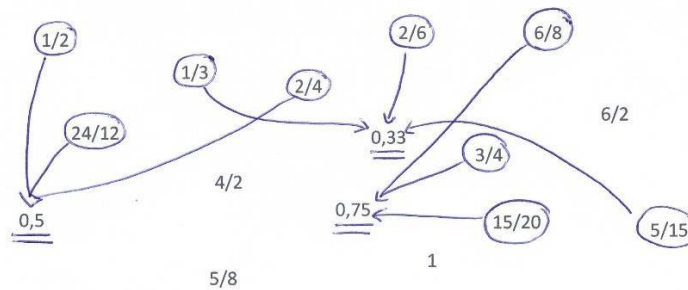
<1	=1	>1
2/3 ; 5/16	7/7	8/6    4/3
10/13 ; 1/2	1	15/5

3. ¿Qué diferencia hay entre equivalencia e igualdad?

Equivalencia: 2 fracciones distintas representan misma cantidad

Igualdad: es el resultado

4. Rodea con colores distintos las fracciones que son equivalentes entre sí, y únelas con su igualdad.





5. Rodea la opción correcta:

•  $5/3 + 8/5 =$   
a)  $13/15$    b)  $15/11$    **c) 49/15**   d)  $13/5$

•  $1/4 + 3/2 =$   
**b) 7/4**   a)  $4/6$    c)  $3/7$    d)  $2/4$

•  $3/5 - 2/4 =$   
a)  $1/1$    b)  $1/4$    **c) 1/10**   d)  $5/9$

6. ¿Qué pasos hay que seguir para sumar y restar fracciones?

m.c.m

Multiplicar el numerador

Realizar la operación

7. Realiza las siguientes operaciones:

1)  $15/3 + 1/4 - 1/2 =$   $23/4$

2)  $10/5 + 3/2 =$   $7/2$

3)  $8/4 - 1/2 =$   $3/2$

4)  $22/3 - 12/6 =$   $16/3$

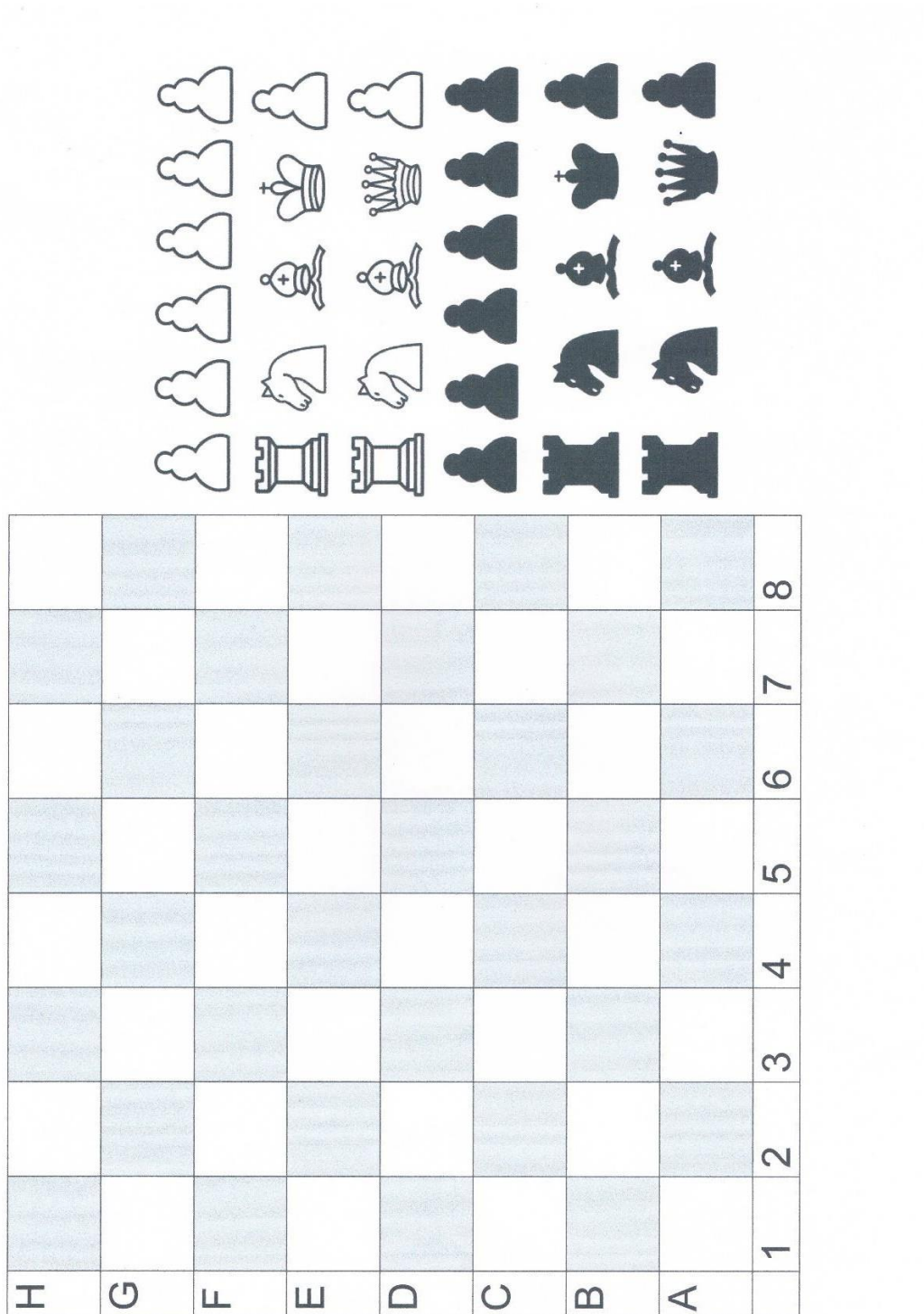
1)  $\frac{15}{3} + \frac{1}{4} + \frac{1}{2} = \frac{60 + 3 + 6}{12} = \frac{69}{12} = \frac{23}{4}$

