Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, *https://doi.org/10.5281/zenodo.5009224*

Creative Magic Squares: Area Representations

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Abstract

It is well known that every magic square can be written as ***perfect square sum of entries****. It is always possible with odd number entries starting from 1. In case of odd order magic squares we can also write with* ***consecutive natural number*** *entries. Still, it is unknown whether it is possible to even order magic squares. In case of odd order magic squares, still we can write them with* ***minimum perfect square*** *sum of entries. Based on this idea of* ***perfect square sum of entries****, we have written a magic square representing areas. This is done for the magic squares of orders 3 to 11. In the case of magic squares of orders 10 and 11 the images are not very clear, as there are a lot of numbers. To have a clear idea, the magic squares are also written in numbers. In all the cases, the area representations are more that one way. It is due to the fact that we can always write magic squares as* ***normal****,* ***bordered*** *and* ***block-bordered*** *ways.*

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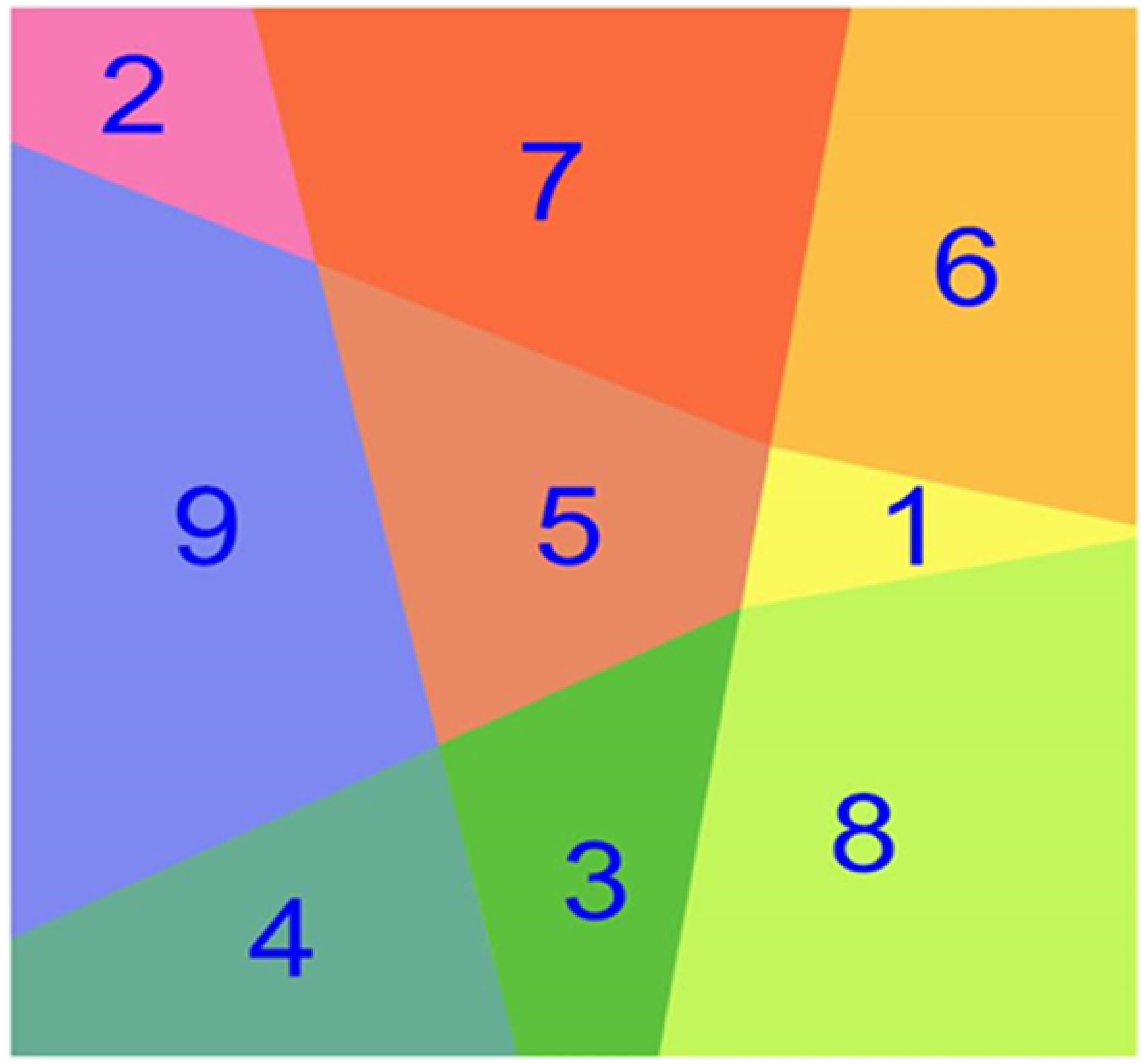
1 Introduction

William Walkington [5] started an interesting discussion as to how to create magic squares with cells that had the same areas as their numbers. Below is a graphic design for a 2017 seasonal greetings card, showing a magic square with approximate areas that was constructed by William Walkington (2016):



William Walkington - 2016

Figure 1

Lee Sallows (2017) [2] also constructed another magic square representing the areas as rectangles. See below:

Lee Sallows - 2017

Figure 2

The sum of all the numbers is given by

$$1+2+3+4+5+6+7+8+9=45. \tag{1}$$

The number 45 is not a perfect square. If we make a slight change, then we can transform the sum into a perfect square:

$$5+6+7+8+9+10+11+12+13=1+3+5+7+9+11+13+15+17=9^2 \qquad (2)$$

Using this number sequence, Walter Trump (2017) [4] was able to construct the following area magic square:

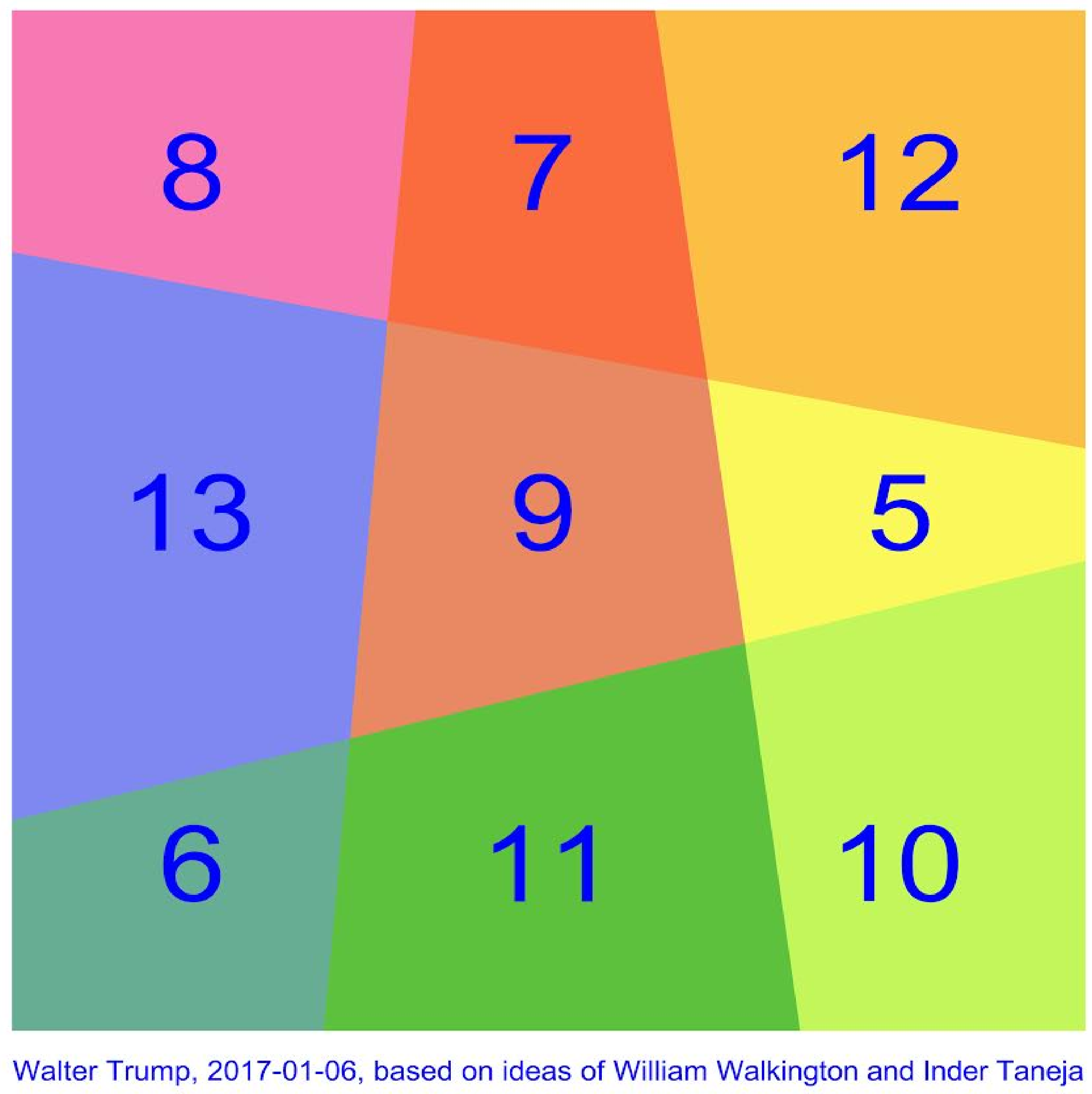


Figure 3

Adding 4 to each number in (1), we obtain the numbers given in (2). Observing area-wise the Figures 1 and 3, there is a considerable difference: For example, from numbers 1 to 2, the cell area is doubled, while from numbers 5 to 6, there is proportionally less increase between the cell areas.

In order to construct a magic square with cell areas that are in proportion to their numbers it is not necessary that the numbers always sum to a perfect square. Below is another example constructed by William Walkington (2017) [5] with sequential numbers from 3 to 11:

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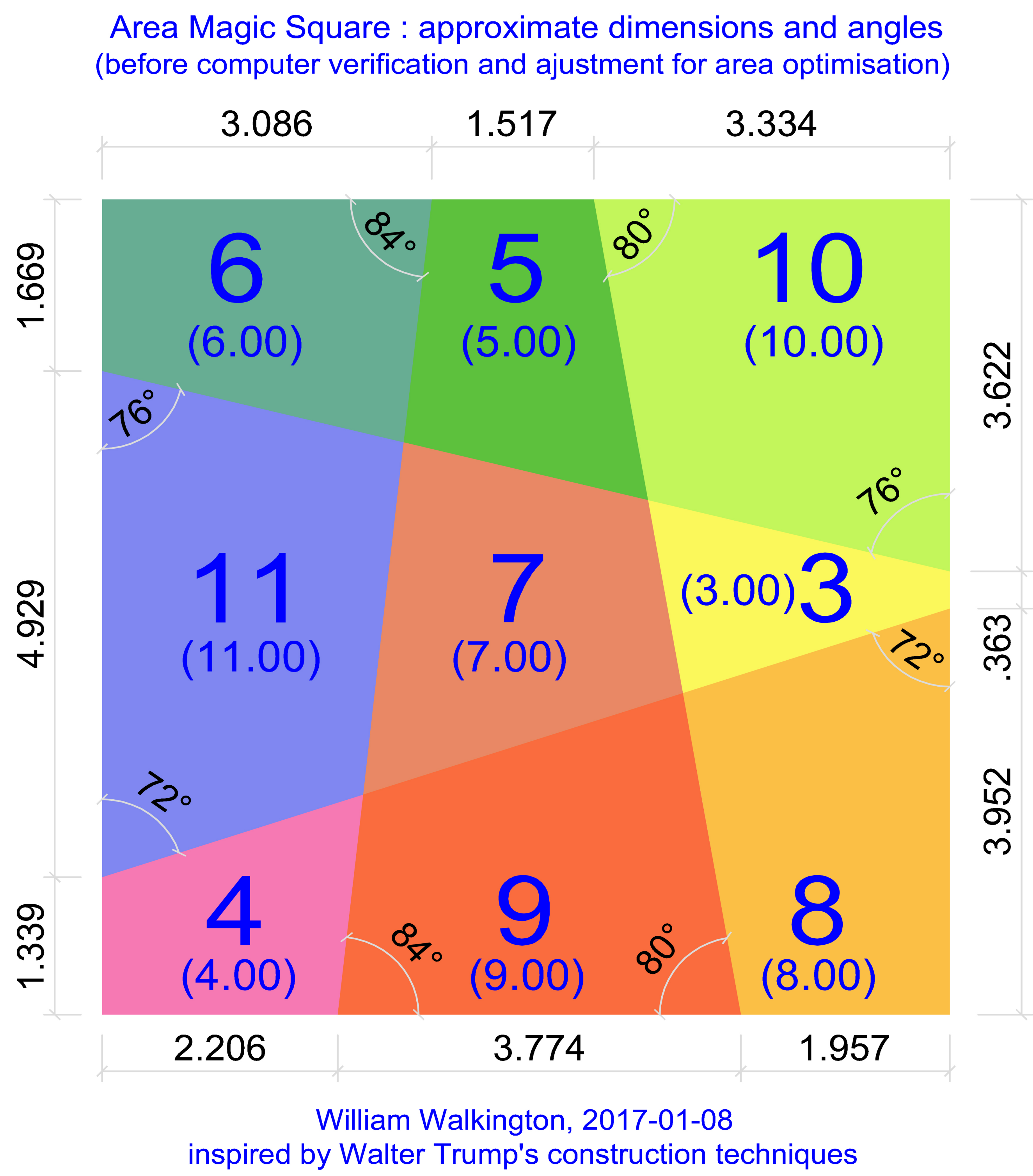
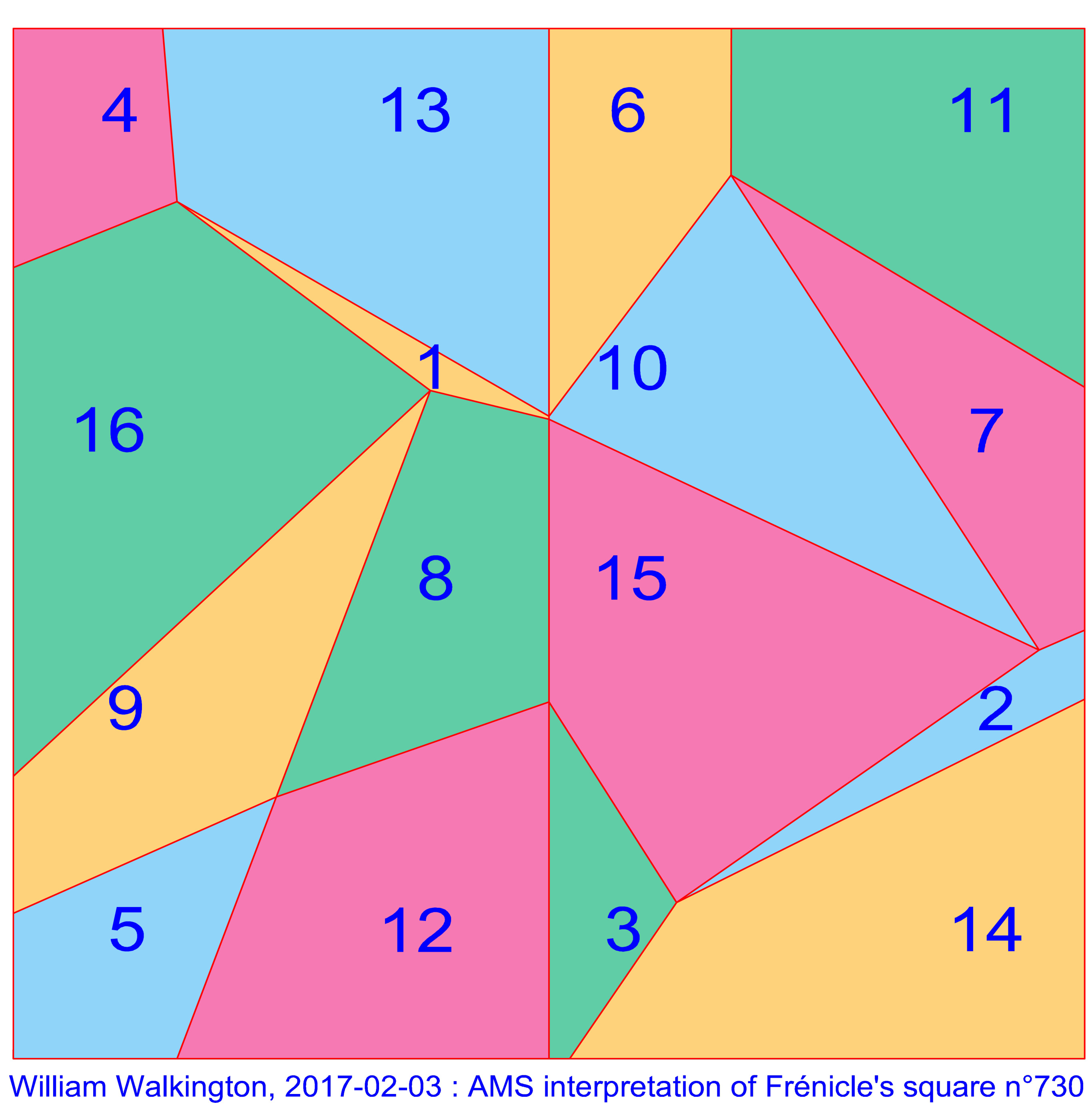
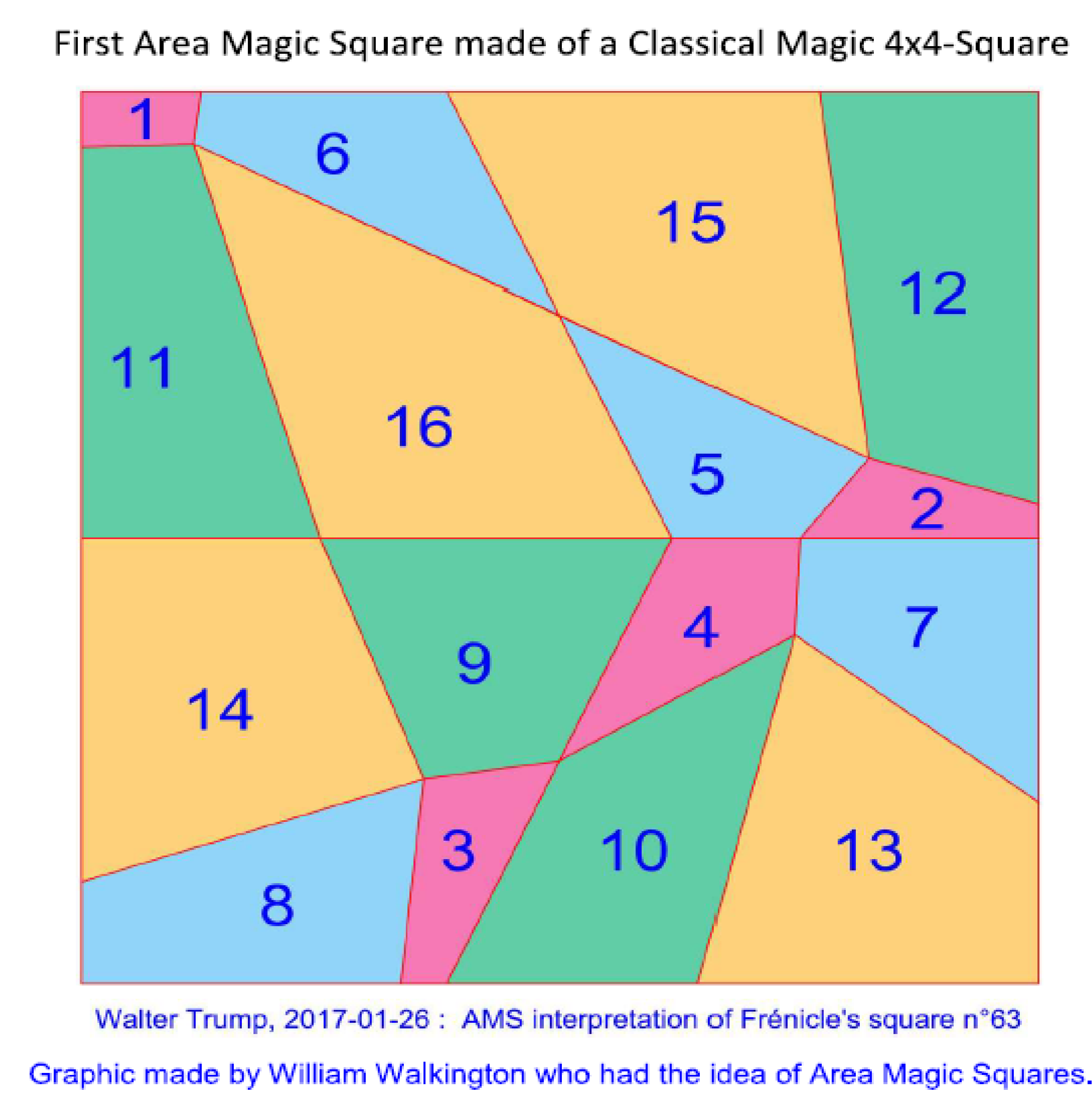


Figure 4

We observe that in Figures 1, 3 and 4, the number cell areas are proportional and aligned in both directions. In Figure 2, the proportionality of the areas is only present in one direction, which is horizontal.

