PROBING CENTRAL STELLAR REGIONS WITH A NEW INDICATOR BASED ON THE INVERSION OF FREQUENCY RATIOS

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Abstract

In the last decade, astonishing progresses were achieved in asteroseismology thanks to the high-quality data from the space-based missions CoRoT, Kepler and TESS and the field will further prosper with the future PLATO mission.

This high precision is however limited by the "so-called" surface effects of solar-like oscillations. Despite several attempts (Kjeldsen et al. 2008, Ball & Gizon 2014, Sonoi et al. 2015, Ball et al. 2016), current approaches remain empirical and constitute a weakness in stellar modelling and inversion techniques. As illustrated in Buldgen et al. (2019) and Bétrisey et al. (2021, submitted), their actual implementation shows biases on the estimated stellar parameters.

For this reason, we developed a new indicator based on the inversion of frequency ratios instead of individual frequencies as it is currently done. This approach is motivated by the works of Roxburgh & Voronsov (2003) and Otí et al. (2005) who pointed out that these frequency ratios and their corresponding kernels are not sensitive to surface regions. In contrast, they are sensitive to deeper stellar layers. Therefore, our new indicator seems promising to better probe central stellar regions of intermediate-mass stars.

Context: relevance of inversion techniques

Following their success in helioseismology, inversion techniques applied in asteroseismology can constrain very precisely the internal structure (e.g. the mean density). The inversions are based on the following equation:

$$\frac{\delta\nu^{n,l}}{\nu^{n,l}} = \int_0^R K_{a,b}^{n,l}\frac{\delta a}{a}dr + \int_0^R K_{b,a}^{n,l}\frac{\delta b}{b}dr + \mathcal{O}(\delta^2) \tag{1}$$

with ν the oscillation frequency, a and b two structural variables (e.g. the density, the sound speed, the entropy proxy, ...), $K_{a,b}^{n,l}$ and $K_{b,a}^{n,l}$ the structural kernels and using the definition $\delta \mathrm{x} = (\mathrm{x}_{\mathrm{obs}} - \mathrm{x}_{\mathrm{ref}})/\mathrm{x}_{\mathrm{ref}}$.

However, frequencies are impacted by surface effects that can only be treated empirically. This motivated Roxburgh & Voronsov (2003) to define frequency ratios that damp these surface effects. Eq. (1) can be adapted for these ratios and the new kernels will have their amplitude suppressed in the surface regions and will be able to better probe central regions.

$$r_{01}(n) = \frac{d_{01}(n)}{\Delta_1(n)} \qquad r_{10}(n) = \frac{d_{10}(n)}{\Delta_0(n+1)} \qquad r_{02}(n) = \frac{d_{02}(n)}{\Delta_1(n)} \tag{2}$$

Defining new structural kernels

Observations: The classical kernels have a high amplitude in the surface region, especially at high radial order (Fig. 1), while the kernels based on ratios of frequencies have their amplitude damped at the surface and are better at probing central regions (Fig. 2, 3).