2)  $\frac{10}{5} + \frac{3}{2} = \frac{20 + 15}{10} = \frac{35}{10} = \frac{7}{2}$

3)  $\frac{8}{4} - \frac{1}{2} = \frac{8 - 2}{4} = \frac{6}{4} = \frac{3}{2}$

4)  $\frac{22}{3} - \frac{12}{6} = \frac{44 - 12}{6} = \frac{32}{6} = \frac{16}{3}$

**ANNEX 3: CHESSBOARD AND PIECES.**



**ANNEX 4: CHESS PLAYER'S PASSBOOK.**

	Cartilla de ajedrecista			
	Nombre:			
	Número:			



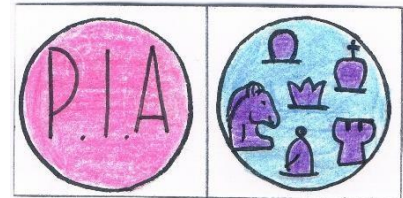
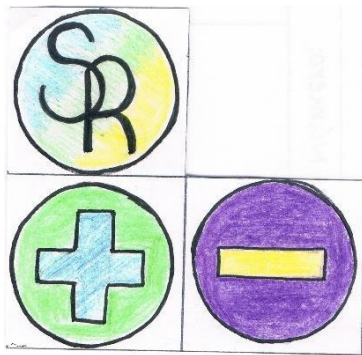
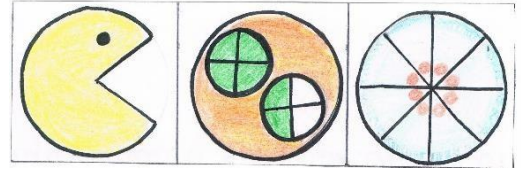
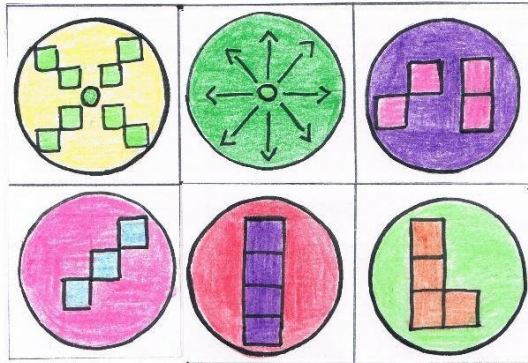
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**ANNEX 5: CHESS NUMERIC BOARD.**

H	1	2	3	4	5	6	7	8
G	2	2	2	2	2	2	2	2
F	3	3	3	3	3	3	3	3
E	4	4	4	4	4	4	4	4
D	4	4	4	4	4	4	4	4
C	3	3	3	3	3	3	3	3
B	2	2	2	2	2	2	2	2
A	1	2	3	4	5	6	7	8
	1	2	3	4	5	6	7	8

---

**ANNEX 6: BADGES**



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**ANNEX 7: FIRST CHALLENGE WITH ANSWERS**

CURSO \_\_\_\_\_  
Nº LISTA \_\_\_\_\_

SEXO: H / M

**1º DESAFÍO**

1. Rodea cuál de las siguientes piezas en su correspondiente posición, son fracciones mayores que la unidad.

Peon B6	Caballo F5	Torre H3
Peon F7	Dama D3	Alfil G5
Alfil C4	Rey D8	Peón H6

¿Cómo se llamaban este tipo de fracciones?

