

Some Stack issues faced during programming exercises for TyöMAA project

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Abstract: This paper describes mathematical online course targeted to upper secondary school students. The main idea on this course is to motivate students to study mathematics with real-life examples and success in calculations. As students can progress at their own pace on this course, automatic feedback is essential. If feedback is incorrect or misleading, it may have negative influence on motivation. Issues mentioned here are faced during the programming and running the first round of online course.

Keywords: STACK, Moodle, MOOC, mathematics.

1 Introduction

TyöMAA is a project targeted to teachers and students of upper secondary schools in Finland. Its main aim is to motivate students on that level to study mathematics by showing actual work-life calculations for both short syllabus and long syllabus mathematics. There is online material for teachers to show examples when lecturing the new topic to answer question “why”. Teachers can also ask students from our campus to visit their lessons and tell more about mathematics needed in their field. One important part of this project is online course, where students can solve mathematical work-life cases.

Students in upper secondary schools must select either short syllabus or long syllabus in mathematics. Students selecting the short syllabus are quite often focusing to the fields, in which they do not see so much need for mathematics, like nursing or tourism. This can negatively affect their motivation for studying mathematics, and that may weaken their success in postgraduate studies. Kärkkäinen and Luojus (2019) discovered that more than 75% of Finnish upper secondary school students were indecisive of their occupational interests and had related concerns and worries. Hence, it can be deduced that the future working life is a meaningful context for many upper secondary school students and could provide a fruitful starting point also for contextualizing mathematics problems. Although contextual framing of mathematics problems has often been suggested as means to motivate and engage students, the related empirical evidence is still somewhat scarce [B11]. Long syllabus in mathematics is nowadays weighted in

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selection of new university students and, therefore, it may have effect on selecting long syllabus instead of short one: even lower grades in long syllabus may give more scores than high grades in short syllabus. This may lead to the situation, where the calculation skills are not as high as needed in long syllabus. The online course offers a possibility to improve mechanical skills and/or concentrate on applying mathematics to real-life examples.

2 Framework of this online course

This course was planned in a close cooperation with local upper secondary school teachers. The teachers gave us insights and tips about difficult topics and pointed out the ones in which students do not see the connection to their future careers. It is also agreed that students get two course marks for graduating from cooperation partner upper secondary schools.

Even when selecting the long syllabus in mathematics, students can still skip physics and chemistry except one compulsory course. Those students, who select the short syllabus, usually select only one compulsory course in physics and chemistry. Hence, mathematical applications in even physics and chemistry remain out of reach for many students. For instance, vectors are mainly connected with forces, and forces are usually applied to technology in mathematics classes. Forces are rarely connected with bodily movements in examples, although they have a big role in areas like physiotherapy. In addition, vectors are studied only in the long syllabus, but students interested in physiotherapy studies mainly select the short syllabus and therefore make no connection between mathematics and their career interests. For that reason, applied exercises cover different fields of our campus, like nursing and health care, tourism and hospitality, business, and technology. There is also provided extra material to cover topics, which are not part of their mathematics in upper secondary school.

Because main point of this course is in motivating, students may study it till they graduate from the upper secondary school. The upper secondary school lasts from three to four years and a school year may be divided from 4 to 6 periods decided locally. This means, that courses are not lectured at the same time in all schools and students may progress at different paces. Exercises for short and long syllabus are presented separately, but students may select any packages. Both parts contain both mechanical exercises and applied exercises. Applied exercises are weighted more than mechanical ones and exercises in long syllabus more than corresponding exercises in short syllabus. Succeeding in easier ones may promote motivation and eager, and it may motivate to solve more in number and in degree of difficulty. On the other hand, the students good at mechanical calculation may concentrate on applying. Based on all these aspects, the use of the MOOC should be as automated as possible. The teachers of this MOOC cannot be available whenever the students want to access it. Therefore, the course provides hints

and advises students during their learning despite the presence of teachers. This kind of support is sometimes called instructional scaffolding [RBK15].