Below are two examples of classical order 4 magic squares with cell areas that are proportional to their numbers:





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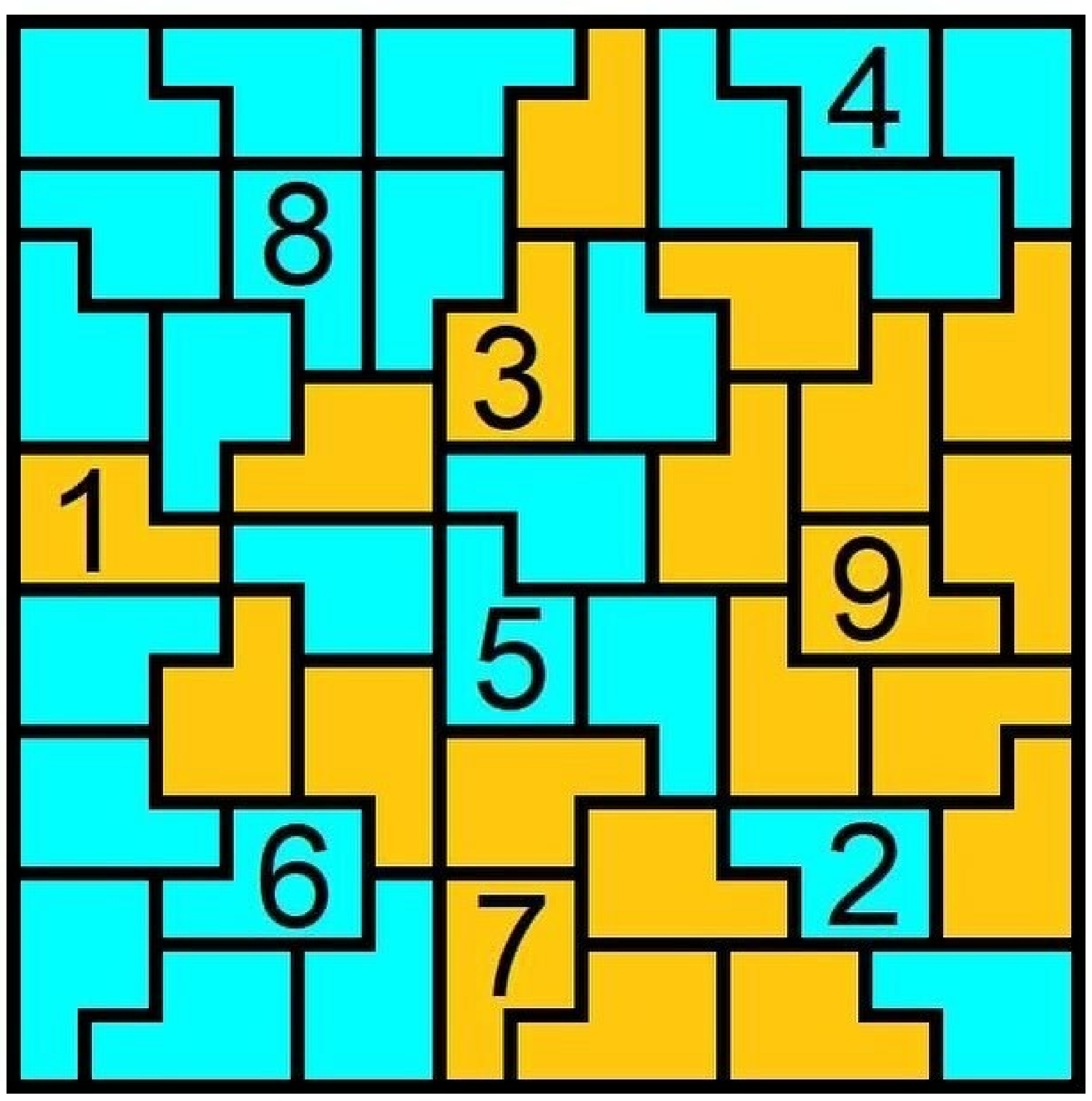
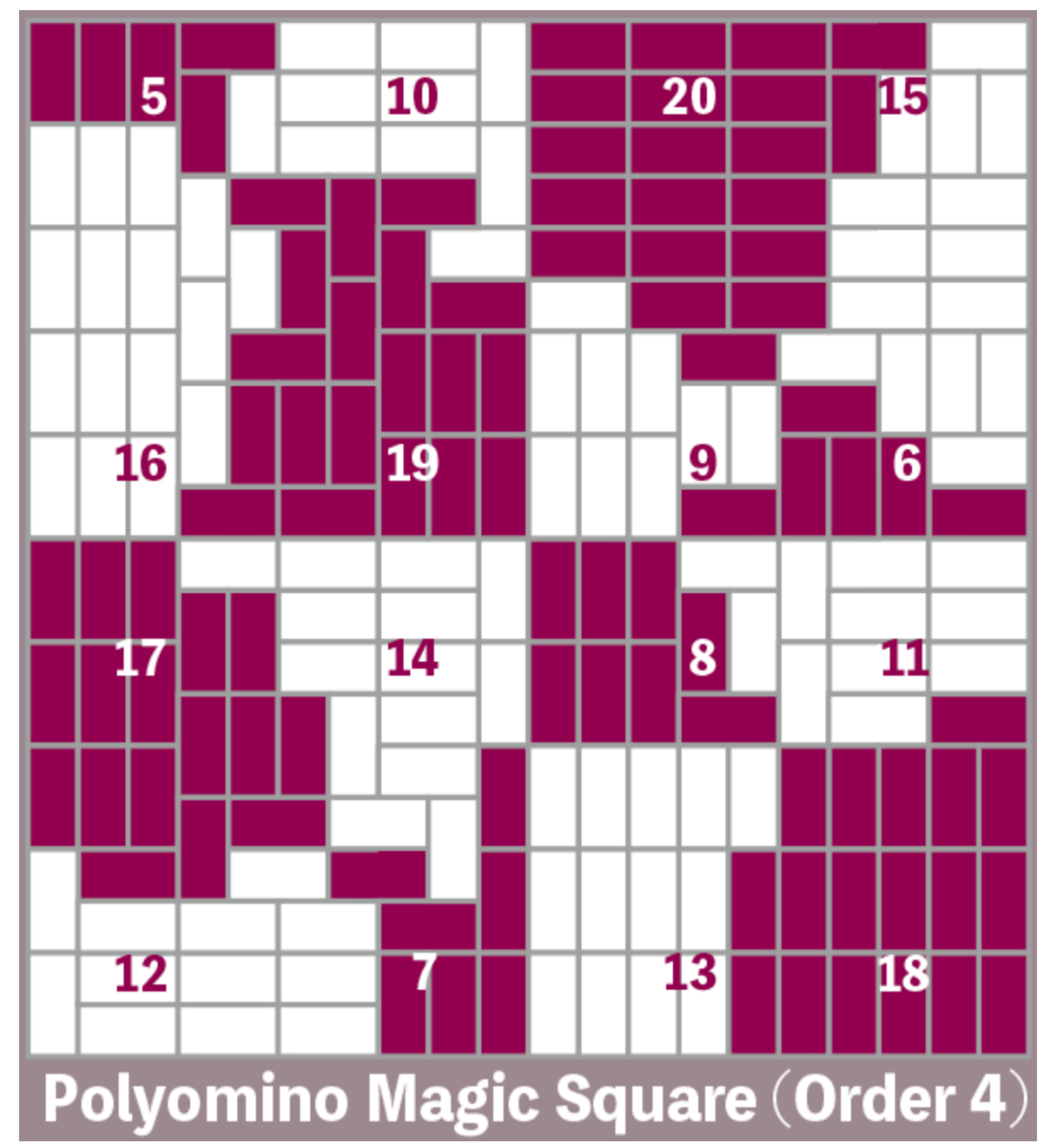
More examples of similar kinds of order 4 area magic squares, together with order 6 area magic squares, can be seen in William Walkington's pages [5]. From equation (2), the question arises, how to create higher order magic squares such that the sum of numbers is always a perfect square. This can be seen in author's work [11].

Below are few examples recently done by Yoshiaki Araki [6] for magic squares of orders 3 and 4.





https://twitter.com/alytile/status/1396758907582779397/photo/1

More examples of similar kind can be seen in Yoshiaki Araki [6] on Facebook or twitter.

Recently, author worked on magic squares of orders 3 to 31 with **perfect square sum** of entries. In case of odd order magic squares, we have two possibilities. One is with **consecutive odd number** entries starting from 1, and another with **consecutive natural number** entries (see equation (2)). In case of even order magic squares, there is only one possibility, i.e., with **consecutive odd number** entries. In case of odd order magic squares, still, we can have **minimum perfect square sum** of **positive** entries. For more details refer Taneja [19]. For more study on magic squares refer author's work [7]-[24].

It is author's fifth work on **creative magic squares**. See below the list of other works:

1. *Single Digit Representations - [20];*
2. *Single Letter Representations - [21];*
3. *Permutable Base-Power Digits Representations - [22];*
4. *Increasing and Decreasing Orders Crazy Representations - [23].*

The aim of this work is to write area representations of magic squares based on the idea of **perfect square sum** of entries. It helps in organizing well the area in case of each number. This we have done only for the magic squares of orders 3 to 11. The same can be done for the higher order magic squares, but in visibility of each number is very less. This can be obviously seen in magic squares of orders 10 and 11.

2 Area Representations Magic Squares

This section brings area representations of magic squares from order 3 to 11. The higher orders can also be written with the same procedure. These are not written here due to visibility problems. These are based on the idea that every magic square can be represented as **perfect square sum** of entries.

2.1 Magic Square of Order 3

Below are two magic square of order 3 with entries as **consecutive odd numbers** and **consecutive natural numbers**.

Example 2.1. *For the* ***consecutive odd numbers*** *entries* $\{1, 3, 5, \ldots, 15, 17\}$*, and for the* ***consecutive natural numbers*** *entries* $\{5, 6, 7, \ldots, 12, 13\}$ *the magic squares of order 3 are respectively given by*

			27
7	17	3	27
5	9	13	27
15	1	11	27
27	27	27	27

			27
8	13	6	27
7	9	11	27
12	5	10	27
27	27	27	27

Both the examples are with same magic sums, i.e., $S_{3\times 3} = 27 = 3^3$*, and the same sum of all entries, i.e.,* $T_9 = 3 \times 27 = 81 = 9^2 = 3^4$*.*

The example below is with **minimum perfect square** sum of entries.

Example 2.2. *A magic square of order 3 with* ***minimum perfect square*** *sum of entries is given by*

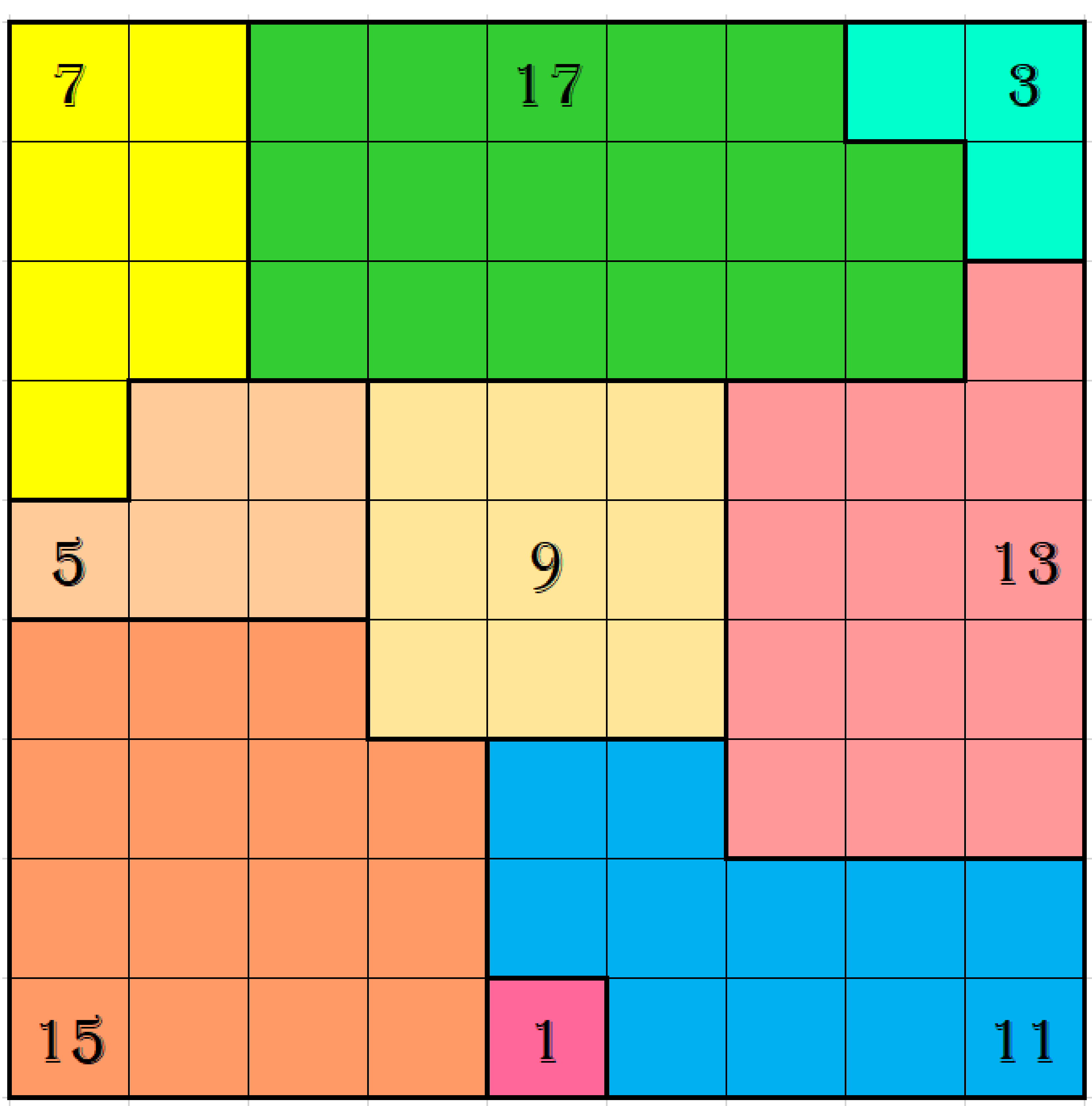
			12
1	6	5	12
8	4	0	12
3	2	7	12
12	12	12	12

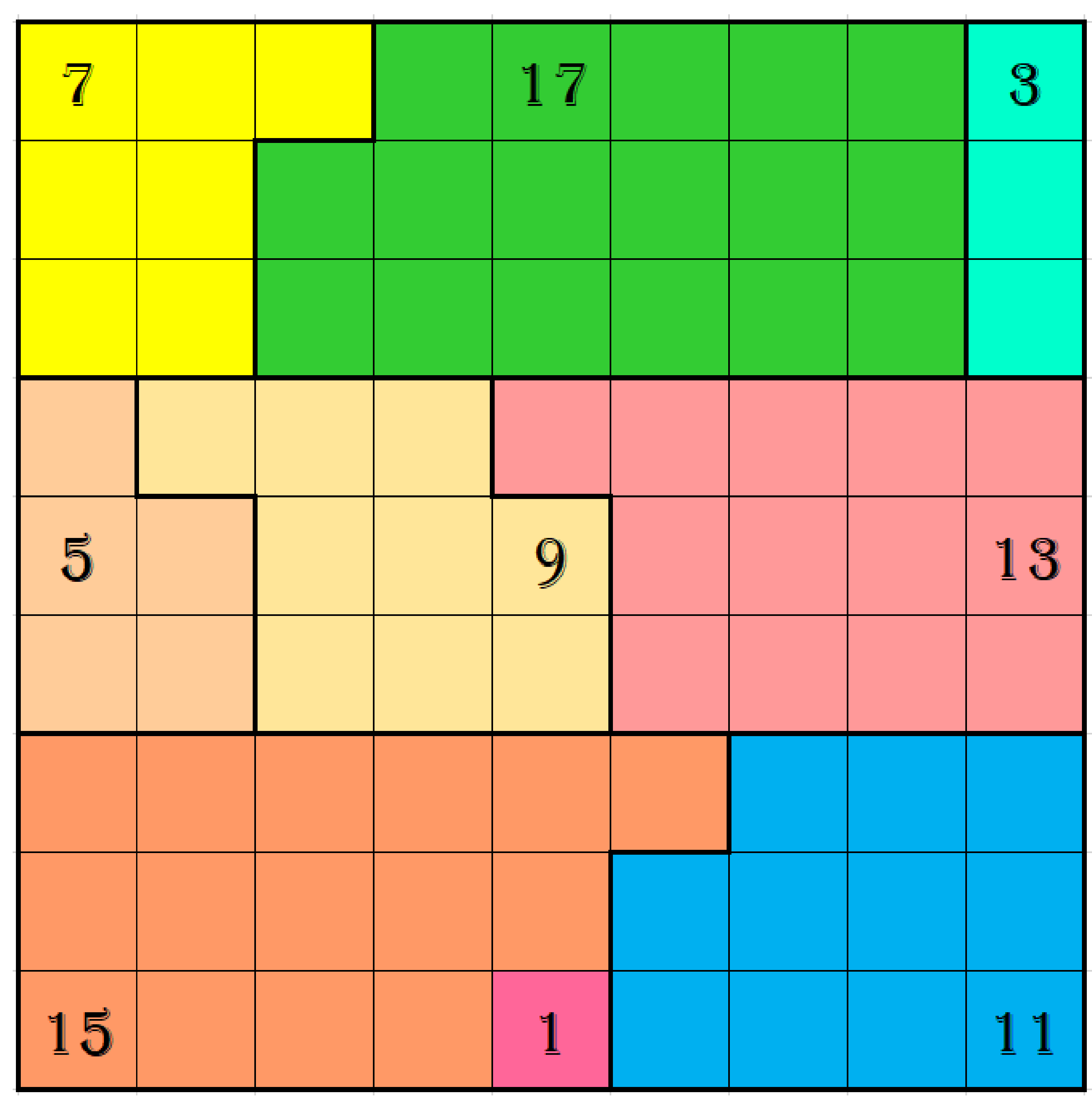
In this case, the magic sum is $S_{3\times 3} = 12$, and the sum of all entries is $T_9 := 36 = 6^2$.

