Hypothesis Scoring and Model Refinement Strategies for FM-based RANSAC*

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Abstract. Robust model estimation is a recurring problem in application areas such as robotics and computer vision. Taking inspiration from a notion of distance that arises in a natural way in fuzzy logic, this paper modifies the well-known robust estimator RANSAC making use of a Fuzzy Metric (FM) within the estimator main loop to encode the compatibility of each sample to the current model/hypothesis. Further, once a number of hypotheses have been explored, this FM-based RANSAC makes use of the same fuzzy metric to refine the winning model. The incorporation of this fuzzy metric permits us to express the distance between two points as a kind of degree of nearness measured with respect to a parameter, which is very appropriate in the presence of the vagueness or imprecision inherent to noisy data. By way of illustration of the performance of the approach, we report on the estimation accuracy achieved by FM-based RANSAC and other RANSAC variants for a benchmark comprising a large number of noisy datasets with varying proportion of outliers and different levels of noise. As it will be shown, FM-based RANSAC outperforms the classical counterparts considered.

Keywords: Model estimation · RANSAC · Fuzzy metric · 2D straight line estimation

1 Introduction

Solving model estimation problems is a fundamental component of numerous applications in robotics, specially when addressing perception tasks. Nowadays, facing this kind of problem requires to cope with new challenges due to an increased use of potentially poor, low-cost sensors, and the ever growing deployment of robotic devices which may operate in potentially unknown environments.

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In general terms, the underlying algorithms need to be capable of being robust against, in particular, strong uncertainty levels. In this regard, a *robust estimator* is able to correctly find the original model that supposedly the input data fits to, even when the data is noisy and contains outliers, i.e. data items which are not consistent with the original model due to an arbitrary bias affecting them. (For the interested reader, [7] details the concepts, techniques and technical issues surrounding robust estimation.)

The Random Sample Consensus algorithm (RANSAC) [4] is one of these robust estimation techniques. Given a dataset comprising both inliers and outliers, the most distinctive feature of RANSAC is the use of random sampling and a voting scheme to find the optimal set of model parameters. RANSAC is widely used nowadays, so much that it has become common in fields such as robotics and computer vision.

Fuzzy methodologies (together with other soft computing paradigms, such as probabilistic methods, machine learning, evolutionary computing and swarm intelligence) have been used since their birth to deal with imprecise data, targeting on the design of systems that are able to cope with uncertainty one way or another and even degrade gracefully if needed [8]. As already mentioned, robotics, and in general perception, is one of the areas where this capability achieves more relevance, particularly when autonomy is a distinctive feature.

In this work, we propose a variant of RANSAC which avoids discriminating between inliers and outliers by means of the use of a Fuzzy Metric (FM) in the sense of I. Kramosil and J. Michalek [10] that provides a degree of compatibility for each data sample with regard to the current model. The aforesaid fuzzy metric is besides used in a final model refinement step that is incorporated after the main hypothesis selection loop.

In the following, Section 2 overviews RANSAC, Section 3 introduces a fuzzy metric for RANSAC, Section 4 describes our approach based on the previous fuzzy metric, while Section 5 reports on a number of experiments to illustrate the performance achieved, and Section 6 concludes the paper.

2 Overview of RANSAC and some variants

Regarding model estimation, a common measure of estimation robustness is the breakdown point (BDP), defined as a percentage threshold on the outlier rate beyond which the technique under consideration is no longer robust to outliers. RANSAC is one of those robust estimators with BDP higher than fifty percent. Fifty percent is the limit of the Least Median of Squares (LMedS) [18], another robust estimator that has also enjoyed high popularity as a high BDP technique. Least Trimmed Squares (LTS) and Minimum Probability of Randomness (MINPRAN) are other high-BDP algorithms [Olu16], although less popular than RANSAC and LMedS. The BDP for others, such as the M-estimators family [HR11], is below 50%. Applications in statistics typically require less than fifty percent BDP, since outliers in this context are anomalies or exceptions in the data. However, the case is often different in robotics and computer vision applications, where outliers are defined with respect to the best among competing models, each describing well a fraction of the input data.