One important aspect of this course is internationality: there are two international upper secondary schools at this area: there are many students, whose native language is not Finnish. Thus, we try to offer exercises in English and in Russian in addition to Finnish. As a student can select the language from Moodle, all exercises should have all three language options available.

3 Issues in STACK exercises

STACK is a powerful tool for practising mathematics at online courses. Randomizing and many possibilities in decision tree helps in producing inspiring and encouraging exercises. Because of randomizing, the number of programmed exercises is almost reasonable, as no multiples of the exact same form needs to be done; still exercises do not look the same for students.

Feedback is important part of a good online exercise [Mäl6]. If a student feels positive and encouraged after feedback, it may affect positively in motivation and in engaging their studies [KC21]. Notwithstanding all the good, there are some cases where STACK acts in a way that is not very motivating tool for a student and/or may even give incorrect feedback. In some cases, it took quite a lot of time to figure out, whether the problem was in programming or in STACK and/or in Maxima.

3.1 Issues in potential response tree

Applied exercises should be the easy cases, as they basically only need some text and the place for an answer. The answer is usually given as a real number, which should be rounded like asked. STACK offers a command *decimalplaces*("num", "dec"), where "num" refers to number and "dec" to desired number of decimals. The problem here is the rounding rule used in Finland:

If the first excluded number is 5 - 9, then the last included number is raised by one.

The rounding rule used by STACK is

When x is a real number, returns the closest integer to x . Multiples of $1/2$ are rounded to the nearest even integer.

As shown in Fig. 1, the number 32.25 is rounded to 32.2 by STACK, where the correct result in Finland would be 32.3. After hard thinking and sometimes long calculations getting "Your answer is incorrect." for this reason is not motivating; especially, as a

student cannot see the reason for incorrect answer. As the packages allows students to retry, they use time to figure out their mistake for nothing.

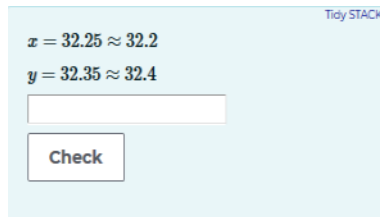


Fig. 1: Rounding in STACK

When multipliers are randomized in exercises, the answers can vary from very close to zero to very big numbers. In these cases, the correctness of the result cannot be checked with accuracy of decimals: for big numbers they may not be even seen and for small numbers accuracy could be zero. An option *at least with n significant numbers* solved the problem.

The idea behind of mechanical exercises is to rehearse basic calculation skills. Students solving these exercises usually have lacks in calculation skills and they are uncertain of their knowledge. It is important in aspect of motivation to give them appropriate feedback. Mathematical expressions are mainly studied at the beginning of the school. There are many ways of giving a correct answer. Sometimes, answer is correct but mathematically not so elegant. At the early steps of learning new things, mathematical elegance should not be the main point; the correct way of handling expressions should be more important.

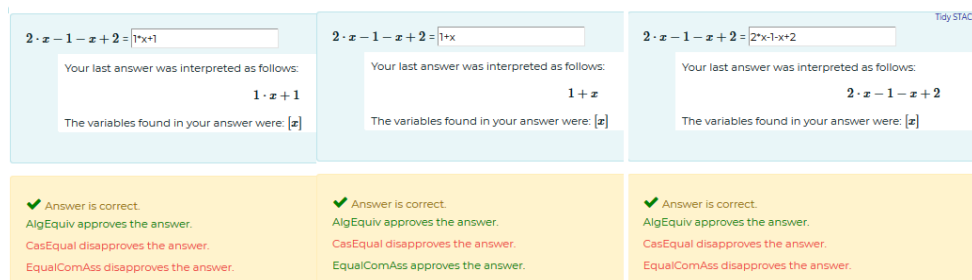


Fig. 2: Expressions handled by potential response tree

In Fig. 2 is given an expression to be simplified and decisions made by potential response tree (PRT). All student's answers are algebraically equivalent (AlgEquiv). Thus, a student could rewrite the original expression, like done in last case, to get full scores. Despite the correct answer, none of these forms is correct by CasEqual, as their parse trees do not equal to the teacher's answer. If the answers are checked based on commutativity and associativity of addition and multiplication, together with their

inverses minus and division (EqualCommAss), only one answer seems to be correct. At the early stages of expression handling, the first two answers should be accepted.