2.1.1 Area Representations

In this subsection, we shall write according to area covered by each number for the Examples 2.1 and 2.2. See below these examples.

Example 2.3. *A magic square of order 3 representing area for each number according to Example 2.1 is given below in two different ways:*





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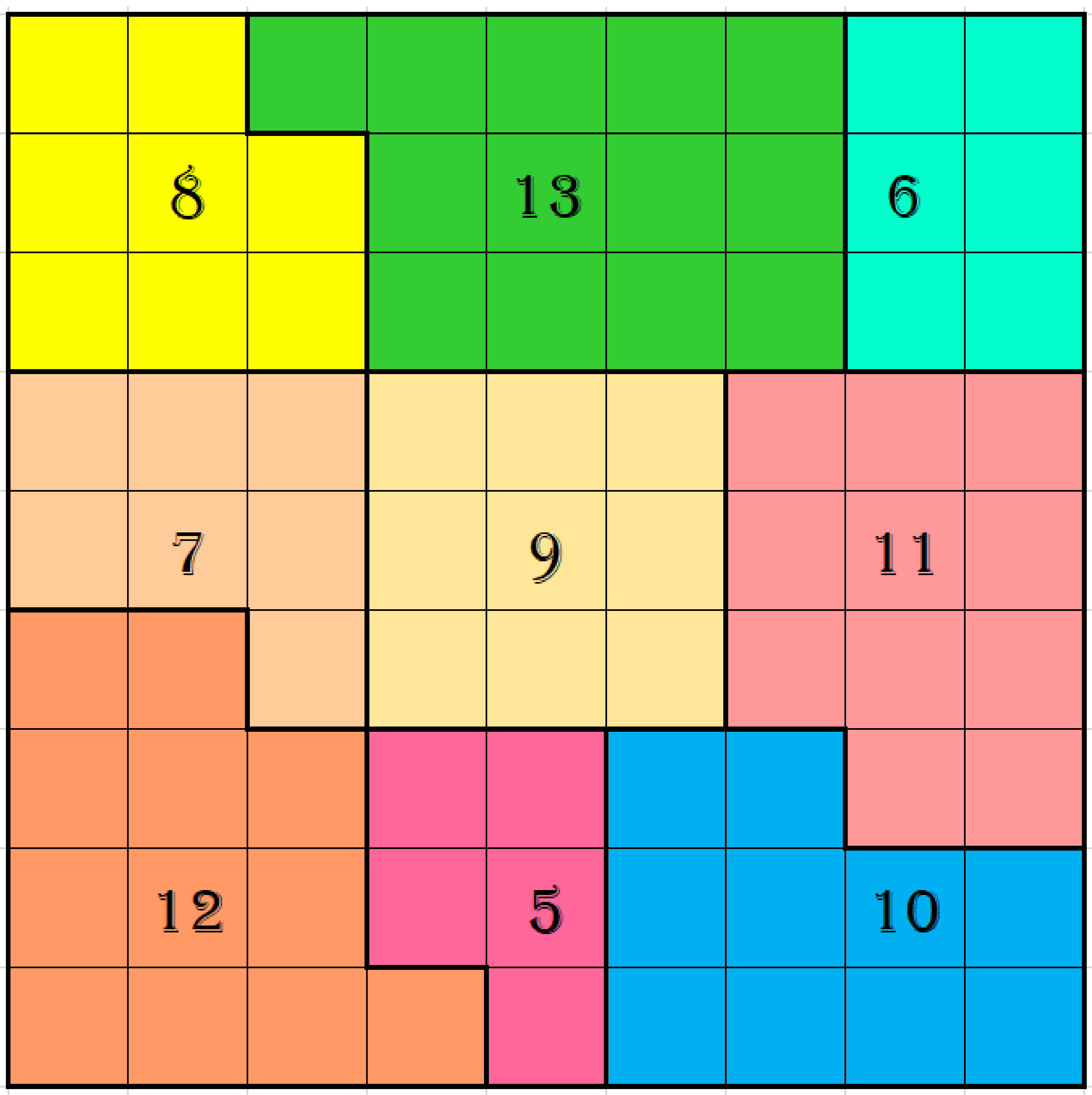
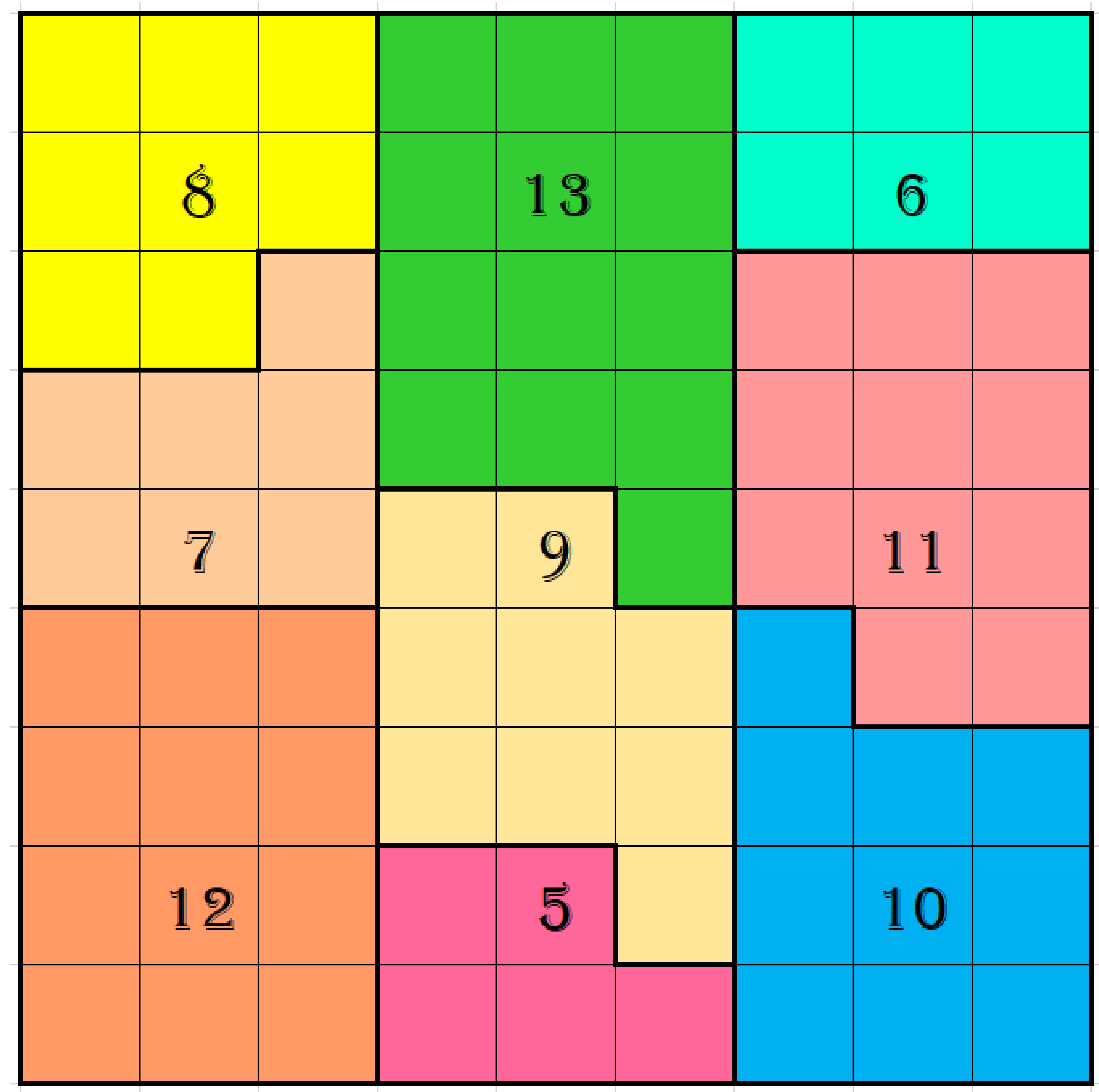
In this case the entries are odd numbers $\{1, 3, 5, \ldots, 15, 17\}$. The sum of all entries is a perfect square, i.e., $T_9 := 81 = 9^2$.

Example 2.4. *A magic square of order 3 representing area for each number according to Example 2.1 is given below in two different ways:*





In this case the entries are natural numbers $\{5, 6, 7, \ldots, 12, 13\}$. The sum of all entries is a perfect square, i.e., $T_9 := 81 = 9^2$.

Remark 2.1. *Even though we can also write an area representation of a magic square with minimum perfect square entries sum given in Example 2.2, but it includes the number 0, that doesn't have any representation. In this case the area magic square comes with 8 numbers. It's not very practical to write.*

2.2 Magic Squares of Order 4

Below is a magic square of order 4 with entries as **consecutive odd numbers** $\{1, 3, \ldots, 29, 31\}$.

Example 2.5. *For the **consecutive odd number** entries $\{1, 3, 5, \ldots, 29, 31\}$, the **pandiagonal** magic square of order 4 is written below in two different ways*

		64	64	64	64
	13	23	1	27	64
64	3	25	15	21	64
64	31	5	19	9	64
64	17	11	29	7	64
	64	64	64	64	64

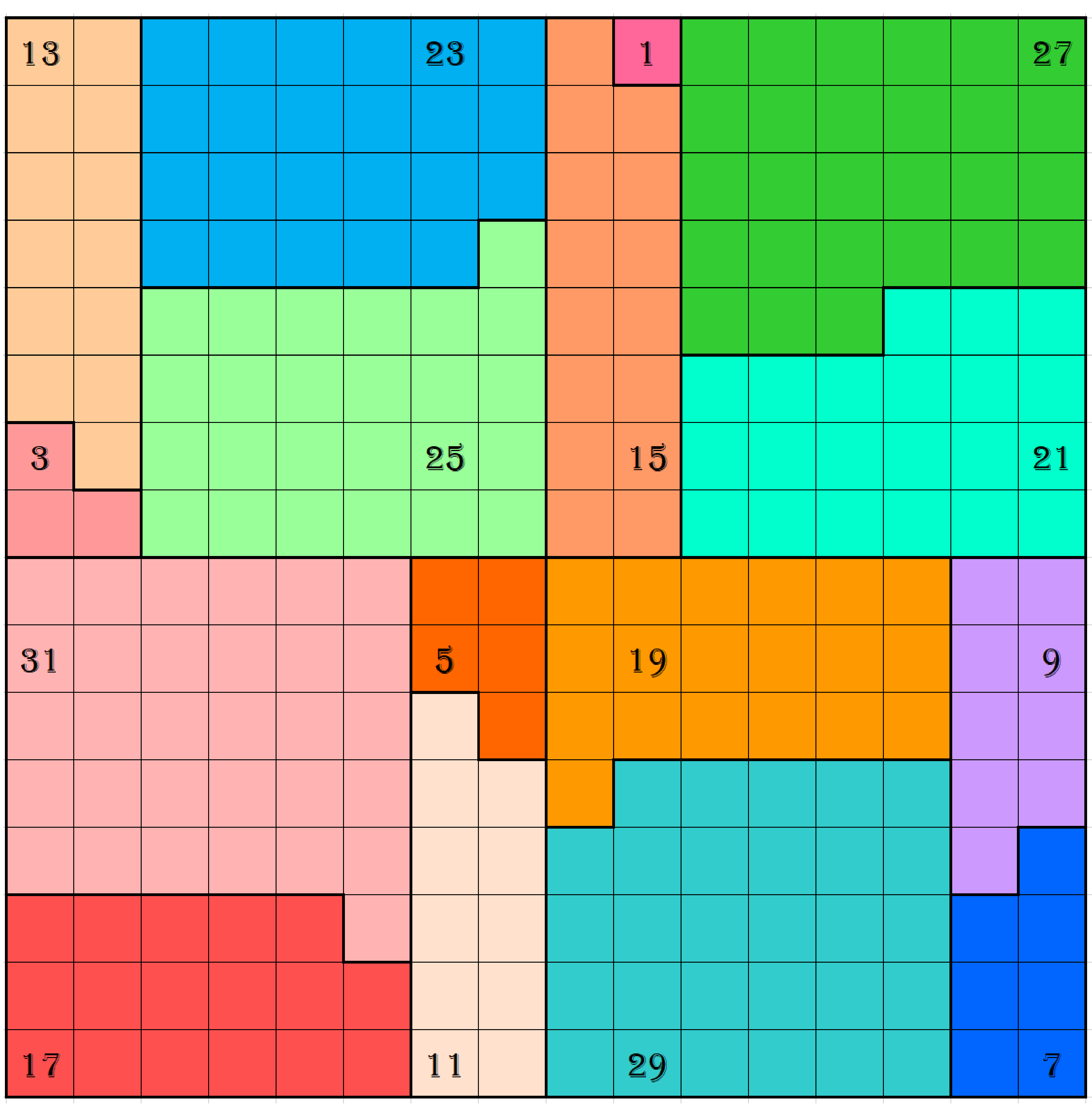
		64	64	64	64
	13	23	1	27	64
64	3	25	15	21	64
64	31	5	19	9	64
64	17	11	29	7	64
	64	64	64	64	64

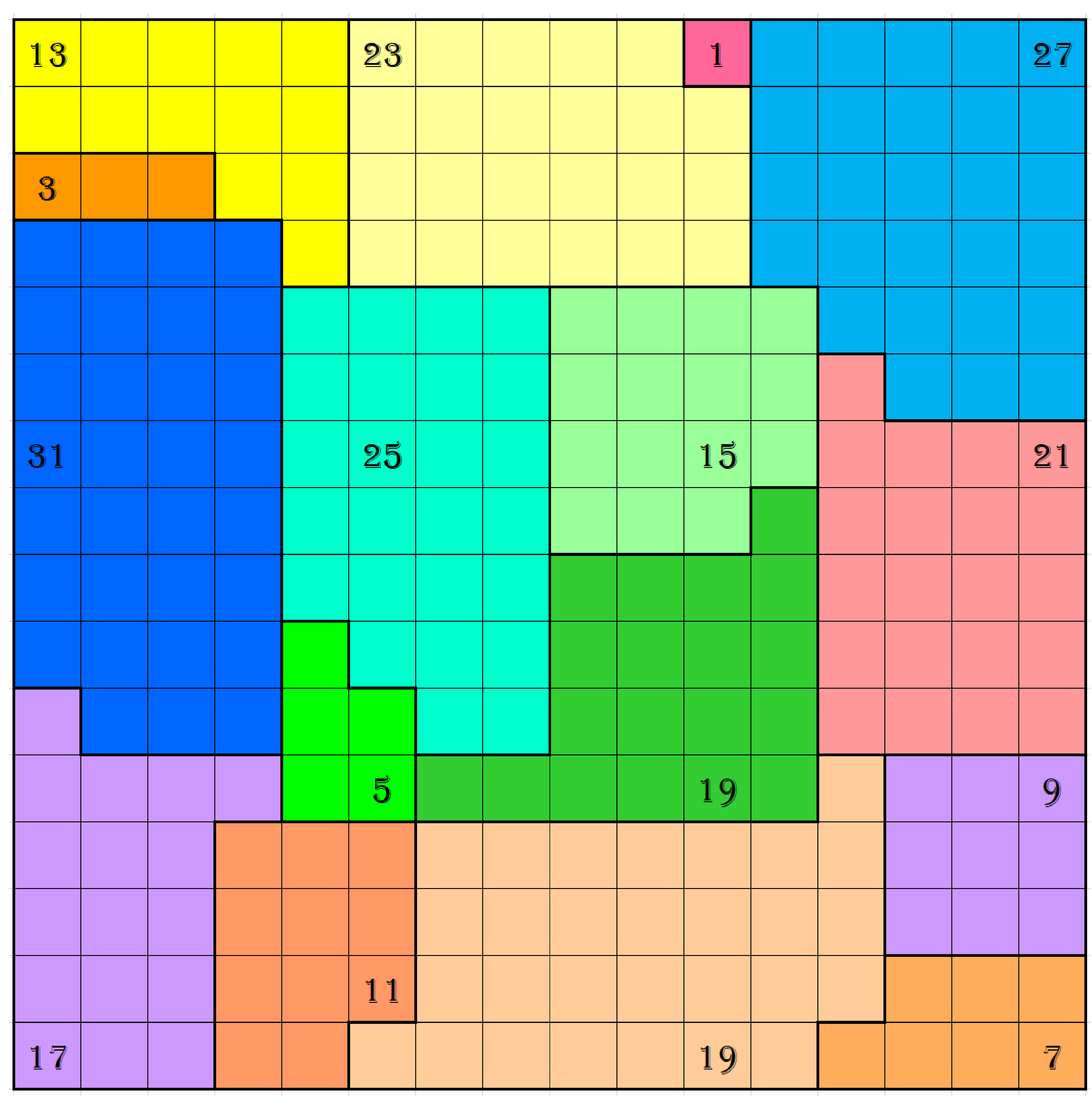
In this case, the magic sum is $S_{4\times 4} := 64 = 4^3$, *and the sum of all entries is* $T_{16} := 256 = 16^2 = 4^4$.

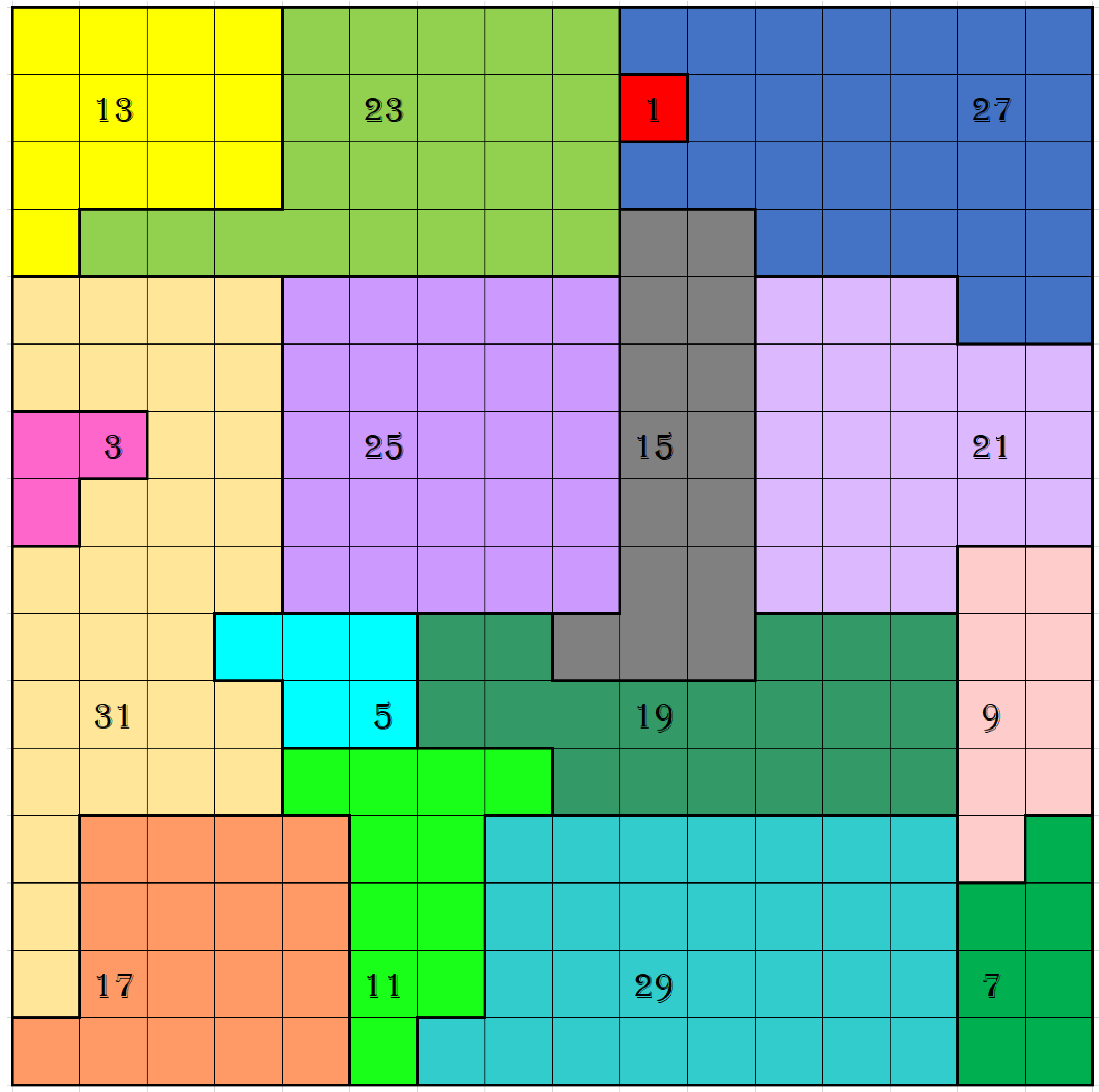
2.2.1 Area Representations

In this subsection, we shall write magic squares according to area covered by each number for the Examples 2.5. See below these examples.