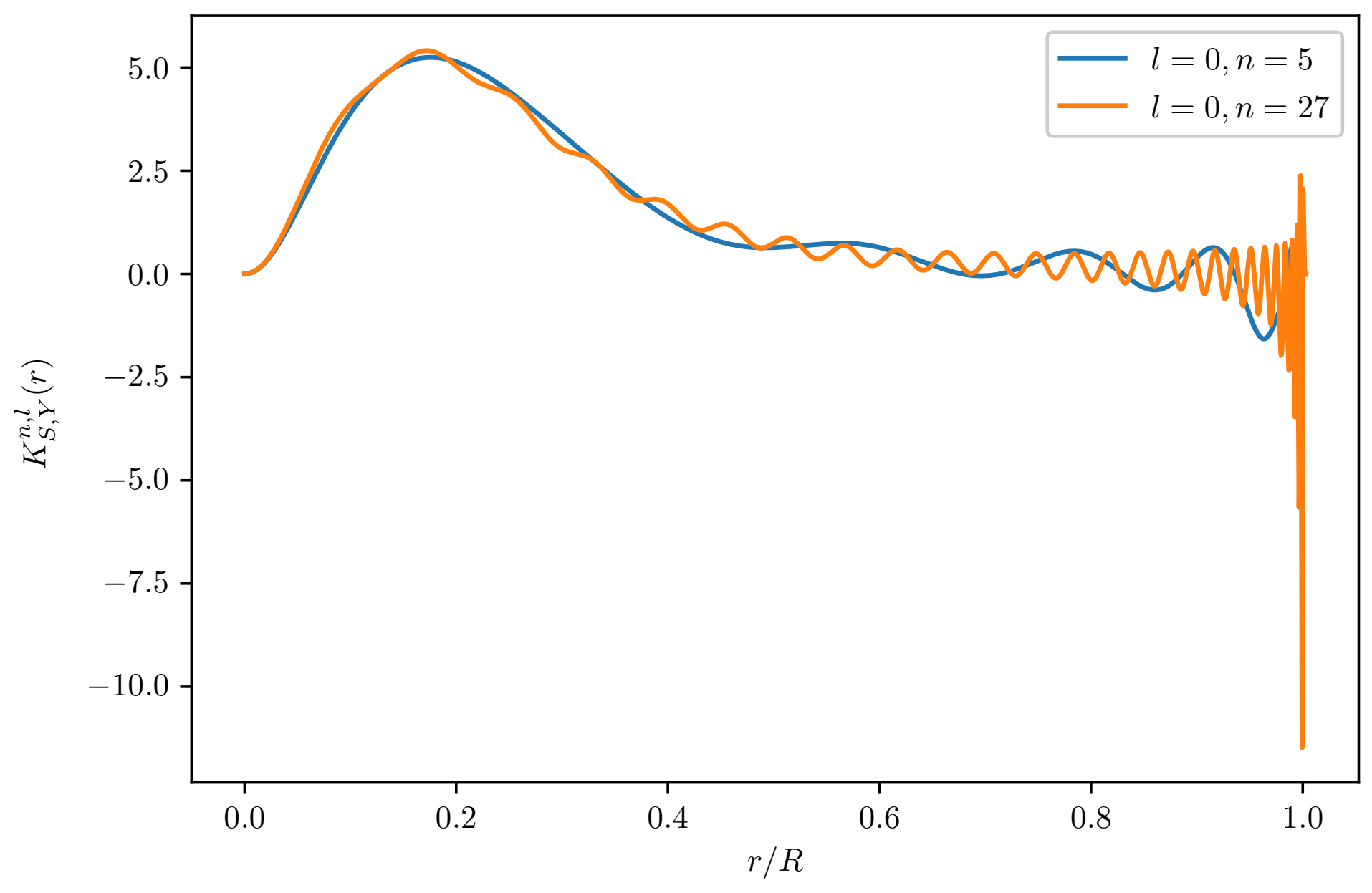

Fig. 1: Structural kernels of the pair (S,Y) based on the frequencies at low (blue) and high (orange) radial order.