By randomly generating hypotheses on the model parameters, RANSAC tries to achieve a maximum consensus in the input dataset in order to deduce the inliers. Once the inliers are discriminated, they are used to estimate the parameters of the underlying model by regression. In more detail, instead of using every sample in the dataset to perform the estimation as in traditional regression techniques, RANSAC tests in turn many random sets of samples. Since picking an extra point decreases exponentially the probability of selecting an outlier-free sample [3], RANSAC takes the Minimum Sample Set size (MSS) to determine a unique candidate model, thus increasing its chances of finding an all-inlier sample set. This model is assigned a score based on the cardinality of its consensus set. Finally, RANSAC returns the hypothesis that has achieved the highest consensus, and the corresponding model is refined through a last minimization step that only involves the inliers found.

Searching for an all-inlier sample, RANSAC typically runs for N iterations:

$$N = \frac{\log(1-\rho)}{\log(1-(1-\omega)^s)} \tag{1}$$

where ρ is the desired probability of success, i.e. at least one of the considered random sets is outlier-free, s is the size of the MSS for the problem at hand and ω is the ratio of outliers. (See [4] for the details on Eq. (1).)

There have been a number of efforts aiming at enhancing the standard RANSAC algorithm, e.g. MSAC, MLESAC, MAPSAC, PROSAC, R-RANSAC, LO-RANSAC and U-RANSAC [2], since it, while robust, has its drawbacks regarding accuracy, efficiency, stability and response time [16, 17]. Among these variants, there is a very reduced set adopting fuzzy methodologies [11, 20]. In both cases, the authors address a homography fitting problem, which, in [11], is solved by discriminating data samples into the good, bad and vague fuzzy sets using a fuzzy classifier, while [20] defines a triangle-type membership function for the set of inliers and combines this with a Monte Carlo method for sample selection. It must be pointed out that the two aforementioned variants of RANSAC differ significantly from the one described in this paper.

3 On fuzzy metrics and RANSAC

In [10], a notion of fuzzy metric was introduced by adapting to the fuzzy approach the concept of statistical metric due to Menger. From now on, we assume that the reader is familiar with the basic notions of fuzzy sets and t-norms. (We refer the reader to [9] for a deep treatment on them.)

Nowadays, by a fuzzy metric space, in the sense of Kramosil and Michalek (see [12]), we are referring to a triple $(X, M, *)$ where X is a non-empty set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times]0, \infty[$ satisfying, for each $x, y, z \in X$ and $\theta, \mu \in]0, \infty[$, the axioms below:

(KM1) $M(x,y,\theta)=1$ for each $\theta\in]0,\infty[$ if and only if $x=y$.
(KM2) $M(x,y,\theta)=M(y,x,\theta)$.
(KM3) $M(x,z,\theta+\mu)\geq M(x,y,\theta)*M(y,z,\mu)$.
(KM4) The assignment $M_{x,y}:]0,\infty[\to[0,1]$ is a left-continuous function, where $M_{x,y}(\theta)=M(x,y,\theta)$ for each $\theta\in]0,\infty[$.

On account of the previous concept, the value $M(x,y,\theta)$ can be interpreted as a degree of nearness between two points $x,y\in X$ with respect to a parameter $\theta\in]0,\infty[$. The larger the value of $M(x,y,\theta)$, the closer the points x and y are, with respect to θ. Observe that, for two distinct points $x,y\in X$, the degree of nearness can be 1 for some $\theta_0\in]0,\infty[$, but such a degree can only be 1 for all $\theta\in]0,1[$ whenever x and y are the same point.

This notion of fuzzy metric has been studied extensively from a mathematical point of view in the literature. Besides, it is worth mentioning that such a kind of measurement has been shown to be useful, for instance, in image filtering and in problems related to perceptual colour difference. For a thorough treatment, we refer the reader to [1, 6, 13–15] and references therein.