3.2 Feature giving incorrect feedback

Equivalence reasoning is excellent tool when practicing mechanical handling. When writing intermediate steps for a solution, it informs if there are problems, for example, in writing or in calculations. This allows students to correct easy mistakes before checking. Unfortunately, it seems to be working only partly correct with systems of equations.

In Fig. 3 is a case, where the system to two equations has infinite number of solutions. As can be seen, it recognizes that the same equation is given twice. This is an important information for students, as they know that there are no writing or calculation mistakes.

Solve

$$\begin{cases} 10 \cdot b + 9 \cdot a = -10 \\ -50 \cdot b - 45 \cdot a = 50 \end{cases}$$

$10 \cdot b + 9 \cdot a = -10$ and $10 \cdot b + 9 \cdot a = 50 / (-5)$
 $0 = 0$ and $10 \cdot b + 9 \cdot a = -10$

$$\Leftrightarrow \begin{cases} 10 \cdot b + 9 \cdot a = -10 \\ 10 \cdot b + 9 \cdot a = \frac{50}{-5} \\ 0 = 0 \\ 10 \cdot b + 9 \cdot a = -10 \end{cases}$$

Write one stage at one row. Combine your equations with 'and' at each row, like $10 \cdot b + 9 \cdot a = -10$ and $-(50 \cdot b) - 45 \cdot a = 50$.

Thus, there is

ei ole ratkaisua | no solution

on yksikäsitteinen ratkaisu | unique solution

on ääretön määrä ratkaisuja | infinite number of solutions

Fig. 3: Infinite number of solutions

In Fig. 4, there is equations of the two parallel lines with no solution to be found. A student has solved the exercise following the instructions given during lectures and the case simplifies to $0 = 5/3$ referring that no solution exists. As STACK gives a red question mark before the final answer, the student may doubt his/her solution for nothing. Although the correct answer is given in the question part, STACK cannot handle it.

Solve

$$\begin{cases} -8 \cdot y - a = -14 \\ 24 \cdot y + 3 \cdot a = 47 \end{cases}$$

$$\begin{aligned} -(8 \cdot y) - a &= -14 \text{ and } -(8 \cdot y) - a = 47 / (-3) \\ 0 &= 5/3 \text{ and } -(8 \cdot y) - a = -14 \end{aligned}$$

$$\begin{cases} -8 \cdot y - a = -14 \\ -8 \cdot y - a = \frac{47}{-3} \\ 0 = \frac{5}{3} \\ -8 \cdot y - a = -14 \end{cases}$$

Write one stage at one row. Combine your equations with 'and' at each row, like $-(8 \cdot y) - a = -14$ and $24 \cdot y + 3 \cdot a = 47$. Thus, there is

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Fig. 4: System of two equations with no solution

3.3 Multilingual exercises

Multilingual questions are an important aspect of this course. Firstly, students can change their language at any point that may upgrade their language skills despite whether there are native Finnish or non-native ones. Secondly, exercises need to be produced only once and, for example, programming mistakes are corrected at the same time for all versions. It must be pointed out here, that these issues are probably more attached with HTML and MathJax than with programming STACK. Nevertheless, a student sees them as STACK problems and a beginner in STACK programming may use a lot of time to figure out where the problem exists.

