Seismic diagnosis for rapidly rotating g-mode upper-main-sequence pulsators: the combined effects of the centrifugal acceleration and differential rotation

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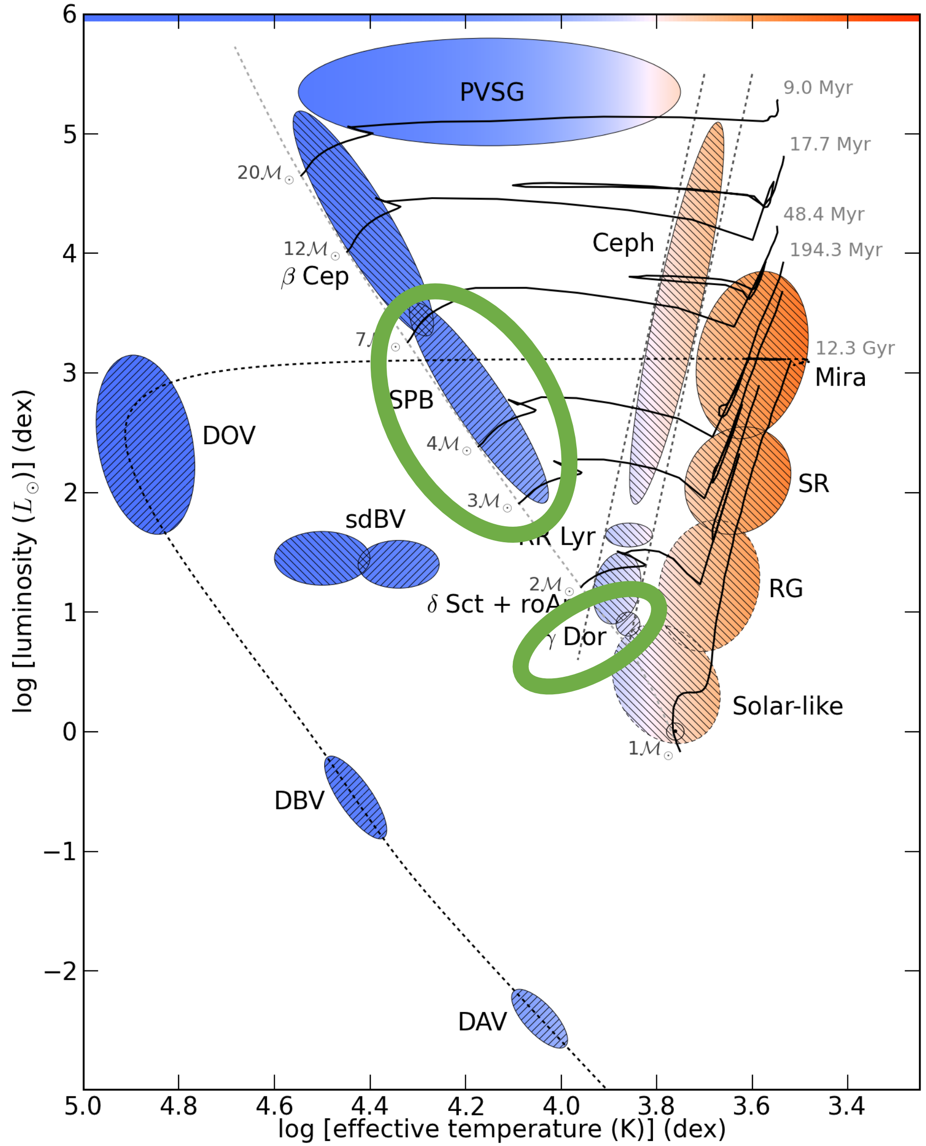
Space-based observations and Asteroseismology

- Asteroseismology is the only way to probe the internal **structural, chemical, rotational and magnetic properties of stars** *(cf. Dominic Bowman talk)*.

- High precision photometry (MOST, CoRoT, *Kepler*/K2, Brite, TESS) allows us to study pulsating stars in the whole HR diagram.

- Our goal: study the signature of rotation in **rapidly rotating stars where rotation is not a perturbation.**

Christensen-Dalsgaard (1999); Van Reeth PhD Thesis (2017)



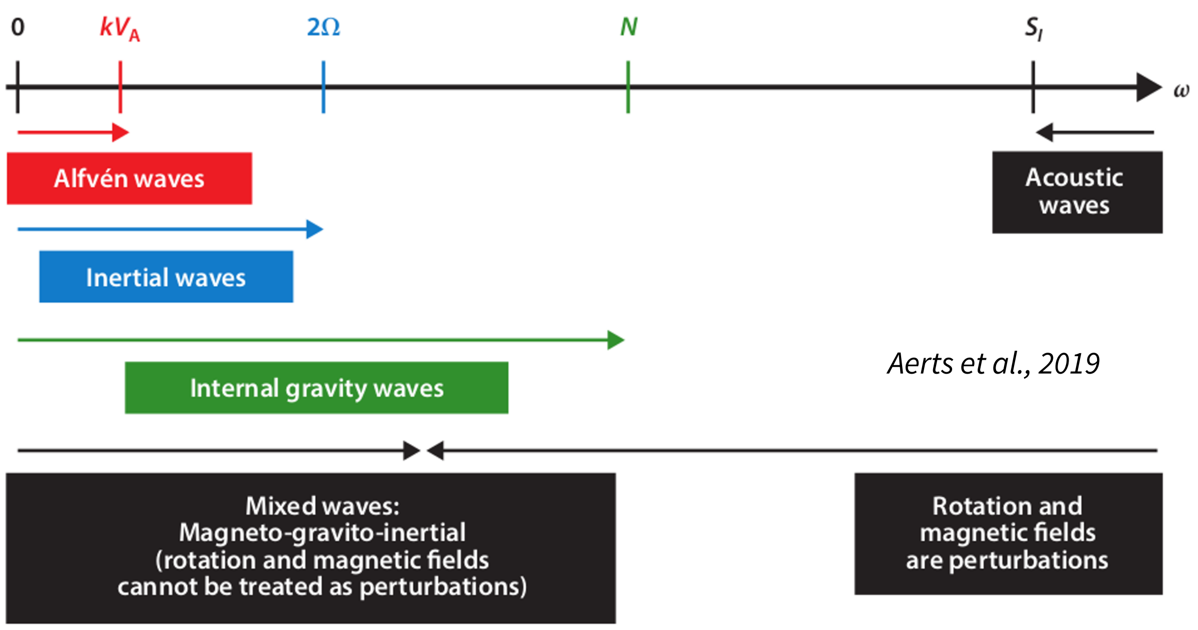
Asteroseismic HR Diagram

Rapidly rotating g mode pulsators

- Gravito-inertial waves (GIWs) are mixed waves driven by
  - buoyancy: chemical and thermal stratifications,
  - rotation: Coriolis and centrifugal accelerations.
- Propagate in stably stratified zones.

↪ Offer a direct probe of their stucture, mixing, and rotation.

(e.g. Neiner et al. 2012; Van Reeth et al. 2016; Li et al. 2020; Saio et al. 2021).



Aerts et al., 2019

The full exploitation of the information provided by detected g-mode pulsations ➞
crucial to improve our understanding of how stellar rotation influences g-modes in rapidly rotating stars.

What can we learn with observed g modes?

- $\Delta P = P_{n+1} - P_n$: period spacing.