A celebrated example of fuzzy metric is the so-called standard fuzzy metric [5], which is induced from a classical metric. Let us recall that, given a metric space (X,d), the triple (X,M_d,min) constitutes the standard fuzzy metric space, where *min* denotes the minimum *t*-norm and M_d is the fuzzy set defined on $X\times X\times]0,\infty[$ given by

$$M_d(x,y,\theta)=\frac{\theta}{\theta+d(x,y)},\ \text{for each } x,y\in X,\theta\in]0,\infty[.$$

Note that for the standard fuzzy metric, the degree of nearness between two points $x,y\in X$ only can be 1, for some $\theta_0\in]0,\infty[$, whenever x and y are the same point. Moreover, the degree of nearness between two points can never be 0.

With the aim of proposing a fuzzy metric that can be a useful tool for RANSAC, and that is to encode the compatibility of each sample to the current model/hypothesis, we introduce, in Theorem 1, a general technique to generate fuzzy metrics from classical metrics. To this end, let us denote by $\mathbb{N}$ the set of positive integer numbers and let us recall that the family of Yager t-norms $(*_Y^\lambda)_{\lambda\in[0,\infty]}$ is given as follows [9]:

$$x*_Y^\lambda y=\begin{cases} x*_D y, & \text{if } \lambda=0;\\ \min\{x,y\}, & \text{if } \lambda=\infty;\\ \max\left\{1-\left((1-x)^\lambda+(1-y)^\lambda\right)^{\frac{1}{\lambda}},0\right\}, & \text{otherwise.}\end{cases}$$

Theorem 1. *Let (X,d) be a metric space and let $n\in\mathbb{N}$. Then $(X,M_d^n,*_Y^{\frac{1}{n}})$ is a fuzzy metric space, where $*_Y^{\frac{1}{n}}$ denotes the Yager t-norm for $\lambda=\frac{1}{n}$ and M_d^n is defined by*

$$M_d^n(x,y,\theta)=\begin{cases}1-\frac{d^n(x,y)}{\theta^n}, & \text{if } x,y\in X,\theta\in]0,\infty[\text{ such that } d(x,y)\leq\theta;\\ 0, & \text{otherwise.}\end{cases}\quad(2)$$

Proof. Next we show that axioms **(KM1)-(KM4)** are satisfied, for each $x, y, z \in X$ and $\theta, \mu \in]0, \infty[$.

(KM1) Let $x, y \in X$ and suppose that $M_d(x, y, \theta) = 1$ for all $\theta \in]0, \infty[$. We have that $d(x, y) \leq \theta$ and $\frac{d^n(x,y)}{\theta^n} = 0$ for all $\theta \in]0, \infty[$. It follows that $d(x, y) = 0$. The fact that d is a metric on X gives that $x = y$. Contrarily, assume that $x = y$. Since d is a metric on X, we obtain that $d(x, y) = 0$, whence $d(x, y) \leq \theta$ for all $\theta \in]0, \infty[$. Moreover, $M_d^n(x, y, \theta) = 1 - \frac{0}{\theta^n} = 1$ for all $\theta \in]0, \infty[$.

(KM2) It is obvious due to the symmetry of d, i.e. $d(x, y) = d(y, x)$ for all $x, y \in X$.

(KM3) Let $x, y, z \in X$ and $\theta, \mu \in]0, \infty[$. We are going to prove that

$$M_d^n(x, z, \theta + \mu) \geq M(x, y, \theta) *_Y^{\frac{1}{n}} M(y, z, \mu).$$

To this end, observe that $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y \in X$.
We distinguish two possible cases:

Case 1. Suppose that $d(x, z) \leq \theta + \mu$. Observe that $M_d^n(x, y) \leq 1 - \frac{d^n(x,y)}{\theta^n} \leq 1 - \frac{d^n(x,y)}{(\theta+\mu)^n}$ and $M_d^n(y, z) \leq 1 - \frac{d^n(y,z)}{\mu^n} \leq 1 - \frac{d^n(x,y)}{(\theta+\mu)^n}$. Moreover, we have that $M_d^n(x, z, \theta + \mu) = 1 - \frac{d^n(x,z)}{(\theta+\mu)^n}$. Therefore,

$$M_d^n(x, z, \theta + \mu) \geq 1 - \frac{(d(x, y) + d(y, z))^n}{(\theta + \mu)^n} \geq$$

$$1 - \left(\left(1 - \left(1 - \frac{d^n(x, y)}{(\theta + \mu)^n} \right) \right)^{\frac{1}{n}} + \left(1 - \left(1 - \frac{d^n(y, z)}{(\theta + \mu)^n} \right) \right)^{\frac{1}{n}} \right)^n =$$

$$\left(1 - \frac{d^n(x, y)}{(\theta + \mu)^n} \right) *_Y^{\frac{1}{n}} \left(1 - \frac{d^n(y, z)}{(\theta + \mu)^n} \right) \geq M_d^n(x, y, \theta) *_Y^{\frac{1}{n}} M_d(y, z, \mu).$$

Case 2. Suppose that $d(x, z) > \theta + \mu$. In such a case, observe that $M_d(x, z, \theta + \mu) = 0$ Moreover, either $d(x, y) > \theta$ or $d(y, z) > \mu$. Indeed, if $d(x, y) \leq \theta$ and $d(y, z) \leq \mu$, then $d(x, z) \leq d(x, y) + d(y, z) \leq \theta + \mu < d(x, z)$, which is a contradiction. Therefore, either $M_d^n(x, y) = 0$ or $M_d(y, z) = 0$. Thus $0 = M_d^n(x, y, \theta) *_Y^{\frac{1}{n}} M_d(y, z, \mu)$. So $M_d(x, z, \theta + \mu) = M_d^n(x, y, \theta) *_Y^{\frac{1}{n}} M_d(y, z, \mu)$.

(KM4) Let $x, y \in X$. Then, the assignment $(M_d^n)_{x,y} :]0, \infty[\to [0, 1]$ given by $(M_d^n)_{x,y}(\theta) = M_d^n(x, y, \theta)$, for each $\theta \in]0, \infty[$, is defined as follows

$$(M_d^n)_{x,y}(\theta) = \begin{cases} 0, & \text{if } \theta < d(x, y) \\ 1 - \frac{d^n(x,y)}{\theta^n}, & \text{if } \theta \geq d(x, y) \end{cases},$$

which, obviously, is (left-)continuous on $]0, \infty[$.

We conclude that $(X, M_d^n, *_Y^{\frac{1}{n}})$ is a fuzzy metric space, as claimed.

□

It must be stressed that one can find particular cases of metric spaces in which the degree of nearness between two points provided by the fuzzy metric in Eq. (2) can be 0. This is a relevant fact, as mentioned before, because the standard fuzzy metric is not able to achieve it.

4 FM-based RANSAC

As already described, RANSAC adopts a *hypothesize-and-verify* approach to fit a model to data contaminated by random noise and outliers: i.e. for every hypothesis/model considered, data samples are classified into inliers and outliers by comparing the fitting error with a threshold τ_I related to data noise, and that model accumulating the largest number of inliers is the one finally chosen as solution of the estimation problem. This simple approach has been systematically used for robust estimation of model parameters in the presence of arbitrary noise, although, along the years, alternative implementations have been proposed to counteract the misbehaviours and shortcomings that have been detected.

In this work, we focus on three facets of RANSAC: (1) *samples classification* into inliers and outliers, which we avoid to prevent the estimator from explicitly, and prematurely, deciding which samples are relevant; (2) *model scoring*, for which we replace the pure cardinality of the inlier set of plain RANSAC by an expression involving the individual fitting errors, similarly to what MSAC and MLESAC do [19]; and (3) *model refinement* once the main hypothesis-checking loop has finished, for which we adopt an iterative re-weighting scheme that makes use of all the available data samples without any distinction between inliers and outliers, contrarily to plain RANSAC, and other variants, that adopt least squares regression for the set of inliers (notice that the distinction between inliers and outliers depends on the current model under consideration, and thus changes with every model).