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</p>
<p><span class="multilang" lang="fi">Hevostallin omistaja
haluaa rakentaa mahdollisimman suuren aitauksen. Mikä olisi
suurin mahdollinen pinta-ala, kun köyttä on käytettävissä
\{#\koysi#\} m?</span>
<span class="multilang" lang="en">An owner of a horse
stable wants to build as large rectangular paddock as
possible. What would be the maximum area, if there is
\{#\koysi#\} metres of fencing in use? </span>
<span class="multilang" lang="ru">Владельцу конюшни
хочет построить как можно больший прямоугольный загон.
Какой должна быть максимальная площадь, если используется
\{#\koysi#\} метров ограждений? </span>
</p>

```

Tidy STACK question tool | Question tests & deployed version

An owner of a horse stable wants to build as large rectangular paddock as possible. What would be the maximum area, if there is 170 metres of fencing?

The maximum area is m².

If you cannot find a solution, give empty brackets.

Fig. 5: An example of a broken question

In multilingual questions, text is easily broken with mathematics. It seems, that all mathematical formulas with $\backslash[\]$ or $\backslash(\)$ should be outside ` ` delimiters. If the question includes lots of information in this form, the question text is anything else but easy to follow by a programmer. This can be handled, although it is annoying. However, if delimiters of mathematical part are inside span delimiters, the translation is truncated. Sometimes the next language is shown (see Fig. 5), sometimes it just truncated, and after last update, the line breaks were just lost.

The problems with radio buttons and check boxes in multilingual questions affect more in appearance and clarity of the exercise. A student should select correct answer(s) and alternatives often have some text. This fixed text is inside a variable and cannot be changed, although language selection in Moodle changes. In practice, all possible languages should be given at the same time, if needed. As seen in Fig. 6, adding extra language to this example would make it too confusing to a student and almost impossible to a programmer to handle.

The screenshot shows a STACK question interface. At the top, it says "Solve $-7 \cdot t^2 - 56 \cdot t - 105 \geq 0$ ". Below this, there are three radio buttons labeled "0", "1", and "2" under the heading "Expression has". A dropdown menu is open, showing options: "(No answer given)", "alaspäin aukeava paraabeli | downward parabola", "ylöspäin aukeava paraabeli | upward parabola", and "ei mikään paraabeli | not a parabola". Below the dropdown, there is a text input field for "zeros, which are" and another for "Give your zeros in brackets, like [-3]". Below that, there is a dropdown menu for "As your expression seems to be" with the option "(No answer given)" selected. Below this are several radio buttons with Finnish and English descriptions of solution types: "ratkaisu on sisävälillä | solution is an interval", "ratkaisu on ulkovälillä | solution is outside an interval", "ratkaisu on kaikki reaaliluvut | solution is all real numbers", "ratkaisu on vain yksi piste | solution is only one point", "ratkaisu on kaikki muut paitsi yksi piste | solution is all real numbers except one point", and "ratkaisua ei ole | solution does not exist". At the bottom, there is a text input field for "Write your solution here" and a section titled "Solution is written as following:" with mathematical expressions: "No solution. => []", " $x \neq 1 \Rightarrow x \neq 1$ ", " $1 \leq x < 2 \Rightarrow x \geq 1$ and $x < 2$ ", " $x < -1 \vee x > 2 \Rightarrow x < -1$ or $x > 2$ ", and "All real numbers Kaikki reaaliluvut kelpaa => [R]".

Fig. 6: Dropdown and radio button

4 Conclusion

STACK is excellent tool for online courses in mathematical subjects. It allows in addition to mechanical knowledge different kinds of exercises to rehearse understanding and, furthermore, enables automatic checking and feedback. These features make online course motivational. Although some of these presented cases may not be an issue

among mathematically motivated students, they have influence on overall experience that is critical for upgrading extrinsic motivation towards intrinsic one.

There are lots of important information and hints in STACK documents. Unfortunately, those documents are not always very clear for beginners. Many of us look for information with search engines, in which case only separated documents are found for this specific case. Documents usually refer to other documents for further knowledge but at some point, the beginner may be so confused that uses the first command found. If the beginner has an opportunity to acquaint oneself with STACK with help of an expert, many problems can be avoided. The short discussion may decrease huge amount of work if done before the first exercise is programmed.

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