EURASIAN JOURNAL OF ACADEMIC RESEARCH
Innovative Academy Research Support Center
www.innacademy.uz

LAW OF INERTION AND ITS APPLICATIONS

Dosanov Murtozaqul[1], Xolboyev Sirojiddin[2], Abduraximov Doniyor[3]
[1] teacher of GulSU
[2] teacher of GulSU
[3] associate professor of GulSU

https://doi.org/10.5281/zenodo.4928779

ARTICLE INFO

Received: 01st June 2021
Accepted: 05th June 2021
Online: 10th June 2021

KEY WORDS

Inertia, quadratic form, vector, determinant, space, proportional.

ABSTRACT

This article describes the law of inertia and its practical applications in detail in terms of mathematical content.

INERSIYA QONUNI VA UNING TATBIQLARI

Dosanov Murtozaqul[1], Xolboyev Sirojiddin[2], Abduraximov Doniyor[3]
[1] GulDU o`qituvchi
[2] GulDU o`qituvchi
[3] GulDU dotsenti

MAQOLA TARIXI

Qabul qilindi: 01-iyun 2021
Ma'qullandi: 05-iyun2021
Chop etildi: 10-iyun 2021

KALIT SO'ZLAR

Inersiya, kvadratik forma, vektor, determinant, fazo, praporsional.

ANNOTATSIYA

Ushbu maqolada inersiya qonuni va uning amaliy tatbiqlari matematik mazmun jihatdan keng yoritilgan.

Kvadratik formani

$$A(x,x) = \lambda_1\xi_1^2 + \lambda_2\xi_2^2 + \cdots + \lambda_n\xi_n^2 \quad (1)$$

ko`rinishga keltiruvchi bazis vektorlarni ularga praporsional vektorlar bilan almashtirish orqali noldan farqli $\lambda_i$ koeffitsentlarni 1 yoki -1 ga teng qilib olish mumkin. Demak, kvadratik formaning kanonik ko`rinishini mos tartibda 0,1 va -1 ga teng bo`lgan koefitsentlar soni bilan xarakterlash mumkin.

Tabiyki, bazisni turlicha tanlab olish mumkinligi uchun, 0,1 va -1 ga teng bo`lgan koeffitsentlar son bazisni tanlab olishga bog`liqmi yoki yo`qmi degan savol tug`iladi. Masalan, $A(x,x)$ kvadratik forma biror $e_1, e_2, \dots, e_n$ bazisda $A = (a_{i,j})$ matritsaga ega bo`lib, matritsaning $\Delta_1, \Delta_2, \dots, \Delta_n$ bosh minorlari noldan farqli bo`lsa, kvadratik formaning kanonik ko`rinishdagi barcha λ_i koeffitsentlar noldan farqli va manfiy koeffitsentlar soni $1, \Delta_1, \Delta_2, \dots, \Delta_n$

determinantlar qatoridagi ishora almashishlar soniga teng bo`ladi.

Ammo boshqa bir $f_1, f_2, \dots, f_n$ boshlang`ich bazis olib, bu bazisga mos keluvchi matritsani $A' = (a'_{i,j})$ orqali belgilab, $\Delta'_1, \Delta'_2, \dots, \Delta'_n$ determinantlarni topsak, hamda kvadratik formani kanonik ko`rinishga keltirsak, nima uchun bu holda ham ishora almashishlar soni yuqoridagi bilan bir xil bo`lishi tushnarli emas.

Biz bu maqolada kvadratik formaning inersiya qonuni deb ataluvchi teoremani isbot qilamiz.

Teorema. Agar kvadratik forma ikki xil usul bilan kanonik ko`rinishga keltirilgan bo`lsa, u holda bu kanonik ko`rinishlarda musbat, manfiy va nolga teng koeffitsentlarning soni ikkala holatda ham bir xil bo`ladi.

Oldin quidagi lemmani isbot qilamiz.

Lemma. n o`lchamli V fazoda mos tartibda $k$ va $l$ o`lchamli V_1 va V_2 qism fazolar berilgan bo`lib, $k + l > n$ bo`lsin.

U holda bu qism fazolarning ikkalasiga ham tegishli bo`lgan noldan farqli x vektor mavjud.