Algorithm 1 describes formally the RANSAC variant that is proposed in this work. The details regarding points (1)-(3) above can be found next:

1. **Samples classification**. As already mentioned, no distinction is made between inliers and outliers, but we make use of the fuzzy metric introduced in Theorem 1 to obtain a compatibility value $\phi \in [0,1]$ between each sample x_i and the current model $\mathrm{M}_{\widehat{\Theta}_k}$, given the fitting error $\epsilon(x_i; \mathrm{M}_{\widehat{\Theta}_k})$. Although the compatibility value is obtained by means of the aforesaid metric and, thus, it depends on the set of parameters (d, Φ) with $\Phi = (n, \theta)$, in the following we will denote it by $\phi(\epsilon; \Phi)$ in order to make clear that such a value refers to the fitting error ϵ.
2. **Model scoring**. The individual compatibility values $\phi(\epsilon; \Phi)$ are aggregated by simple summation to obtain the model score (step 6 in Algorithm 1) and hence the *so-far-the-best-model* is given by the maximum score found up to the current iteration (steps 7 - 9 of Algorithm 1).
3. **Model refinement**. Once a sufficient number of hypotheses/models have been considered, we re-estimate the winning model using iterative weighted least squares, where the compatibility values $\phi(\epsilon; \Phi)$, calculated for the fitting

Algorithm 1 FM-based RANSAC

Input: D - dataset comprising samples $\{x_i\}$
$\phi(\epsilon\,;\,\Phi)$ - FM compatibility value for fitting error ϵ and parameters Φ
$k_{\max}$ - maximum number of iterations of the main loop, as given by Eq. (1)
$t_{\max}$ - maximum number of iterations of the refinement stage
Output: $\mathrm{M}_{\widehat{\Theta}}$ - estimated model, whose parameters are compactly represented by $\widehat{\Theta}$

1: $k := 0,\ \varphi_{\max} := -\infty$
2: **for** $k := 1$ **to** $k_{\max}$ **do** ▷ find maximum consensus model $\mathrm{M}_{\widehat{\Theta}}$
3: select randomly a minimal sample set S_k of size s from D
4: estimate model $\mathrm{M}_{\widehat{\Theta}_k}$ from S_k
5: calculate fitting errors $\epsilon(x_i; \mathrm{M}_{\widehat{\Theta}_k}), \forall x_i \in \mathrm{D}$
6: find model score $\varphi_k := \sum_{x_i \in \mathrm{D}} \phi(\,\epsilon(x_i; \mathrm{M}_{\widehat{\Theta}_k})\,;\,\Phi\,)$
7: **if** $\varphi_k > \varphi_{\max}$ **then**
8: $\varphi_{\max} := \varphi_k$, $\mathrm{M}^0_{\widehat{\Theta}} := \mathrm{M}_{\widehat{\Theta}_k}$
9: **end if**
10: **end for**
11: $t := 0$
12: **repeat** ▷ refine model $\mathrm{M}_{\widehat{\Theta}}$
13: calculate fitting errors $\epsilon(x_i; \mathrm{M}^t_{\widehat{\Theta}}), \forall x_i \in \mathrm{D}$
14: estimate model $\mathrm{M}^{t+1}_{\widehat{\Theta}}$ using weights $\phi(\epsilon(x_i; \mathrm{M}^t_{\widehat{\Theta}})\,;\,\Phi\,)$
15: $t := t + 1$
16: **until** convergence **or** $t \geq t_{\max}$
17: **return** $\mathrm{M}^t_{\widehat{\Theta}}$

errors resulting from the current model, are used as weights for the new, refined model (steps 12 - 16 of Algorithm 1). The loop iterates until changes in Θ are negligible (or after a maximum number of iterations).