Example 2.6. *Below are three different ways of writing magic square of order 4 representing area for each number according to Example 2.5 is given below:*







In all the cases, the entries are odd numbers $\{1,3,5,\ldots,29,31\}$*. The sum of all entries is a perfect square, i.e.,* $T_{16} := 256 = 16^2$*.*

Remark 2.2. *Above there are three different representations of magic square of order 4. The first ways is with 4* ***blocks*** *of 4 elements each with equal sums, i.e.,* $T_{64} := 8^2$*. The second way is* ***bordered*** *magic square. In this case, the inner block of 16 elements is a magic square of order 4 with entries sum a* ***perfect square****, i.e.,* $T_{16} := 64 = 8^2$*. Also the sum of innter four elements is a perfect square, i.e.,* $T_4 := 16 = 4^2$*. The third way is just normal.*

2.3 Magic Squares of Order 5

Below are two magic square of order 5 with entries as **consecutive odd numbers** and **consecutive natural numbers**.

Example 2.7. *For the* ***consecutive odd number*** *entries* $\{1,3,5,\ldots,47,49\}$*, and* ***consecutive natural number*** *entries* $\{13,14,15,\ldots,36,37\}$ ***pandiagonal*** *magic squares of order 5 are respectively given by*

		125	125	125	125	125
	1	17	23	39	45	125
125	33	49	5	11	27	125
125	15	21	37	43	9	125
125	47	3	19	25	31	125
125	29	35	41	7	13	125
	125	125	125	125	125	125

		125	125	125	125	125
	13	21	24	32	35	125
125	29	37	15	18	26	125
125	20	23	31	34	17	125
125	36	14	22	25	18	125
125	27	30	33	16	19	125
	125	125	125	125	125	125

Both the examples written above are with same magic sums, i.e., $S_{5\times 5} = 125 = 5^3$, *and the same sum of all entries, i.e.,* $T_{25} = 5 \times 125 = 625 = 25^2 = 5^4$. *The example below is with* ***minimum perfect square*** *sum of entries.*

Example 2.8. *For the* ***consecutive natural number*** *entries* $\{4, 5, 6, \ldots, 27, 28\}$, *the* ***pandiagonal*** *magic square of order 5 is given by*

		80	80	80	80	80
	4	10	16	22	28	80
80	21	27	8	9	15	80
80	13	14	20	26	7	80
80	25	6	12	18	19	80
80	17	23	24	5	11	80
	80	80	80	80	80	80

In this case the magic sum is $S_{5\times 5} = 80$, and the sum of all entries is $T_{25} := 400 = 20^2$. It is **minimum perfect square** sum of entries.

The magic squares given in Example 2.7 are with **consecutive odd numbers**, and **consecutive natural numbers** entries. Let's write them as **bordered magic squares**.

Example 2.9. *The* ***bordered*** *magic squares of order 5 for the* ***consecutive odd number*** *entries* $\{1, 3, 5, \ldots, 47, 49\}$, *and* ***consecutive natural number*** *entries* $\{13, 14, 15, \ldots, 36, 37\}$ *are respectively given by*

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43	49	9	13	11
3	23	33	19	47
5	21	25	29	45
35	31	17	27	15
39	1	41	37	7

34	37	17	19	18
14	24	29	22	36
15	23	25	27	35
30	28	21	26	20
32	13	33	31	16

In both the cases, the magic sums are same, i.e., $S_{5\times 5} = 125$*, and the sum of all entries are* $T_{25} := 625 = 25^2$*. Moreover, magic squares of order 3 are also with perfect square sum of entries, i.e.,* $S_{3\times 3} = 75$ *and* $T_9 := 225 = 15^2$*. The central element is also a perfect square, i.e.,* $T_1 := 25 = 5^2$*.*

Example 2.10. *A **bordered** magic square of order 5 for the entries* $\{4, 5, 6, \ldots, 27, 28\}$ *is given by*

25	28	8	10	9
5	15	20	13	27
6	14	16	18	26
21	19	12	17	11
23	4	24	22	7

In this case the magic sum is $S_{5\times 5} = 80$*, and the sum of all entries is* $T_{25} := 400 = 20^2$*. It is **minimum perfect square sum** of entries. Moreover, magic squares of order 3 are also with perfect square sum of entries, i.e.,* $S_{3\times 3} = 48$ *and* $T_9 := 144 = 12^2$*. The central element is also a perfect square, i.e.,* $T_1 := 16 = 4^2$*.*

2.3.1 Area Representations

In this subsection, we shall write magic squares of order 5 according to area covered by each number for the Examples 2.7, 2.8 and 2.9. See below these examples.

Example 2.11. *A **Bordered** magic square of order 5 representing area for each number according to Example 2.9 for **consecutive odd number** entries is given by*

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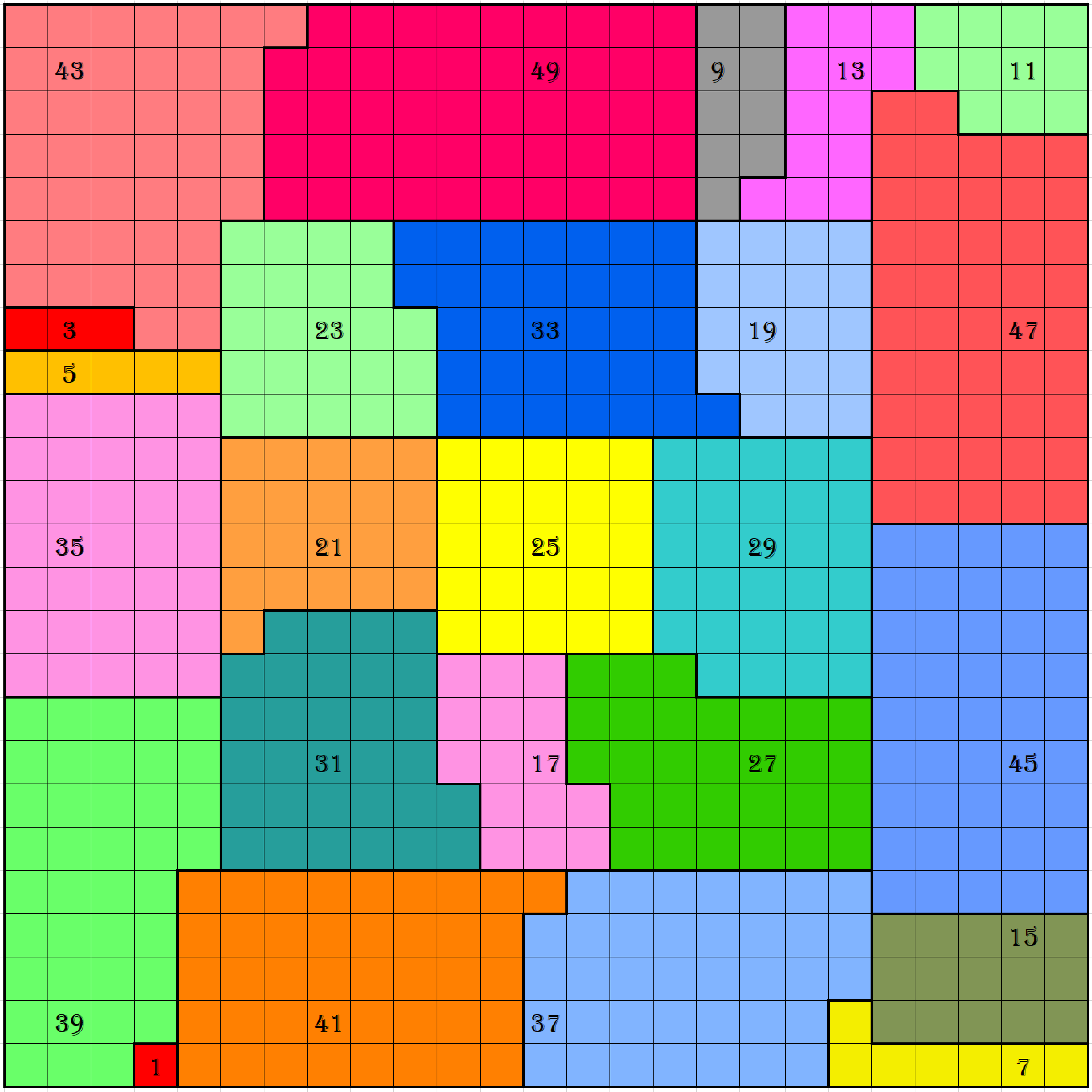

Example 2.12. *A* ***Bordered*** *magic square of order 5 representing area for each number according to Example 2.9 for* ***consecutive natural number*** *entries is given by*

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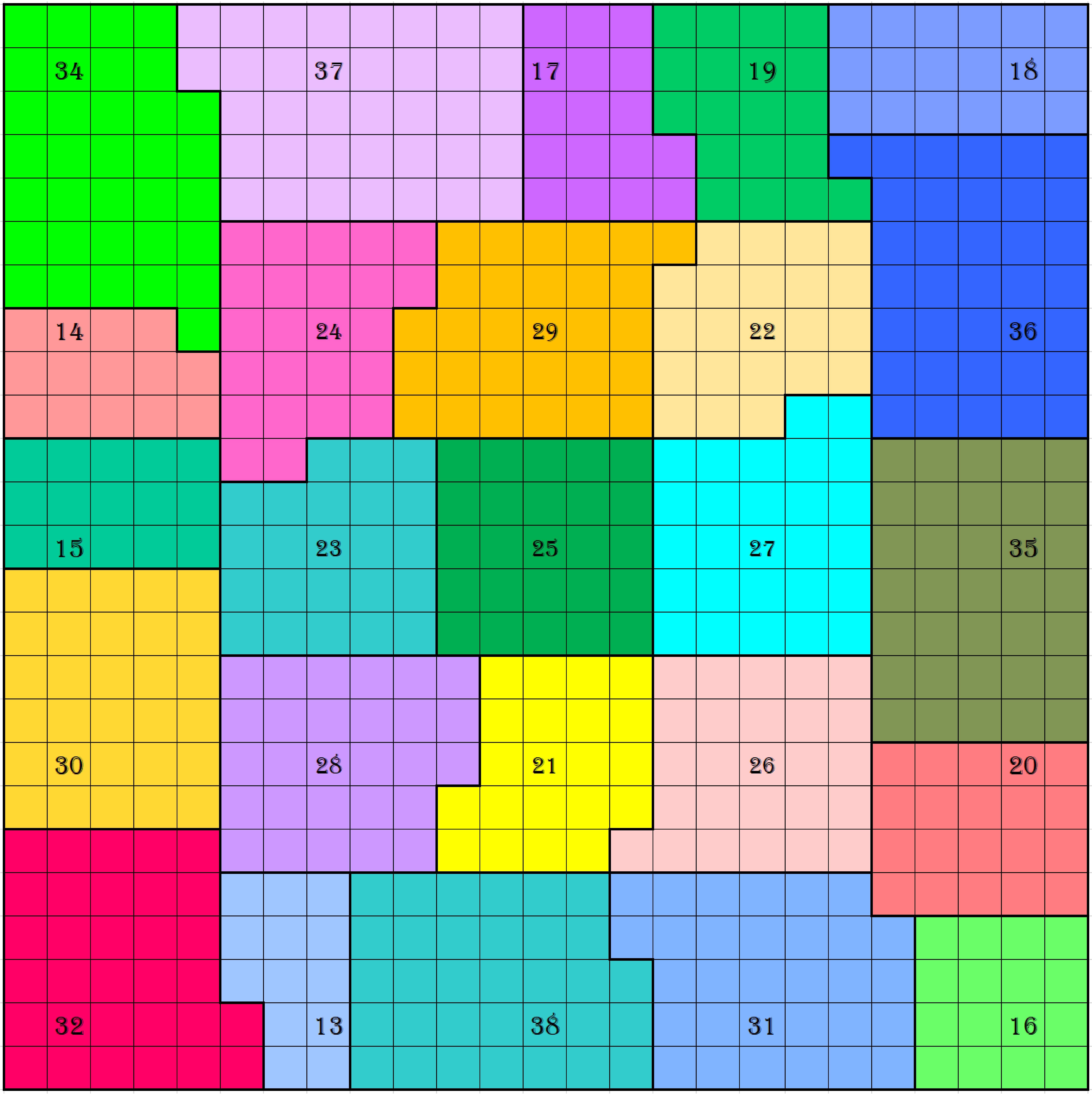

Example 2.13. *A* ***bordered*** *magic square of order 5 representing area for each number according to Example 2.10 for the* ***consecutive natural number*** *entries is given by*

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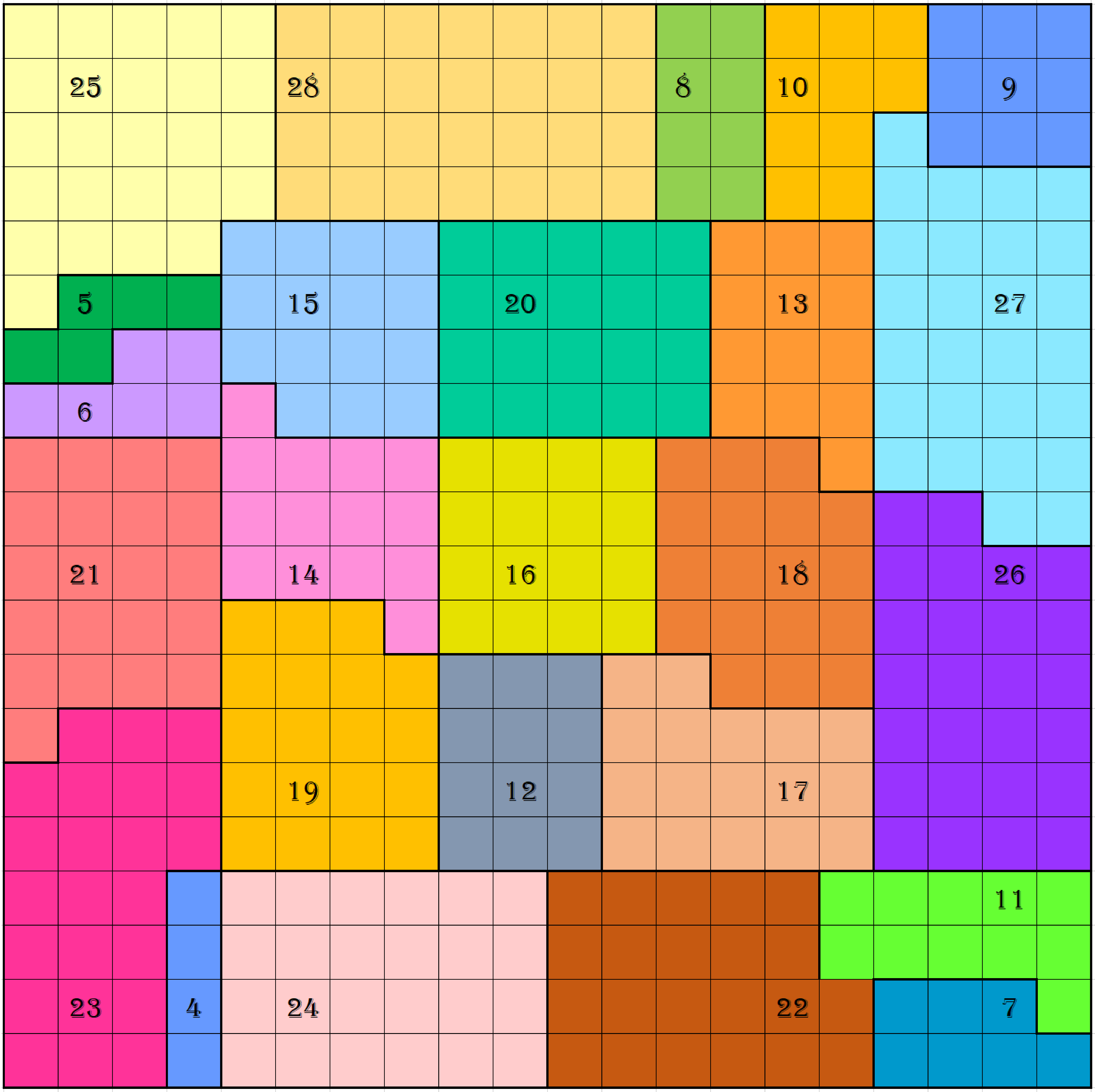
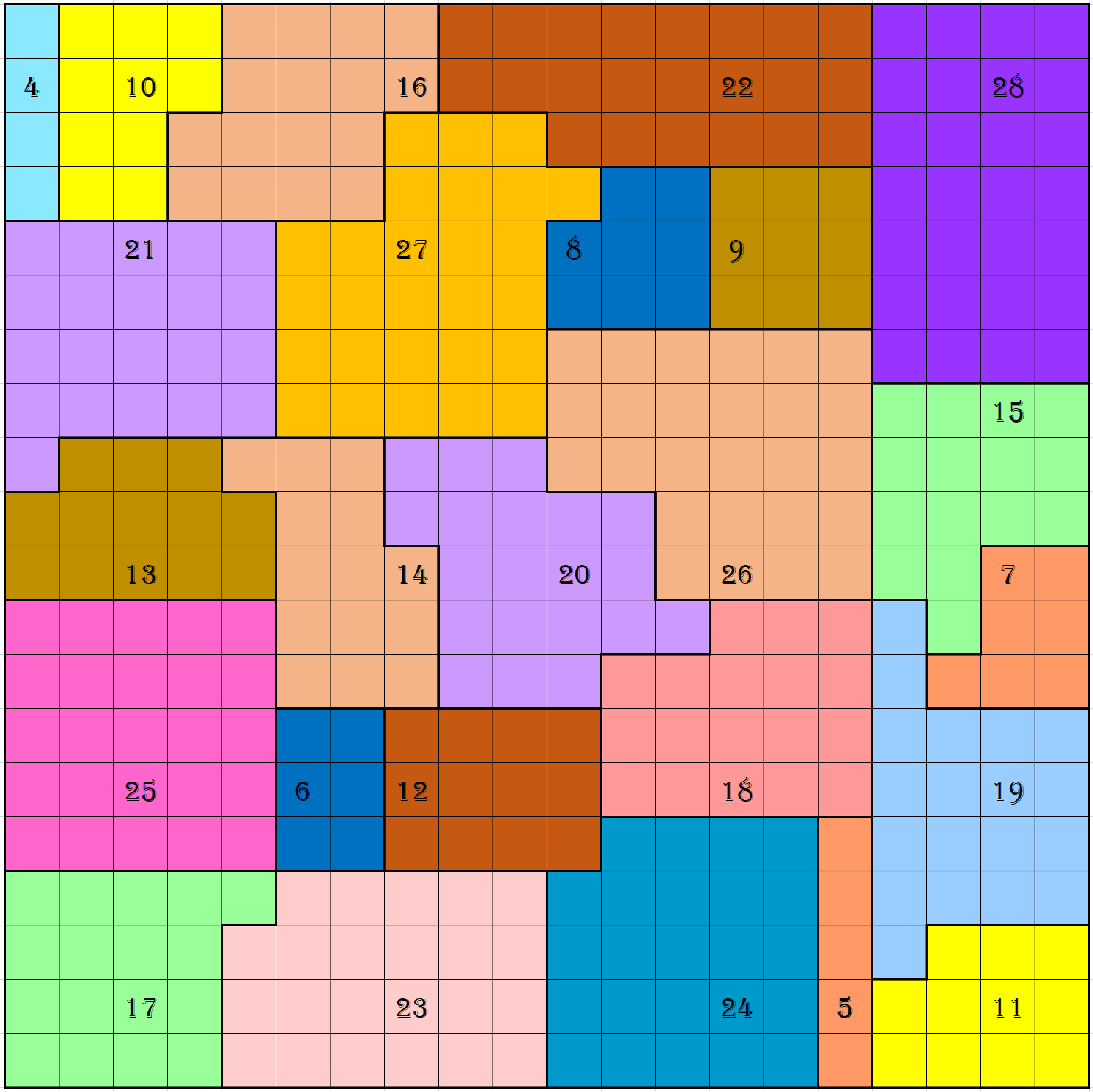