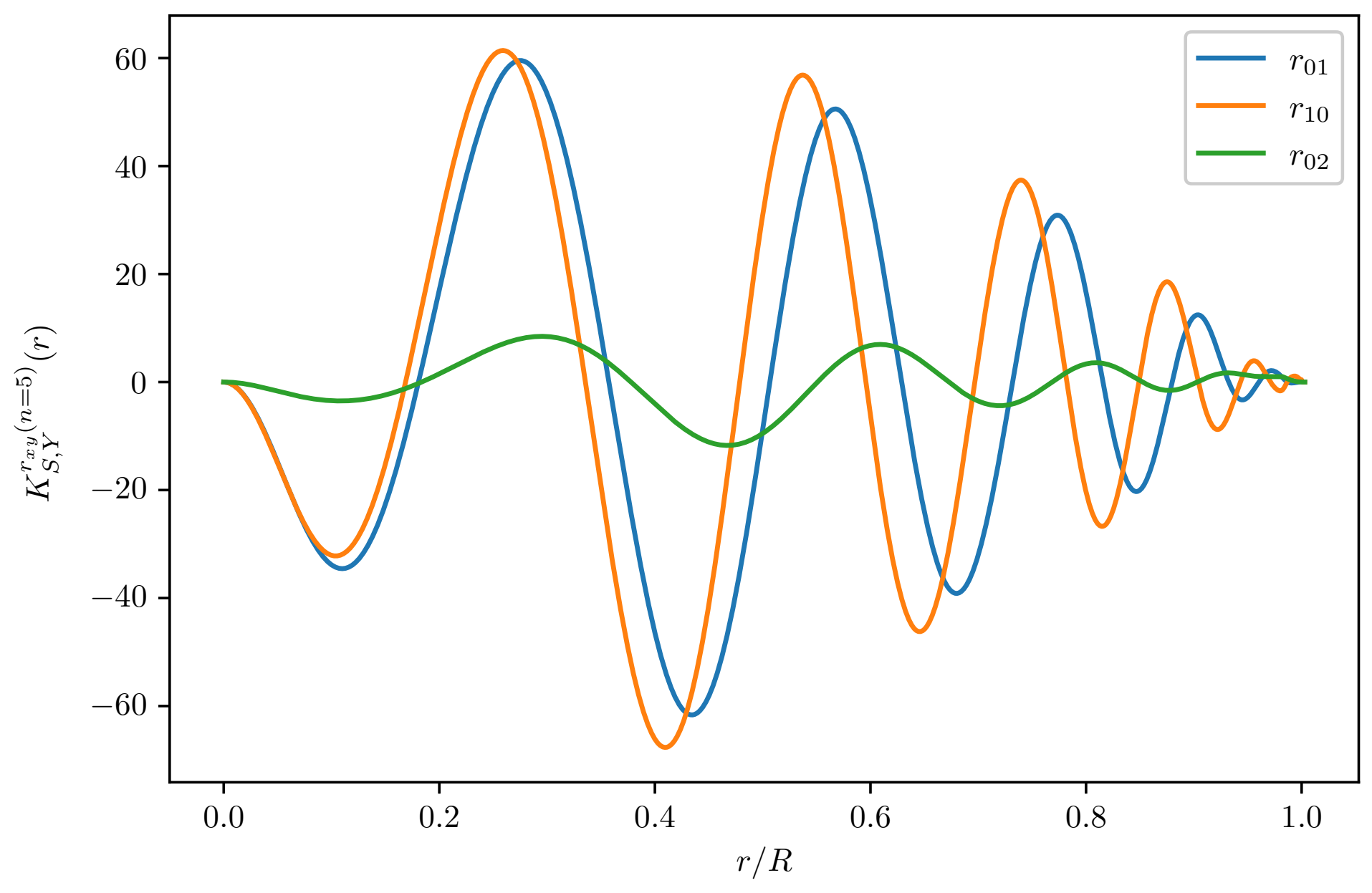

Fig. 2: SY-kernels of ratios with low radial order ($n = 5$).