5 Experimental results

In this section, we report on the performance of FM-based RANSAC for a number of experiments that include a comparison with plain RANSAC and MSAC (their computational requirements are similar to ours). For illustration purposes, all experiments involve the estimation of 2D lines described by $\Theta = (a, b, c)$, corresponding to a straight line in general form $ax + by + c = 0$.

5.1 Experimental setup

For testing purposes, we generate synthetic datasets with points stemming from 2D lines in different orientations and positions. Each dataset contains a total of 300 points which comprise both inliers and outliers, the latter in a proportion equal to ω. Given a random point p over a line $\Theta = (a, b, c)$, i.e. $ap_x + bp_y + c = 0$, whose normal vector is $\vec{n}$, an inlier p_I of the dataset is generated by shifting p along $\vec{n}$ using a 0-mean Gaussian distribution with standard deviation σ, i.e.

Table 1: Estimation accuracy and number of iterations of the refinement stage for (a) different outlier ratios ω, (b) different noise magnitudes σ and (c) different settings for $\tau_I, \theta = \kappa \cdot \sigma$. Whenever they are kept constant, $\sigma = 1$, $\omega = 0.4$ and $\kappa = 3$. Lighter background means higher performance.

		$\mu[\varepsilon]$ (°)					$\mu[t]$	
	ω	**RANSAC**	**MSAC**	**ours** $n=1$	**ours** $n=2$	**ω**	**ours** $n=1$	**ours** $n=2$
(a)	**0.60**	4.43	3.14	1.51	1.55	**0.60**	11.83	10.81
	0.50	3.03	2.33	1.02	1.07	**0.50**	9.63	8.43
	0.40	2.13	1.81	0.86	0.88	**0.40**	8.55	7.23
	0.20	1.58	1.53	0.67	0.66	**0.20**	7.64	6.12
	σ	**RANSAC**	**MSAC**	**ours** $n=1$	**ours** $n=2$	**σ**	**ours** $n=1$	**ours** $n=2$
(b)	**2.00**	6.76	6.87	3.76	3.78	**2.00**	21.02	17.68
	1.00	2.13	1.81	0.86	0.88	**1.00**	8.55	7.23
	0.50	1.32	0.89	0.39	0.44	**0.50**	5.65	5.04
	0.25	1.05	0.62	0.23	0.29	**0.25**	4.60	4.31
	κ	**RANSAC**	**MSAC**	**ours** $n=1$	**ours** $n=2$	**κ**	**ours** $n=1$	**ours** $n=2$
(c)	**4.00**	2.85	2.09	1.01	1.10	**4.00**	7.75	6.88
	3.00	2.13	1.81	0.86	0.88	**3.00**	8.55	7.23
	2.50	2.03	1.88	0.82	0.81	**2.50**	9.56	7.91
	2.00	2.18	2.18	0.85	0.82	**2.00**	12.22	10.04
	1.00	3.60	3.58	1.82	1.82	**1.00**	33.16	27.79

$p_I = p + \mathcal{N}(0, \sigma) \cdot \vec{n}$. Outliers p_O are uniformly generated within a rectangular area containing the straight line, ensuring that they lie out of a $\pm 3\sigma$ stripe along the line. For every combination (σ, ω), we generate a total of 500 datasets.

Regarding hypothesis generation within the main loop, in all experiments, the size of the MSS is always $s = 2$ points. Besides, the number of iterations $k_{\max}$ is calculated according to Eq. (1), with $\rho = 99\%$. The parameters of $\phi(\epsilon; \Phi)$, $\Phi = (\theta, n)$, are set as follows: $\theta = \kappa \cdot \sigma$, as well as τ_I for RANSAC/MSAC, considering different values for κ; $n = 1$ or 2, as indicated for each experiment. Finally, to compare properly RANSAC, MSAC and the FM-based RANSAC, we make use of the same sequence of MSS's to avoid the effect of randomness.