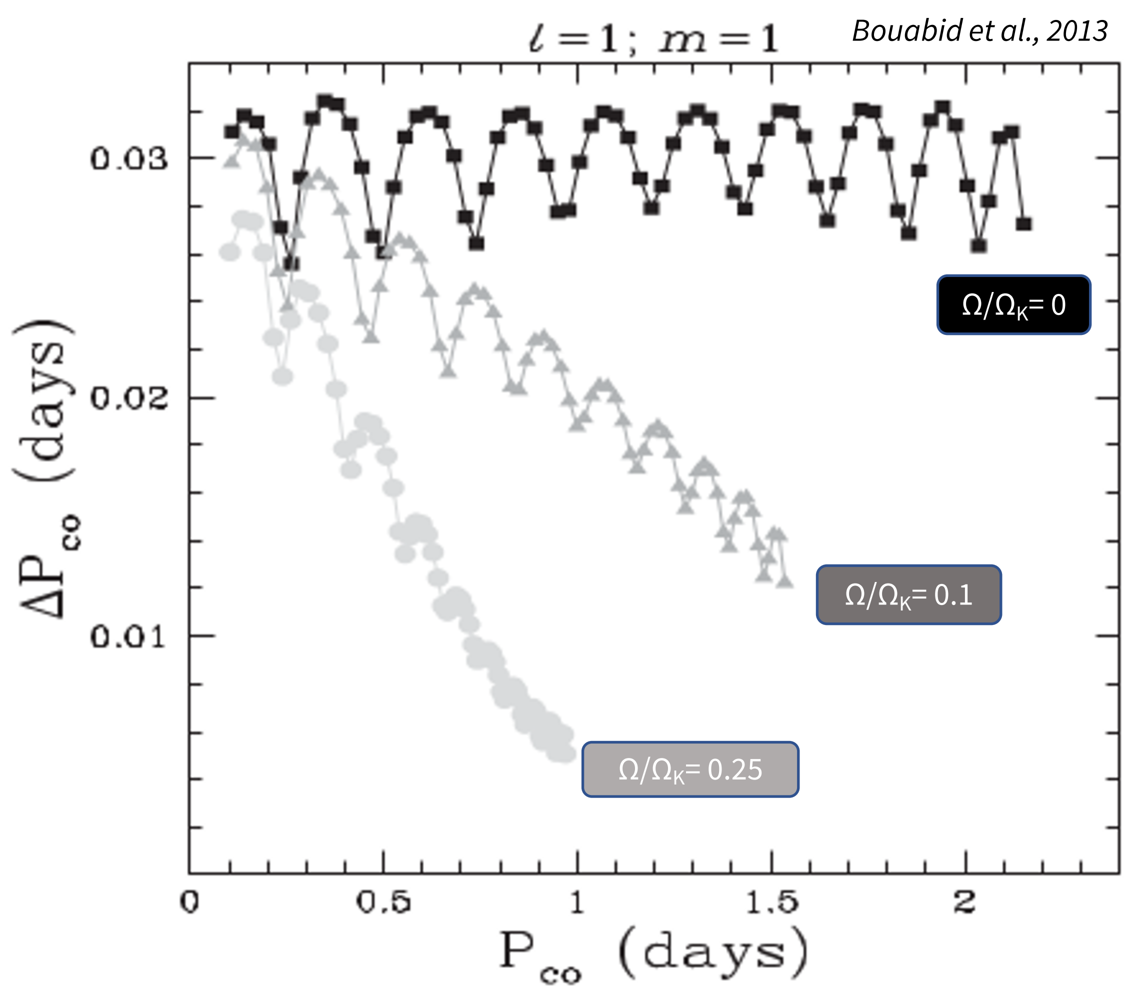
- In the non-rotating case: $\Delta P(P)$ is a constant.
- In the uniformly rotating case: a slope appear in ΔP–P diagram.
  - ✓ Ω ↗ ⇒ the slope ↗.
  - ✓ The measurement of the slope ⇒ a measure of Ω. (Bouabid et al. 2013; Ouazzani et al. 2017)

  ↪ a probe of the inner radiative regions of early type stars.

- The dips are caused by:
  - ✓ The chemical composition and rotation gradients.
  - ✓ The resonant coupling between inertial modes in the convective core and g modes (Ouazzani et al. 2020, Saio et al. 2021).

  ↪ a probe of the convective core rotation of early-type stars.

↪ Constrains on the transport of angular momentum and the internal mixing along the evolution of stars.

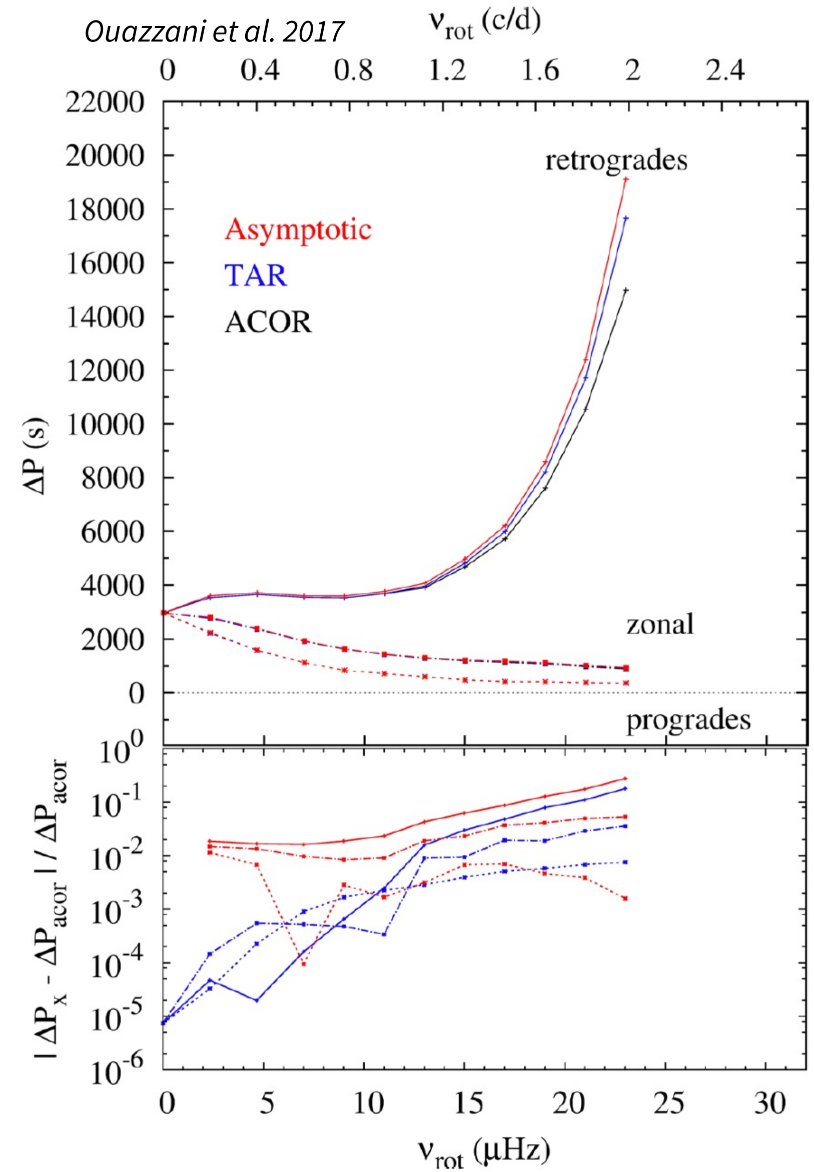


How we compute ΔP versus P diagram?