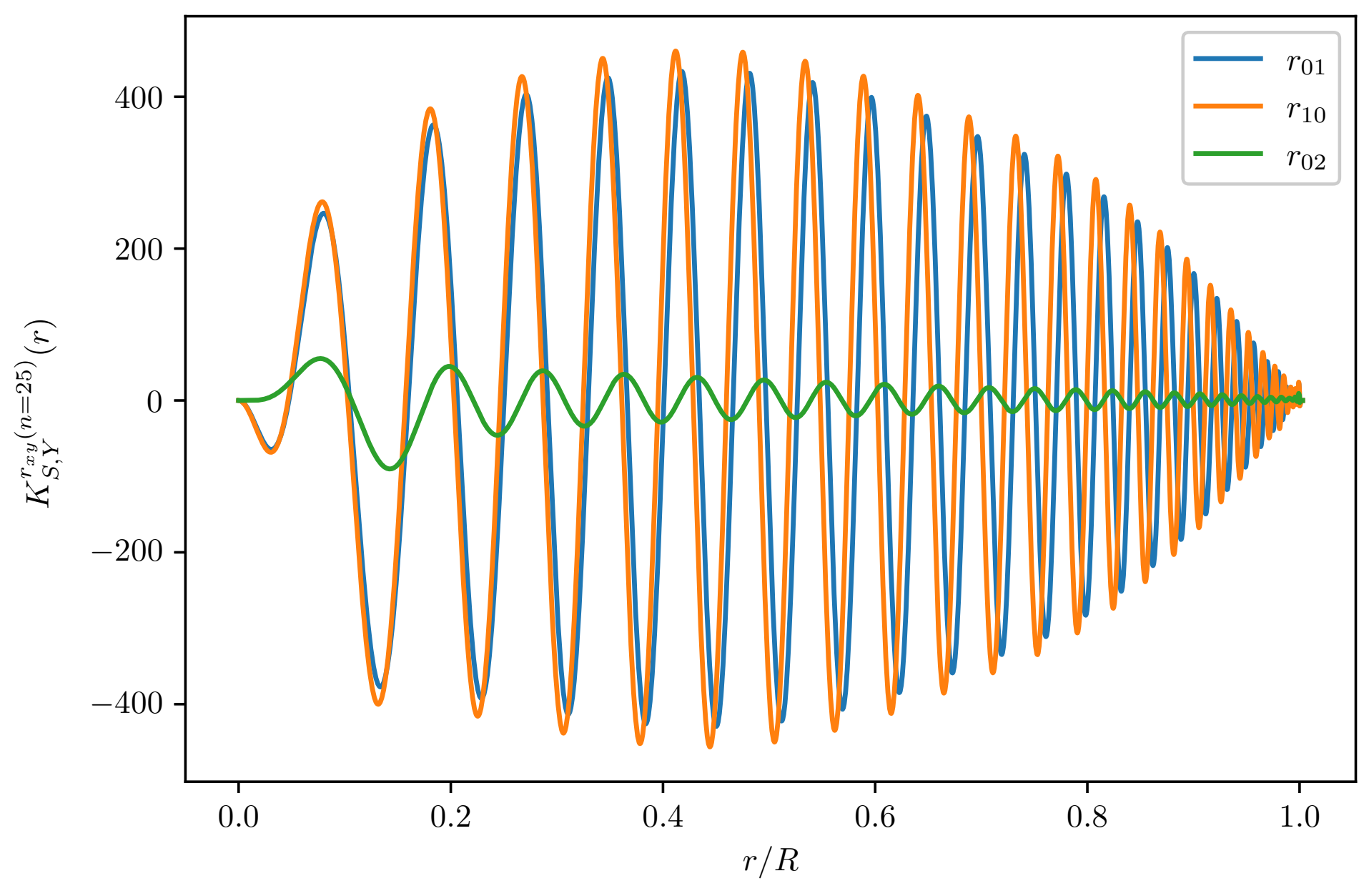

Fig. 3: SY-kernels of ratios with high radial order ($n = 25$).

Investigating the domain of application

Analysis: We compare the left-hand side (LHS) and right-hand side (RHS) of Eq. (1) adapted for the r_{02} ratios. For this purpose, we define an error function $\mathcal{E}_{a,b}^{r_{02}}$ that is the relative difference between the LHS and RHS. Eq. (1) only considers linear terms and non-linearities are reflected in $\mathcal{E}_{a,b}^{r_{02}}$. We investigated several structural pairs.

Observation 1: Some structural pairs are less suited, especially if the density is one of structural variables (see e.g. blue curves). This was expected since we divide by the large separation to get the ratios and thus suppress the information about the density.

Observation 2: Extreme radial order modes seem to be intrinsically non-linear. This point should be investigated further.

Observation 3: The error function can point out non-linearities but has limitations. In fact, if the difference δr_{02} is very small, $\mathcal{E}_{a,b}^{r_{02}}$ shows numerical noise. This is what happens in Fig. 5 for the modes $n = 16$ and $n = 17$.

