Contribution of Coherent and Incoherent Vorticity Fields to High Reynolds Number Homogeneous Isotropic Turbulence : a Wavelet Viewpoint

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Summary. A wavelet-based method to extract coherent vortices is applied to data obtained by direct numerical simulations of three-dimensional incompressible homogeneous isotropic turbulence for different Taylor microscale Reynolds numbers, ranging from $R_\lambda = 167$ to 732. We find that the coherent vortices well preserve statistics of the total flow. The incoherent flow is structureless and noise like. The percentage of wavelet coefficients representing the coherent vortices decreases as the Reynolds number increases.

A wavelet-based method to extract coherent vortices, proposed in [1, 2], is applied to data obtained by direct numerical simulations (DNSs) of three-dimensional incompressible homogeneous isotropic turbulence performed on the Earth Simulator [3, 4]. We use four datasets for $k_{\max}\eta \simeq 1$ at different resolutions, from 256^3 up to 2048^3, which correspond to different Taylor microscale Reynolds numbers, from $R_\lambda = 167$ to 732. Here $k_{\max}$ is the maximum wavenumber retained in each of the DNSs, and η is the Kolmogorov length scale.

The wavelet-based extraction method assumes that coherent vortices are what remain after denoising, without requiring any template of their shape. Hypotheses are only made on the noise which, as the simplest guess, is considered to be additive, Gaussian and white. An orthogonal wavelet decomposition is applied to the vorticity field. A threshold depending on the enstrophy and the resolution of the field, which are both known *a priori* splits the wavelet coefficients into two sets. The coherent (incoherent) vorticity is reconstructed from few (most) wavelet coefficients whose moduli are larger (smaller) than the threshold.

The aim of this work is to examine the coherent and incoherent contribution to statistics of the turbulent flow, and also the dependence of compression rate (the percentage of wavelet coefficients representing the coherent vorticity) on the Reynolds number.

In Figs. 1, 2 and 3, we present the case $R_\lambda = 471$. We observe that the coherent vorticity represented by 2.9% of the wavelet coefficients well retains the vortex tubes observed in the total vorticity field. In [5], we observed a strong scale-by-scale correlation between the coherent and total velocity fields over the scales retained by the data with $R_\lambda = 471$. In contrast, the incoherent vorticity is structureless without any vortex tubes left.

The probability density functions (PDFs) of velocity and vorticity of the total, coherent and incoherent flows are depicted in Fig. 2. All velocity PDFs exhibit parabola-like shapes. The total and coherent velocity PDFs coincide well, while the incoherent one has a strongly reduced variance. The PDFs of the total and coherent vorticity, both almost superimpose, show a stretched exponential behavior which illustrates the intermittency due to the presence of coherent vortices. The PDF of the incoherent vorticity has an exponential shape with a reduced variance compared to the PDFs of the total and coherent vorticity.

Figure 3 shows the compensated energy spectra of the total, coherent and incoherent flows. The spectrum of the coherent flow is identical to that of the total flow all along the inertial range. The shoulder of the energy spectrum of the total flow with a maximum around $k\eta \sim 0.13$ is well retained in the coherent flow. For the incoherent flow, we observe that $E(k)$ scales as k^2, which corresponds to an energy equipartition.

In [6], we found that as the Reynolds number increases, the compression rate improves from 3.6% for $R_\lambda = 167$ to 2.6% for $R_\lambda = 732$ and that the coherent velocity fields preserve the nonlinear dynamics of the flow in the inertial range. It is conjectured that the number of degrees of freedom N to compute fully-developed turbulent flows could be reduced in comparison to the estimation based on the Kolmogorov theory, i.e. from $N \propto R_\lambda^{9/2}$, to $R_\lambda^{3.9}$.

References

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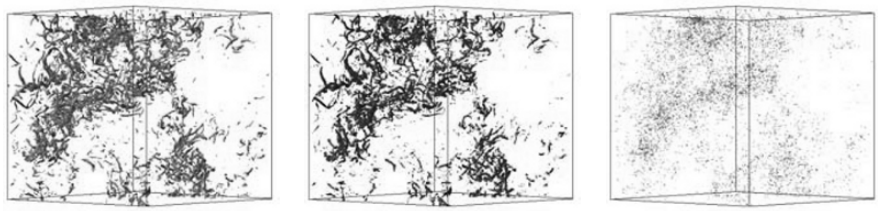

Fig. 1. Isosurfaces of the vorticity modulus for the total (left), coherent (middle) and incoherent (right) flow fields (from Ref. [5]). The values of the isosurfaces are $|\boldsymbol{\omega}| = \omega_m + 3\sigma_\omega$ for the total and coherent vorticity and $2(\omega_m + 3\sigma_\omega)/5$ for the incoherent one. ω_m and σ_ω are the mean value of $|\boldsymbol{\omega}|$ and the standard deviation of $|\boldsymbol{\omega}|$, respectively. Subcubes of size 256^3 are visualized.

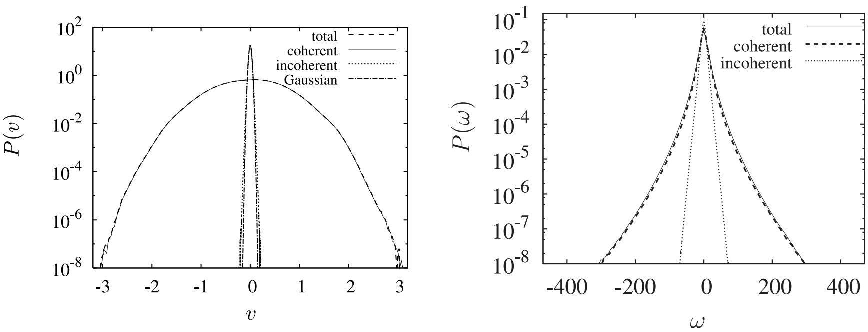

Fig. 2. PDFs of velocity (left) and vorticity (right).

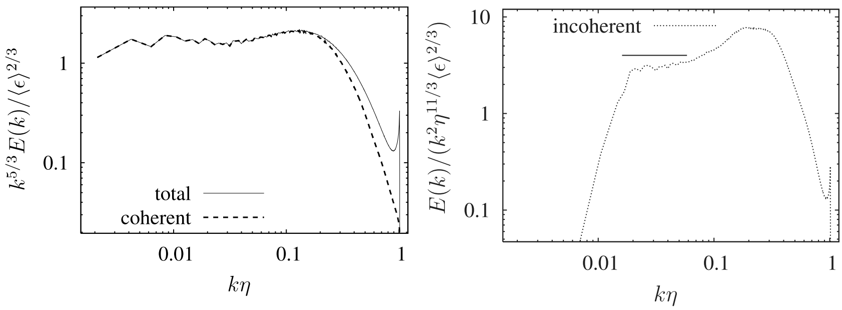

Fig. 3. Compensated energy spectra of the total and coherent flows (left), and of the incoherent flow (right). Here $\langle \epsilon \rangle$ is the mean energy dissipation rate per unit mass of the total flow.

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