- Direct computations using 2D stellar oscillation codes TOP (Reese et al. 2021) and ACOR (Ouazzani et al. 2017).
  - ➢ Impossibility to perform intensive detailed seismic modelling.

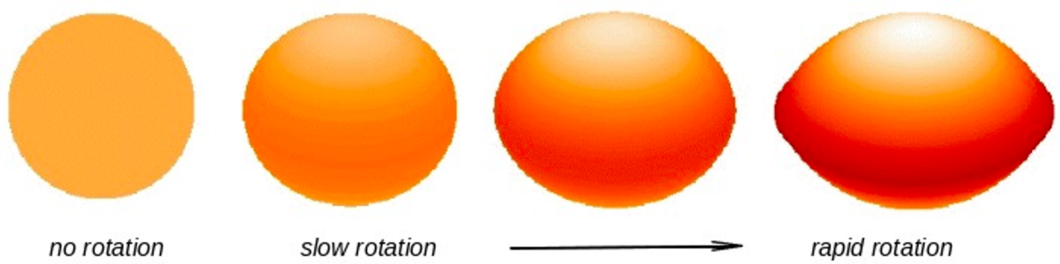
- The traditional approximation of rotation (TAR) (e.g. Eckart 1960):
  - ➢ Flexible and robust; allow us to derive powerful seismic diagnostics.
  - ✓ Applicable only in the stably stratified regions.
  - ✓ Hypothesis:
    - ❑ Spherical symmetry.
    - ❑ Uniform rotation.
    - ❑ Adiabatic treatment.
    - ❑ Cowling approximation.

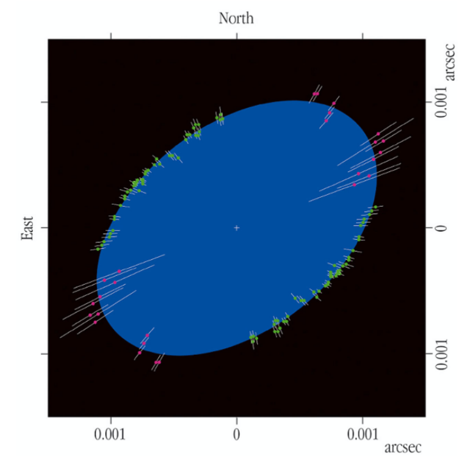
Ouazzani et al. 2017



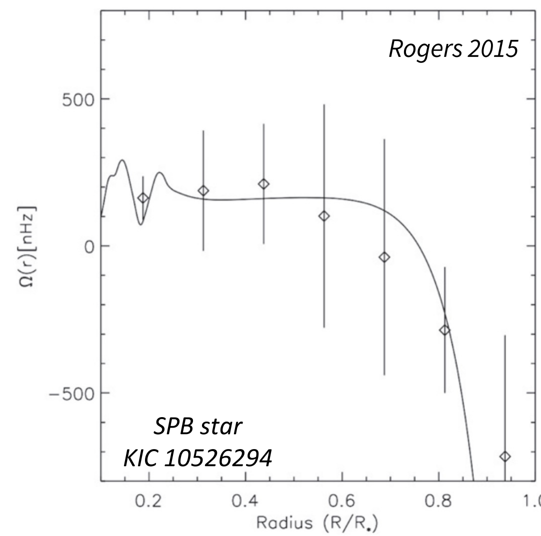
Generalisation of the TAR

But early type stars can be strongly deformed and differentially rotating



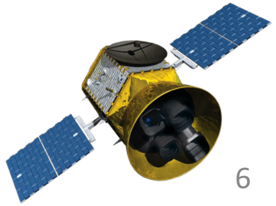


Achernar
(Domiciano de Souza et al. 2003)



- First steps:
  - ✓ Mathis (2009) and Van Reeth et al. (2018): including differential rotation in the spherical case.
  - ✓ Mathis & Prat (2019) and Henneco et al. (2021): generalisation of the TAR in the case of slightly deformed spheres (perturbative approach).
- Our work:
  - ✓ **Generalise the TAR to take into account simultaneously the centrifugal deformation and differential rotation.**

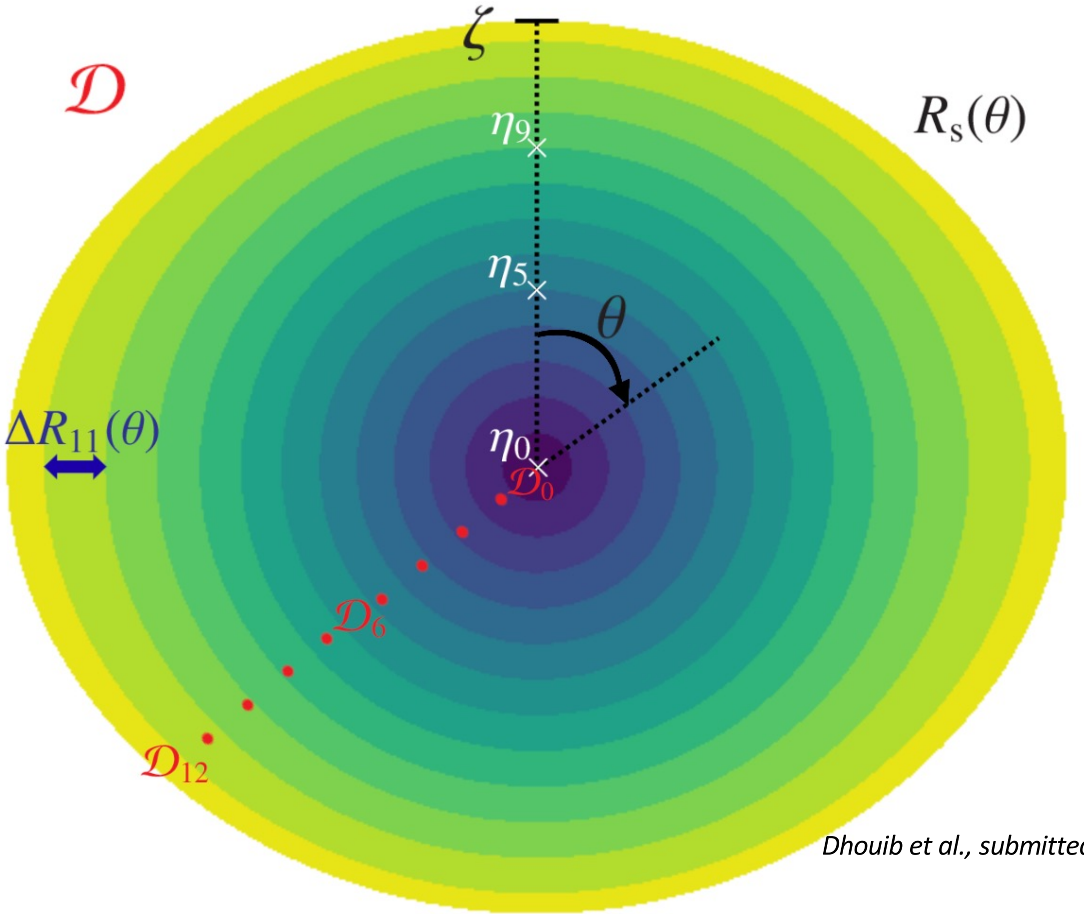
Intensive seismic modelling in support of data from NASA *Kepler* / K2 and TESS space missions.