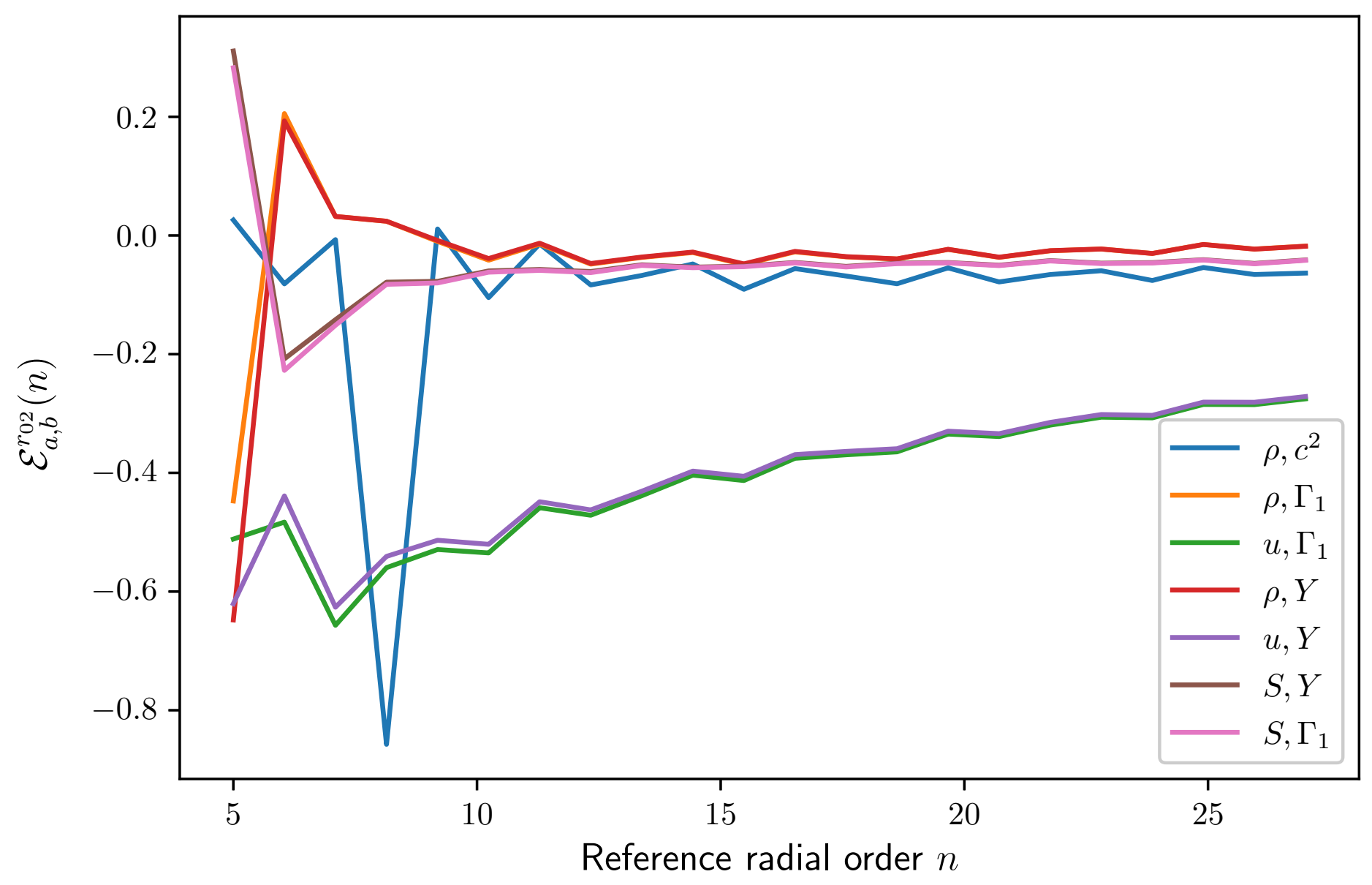

Fig. 4: Comparison between two solar models with different abundances.