\_\_\_\_\_

2. ¿Qué pieza no puede ser nunca una fracción mayor que la unidad? ¿Por qué?

\_\_\_\_\_

\_\_\_\_\_

3. Escribe la representación de las siguientes fracciones, y di si son: propias (Pro), aparentes (apa) o impropias (Imp).

$\frac{5}{3}$

$\frac{4}{3}$

$\frac{5}{8}$

$\frac{2}{3}$

$\frac{3}{5}$

$\frac{6}{6}$

$\frac{2}{4}$

$\frac{1}{1}$

$\frac{2}{1}$

4. Ordena de menor a mayor las siguientes piezas: Rey A1, alfil G4, Torre H4, Dama D3, Peón C5.

\_\_\_\_\_



ANSWERS

CURSO \_\_\_\_\_

Nº LISTA \_\_\_\_\_

SEXO: H / M

1º DESAFÍO

1. Rodea cuál de las siguientes piezas en su correspondiente posición, son fracciones mayores que la unidad.

Peon B6	Caballo F5 $\frac{4}{3}$	Torre H3 $\frac{5}{3}$
Peon F7	Dama D3 $\frac{6}{4}$	Alfil G5 $\frac{3}{2}$
Alfil C4	Rey D8	Peón H6

¿Cómo se llamaban este tipo de fracciones?

Impropias

2. ¿Qué pieza no puede ser nunca una fracción mayor que la unidad? ¿Por qué?

El rey, porque su valor es 1, y en el tablero no hay ningún número < de 1

3. Escribe la representación de las siguientes fracciones, y di si son: propias (Pro), aparentes (apa) o impropias (Imp). *(Hay más opciones)*

$\frac{5}{3}$ <span style="border: 1px solid black; padding: 2px;">To C4 / Imp</span>	$\frac{4}{3}$ <span style="border: 1px solid black; padding: 2px;">Cb DG / Imp</span>	$\frac{5}{8}$ <span style="border: 1px solid black; padding: 2px;">To A8 / Pro</span>
$\frac{2}{3}$ <span style="border: 1px solid black; padding: 2px;">Pe F7 / Pro</span>	$\frac{3}{5}$ <span style="border: 1px solid black; padding: 2px;">Al H5 / Pro</span>	$\frac{6}{6}$ <span style="border: 1px solid black; padding: 2px;">Re H6 / Apa</span>
$\frac{2}{4}$ <span style="border: 1px solid black; padding: 2px;">Re E2 / Pro</span>	$\frac{1}{1}$ <span style="border: 1px solid black; padding: 2px;">Ry A1 / Apa</span>	$\frac{2}{1}$ <span style="border: 1px solid black; padding: 2px;">Pe H1 / Imp</span>

4. Ordena de menor a mayor las siguientes piezas: Rey A1, alfil G4, Torre H4, Dama D3, Peón C5.

Peón < Alfil < Rey < Torre < Dama

Rey A1 =  $\frac{1}{1}$

Dama D3 =  $\frac{6}{4}$

Alfil G4 =  $\frac{3}{4}$

Peón E7 =  $\frac{2}{4}$

Torre H4 =  $\frac{5}{4}$

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**ANNEX 8: SECOND CHALLENGE WITH ANSWERS.**

CURSO \_\_\_\_\_

Nº LISTA \_\_\_\_\_

SEXO: H / M

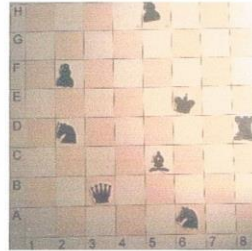
**2º DESAFIO**

1. ¿Cuáles de las siguientes piezas posicionadas en el tablero son equivalentes?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



2. Podrías poner un ejemplo de dos piezas equivalentes al alfil en el G7

\_\_\_\_\_

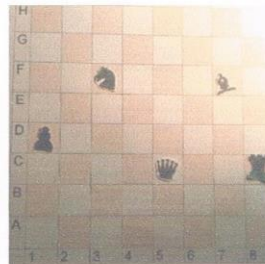
3. ¿A que son iguales las siguientes piezas?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

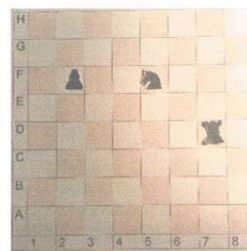
\_\_\_\_\_



4. ¿A qué casilla debería moverse el caballo para ser equivalente al peón? ¿y para ser equivalente a la torre?