Spheroidal geometry

- New coordinate system (ζ, θ, ϕ) linked to the spherical coordinate system (r, θ, ϕ).

$$\zeta = 0 \rightarrow r = 0 : \text{the center of the star.}$$
$$\zeta = 1 \rightarrow r = R_s(\theta) : \text{the deformed surface of the star.}$$



Dhouib et al., submitted

Spheroidal coordinate system with n=13 subdomain of a star rotating at 60% of its break-up velocity ($3M_\odot$ ESTER model with X_c=0.7).

Generalised TAR

General hydrodynamic system

+

Frequency hierarchy		Approximation on the stratification profile
$\omega \ll N$	$2\Omega \ll N$	$N^2(\zeta,\theta) \approx N^2(\zeta)$

↓

Simplified system

$$\begin{cases}
(\partial_t + \Omega\partial_\varphi)\rho' = -\dfrac{\zeta^2\partial_\zeta\rho_0}{r^2 r_\zeta} v^\zeta - \dfrac{\zeta\partial_\theta\rho_0}{r^2 r_\zeta} v^\theta - \dfrac{\zeta^2\rho_0}{r^2 r_\zeta}\left[\dfrac{\partial_\zeta\left(\zeta^2 v^\zeta\right)}{\zeta^2} + \dfrac{\partial_\theta\left(\sin\theta v^\theta\right)}{\zeta\sin\theta} + \dfrac{\partial_\varphi v^\varphi}{\zeta\sin\theta}\right] \\
(\partial_t + \Omega\partial_\varphi)\left[\dfrac{\zeta^2 r_\zeta}{r^2} v^\zeta + \dfrac{\zeta r_\theta}{r^2} v^\theta\right] = 2\Omega\dfrac{\zeta\sin\theta}{r} v^\varphi - \dfrac{1}{\rho_0}\partial_\zeta P' + \dfrac{\rho'}{\rho_0^2}\partial_\zeta P_0 \\
(\partial_t + \Omega\partial_\varphi)\left[\dfrac{\zeta^2 r_\theta}{r^2} v^\zeta + \dfrac{\zeta\left(r^2 + r_\theta^2\right)}{r^2 r_\zeta} v^\theta\right] = 2\Omega\dfrac{\zeta(r_\theta\sin\theta + r\cos\theta)}{r r_\zeta} v^\varphi - \dfrac{1}{\rho_0}\partial_\theta P' + \dfrac{\rho'}{\rho_0^2}\partial_\theta P_0 \\
\dfrac{\zeta}{r_\zeta}(\partial_t + \Omega\partial_\varphi)v^\varphi = -2\Omega\dfrac{\zeta^2\sin\theta}{r} v^\zeta - 2\Omega\dfrac{\zeta(r_\theta\sin\theta + r\cos\theta)}{r r_\zeta} v^\theta - \dfrac{1}{\rho_0\sin\theta}\partial_\varphi P' - \dfrac{\zeta\sin\theta}{r_\zeta}\left(\zeta\partial_\zeta\Omega u^\zeta + \partial_\theta\Omega u^\theta\right) \\
(\partial_t + \Omega\partial_\varphi)\left(\dfrac{1}{\Gamma_1}\dfrac{P'}{P_0} - \dfrac{\rho'}{\rho_0}\right) = \dfrac{N^2}{\|\vec{g}_{\rm eff}\|^2}\vec{v}\cdot\vec{g}_{\rm eff}
\end{cases}$$

$$\begin{cases}
\zeta\partial_\zeta\rho_0 u^\zeta + \rho_0\left[\dfrac{\partial_\zeta\left(\zeta^2 u^\zeta\right)}{\zeta} + \dfrac{1}{\sin\theta}\partial_\theta\left(\sin\theta u^\theta\right) - \dfrac{im}{\sin\theta}u^\varphi\right] = 0 \\
i\omega\partial_\zeta\widetilde{W} + N^2\zeta^2\mathcal{A}u^\zeta = 0 \\
i\omega\zeta\mathcal{B}u^\theta - 2\Omega\zeta\mathcal{C}u^\varphi + \partial_\theta\widetilde{W} = 0 \\
i\omega\zeta\mathcal{D}u^\varphi + \left(2\Omega\zeta\mathcal{C} + \dfrac{\sin\theta}{r_\zeta}\partial_\theta\Omega\right)u^\theta - \dfrac{im}{\sin\theta}\widetilde{W} = 0
\end{cases}$$

↪ **The vertical and horizontal components of the velocity are partially decoupled as in the non-rotating case.**

Generalised Laplace tidal equation (GLTE)

2D JWKB Approximation	Approximation on the coefficient A	Approximation on the angular velocity
$X(\zeta,\theta)=\sum_k\left\{x_{\nu km}(\zeta,\theta)\frac{A_{\nu km}}{k^{1/2}_{V;\nu km}}\exp\left[i\int^{\zeta}k_{V;\nu km}\mathrm{d}\zeta\right]\right\},\quad X=\{\widetilde{W},u^j\}.$	$\mathcal{A}(\zeta,\theta)\approx\mathcal{A}(\zeta)$	$\Omega(\zeta,\theta)\approx\Omega(\zeta)$

$$\mathcal{L}_{\omega^{\rm in}m}[w_{\omega^{\rm in}km}]=\omega\partial_x\left[\frac{1}{\omega}\frac{1}{\mathcal{B}(\mathcal{E}-\mathcal{F})}\left(\mathcal{E}+\frac{\nu^2\mathcal{C}^2}{\mathcal{B}}\right)(1-x^2)\partial_x\right]-\frac{m\mathcal{F}}{\nu\mathcal{C}(\mathcal{E}-\mathcal{F})}\partial_x w_{\omega^{\rm in}km}+\left(m\omega\partial_x\left(\frac{\nu\mathcal{C}}{\omega\mathcal{B}(\mathcal{E}-\mathcal{F})}\right)-\frac{m^2}{(\mathcal{E}-\mathcal{F})(1-x^2)}\right)w_{\omega^{\rm in}km}$$
$$=-\Lambda_{\omega^{\rm in}km}(\zeta)w_{\omega^{\rm in}km}$$

Eigenvalues

Hough functions

Terms	Spheroidal	Spherical
$\mathcal{A}$	$\frac{1}{r^2r_\zeta}$	$\frac{1}{\zeta^2}$
$\mathcal{B}$	$\frac{r^2+(1-x^2)r_x^2}{r^2r_\zeta}$	1
$\mathcal{C}$	$\frac{-(1-x^2)r_x+rx}{rr_\zeta}$	x
$\mathcal{D}$	$\frac{1}{r_\zeta}$	1
$\mathcal{E}$	$\mathcal{D}-\frac{\nu^2\mathcal{C}^2}{\mathcal{B}}$	$1-\nu^2x^2$
$\mathcal{F}$	$-\frac{\nu}{\omega}\frac{\mathcal{C}\mathcal{D}}{\mathcal{B}}(1-x^2)\partial_x\Omega$	$-\frac{\nu x}{\omega}(1-x^2)\partial_x\Omega$

Centrifugal deformation ($\mathcal{A}$–$\mathcal{E}$)

Differential rotation ($\mathcal{F}$)