In this case the entries are ***minimum perfect square sum****.*

Example 2.14. *A magic square of order 5 representing area for each number according to Example 2.9 is given below in two different ways:*


4
10
16
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19
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23
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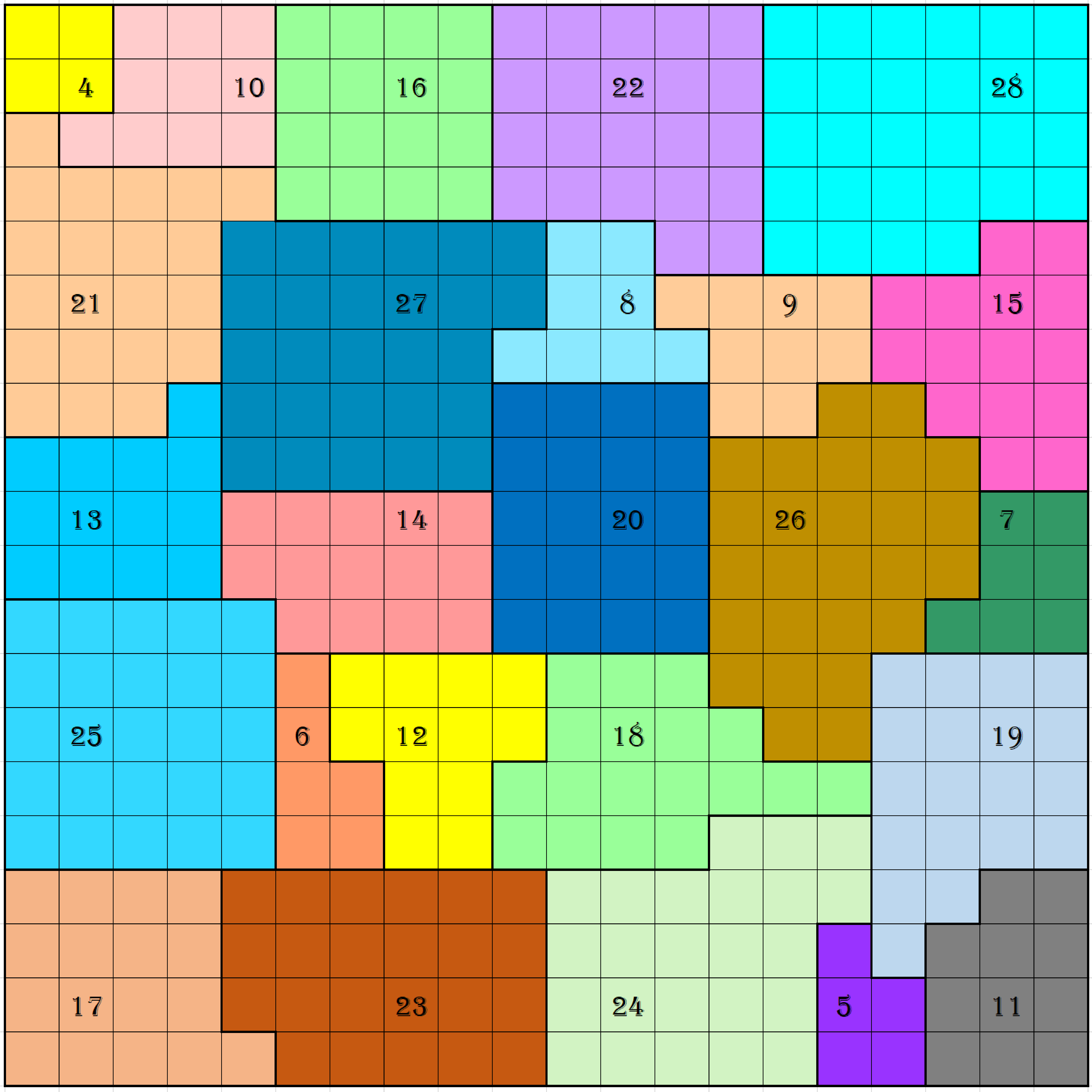
11



In this case the magic squares are written in general way without rule just following the numbers given in Example 2.9.

2.4 Magic Squares of Order 6

Example 2.15. *For the **consecutive odd number** entries* $\{1, 3, 5, \ldots, 69, 71\}$*, a magic square of order 6 is given by*

						216
1	45	55	67	33	15	216
57	13	69	27	41	9	216
23	11	25	53	61	43	216
63	31	7	47	19	49	216
37	65	21	5	59	29	216
35	51	39	17	3	71	216
216	216	216	216	216	216	216

In this case, the magic sum is $S_{6\times 6} := 216 = 6^3$, and the sum of the entries is $T_{36} := 1296 = 36^2 = 6^4$.

Let's write a magic square of order 6 given in Example 2.15 as **bordered magic squares**.

Example 2.16. *A **bordered** magic square of order 6 for the entries* $\{1,3,5,\ldots,69,71\}$ *is given by*

63	59	5	71	7	11
3	33	43	21	47	69
15	23	45	35	41	57
19	51	25	39	29	53
55	37	31	49	27	17
61	13	67	1	65	9

In this case the magic sums are $S_{6\times 6} = 216$ *and* $S_{4\times 4} = 144$ *and the sum of all entries is* $T_{36} := 1296 = 36^2 = 6^4$ *and* $T_{16} := 674 = 24^2$*. The sum of inner four elements is* $T_4 := 144 = 12^2$*.*

2.4.1 Area Representations

In this subsection, we shall write magic square according to area covered by each number for the Examples 2.15 and 2.16.

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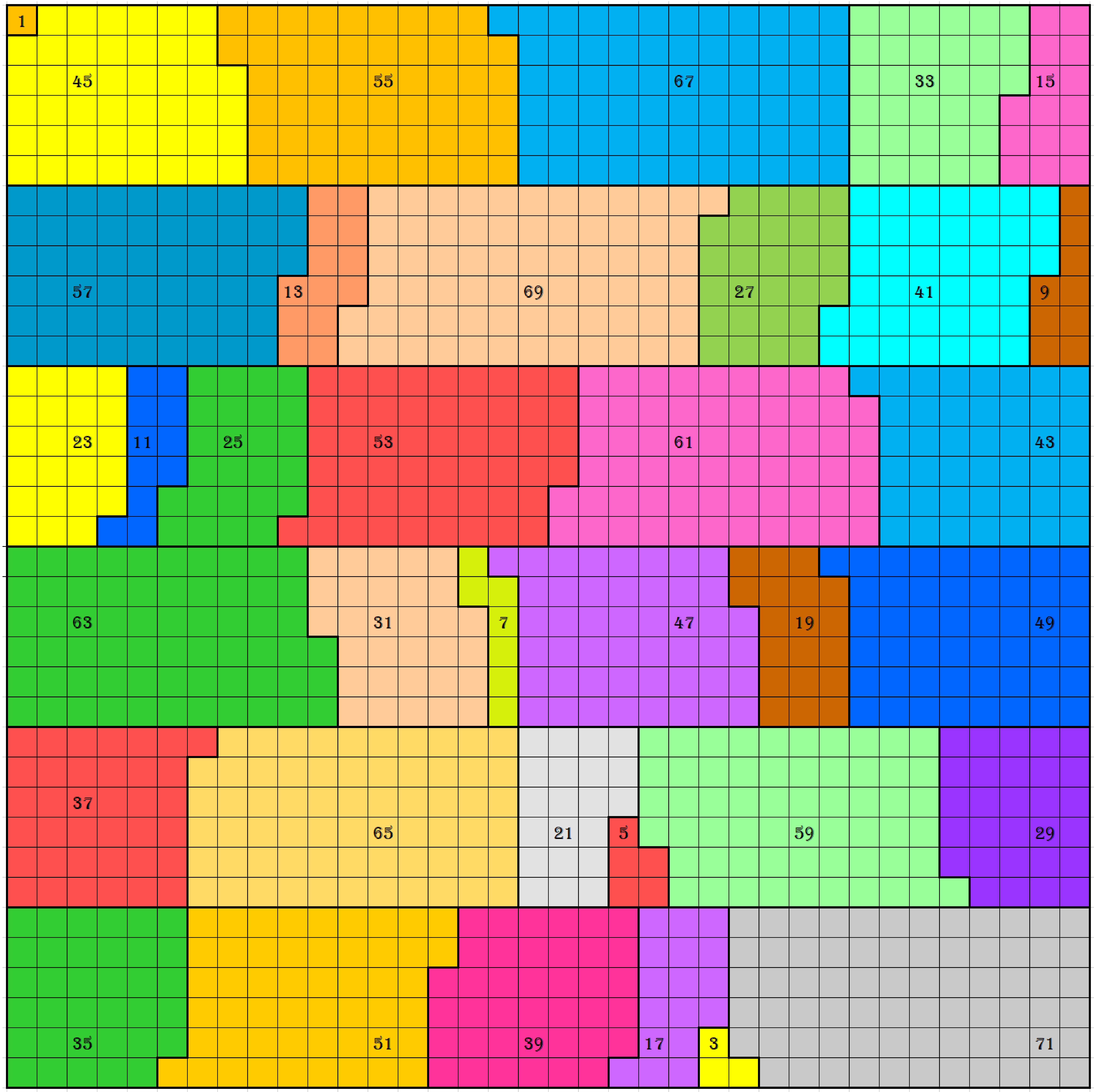
Example 2.17. *A magic square of order 6 representing area for each number according to Example 2.15 is given below:*



In this case, the entries are odd numbers $\{1, 3, 5, \ldots, 69, 71\}$. *The sum of all entries is a perfect square, i.e.,* $T_{36} := 1296 = 36^2$. *It is written according to each line of Example 2.15.*

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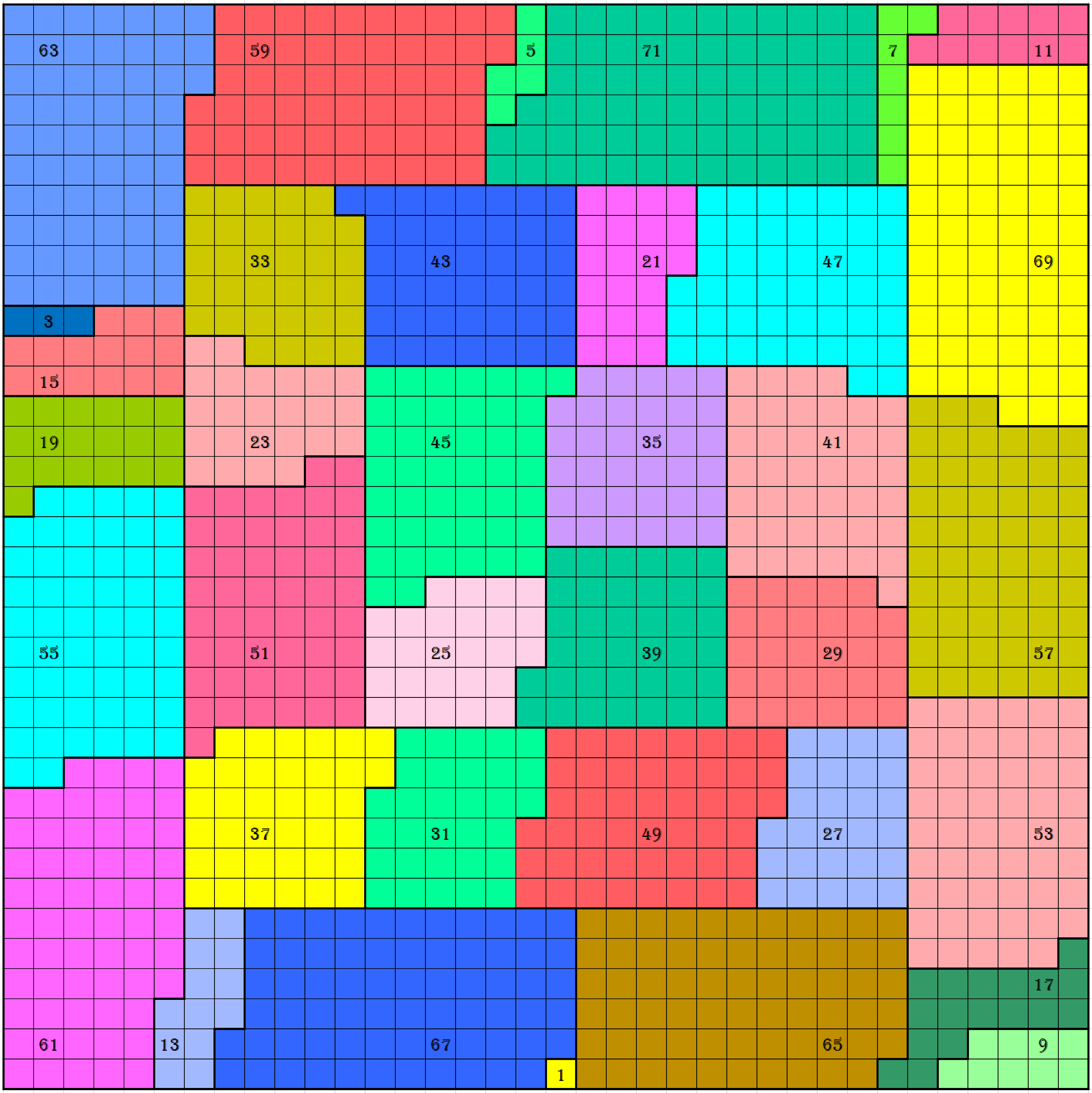
Example 2.18. *A **bordered** magic square of order 6 representing area for each number according to Example 2.16 is given below:*



In this case, the entries are odd numbers $\{1, 3, 5, \ldots, 69, 71\}$. *The sum of all entries is a perfect square, i.e.,* $T_{36} := 1296 = 36^2$. *Moreover the inner magic square is also with similar*

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properties, i.e., $T_{16} := 576 = 24^2$*. The sum of inner four elements is also a perfect square, i.e.,* $T_4 := 144 = 12^2$

2.5 Magic Squares of Order 7

In this case let's write directly a magic square of order 7 with entries sum a **minimum perfect square**.

Example 2.19. *For the* ***consecutive natural number*** *entries* $\{1, 2, 3, \ldots, 48, 49\}$*, a* ***pandiagonal*** *magic square of order 7 is given by*

		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

In this case the magic sum is $S_{7\times 7} = 175$, and the sum of all entries is $T_{49} := 7 \times 175 = 1225 = 35^2$. It is the first example of a **minimum perfect square** sum of entries starting from the number 1. The next example of this kind is of order 239. For details see [3]. Below is same magic square written as **bordered** magic square.

Example 2.20. *A* ***bordered*** *magic square of order 7 for the* ***consecutive natural numbers*** $\{1, 2, 3, \ldots, 48, 49\}$ *is given by*

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42	38	40	5	4	2	44
1	34	37	17	19	18	49
3	14	24	29	22	36	47
43	15	23	25	27	35	7
41	30	28	21	26	20	9
39	32	13	33	31	16	11
6	12	10	45	46	48	8

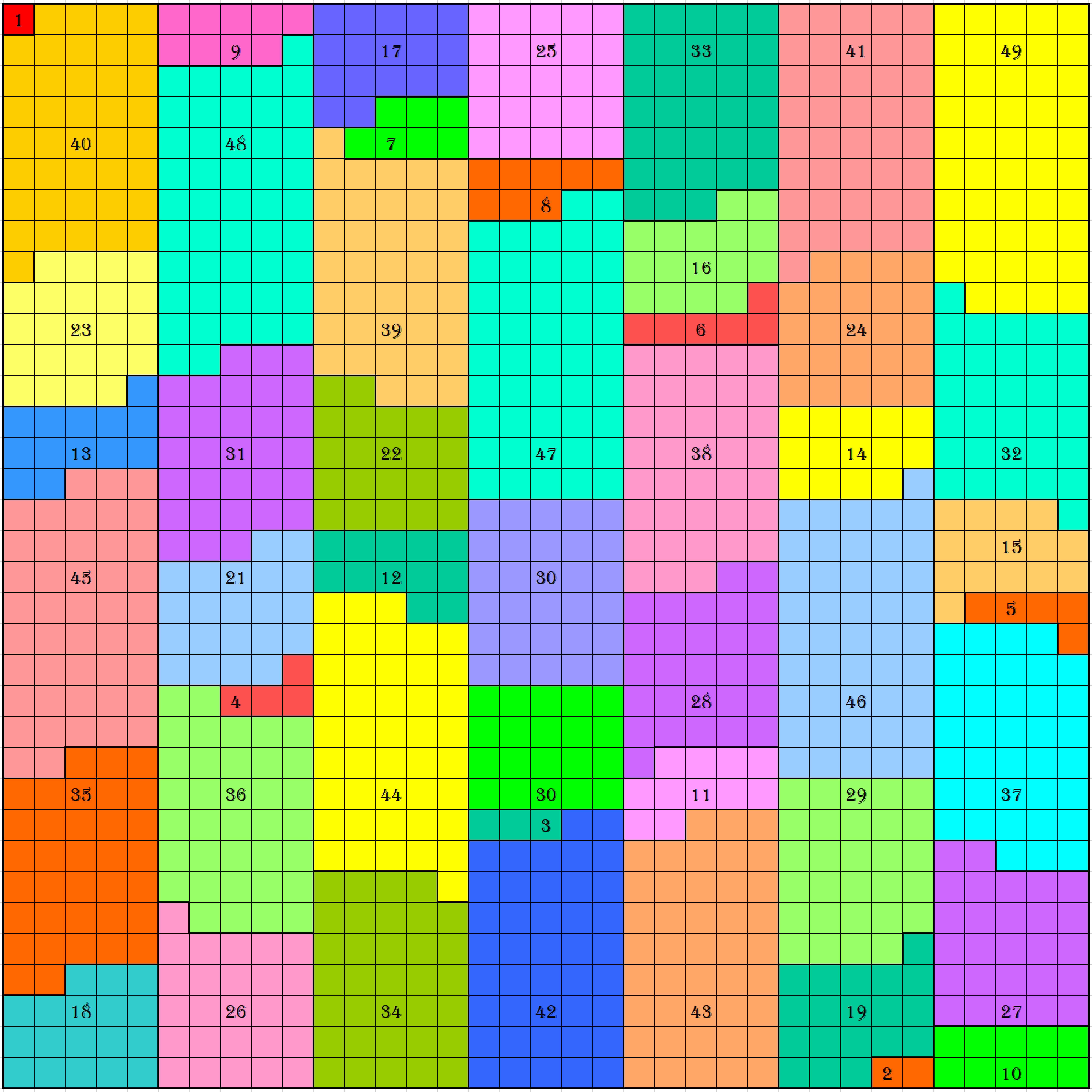
In this case the magic sum is $S_{7\times7} = 175$, and the sum of all entries is $T_{49} := 1225 = 35^2$. Moreover, blocks of orders 5 and 3 are also magic squares. In these cases the total sum of entries are also perfect squares, i.e., $T_{25} := 625 = 25^2$, $T_9 := 225 = 15^2$ and $T_1 := 25 = 5^2$.