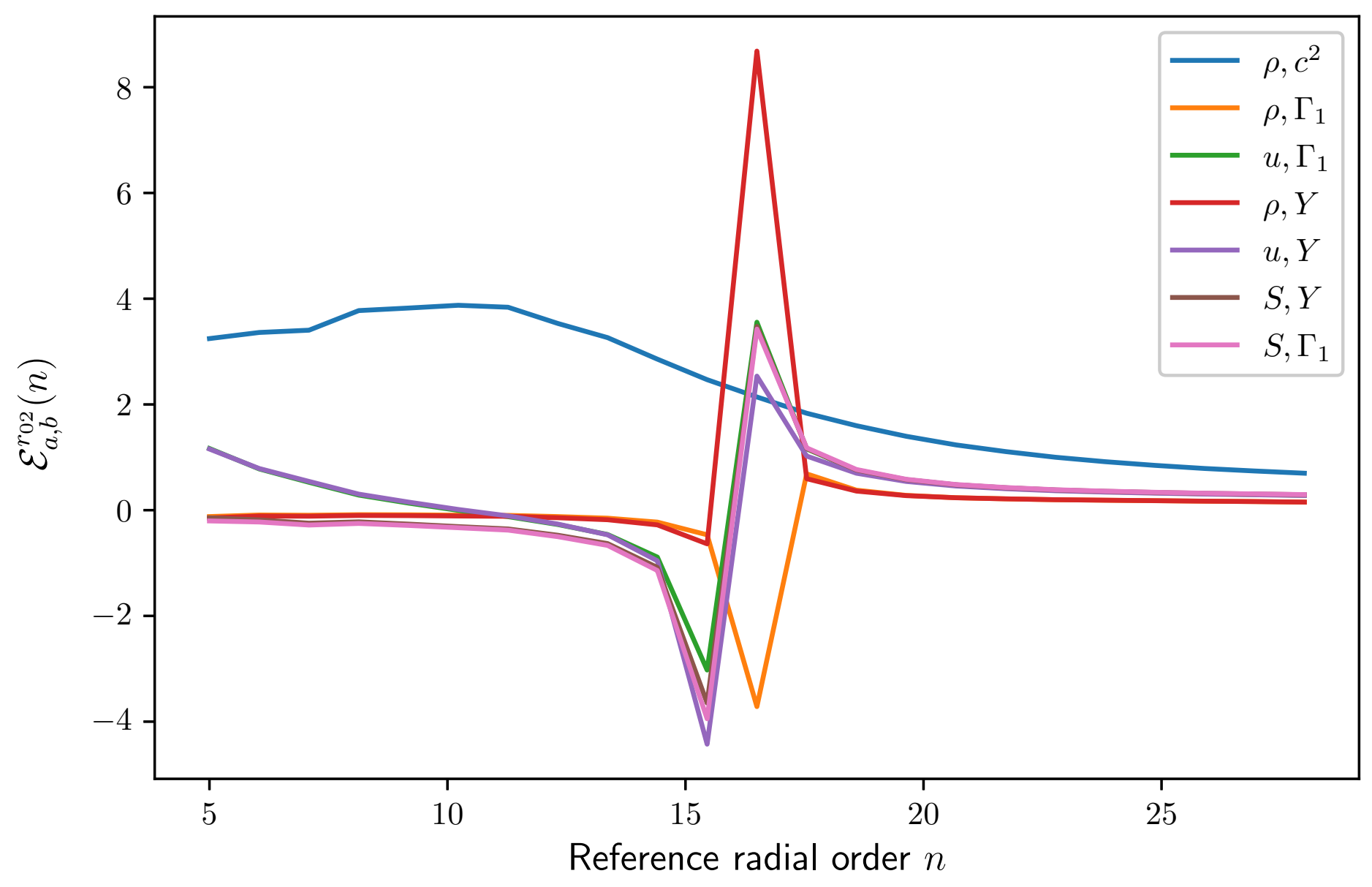

Fig. 5: Comparison between two models of Kepler-93 with different overshoot values.

A new indicator for the central regions

The indicator: We define our new indicator as following:

$$S_{core} = \int_0^R f(r) \exp\left(-\frac{1}{S_{5/3}(r)}\right) dr \tag{3}$$

with $f(r)$ a function that depends on the radius and $S_{5/3} = P/\rho^{5/3}$, the entropy proxy.

Analysis: In Fig. 6, we conducted hare and hounds trials by generating an "observed" model (purple star) that we tried to reproduce (red plus) by changing the abundances. We then conducted inversions (crosses) with the new indicator to test if we could retrieve the "observed" value. We selected different sets of frequencies to see what could be expected for excellent targets.

Observations: The new indicator provides a correction improving the reference value for all the models considered. However, as shown in Fig. 7, there are compensations if we include low radial order modes. This effect should be investigated further.