Asymptotic frequency of low-frequency GIWs

- The asymptotic eigenfrequencies (quantisation in the pseudo-radial direction):

$$\omega_{nkm} = \frac{\int_{\zeta_1}^{\zeta_2} N(\zeta)\sqrt{\mathcal{A}(\zeta)\Lambda_{\omega_n^{\rm in}km}(\zeta)}\,\mathrm{d}\zeta}{(n+1/2)\pi}$$

Stratification — $N(\zeta)$; Rotation — $\Lambda_{\omega_n^{\rm in}km}(\zeta)$

- The corresponding period spacing:

$$\Delta P_{km} \approx \frac{2\pi^2}{\int_{\zeta_1}^{\zeta_2} N\sqrt{\mathcal{A}\Lambda_{\omega_{n+1}^{\rm in}km}}\mathrm{d}\zeta\left(1+\frac{1}{2}\frac{\int_{\zeta_1}^{\zeta_2} N\sqrt{\mathcal{A}\Lambda_{\omega_n^{\rm in}km}}\frac{d\ln\Lambda_{\omega_n^{\rm in}km}}{d\ln\omega^{\rm in}}\mathrm{d}\zeta}{\int_{\zeta_1}^{\zeta_2} N\sqrt{\mathcal{A}\Lambda_{\omega_n^{\rm in}km}}\mathrm{d}\zeta}\right)}$$

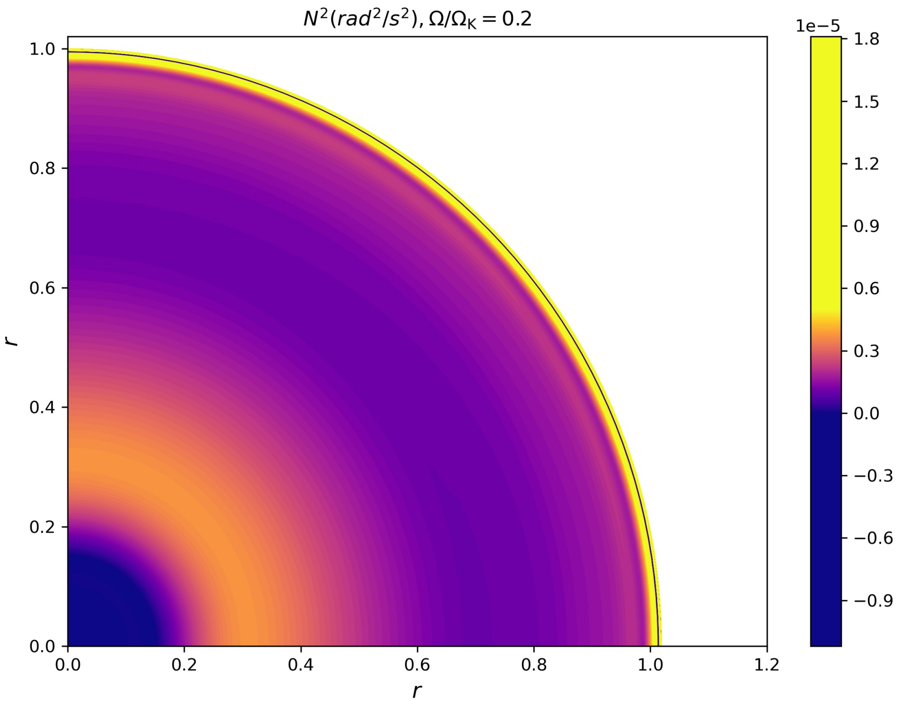
- Easily applicable for a large number of stars.
- Great advantage when full grids of stellar models need to be calculated for detailed seismic modelling.

Validity domain of the generalised TAR

- **Application to rapidly rotating early-type stars.**
- ESTER model:
  - 2D stellar structure code.
  - computes an equilibrium model.
  - unique of its kind.
  - takes into account the effect of the centrifugal acceleration in a non-perturbative way at any rotation rate.

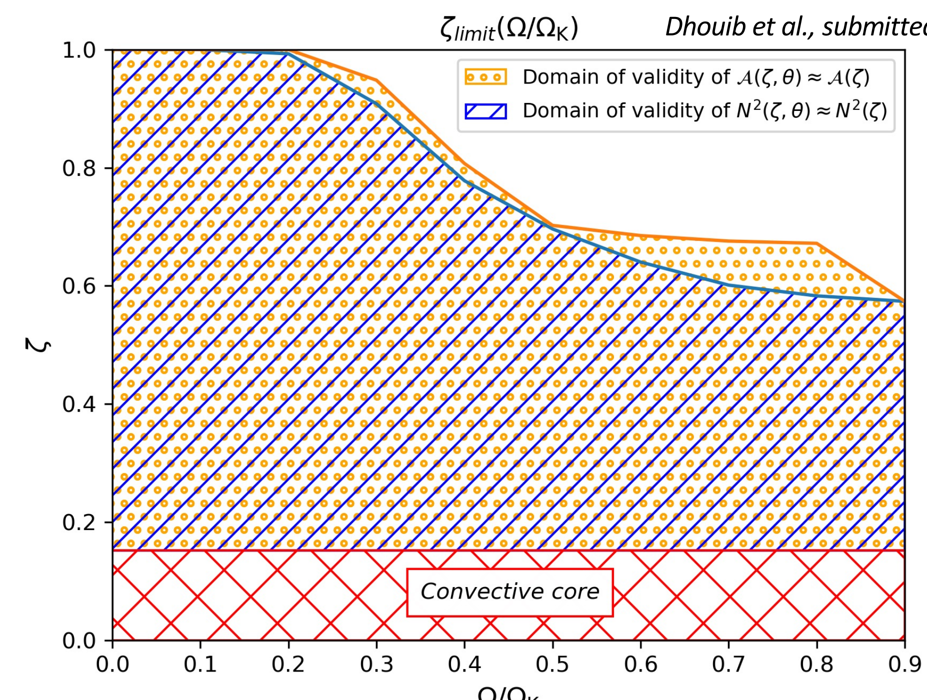
- The generalised TAR is applicable to early-type stars rotating up to Ω/Ω_K=20%.
- This limit proposed by Mathis & Prat (2019) using a perturbative approach was 40%.

➢ 20% is a more realistic limit : derived from a 2D model which takes into account the centrifugal acceleration in a non-perturbative way.