2.5.1 Area Representations

In this subsection, we shall write magic square of order 7 according to area covered by each number for the Examples 2.19 and 2.20.

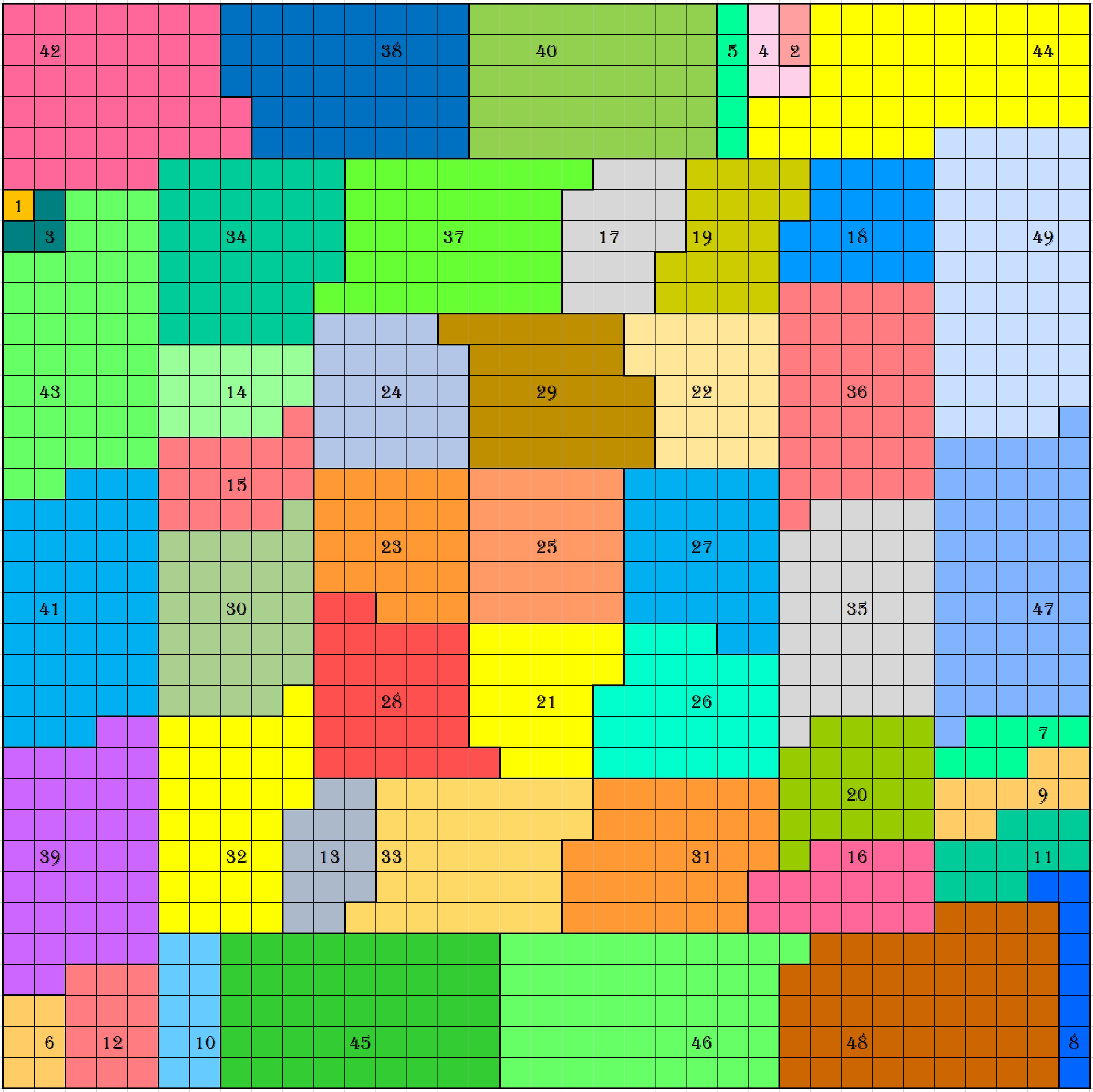
Example 2.21. *A magic square of order 7 representing area for each number according to Example 2.19 is given by*

Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, *https://doi.org/10.5281/zenodo.5009224*



Example 2.22. *A* ***bordered*** *magic square of order 7 representing area for each number according to Example 2.20 is given by*

Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, *https://doi.org/10.5281/zenodo.5009224*



Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, https://doi.org/10.5281/zenodo.5009224

2.6 Magic Squares of Order 8

This subsection bring magic squares of order 8 in three different ways for the consecutive odd number entries. Two ways are based on **pandiagonal** magic squares and the third way is based on **bordered** magic square.

Example 2.23. *For the* ***consecutive odd number*** *entries* $\{1,3,5,...,125,127\}$*, let's write a* ***pandiagonal*** *magic square of order 8 in two different ways*

		512	512	512	512	512	512	512	512
	57	79	1	119	41	95	17	103	512
512	7	113	63	73	23	97	47	89	512
512	127	9	71	49	111	25	87	33	512
512	65	55	121	15	81	39	105	31	512
512	59	77	3	117	43	93	19	101	512
512	5	115	61	75	21	99	45	91	512
512	125	11	69	51	109	27	85	35	512
512	67	53	123	13	83	37	107	29	512
	512	512	512	512	512	512	512	512	512

		512	512	512	512	512	512	512	512
	57	79	1	119	41	95	17	103	512
512	7	113	63	73	23	97	47	89	512
512	127	9	71	49	111	25	87	33	512
512	65	55	121	15	81	39	105	31	512
512	59	77	3	117	43	93	19	101	512
512	5	115	61	75	21	99	45	91	512
512	125	11	69	51	109	27	85	35	512
512	67	53	123	13	83	37	107	29	512
	512	512	512	512	512	512	512	512	512

In both the examples the magic sum is $S_{8\times 8} = 512$*, and the sum of all the entries is* $T_{64} = 4096 = 64^2 = 8^4$*. Each block of order 4 is also a* ***pandiagonal*** *with equal magic sums, i.e.,* $S_{4\times 4} = 256$ *with entries sum as* $T_{16} = 1024 = 32^2$*. Moreover, each block of 4 elements are of equal sums, i.e.,* $T_4 = 256 = 16^2$*.*

Below is a **bordered** magic square of order 8 with same entries as of Example 2.23

Example 2.24. *A* ***bordered*** *magic square of order 8 for the* $\{1,3,5,...,125,127\}$*, is given by*

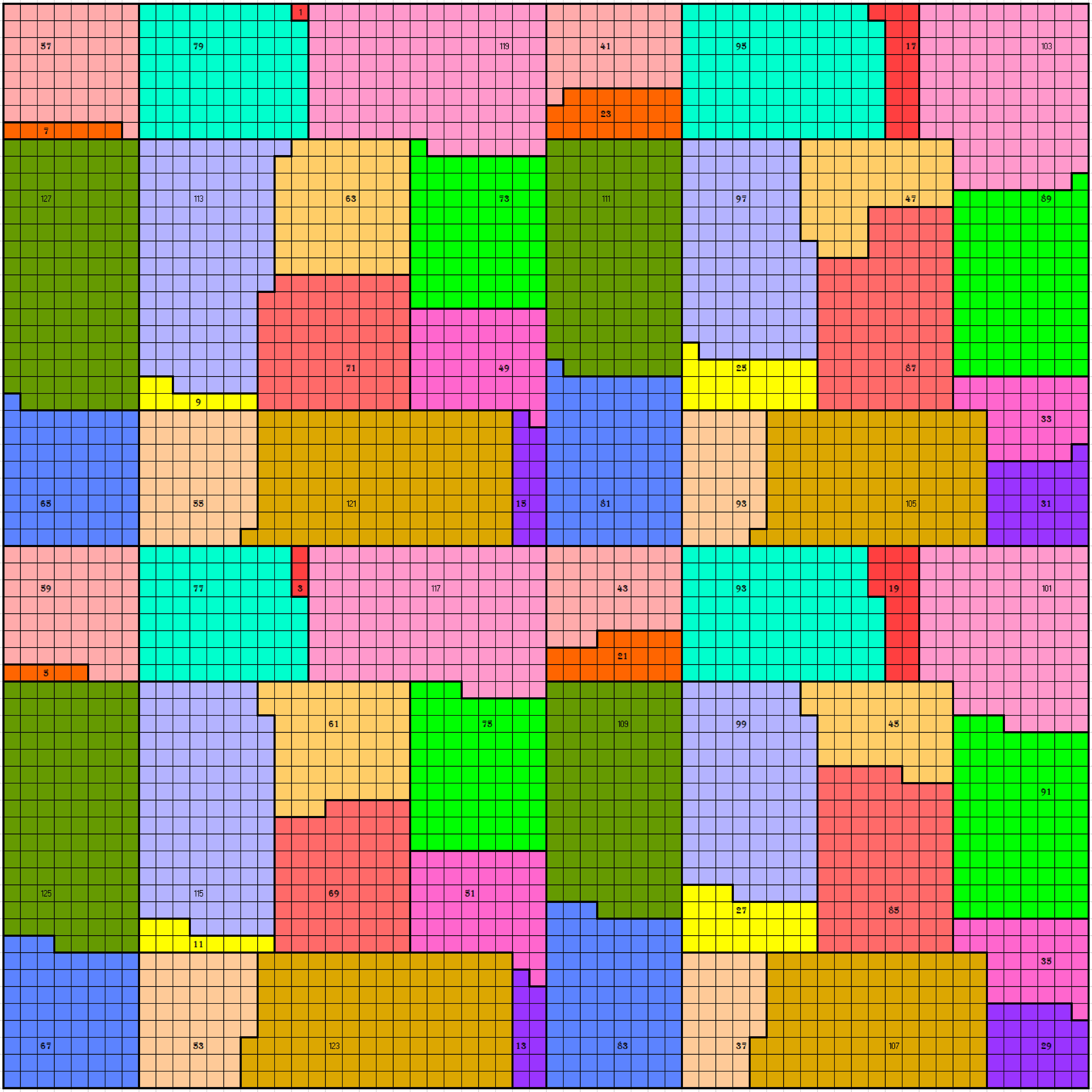
15	3	123	127	101	25	105	13
9	91	87	33	99	35	39	119
11	31	75	49	55	77	97	117
21	43	61	71	65	59	85	107
121	47	69	63	57	67	81	7
111	83	51	73	79	53	45	17
109	89	41	95	29	93	37	19
115	125	5	1	27	103	23	113

In this case the blocks of order 6 and 4 are also magic squares, i.e., $S_{6\times 6} = 512$ *and* $S_{4\times 4} = 256$ *The sums of entries are also prefect squares, i.e.,* $T_{36} = 2304 = 48^2 = 8^4$, $T_{16} = 1024 = 32^2$ *and* $T_4 = 256 = 16^2 = 4^4$.

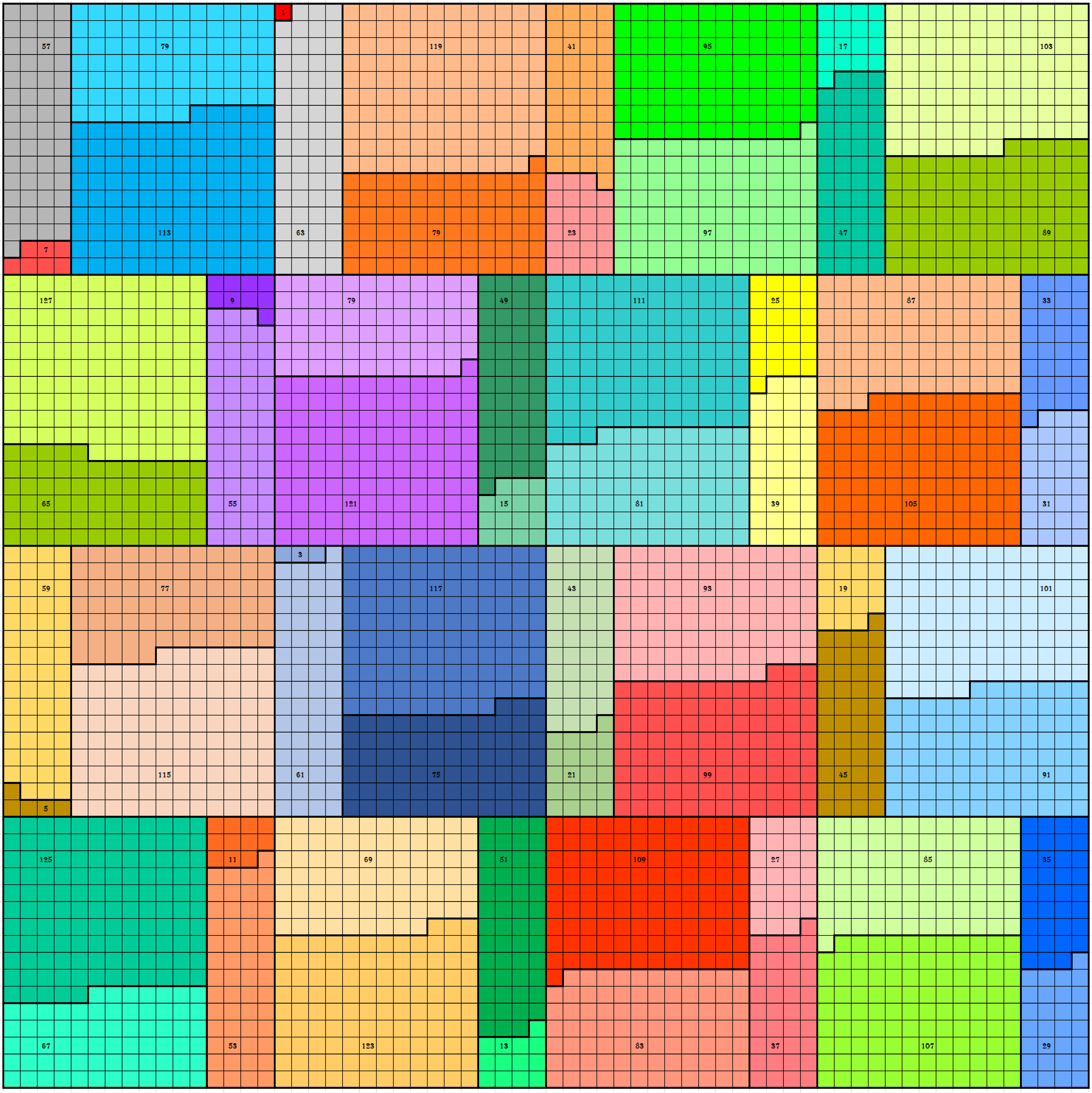
2.6.1 Area Representations

In this subsection, we shall write magic squares of order 8 according to area covered by each number for the Examples 2.23 and 2.24. In all the examples the entries are with odd numbers, i.e., $\{1, 3, 5, \ldots, 125, 127\}$. The sum of all entries is a perfect square, i.e., $T_{64} = 4096 = 64^2 = 8^4$. See below these examples.

Example 2.25. *A magic square of order 8 representing area for each number according to first example of Example 2.23 is given by*

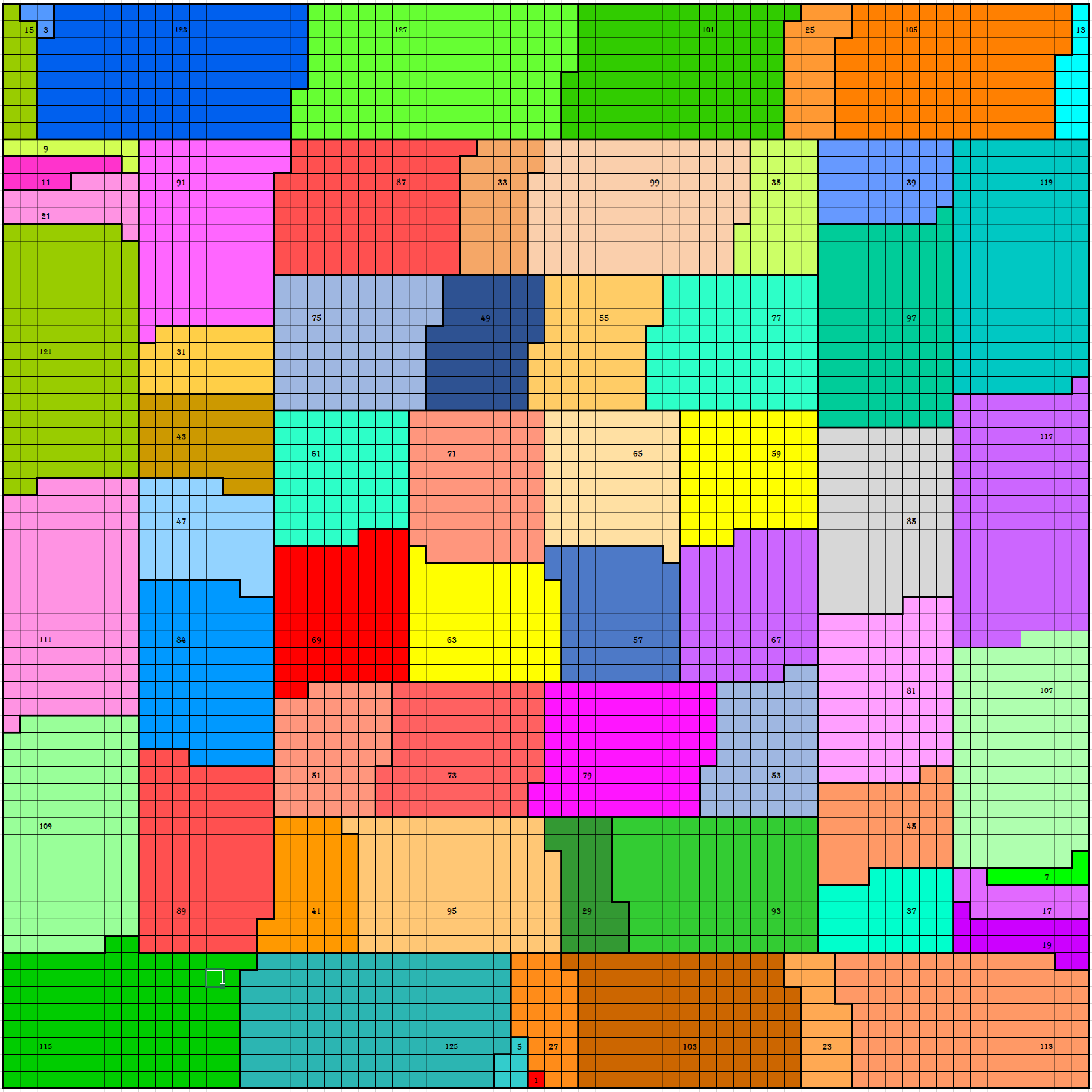


Example 2.26. *A magic square of order 8 representing area for each number according to second example of Example 2.23 is given by*



Example 2.27. *A* ***bordered*** *magic square of order 8 representing area for each number according to Example 2.24 is given by*

Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, https://doi.org/10.5281/zenodo.5009224



 Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, *https://doi.org/10.5281/zenodo.5009224*

2.7 Magic Square of Order 9

In this case let's write directly a magic square of order 9 with entries sum a **minimum perfect square**. In this case, we shall work only with consecutive natural number entries.