5.2 Results and discussion

In the following, to measure the estimation accuracy, we make use of the average $\mu[\varepsilon]$ of the angle ε between the true and the estimated normal vector; we as well report on the average number of iterations spent during model refinement $\mu[t]$.

On the one hand, Table 1 shows performance results for several outlier ratios ω and Gaussian noise magnitudes σ. In sight of these results, it is worth noting that: (1) the estimation accuracy of the FM-based RANSAC is above that of plain RANSAC and MSAC in all cases; (2) the most substantial differences are found for higher values of ω and σ; (3) the value of θ in ϕ does not seem to be critical, since very similar errors result for $\kappa = 2$ - 4; (4) estimation accuracy does not differ significantly between $n = 1$ and $n = 2$; (5) as for the number of

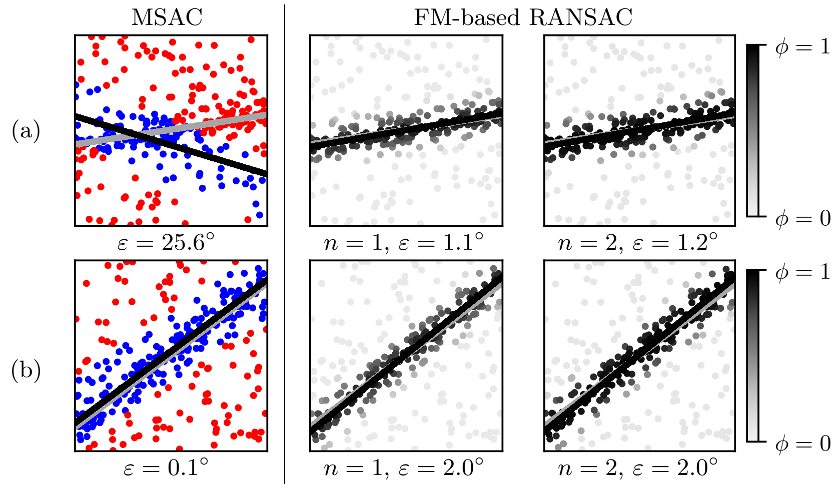

Fig. 1: (a) Best and (b) worst estimations found in 500 datasets for FM-based RANSAC in comparison with MSAC. The true models M_{Θ^*} are (a) $0.15x - 0.99y + 0.00 = 0$ and (b) $0.60x - 0.80y + 0.00 = 0$. The noise parameters in both cases are $(\sigma, \omega) = (1, 0.4)$ and $\kappa = 3$. The colour code is as follows: true/estimated model as gray/black lines, MSAC: inliers/outliers as blue/red dots, FM-based RANSAC: $\phi(\epsilon(x_i; \mathrm{M}_{\widehat{\Theta}})\,;\, \Phi\,)$ coded in gray scale.

iterations of the refinement stage t, it tends to be lower for $n = 2$ with regard to $n = 1$, and the difference becomes larger when the magnitude of noise is higher, i.e. $\sigma = 2$; (6) a correct setting of κ also reduces t.

On the other hand, Fig. 1 reports on the best- and the worst-case estimations for the FM-based RANSAC in comparison with MSAC for 500 datasets; that is to say, the best case is the case for which FM-based RANSAC outperforms MSAC the most, and the worst case is the case in which MSAC outperforms FM-based RANSAC the most. As can be observed, in both cases, data samples are correctly scored by the FM-based RANSAC, and the estimated and true models are almost identical even for the worst case.

6 Conclusions

In this paper, we have introduced a new Fuzzy Metric (FM) and proposed a variant of RANSAC which avoids discriminating between inliers and outliers by means of the use of such an FM, which provides a compatibility value for each data sample with respect to the current model. These compatibility values are aggregated next to score the model against other hypotheses generated inside the main RANSAC loop. The output model is refined at the latest stage by means of an iterated re-weighting least-squares scheme making use of the same FM. Experimental results show good performance for the FM-based RANSAC against other implementations of RANSAC, actually outperforming its classical counterparts.

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