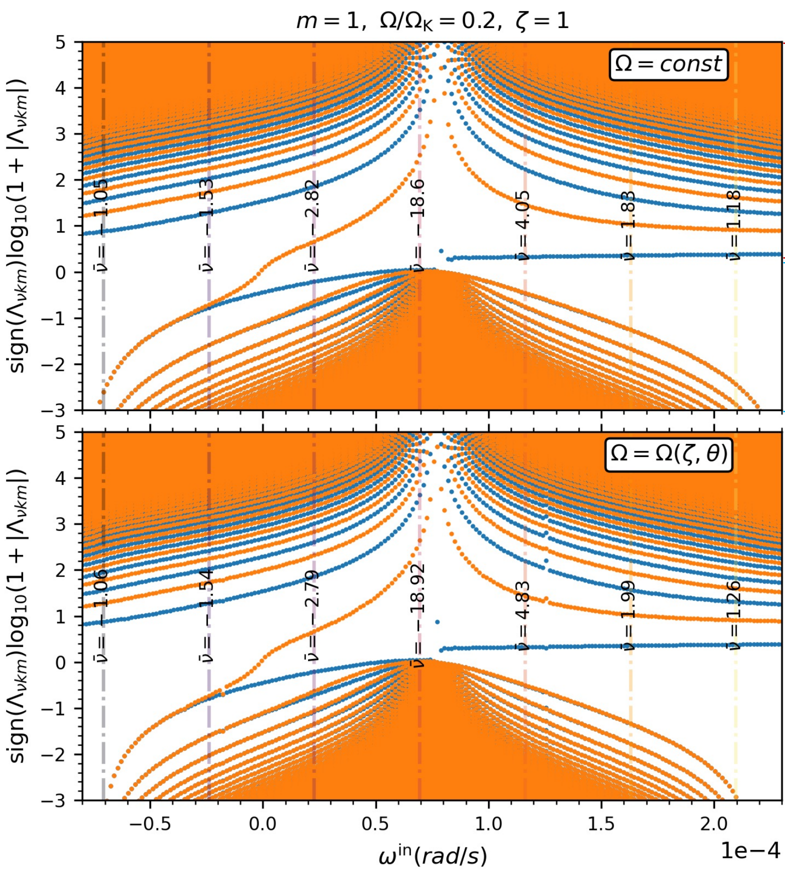
The Brunt–Väisälä frequency profile

ESTER model ($3M_\odot$, X_c = 0.7)



The validity domain of the TAR

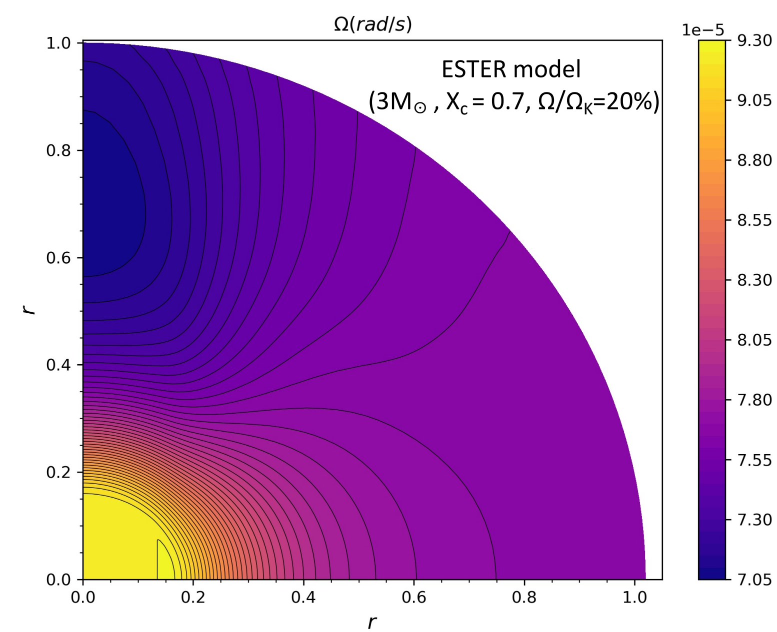
Spectrum (eigenvalues) of the GLTE

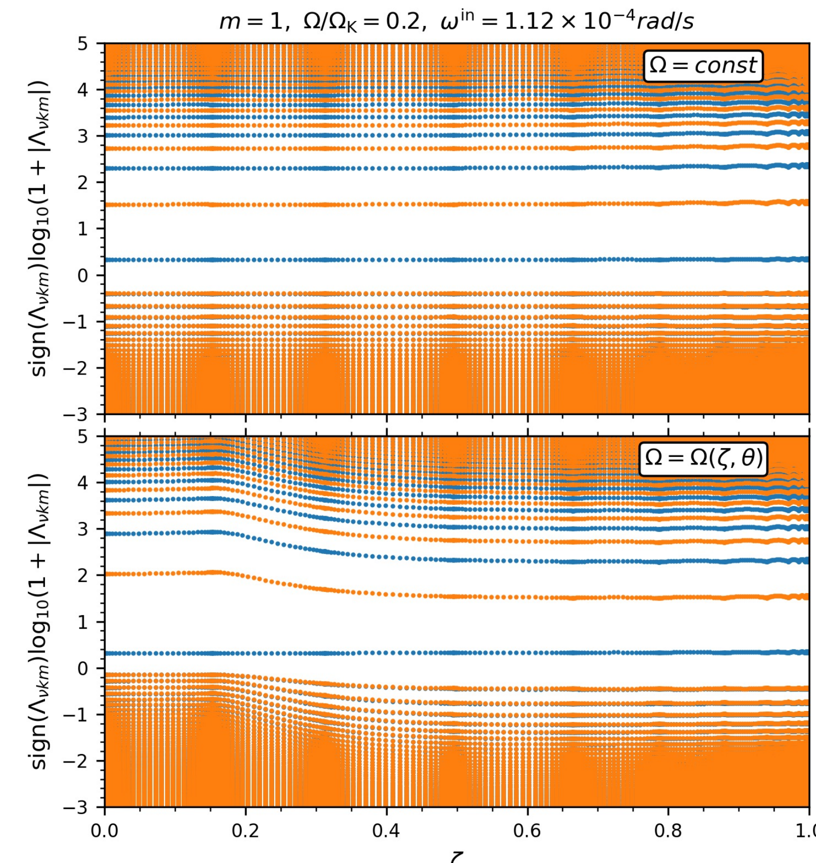


Dhouib et al., submitted

Gravity-like solutions g modes modified by rotation.

Rossby-like solutions appear only in a rotating star.





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Blue (respectively, orange) dots correspond to even (respectively, odd) Hough functions.