Example 2.28. *For the **consecutive natural number** entries* $\{9, 10, 11, \ldots, 88, 89\}$, *a **pan-diagonal** magic square of order 9 is given by*

		441	441	441	441	441	441	441	441	441
	30	79	38	35	72	40	28	77	42	441
441	43	29	75	36	31	80	41	33	73	441
441	74	39	34	76	44	27	78	37	32	441
441	48	16	83	53	9	85	46	14	87	441
441	88	47	12	81	49	17	86	51	10	441
441	11	84	52	13	89	45	15	82	50	441
441	66	61	20	71	54	22	64	59	24	441
441	25	65	57	18	67	62	23	69	55	441
441	56	21	70	58	26	63	60	19	68	441
	441	441	441	441	441	441	441	441	441	441

The above Example 2.28 is with magic sum $S_{9\times 9} = 441$, and the sum of all entries is $T_{81} := 9 \times 441 = 3969 = 63^2$. It is **pandiagonal minimum perfect square entries sum** magic square. Blocks of order 3 are **semi-magic** squares with equal **semi-magic** sums, i.e., $Sm_{3\times 3} = 147$, and the sum of all 9 entries in each case are the same, i.e., $T_9 := 441 = 21^2$.

The magic square given in Example 2.28 is with **consecutive natural numbers**. Let's write it as **bordered** magic square.

Example 2.29. *A **bordered magic square** of order 9 for the entries* $\{9, 10, 11, \ldots, 88, 89\}$ *is given by*

16	88	86	84	83	20	22	24	18
9	66	62	64	29	28	26	68	89
11	25	58	61	41	43	42	73	87
13	27	38	48	53	46	60	71	85
81	67	39	47	49	51	59	31	17
79	65	54	52	45	50	44	33	19
77	63	56	37	57	55	40	35	21
75	30	36	34	69	70	72	32	23
80	10	12	14	15	78	76	74	82

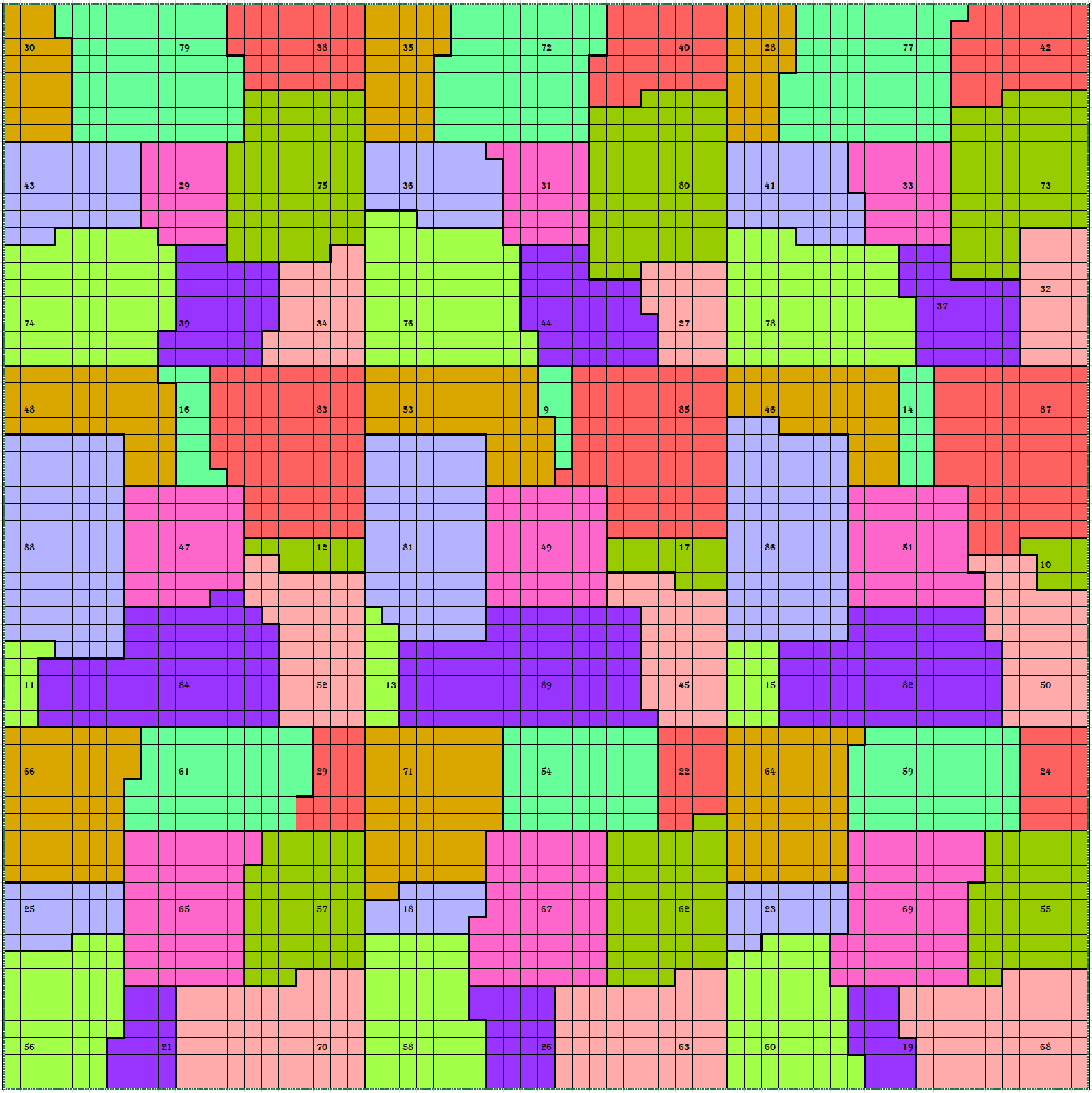
The magic sums are sum of entries are as follows:

$$\begin{array}{ll} S_{9\times 9} := 441 & T_{81} := 9\times 441 = 3963 = 63^2 \\ S_{7\times 7} := 343 & T_{49} := 7\times 343 = 2401 = 49^2 \\ S_{5\times 5} := 245 & T_{25} := 5\times 245 = 1225 = 35^2 \\ S_{3\times 3} := 147 & T_{9} := 3\times 147 = 441 = 21^2 \\ & T_{1} := 49 = 7^2 \end{array}$$

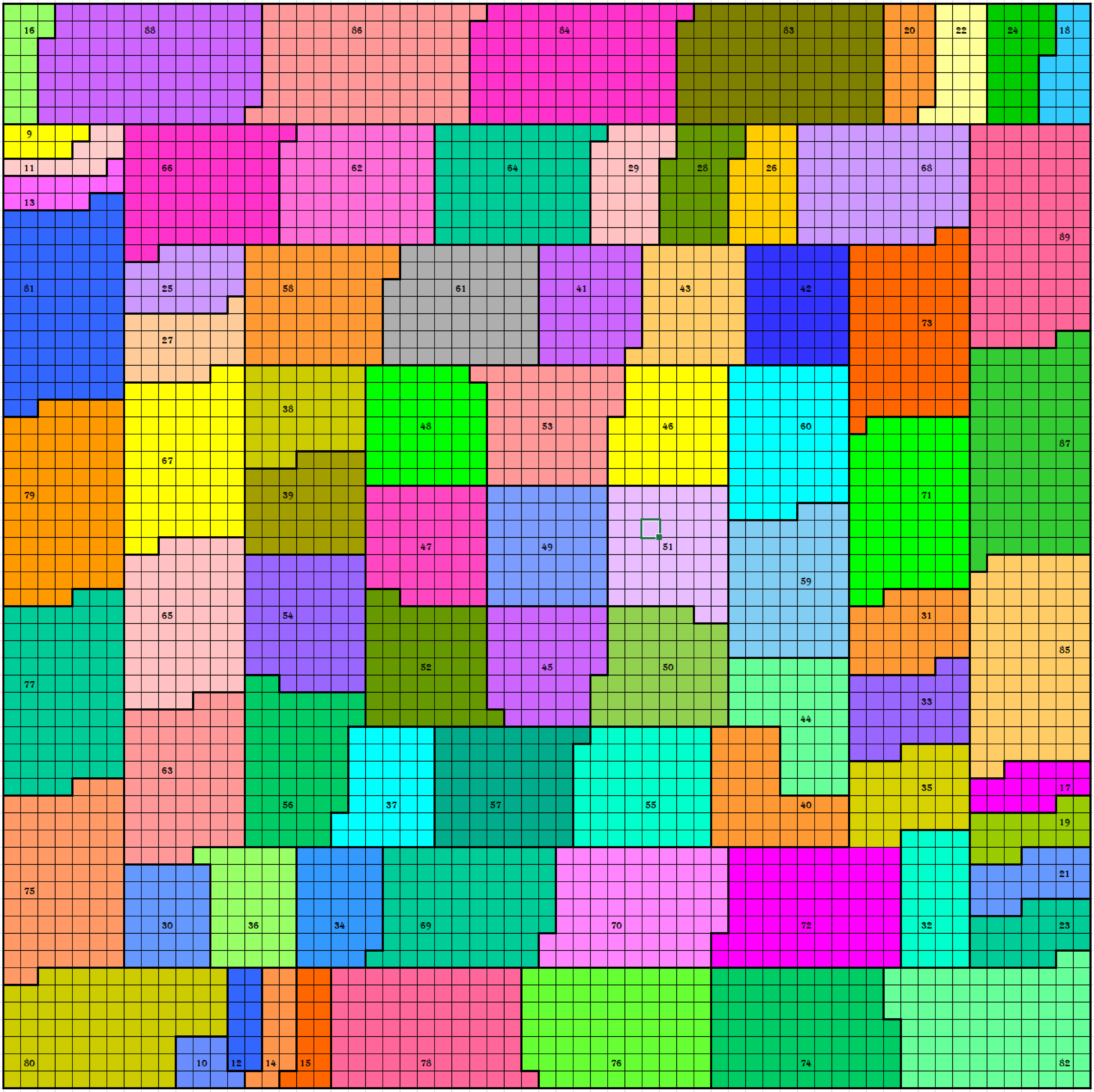
2.7.1 Area Representations

In this subsection, we shall write magic squares of order 9 according to area covered by each number for the Examples 2.28 and 2.29 Both the examples are with natural numbers entries, i.e., $\{9, 10, 11, \ldots, 88, 89\}$. The sum of all entries is a perfect square, i.e., $T_{81} = 3969 = 63^2$. See below these examples.

Example 2.30. *A magic square of order 9 representing area for each number according to Example 2.28 is given by*



Example 2.31. *A **bordered** magic square of order 9 representing area for each number according to Example 2.29 is given by*



The Examples 2.30 and 2.31 are with same properties as of Examples 2.28 and 2.29 respectively.

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2.8 Magic Squares of Order 10

In this subsection, we shall write **block-bordered** and **bordered** magic squares of order 10 for the **consecutive odd number** entries $\{1, 3, 5, \ldots, 197, 199\}$. See below both the examples

Example 2.32. *For the* ***consecutive odd number*** *entries* $\{1, 3, 5, \ldots, 197, 199\}$, *a* ***block-bordered*** *magic square of order 10 is given by*

181	171	31	167	35	27	7	195	3	183
25	93	115	37	155	77	131	53	139	175
177	43	149	99	109	59	133	83	125	23
21	163	45	107	85	147	61	123	69	179
191	101	91	157	51	117	75	141	67	9
1	95	113	39	153	79	129	55	137	199
185	41	151	97	111	57	135	81	127	15
13	161	47	105	87	145	63	121	71	187
189	103	89	159	49	119	73	143	65	11
17	29	169	33	165	173	193	5	197	19

The magic sum of Example 2.32 is $S_{10\times10} = 1000$, and the sum of all entries is $T_{100} := 10 \times 1000 = 10000 = 100^2 = 10^4$. Moreover, the inner magic square is **pandiagonal** magic square of order 8 with equal sum blocks of **pandiagonal** magic square of order 4. The magic sums are $S_{8\times8} = 800$ and $S_{4\times4} = 400$.

Example 2.33. *For the* ***consecutive odd number*** *entries* $\{1, 3, 5, \ldots, 197, 199\}$, *a* ***block-bordered*** *magic square of order 10 is given by*

181	171	31	167	35	27	7	195	3	183
25	51	39	159	163	137	61	141	49	175
177	45	127	123	69	135	71	75	155	23
21	47	67	111	85	91	113	133	153	179
191	57	79	97	107	101	95	121	143	9
1	157	83	105	99	93	103	117	43	199
185	147	119	87	109	115	89	81	53	15
13	145	125	77	131	65	129	73	55	187
189	151	161	41	37	63	139	59	149	11
17	29	169	33	165	173	193	5	197	19

It is the same magic square as given in Example 2.32 with the same distribution of entries. It is written as **bordered magic square**. It has the following interesting sums:

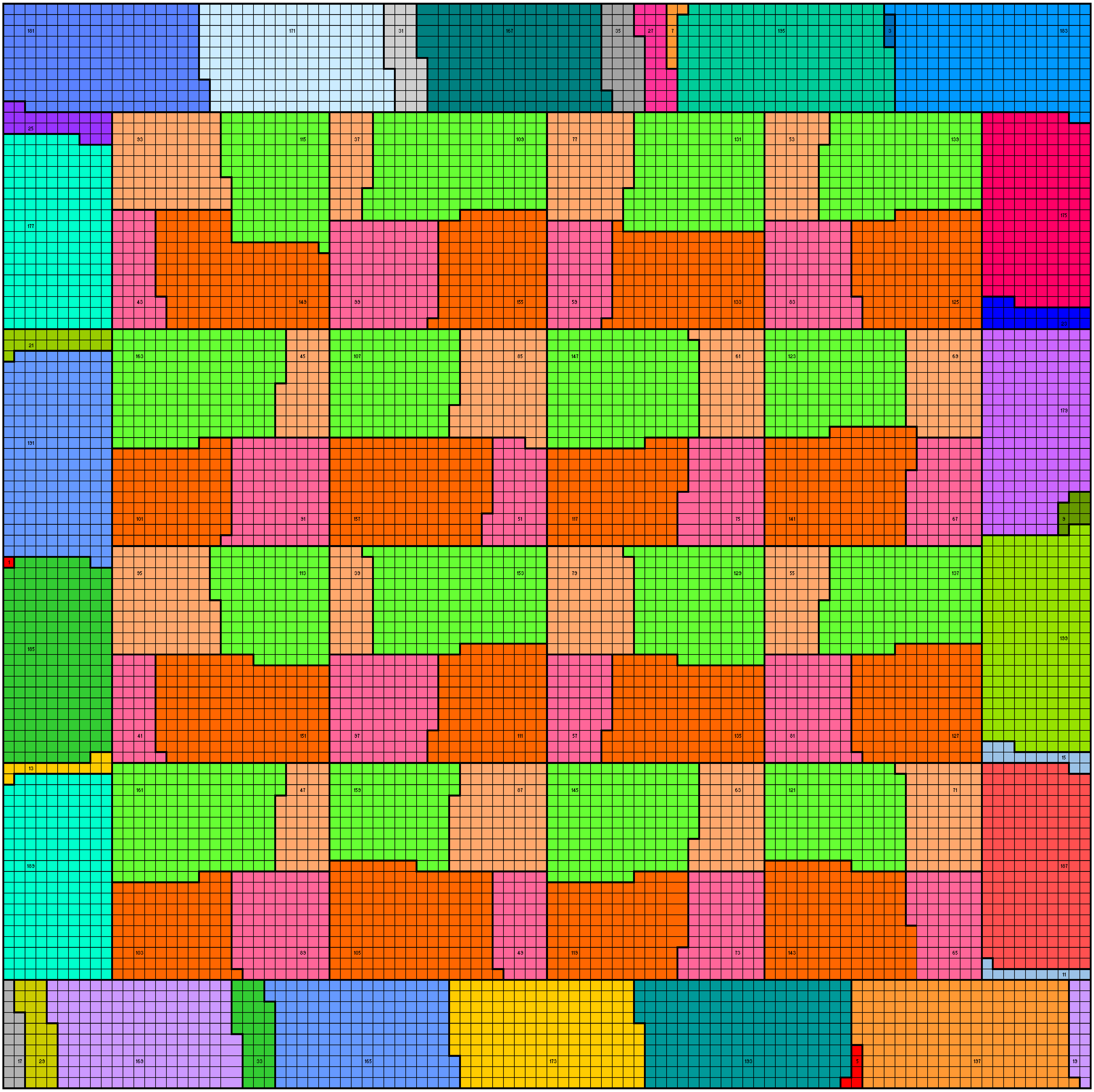
$$\begin{aligned}
S_{10\times 10} &:= 1000 & T_{100} &:= 10 \times 1000 = 10000 = 100^2 \\
S_{8\times 8} &:= 800 & T_{64} &:= 8 \times 800 = 6400 = 80^2 \\
S_{6\times 6} &:= 600 & T_{36} &:= 6 \times 600 = 3600 = 60^2 \\
S_{4\times 4} &:= 400 & T_{16} &:= 4 \times 400 = 1600 = 40^2 \\
& & T_{4} &:= 400 = 20^2
\end{aligned}$$

The last line is the sum of central 4 elements written in pink color.