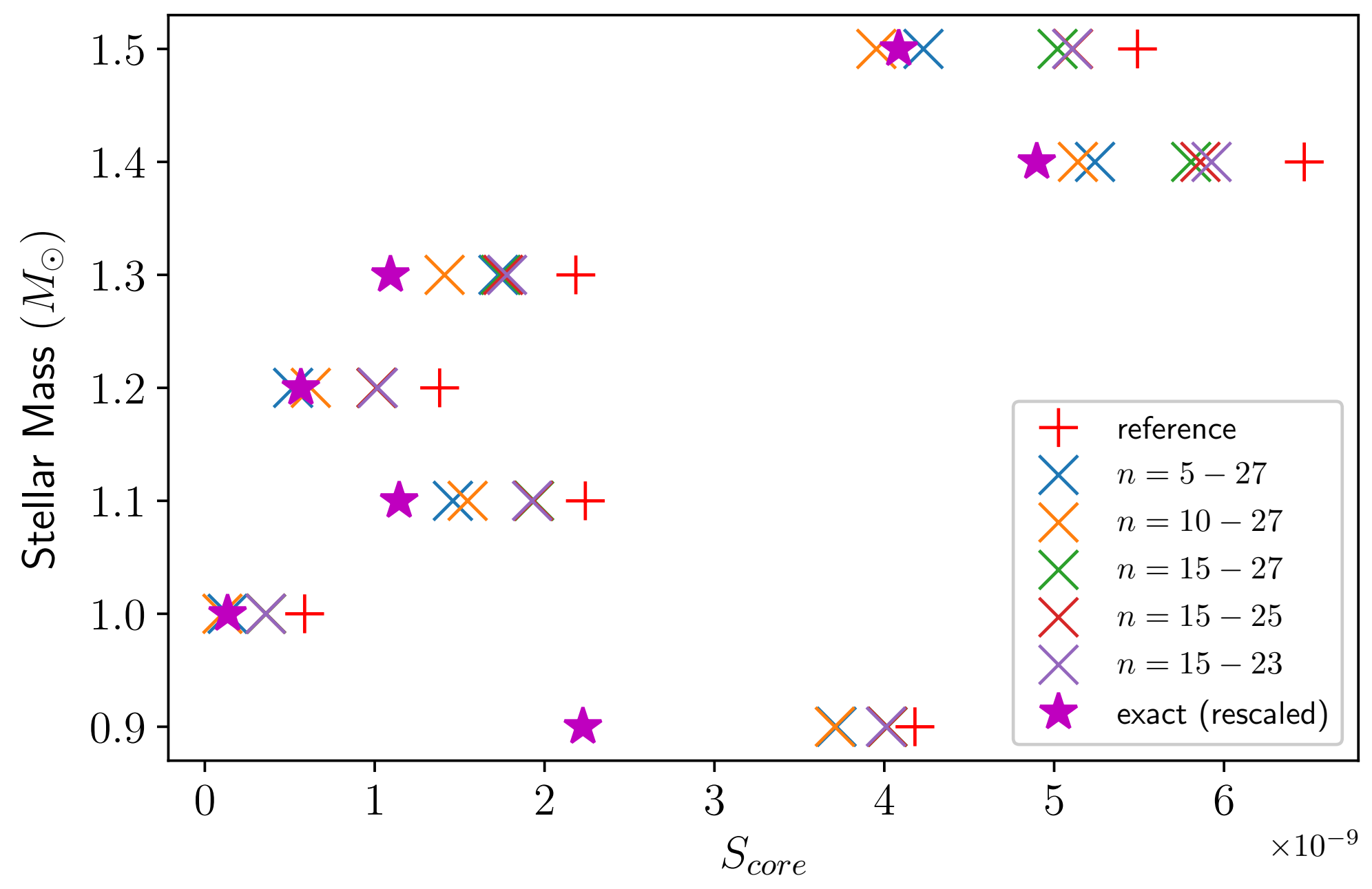

Fig. 6: Hare and hounds for the new indicator. We start with the reference model (red plus), the inversion proposes a corrected value (crosses) that is closer to the "observed" model (purple star).