Generalised Hough functions (eigenfunctions) of the GLTE

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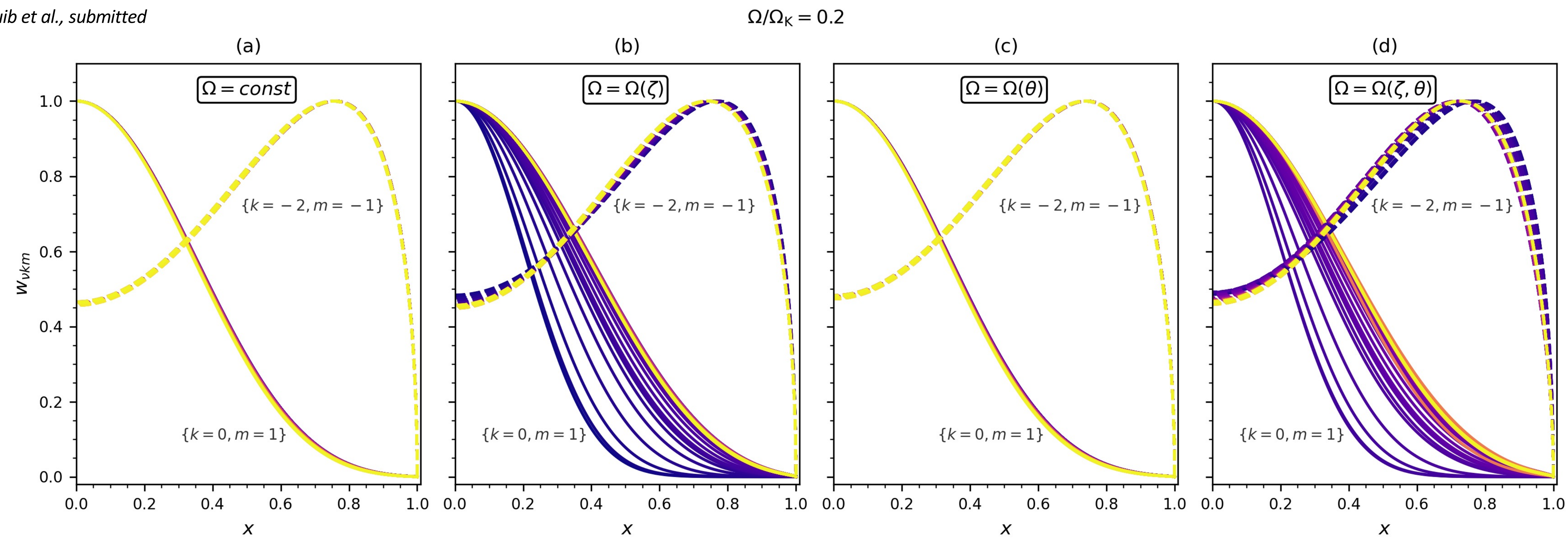
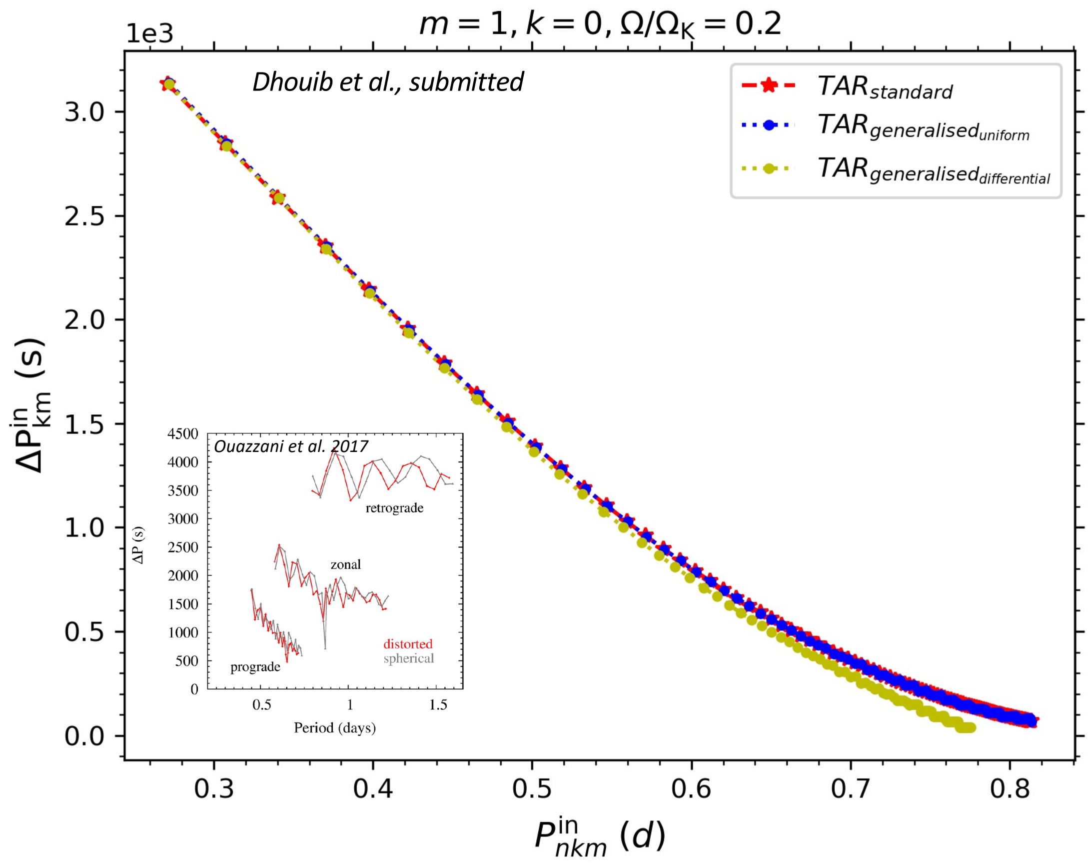


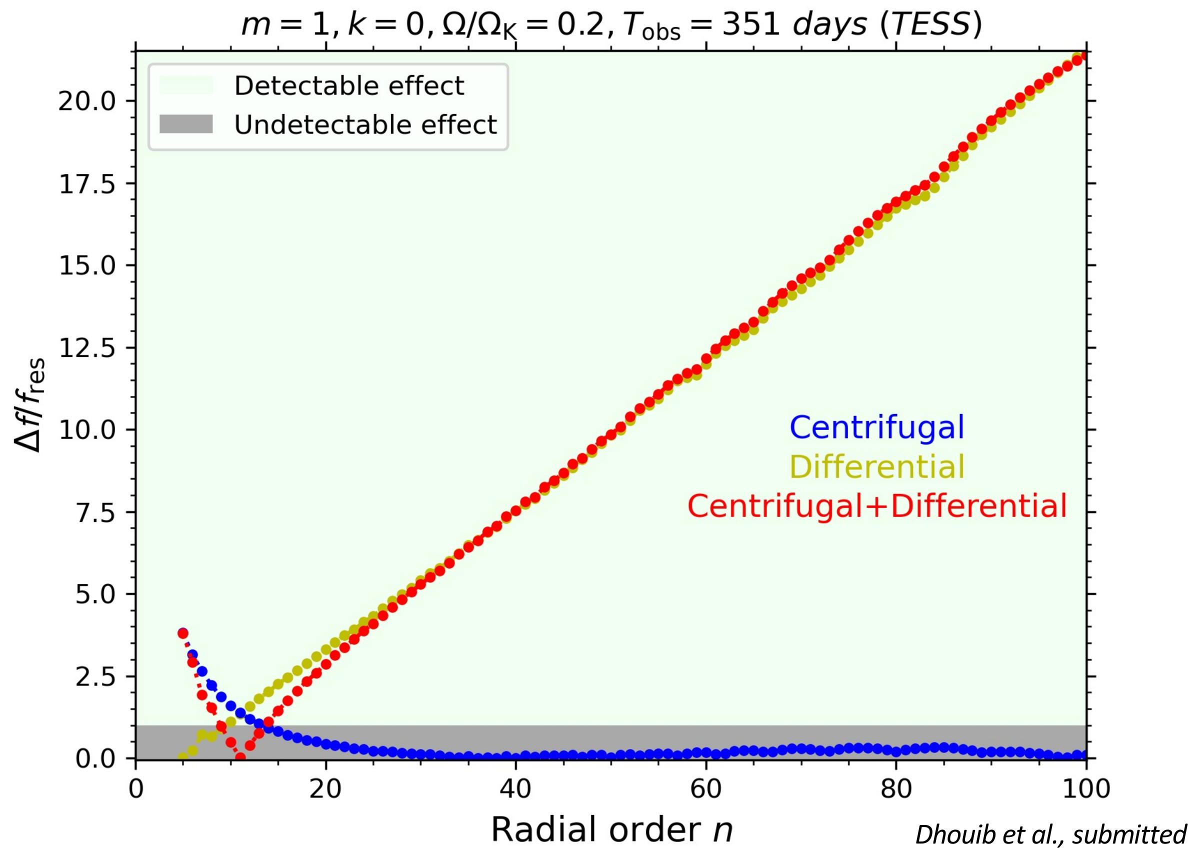
Blue: ζ=0.153 (convective core boundary) / Yellow: deformed stellar surface

Solid lines: gravity-like solutions {k=0, m = 1} / Dotted lines: Rossby-like solutions {k=-2,m=-1}

Asymptotic period spacing pattern and detectability of the differential rotation and the centrifugal effects



Period spacing pattern in the inertial frame computed in a spherical (red) and a deformed (blue) star uniformly rotating and in a deformed star differentially rotating (yellow).