2.8.1 Area Representations

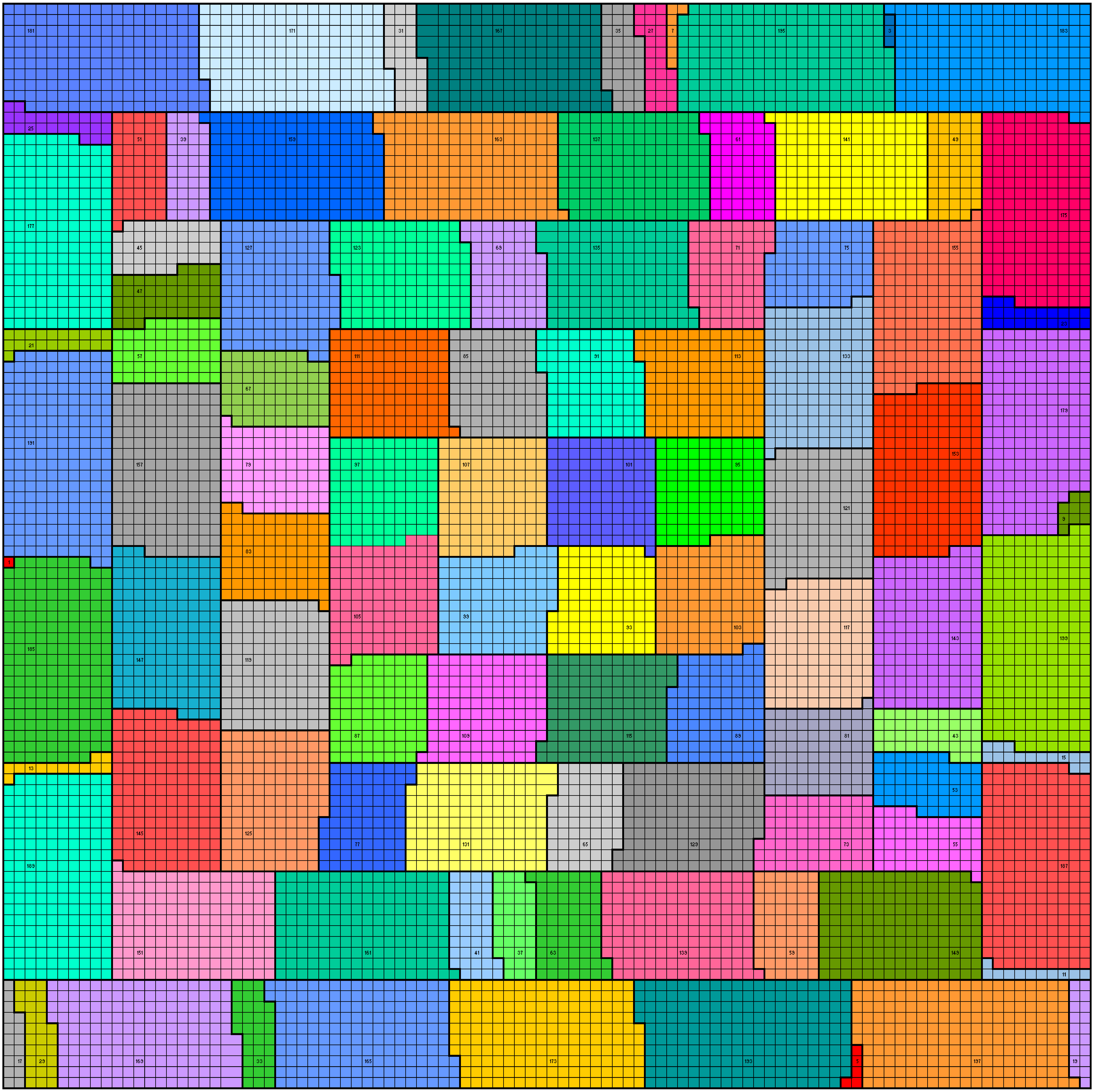
In this subsection, we shall write magic squares of order 10 according to area covered by each number for the Example 2.32. In this case the entries are **consecutive odd number** entries $\{1, 3, 5, \ldots, 197, 199\}$. The inner block is **pandiagonal** magic square of order 8, where the blocks of order are also **pandiagonal** magic square of order 4 with equal magic sums.

Example 2.34. *A **block-bordered** magic square of order 10 representing area for each number according to Example 2.32 is given by*



Example 2.35. *A* ***bordered*** *magic square of order 10 representing area for each number according to Example 2.33 is given by*

Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, https://doi.org/10.5281/zenodo.5009224



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2.9 Magic Squares of Order 11

In this case let's write directly a magic square of order 11 with entries sum a **minimum perfect square**. In this case the entries are **consecutive natural numbers**, i.e., $\{4, 5, 6, \ldots, 123, 124\}$.

Example 2.36. *For the* ***consecutive natural number*** *entries* $\{4, 5, 6, \ldots, 123, 124\}$*, a* ***block-bordered*** *magic square of order 11 is given by*

15	23	21	19	17	116	117	119	121	123	13
124	45	94	53	50	87	55	43	92	57	4
122	58	44	90	51	46	95	56	48	88	6
120	89	54	49	91	59	42	93	52	47	8
118	63	31	98	68	24	100	61	29	102	10
14	103	62	27	96	64	32	101	66	25	114
16	26	99	67	28	104	60	30	97	65	112
18	81	76	35	86	69	37	79	74	39	110
20	40	80	72	33	82	77	38	84	70	108
22	71	36	85	73	41	78	75	34	83	106
115	105	107	109	111	12	11	9	7	5	113

The magic sum of Example 2.38 is $S_{11\times 11} = 704$, and the sum of all entries is $T_{121} := 11 \times 704 = 7744 = 88^2$. It is **minimum perfect square entries sum** magic square of order 11. Moreover, the inner magic square of order 9 is **pandiagonal** with blocks of **semi-magic** squares of order 3 with equal **semi-magic** sums. The magic sums are $S_{9\times 9} = 576$ and $Sm_{3\times 3} = 192$. In this case the entries sums are $T_{81} := 9 \times 576 = 5184 = 72^2$ and $T_9 := 3 \times 192 = 576 = 24^2$.

Example 2.37. *For the* ***consecutive natural number*** *entries* $\{4, 5, 6, \ldots, 123, 124\}$*, a* ***bordered*** *magic square of order 11 is given by*

Creative Magic Squares: Area Representations, June 22, 2021, pp. 1-45, https://doi.org/10.5281/zenodo.5009224

15	23	21	19	17	116	117	119	121	123	13
124	31	103	101	99	98	35	37	39	33	4
122	24	81	77	79	44	43	41	83	104	6
120	26	40	73	76	56	58	57	88	102	8
118	28	42	53	63	68	61	75	86	100	10
14	96	82	54	62	64	66	74	46	32	114
16	94	80	69	67	60	65	59	48	34	112
18	92	78	71	52	72	70	55	50	36	110
20	90	45	51	49	84	85	87	47	38	108
22	95	25	27	29	30	93	91	89	97	106
115	105	107	109	111	12	11	9	7	5	113

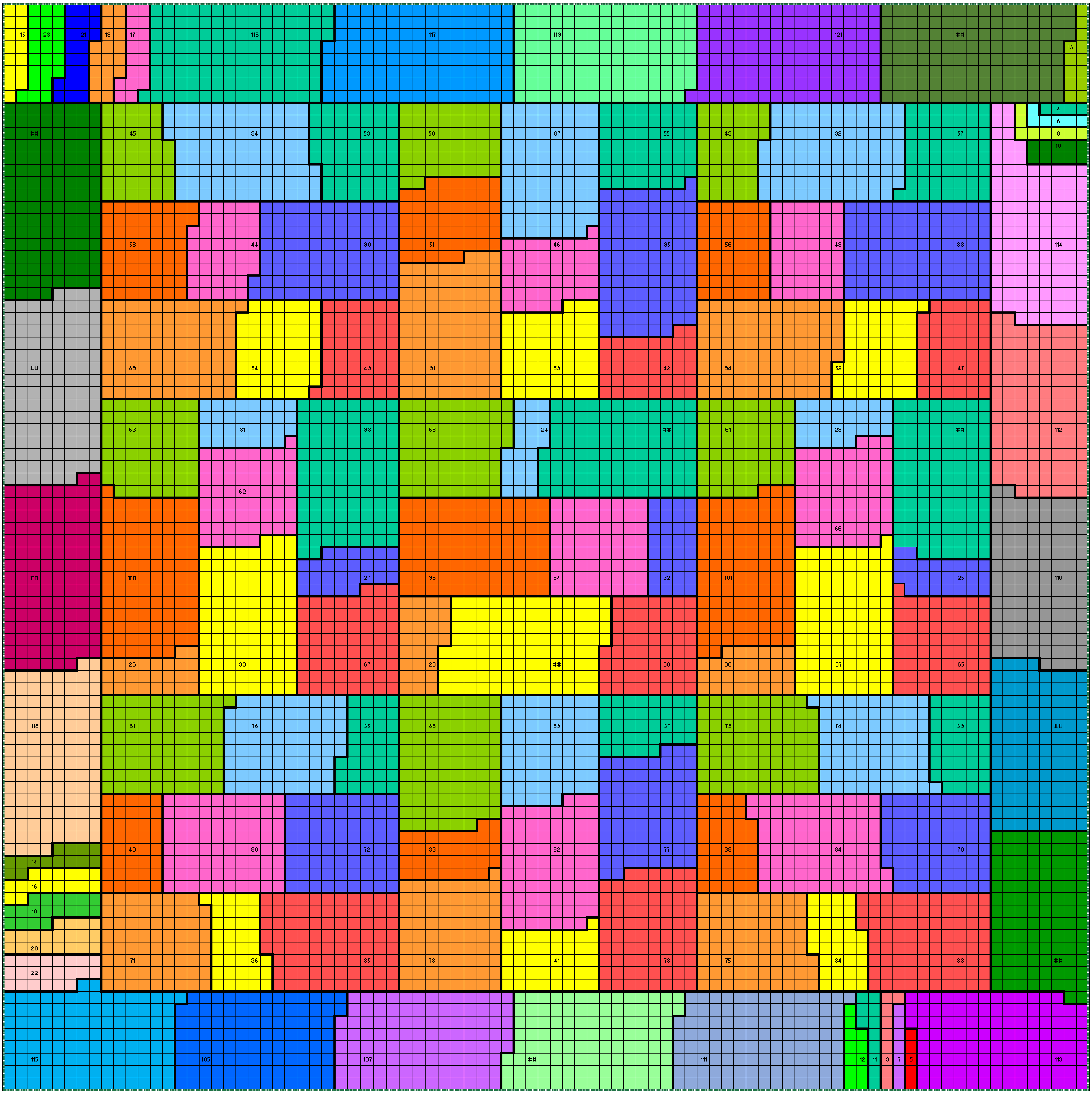
It is the same magic square as given in Example 2.36 with the same distribution of entries written as ***bordered*** *magic square. It has the following interesting sums:*

$$
\begin{aligned}
S_{11\times 11} &:= 704 & T_{121} &:= 11 \times 704 = 7744 = 88^2\\
S_{9\times 9} &:= 576 & T_{81} &:= 9 \times 576 = 5184 = 72^2\\
S_{7\times 7} &:= 448 & T_{49} &:= 7 \times 448 = 3136 = 56^2\\
S_{5\times 5} &:= 320 & T_{25} &:= 5 \times 320 = 1600 = 40^2\\
S_{3\times 3} &:= 192 & T_{9} &:= 3 \times 192 = 576 = 24^2\\
& & T_{1} &:= 64 = 8^2
\end{aligned}
$$

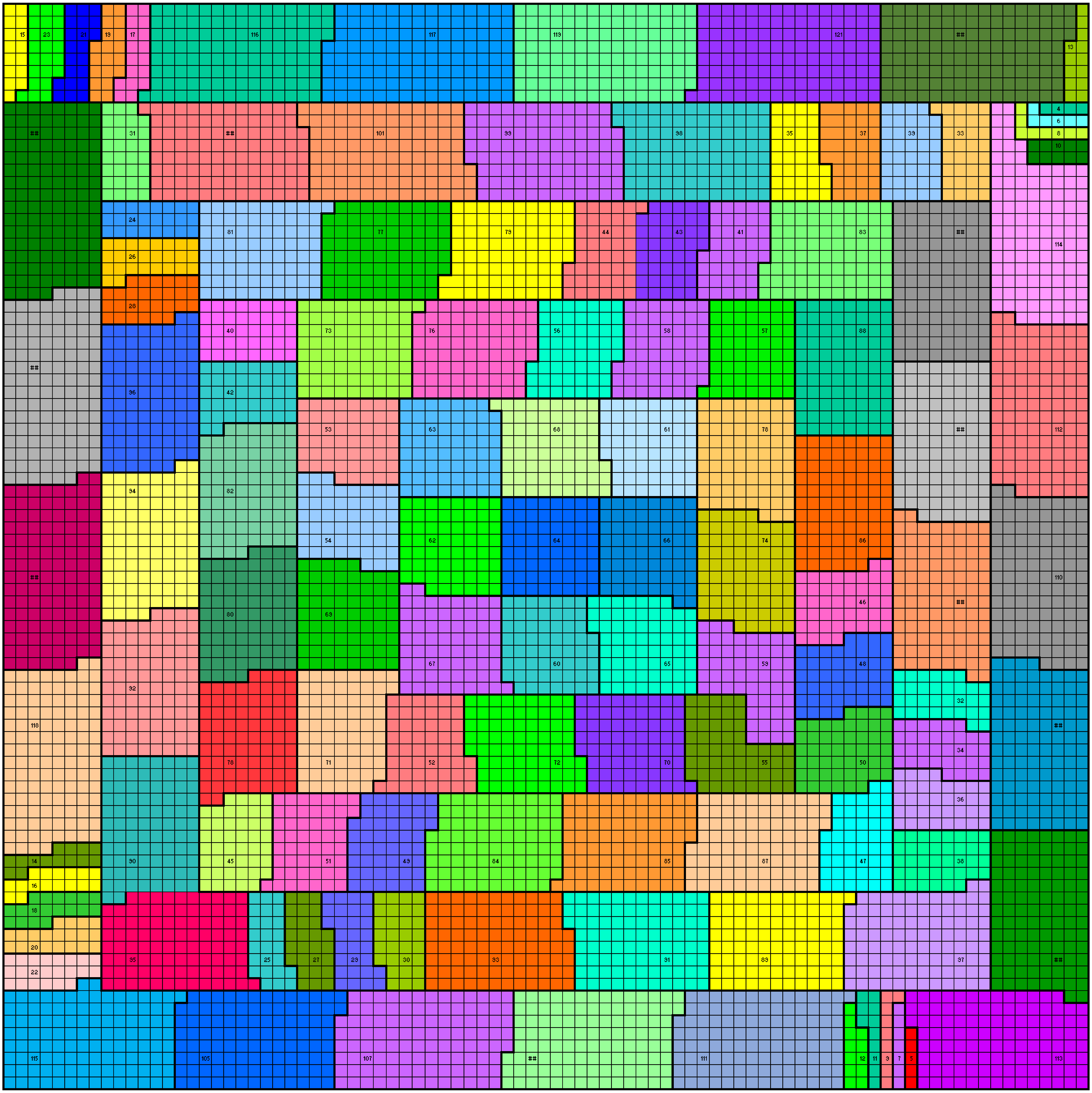
2.9.1 Area Representations

In this subsection, we shall write magic squares of order 11 according to area covered by each number for the Examples 2.36 and 2.37. In this case the entries are **consecutive natural numbers** entries $\{4, 5, 6, \ldots, 123, 124\}$. In the first case, the inner block is **pandiagonal** magic square of order 9, where the blocks of order 3 are semi-magic squares with equal sums entries. In the second case, the magic square is **bordered** magic square.

Example 2.38. *A* ***block-bordered*** *magic square of order 11 representing area for each number according to Example 2.36 is given by*



Example 2.39. *A* ***bordered*** *magic square of order 11 representing area for each number according to Example 2.37 is given by*



References

[1] H. Heinz, Magic Squares, Magic Stars and Other Patterns *http://www.magic-squares.net*.

[2] L. Sallows, Online discussion, January, 2017.

[3] N. J. A. Sloane, NSW numbers: $a(n) = 6 \times a(n-1) - a(n-2)$; also $a(n)^2 - 2 \times b(n)^2 = -1$ with $b(n) = A001653(n)$, ***https://oeis.org/A002315***, *https://oeis.org/A001653*, Numbers k such that $2 \times k^2 - 1$ is a square

[4] W. Trump, Area Magic Squares, *http://www.trump.de/magic-squares/area-magic/index.html*

[5] W. Walkington, Online discussion, January, 2017. For more details:

a. *https://carresmagiques.blogspot.com.br/2017/01/area-magic-squares-and-tori-of-order-3.html*.

b. *https://carresmagiques.blogspot.com.br/2017/01/area-magic-squares-of-order-6.html*.

c. *http://www.primepuzzles.net/puzzles/puzz_865.htm - https://goo.gl/k7n8RB*.

d. *https://www.futilitycloset.com/2017/01/19/area-magic-squares/*

e. *https://en.wikipedia.org/wiki/Magic_square - https://goo.gl/jyUqnA*

[6] Yoshiaki Araki, Polyomino version of Area Magic Square, *https://www.tessellation.jp/*; *https://www.facebook.com/yoshiaki.araki.3*; *https://twitter.com/alytile*.

[7] Inder J. Taneja, Magic Squares with Perfect Square Number Sums, Research Report Collection, **20**(2017), Article 11, pp.1-24, *http://rgmia.org/papers/v20/v20a11.pdf*.

[8] Inder J. Taneja, Pythagorean Triples and Perfect Square Entries Sum Magic Squares, RGMIA Research Report Collection, **20**(2017), Art. 128, pp. 1-22, *http://rgmia.org/papers/v20/v20a128.pdf*.

[9] Inder J. Taneja, Block-Wise Magic and Bimagic Squares of Orders 12 to 36, **Zenodo**, February 1, 2019, pp. 1-53, *http://doi.org/10.5281/zenodo.2555343*.

[10] Inder J. Taneja, Block-Wise Magic and Bimagic Squares of Orders 39 to 45, **Zenodo**, February 2, 2019, pp. 1-73, *http://doi.org/10.5281/zenodo.2555889*.

[11] Inder J. Taneja, Perfect Square Sum Magic Squares, **Zenodo**, April 29, 2019, pp. 1-65, *http://doi.org/10.5281/zenodo.2653927*.

[12] Inder J. Taneja, Block-Bordered Magic Squares of Prime and Double Prime Numbers - I, **Zenodo**, August 18, 2020, *http://doi.org/10.5281/zenodo.3990291*, pp. 1-81

[13] Inder J. Taneja, Block-Bordered Magic Squares of Prime and Double Prime Numbers - II, **Zenodo**, August 18, 2020, *http://doi.org/10.5281/zenodo.3990293*, pp. 1-90

[14] Inder J. Taneja, Block-Bordered Magic Squares of Prime and Double Prime Numbers - III, **Zenodo**, September 01, 2020, *http://doi.org/10.5281/zenodo.4011213*, pp. 1-93

[15] Inder J. Taneja, Block-Wise and Block-Bordered Magic and Bimagic Squares With Magic Sums 21, 21^2 and 2021, **Zenodo**, December 16, 2020, *http://doi.org/10.5281/zenodo.4380343*. pp. 1-118.

[16] Inder J. Taneja, Block-Wise and Block-Bordered Magic and Bimagic Squares of Orders 10 to 47, **Zenodo**, January 14, 2021, pp. 1-185, *http://doi.org/10.5281/zenodo.4437783*.

[17] Inder J. Taneja, Bordered and Block-Wise Bordered Magic Squares: Odd Order Multiples, **Zenodo**, February 10, 2021, pp. 1-75, *http://doi.org/10.5281/zenodo.4527739*.

[18] Inder J. Taneja, Bordered and Block-Wise Bordered Magic Squares: Even Order Multiples, **Zenodo**, February 10, 2021, pp. 1-96, *http://doi.org/10.5281/zenodo.4527746*.

[19] Inder J. Taneja, Generating Pythagorean Triples and Magic Squares: Orders 3 to 31, **Zenodo**, May 28, 2021, pp. 1-153, *http://doi.org/10.5281/zenodo.4837491*.

[20] Inder J. Taneja, Creative Magic Squares: Single Digit Representations, **Zenodo**, March 25, 2021, pp. 1-165, *http://doi.org/10.5281/zenodo.4637121*.

[21] Inder J. Taneja, Creative Magic Squares: Single Letter Representations, **Zenodo**, March 25, 2021, pp. 1-41, *http://doi.org/10.5281/zenodo.4637125*.

[22] Inder J. Taneja, Creative Magic Squares: Permutable Base-Power Digits Representations, **Zenodo**, April 03, 2021, pp. 1-44, *http://doi.org/10.5281/zenodo.4661586*.

[23] Inder J. Taneja, Creative Magic Squares: Increasing and Decreasing Orders Crazy Representations, **Zenodo**, May 26, pp. 1-54, *http://doi.org/10.5281/zenodo.4813030*.

[24] Inder J. Taneja. (2021). Sequential Pythagorean Triples and Perfect Square Sum Magic Squares, **Zenodo**, June 21, pp. 1-595, *http://doi.org/10.5281/zenodo.5009204*.

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