\_\_\_\_\_

\_\_\_\_\_





ANSWERS

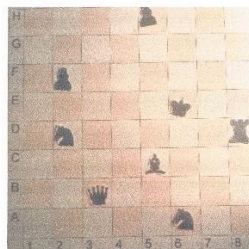
CURSO \_\_\_\_\_

Nº LISTA \_\_\_\_\_

SEXO: H / M

2º DESAFIO

1. ¿Cuáles de las siguientes piezas posicionadas en el tablero son equivalentes?



Ca D2 = Al C5 (4/4 = 3/3)  
Pf2 = CA6 (2/3 = 4/6)

2. Podrías poner un ejemplo de dos piezas equivalentes al alfil en el G7

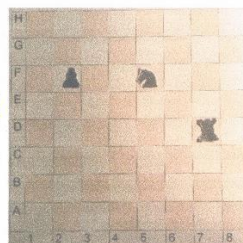
Al G7 = 3/2 // en

3. ¿A que son iguales las siguientes piezas?



Pe D1 (2/4) = 0'5  
R4 C5 (1/3) = 0'33  
Al F7 (3/3) = 1  
To C8 (5/3) = 1'6  
Ca C3 (4/3) = 1'3

4. ¿A qué casilla debería moverse el caballo para ser equivalente al peón? ¿y para ser equivalente a la torre?



Ca C5 (4/3) → Ca H6 (4/6) (equiv peón)  
Caballo no puede ser equivalente a la torre.

---

**ANNEX 9: THIRD CHALLENGE WITH ANSWERS.**

CURSO \_\_\_\_\_  
Nº LISTA \_\_\_\_\_

SEXO: H / M

3º DESAFIO

1. ¿Qué es lo primero que hay que hacer para sumar o restar fracciones?

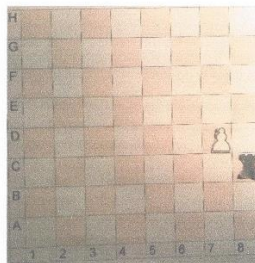
\_\_\_\_\_

2. Si el alfil en la posición E4, se mueve hacia delante hasta la G6, ¿qué resultado obtendrá en su movimiento?

\_\_\_\_\_

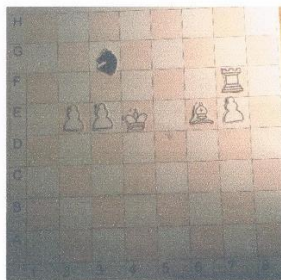
3. La torre en el C8 se ve atacada por el peón del D4, y retrocede dos casillas, ¿Cuál es el resultado del movimiento?

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_



4. El caballo se ve envuelto en un laberinto del cual debe salir, dibuja los movimientos y realiza las operaciones oportunas. Recuerda será + si se desplaza lateralmente o hacia delante, y será - si es para detrás.

\_\_\_\_\_  
\_\_\_\_\_



ANSWERS

CURSO \_\_\_\_\_  
Nº LISTA \_\_\_\_\_

SÉXO: H / M

3º DESAFIO

1. ¿Qué es lo primero que hay que hacer para sumar o restar fracciones?

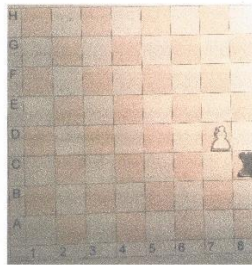
m.c.m del denominador

2. Si el alfil en la posición E4, se mueve hacia delante hasta la G6, ¿qué resultado obtendrá en su movimiento?

$$\frac{3}{4} + \frac{3}{2} = \frac{3+6}{4} = \frac{9}{4}$$

3. La torre en el C8 se ve atacada por el peón del D4, y retrocede dos casillas, ¿Cuál es el resultado del movimiento?

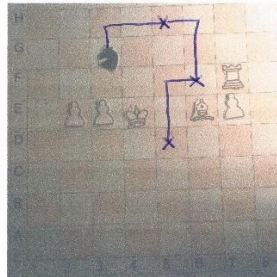
$$\frac{5}{4} - \frac{5}{8} = \frac{10-5}{8} = \frac{5}{8}$$



4. El caballo se ve envuelto en un laberinto del cual debe salir, dibuja los movimientos y realiza las operaciones oportunas. Recuerda será + si se desplaza lateralmente o hacia delante, y será - si es para atrás.

Hay varias formas es:

G3 → H5 → F6 → D5



$$\frac{4}{2} \ominus \frac{4}{5} \oplus \frac{4}{3} \oplus 1$$