The detectable radial orders n for a {k=0,m=1} mode based on the frequency resolution of TESS.

Conclusion and Take Home Messages

- A new generalisation of the TAR, which takes into account **simultaneously the differential rotation and the centrifugal acceleration** in a non-perturbative way, is derived. **It is designed for 2D stellar models like ESTER.**

- This generalisation allows us to study the **detectability** and the **signature** of the centrifugal effects on GIWs in differentially rotating deformed stars until 20% of the critical angular velocity.

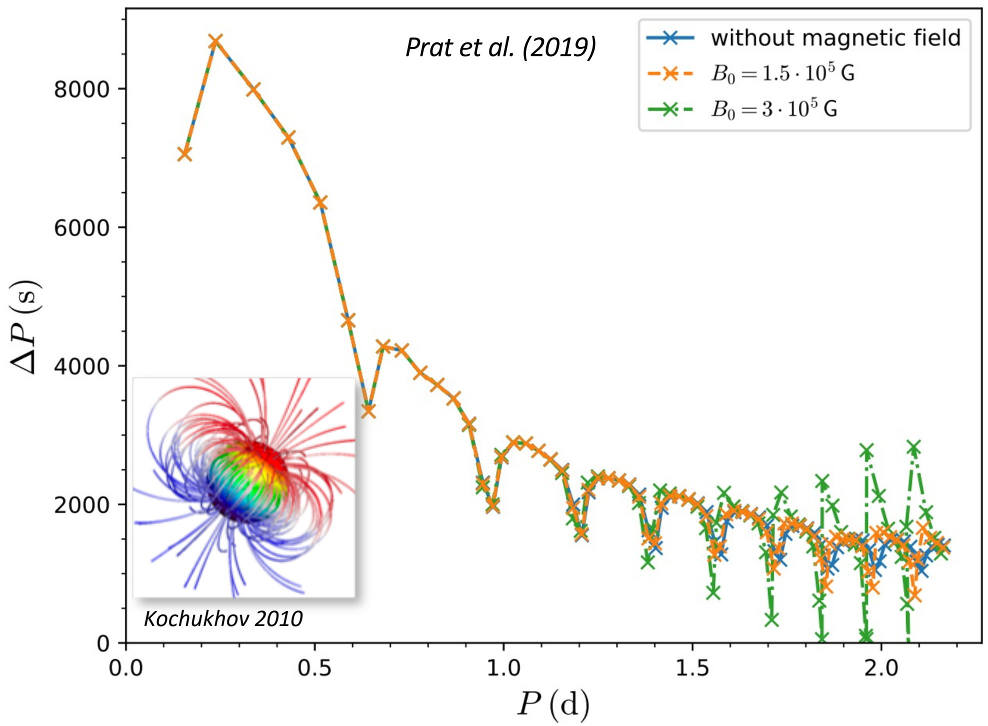
- These effects are **theoretically detectable in early-type stars using observations from Kepler and TESS.**

- This new generalisation of the TAR can also be used to study:
  - ✓ the tidally excited oscillations in rapidly rotating early-type stars,
  - ✓ the angular momentum transport by GIWs.

Future work: Magnetism

- Early-type stars can host magnetic fields *(cf. Lisa Bugnet, Matteo Cantiello and Laurane Freour talks).*
- Prat et al. (2019, 2020) and Van Beeck et al. (2020) studied the effects of a magnetic field on the periods of g modes using a perturbative treatment:
  - the slope is not affected by the magnetic field,
  - the depth and the spacing of the dips are modified.
- Mathis & de Brye (2011, 2012) have taken into account the toroidal magnetic fields in a non-perturbative way.

⇒ Generalise the TAR to account for general magnetic field in differentially rotating stars.



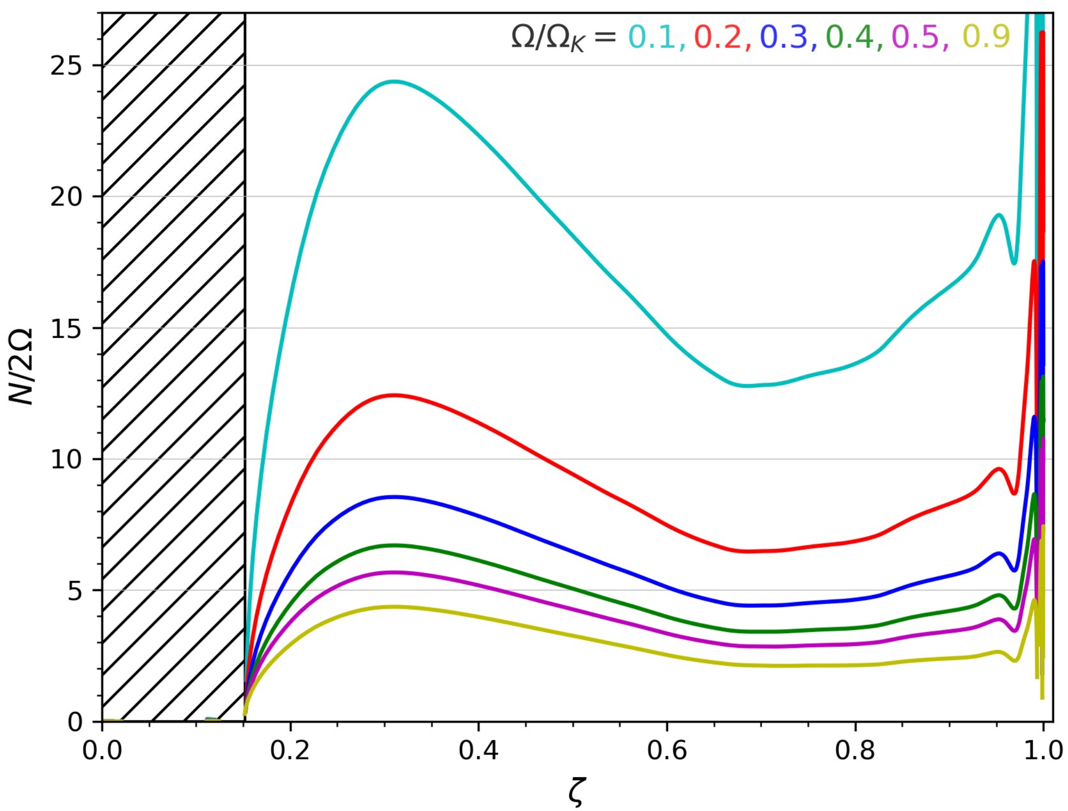
Extra slides

Evaluation of the frequency hierarchy imposed by the generalised TAR

- Frequency hierarchy :
  - Low frequency waves ⇒ $\omega \ll N$
  - Waves in a strongly stratified region ⇒ $2\Omega \ll N$

Dhouib et al., submitted

The N/2Ω term



The N/ω term

$m = 1, k = 0, \Omega/\Omega_K = 0.2$

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