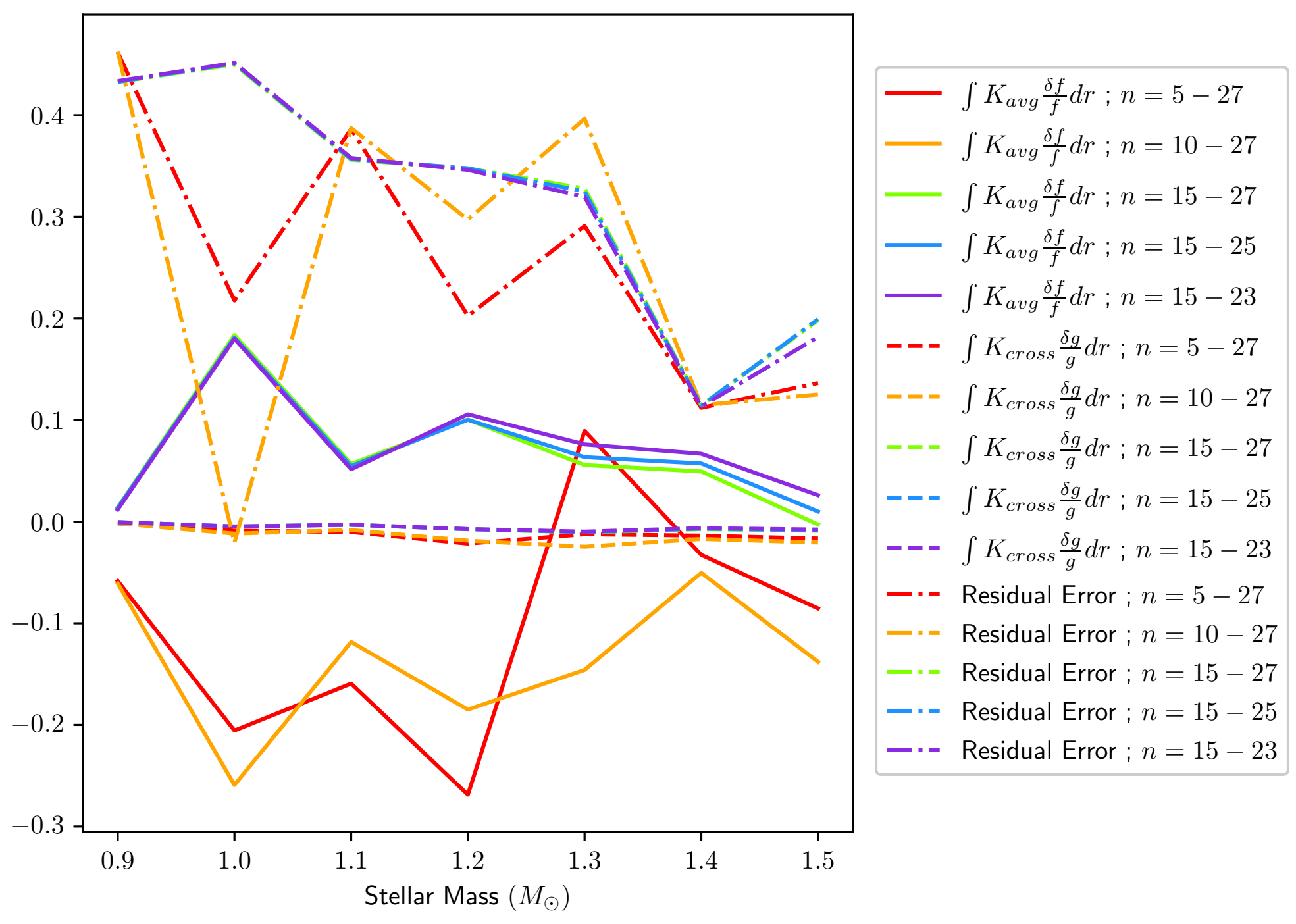

Fig. 7: Averaging kernel error (solid lines), cross-term kernel error (dashed lines) and residual error (dot-dashed lines) for different sets of frequencies.

Prospects and discussions

A promising new technique to probe the stellar cores:

- The indicator allows to constrain the physical ingredients of stellar models.
- It provides meaningful corrections even with a limited dataset.
- It is almost insensitive to surface effects by construction.

Necessary investigations:

- Test multiple changes of ingredients (especially overshoot in F type stars)
- Study in details the limits of the linear regime
- Define new indicators based on other frequency ratios (r_{01} or r_{10}).

The preliminary results for the inversion of the kernels of frequency ratios indicate that it is a promising path to circumvent surface effects and efficiently constrain the physics of deep stellar cores of solar like oscillators.

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