Isbot. Berilgan V_1 va V_2 qism fazolarda mos ravishda $e_1, e_2, \dots, e_k$ va $f_1, f_2, \dots, f_l$ bazislar olaylik. $k + l > n$ ekanligi uchun $e_1, e_2, \dots, e_k$, $f_1, f_2, \dots, f_l$ vektorlar chiziqli bog`liq bo`ladi. Demak, kamida bittasi noldan farqli bo`lgan $\lambda_1, \lambda_2, \dots, \lambda_k, \mu_1, \mu_2, \dots, \mu_l$ sonlar topilib,

$$\lambda_1 e_1 + \lambda_2 e_2 + \dots + \lambda_k e_k + \mu_1 f_1 + \mu_2 f_2 + \dots + \mu_l f_l = 0$$

ya'ni

$$\lambda_1 e_1 + \lambda_2 e_2 + \dots + \lambda_k e_k = -\mu_1 f_1 - \mu_2 f_2 - \dots - \mu_l f_l$$

Agar

$$x = \lambda_1 e_1 + \lambda_2 e_2 + \dots + \lambda_k e_k = -\mu_1 f_1 - \mu_2 f_2 - \dots - \mu_l f_l$$

deb faraz qilsak, x vektor bir tomondan $e_1, e_2, \dots, e_k$ vektorlarning chiziqli kombinatsiyasi, ikkinchi tomondan esa $f_1, f_2, \dots, f_l$ vektorlarning chiziqli kombinatsiyasi sifatida tasvirlanishini ko`rishimiz mumkin. Demak, x vektor $V_1$ va $V_2$ qism fazolarning ikkalasiga ham tegishli bo`ladi.

Endi ushbu x vektorni noldan farqli ekanligini ko`rsatamiz. Agar $x = 0$ bo`lsa,

$$\lambda_1 e_1 + \lambda_2 e_2 + \dots + \lambda_k e_k = 0, \quad \mu_1 f_1 + \mu_2 f_2 + \dots + \mu_l f_l = 0.$$

$e_1, e_2, \dots, e_k$ va $f_1, f_2, \dots, f_l$ vektorlar sistemalari mos ravishda V_1 va V_2 qism fazolarning bazislari bo`lganligi uchun, bu vektorlar sistemalari chiziqli erkli. Bundan esa $\lambda_1 = \lambda_2 = \dots = \lambda_k = 0$, $\mu_1 = \mu_2 = \dots = \mu_l = 0$ ekanligi kelib chiqadi. Bu esa $\lambda_1, \lambda_2, \dots, \lambda_k, \mu_1, \mu_2, \dots, \mu_l$ sonlarning kamida bittasi noldan farqli ekanligiga ziddir. Demak, $x \neq 0$

Endi teoremaning isbotiga o`tamiz. Aytaylik, $A(x, x)$ kvadratik forma $e_1, e_2, \dots, e_n$ bazisda

$$A(x, x) = \xi_1^2 + \xi_2^2 + \dots + \xi_p^2 - \xi_{p+1}^2 - \xi_{p+2}^2 - \dots - \xi_{p+q}^2 \quad (2)$$

ko`rinishga, $f_1, f_2, \dots, f_n$ bazisda esa

$$A(x, x) = \eta_1^2 + \eta_2^2 + \dots + \eta_{p'}^2 - \eta_{p'+1}^2 - \eta_{p'+2}^2 - \dots - \eta_{p'+q'}^2 \quad (3)$$

ko`rinishga ega bo`lsin, bu yerda $\xi_1, \xi_2, \dots, \xi_n$ va $\eta_1, \eta_2, \dots, \eta_n$ lar mos ravishda x vektorning $e_1, e_2, \dots, e_n$ va $f_1, f_2, \dots, f_n$ bazislardagi koordinatalari.

$p = p'$ va $q = q'$ ekanligini isbot qilishimiz kerak. Faraz qilaylik, $p > p'$ bo`lsin.

$e_1, e_2, \dots, e_p$ vektorlarning chiziqli kombinatsiyasidan iborat bo`lgan $V_1$ va $f_{p'+1}, f_{p'+2}, \dots, f_n$ vektorlarning chiziqli kombinatsiyasidan iborat bo`lgan V_2 qism fazolarni qaraymiz. Ma'lumki, $\dim(V_1) = p, \dim(V_2) = n - p'$ bo`lib, $n - p' + p > n$ ekanligi uchun lemmaga asosan V_1 va V_2

qism fazolarning kesishmasida noldan farqli $x \in V_1 \cap V_2$ vektor mavjud. U holda

$$x = \xi_1 e_1 + \xi_2 e_2 + \cdots + \xi_p e_p$$

va

$$x = \eta_{p'+1} f_{p'+1} + \eta_{p'+2} f_{p'+2} + \cdots + \eta_n f_n$$

bo`ladi, ya'ni $x$ vektor $e_1, e_2, \dots, e_n$ bazisda $(\xi_1, \xi_2, \dots, \xi_p, 0, \dots, 0)$ koordinatalarga, $f_1, f_2, \dots, f_n$ bazisda esa $(0, \dots, 0, \eta_{p'+1}, \eta_{p'+2}, \dots, \eta_n)$ koordinatalarga ega bo`ladi.

Bu koordinatalarni (2) va (3) tengliklarga qo`yib, bir tomondan

$$A(x,x) = \xi_1^2 + \xi_2^2 + \cdots + \xi_p^2 > 0,$$

ikkinchi tomondan esa

$$A(x,x) = -\eta_{p'+1}^2 - \eta_{p'+2}^2 - \cdots - \eta_{p'+q'}^2 < 0$$

munosabat hosil qilamiz. Bu esa $p > p'$ deb olingan farazga zid, ya'ni $p \le p'$ bo`ladi.

Endi faraz qilaylik $p < p'$ bo`lsin. $e_{p+1}, e_{p+2}, \dots, e_n$ vektorlarning chiziqli kombinatsiyasidan iborat bo`lgan V_1 va $f_1, f_2, \dots, f_{p'}$ vektorlrning chiziqli kombinatsiyasidan iborat bo`lgan $V_2$ qism fazolarni qaraymiz. Ma'lumki $\dim(V_1) = n - p, \dim(V_2) = p'$ bo`lib $p' + n - p > n$ ekanligi uchun lemmaga asosan V_1 va V_2 qism fazolarning kesishmasida no`ldan farqli $x \in V_1 \cap V_2$ vektor mavjud. U holda

$$x = \xi_{p+1} e_{p+1} + \xi_{p+2} e_{p+2} + \cdots + \xi_n e_n$$

va

$$x = \eta_1 f_1 + \eta_2 f_2 + \cdots + \eta_{p'} f_{p'}$$

bo`ladi, ya'ni $x$ vektor $e_1, e_2, \dots, e_n$ bazisda $(0, \dots, 0, \xi_{p'+1}, \xi_{p'+2}, \dots, \xi_n)$ koordinatalarga, $f_1, f_2, \dots, f_n$ bazisda esa $(\eta_1, \eta_2, \dots, \eta_p, 0, \dots, 0)$ koordinatalarga ega bo`ladi.

Bu koordinatalarni (2) va (3) tengliklarga qo`yib, bir tomondan

$$A(x,x) = -\xi_{p+1}^2 - \xi_{p+2}^2 - \cdots - \xi_{p+q}^2 < 0,$$

ikkinchi tomondan esa

$$A(x,x) = \eta_1^2 + \eta_2^2 + \cdots + \eta_{p'}^2 < 0$$

munosabat hosil qilamiz. Bu esa $p < p'$ deb olingan farazga zid, ya'ni $p \ge p'$

demak $p \le p'$ va $p \ge p'$ lardan $p = p'$ ni hosil qilamiz.

Unda $A(x,x)$ kvadratik forma $e_1, e_2, \dots, e_n$ bazisda

$$A(x,x) = \xi_1^2 + \xi_2^2 + \cdots + \xi_p^2 - \xi_{p+1}^2 - \xi_{p+2}^2 - \cdots - \xi_{p+q}^2$$

ko`rinishga, $f_1, f_2, \dots, f_n$ bazisda esa

$$A(x,x) = \eta_1^2 + \eta_2^2 + \cdots + \eta_{p'}^2 - \eta_{p'+1}^2 - \eta_{p'+2}^2 - \cdots - \eta_{p'+q'}^2$$

ko`rinishga ega bo`ladi.

Endi yuqorida keltirilgan mulohazalarni $-A(x,x)$ kvadratik formaga tadbiq qilib $-A(x,x)$ kvadratik forma $e_1, e_2, \dots, e_n$ bazisda

$$-A(x,x) = -\xi_1^2 - \xi_2^2 - \cdots - \xi_p^2 + \xi_{p+1}^2 + \xi_{p+2}^2 + \cdots + \xi_{p+q}^2$$

ko`rinishga $f_1, f_2, \dots, f_n$ bazisda esa

$$-A(x,x) = -\eta_1^2 - \eta_2^2 - \cdots - \eta_{p'}^2 + \eta_{p'+1}^2 + \eta_{p'+2}^2 + \cdots + \eta_{p'+q'}^2$$

ko`rinishga ega bo`lamiz $e_1, e_2, \dots, e_n$ da musbat qiymatlar soni q ga $f_1, f_2, \dots, f_n$ bazisda esa q' ta bo`ladi, ya'ni $q = q'$ ekan isbot tugadi.

Foydalanilgan adabiyotlar:

1. **Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev, F.H.Haydarov "ALGEBRA VA SONLAR NAZARIYASI" Toshkent 2019 yil 319 bet.**
2. Dixon M.R., Kurdachenko L.A., Subbotin I.Ya., Algeba and Number theory. 2010. – 523 p
3. Кострикин А.И. Введение в алгебру. Часть II. Линейная алгебра. 2000. – 368 с.
4. Куликов Л.Я. Алгебра и теория чисел. Москва. 1979. –559 с.