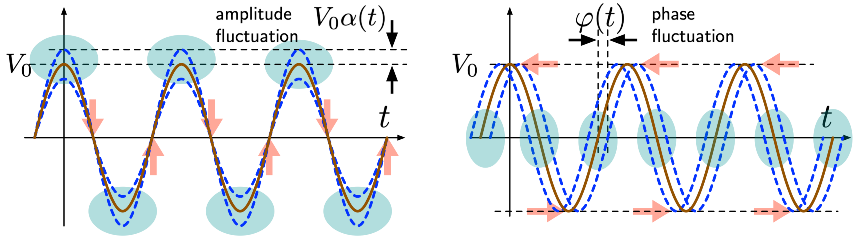
Enrico's Chart of Phase Noise and Two-Sample Variances

Enrico Rubiola – http://rubiola.org
European Frequency and Time Seminar – http://efts.eu
Oscillator Instability Measurement Platform http://oscillator-imp.com

Thanks to FIRST-TF https://first-tf.com

Clock signal $v(t) = V_0[1+\alpha(t)]\cos[2\pi\nu_0 t + \varphi(t)]$

Download from Zenodo (CERN)



Boldface notation

total = nominal + fluctuation

$\boldsymbol{\varphi}(t) = 2\pi\nu_0 t + \varphi(t)$		phase
$\boldsymbol{\nu}(t) = \nu_0 + (\Delta\nu)(t)$		frequency
$\mathbf{x}(t) = t + \mathrm{x}(t)$		time
$\mathbf{y}(t) = 1 + \mathrm{y}(t)$		fractional frequency

Phase noise spectrum

Definition

$S_\varphi(f)$ [rad^2/Hz] is the one-sided PSD ($f>0$) of $\varphi(t)$

$$S_\varphi(f) = 2\mathcal{F}\{\mathbb{E}\{\varphi(t)\varphi(t+\tau)\}\}, \quad f>0$$

Evaluation

$$S_\varphi(f) = \tfrac{2}{T}\langle \Phi_T(f)\,\Phi_T^*(f)\rangle_m$$

avg on m data, $\Phi_T(f)$ = DFT of $\varphi(t)$ truncated on T

Usage

most often, 'phase noise' refers to $\mathscr{L}(f)$

Only $10\log_{10}[\mathscr{L}(f)]$ is used, given in dBc/Hz

Definition: $\mathscr{L}(f) = \frac{1}{2}S_\varphi(f)$ [the unit c/Hz never used]

The unit 'c' is a squared angle, $\sqrt{\mathrm{c}} = \sqrt{2}$ rad $\simeq 81°$

Two-sample (Allan-like) variances

Definition

$$\sigma^2_{\mathrm{y}}(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{\mathrm{y}}_2 - \bar{\mathrm{y}}_1\right]^2\right\}$$

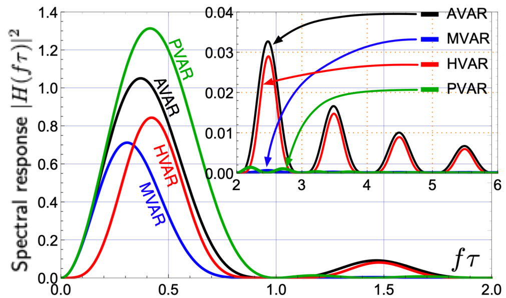
$\mathrm{y}(t) \to \bar{\mathrm{y}}$ averaged over τ
$\bar{\mathrm{y}}_2$ and $\bar{\mathrm{y}}_1$ are contiguous

Bare mean $\bar{\mathrm{y}} \to$ Allan variance AVAR
Weighted averages $\to$ MVAR, PVAR, etc.

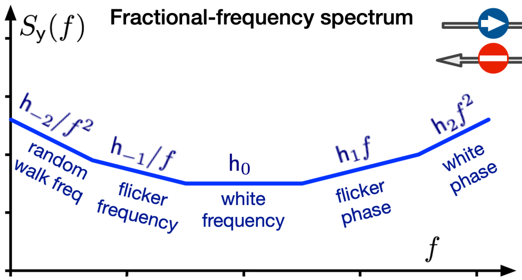
Evaluation

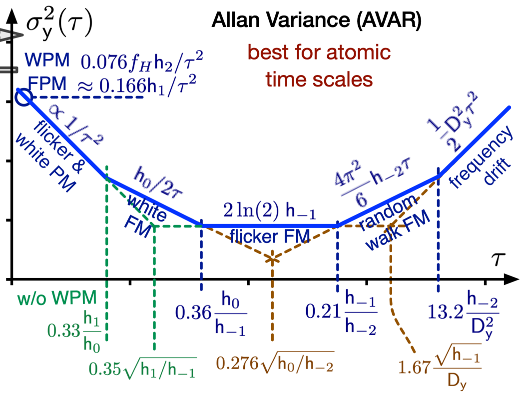
$$\sigma^2_{\mathrm{y}}(\tau) = \frac{1}{2(M-1)}\sum_{k=1}^{M-1}\left[\bar{\mathrm{y}}_{k+1} - \bar{\mathrm{y}}_k\right]^2$$

M contiguous samples of $\bar{\mathrm{y}}$



Frequency fluctuation PSD <—> Allan Variance

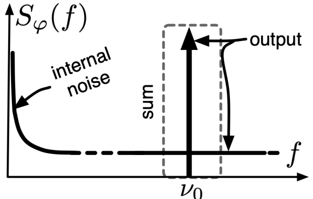




DOI: 10.5281/zenodo.4399218 (latest version)

ORCID 0000-0002-5364-1835

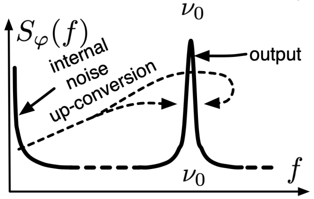
Last update
June 8, 2021



Additive Noise

RF noise added to the carrier

Statistically independent AM & PM

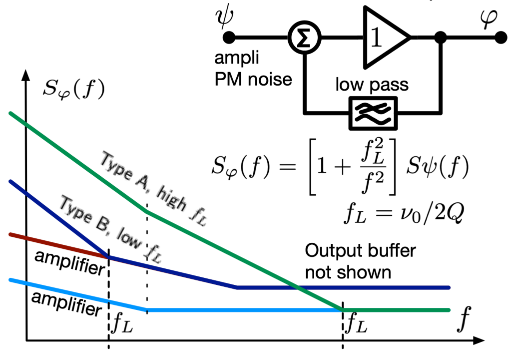


Parametric Noise

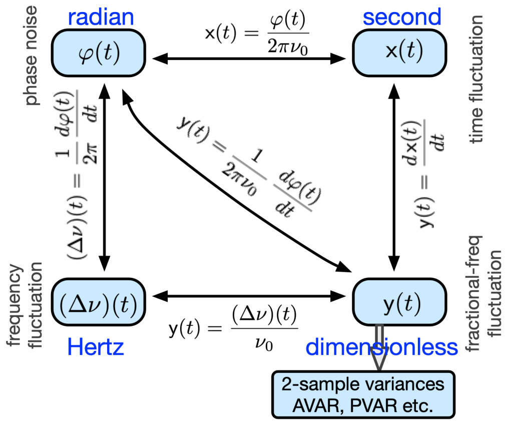
Near-dc noise modulates the carrier

AM & PM related and narrowband

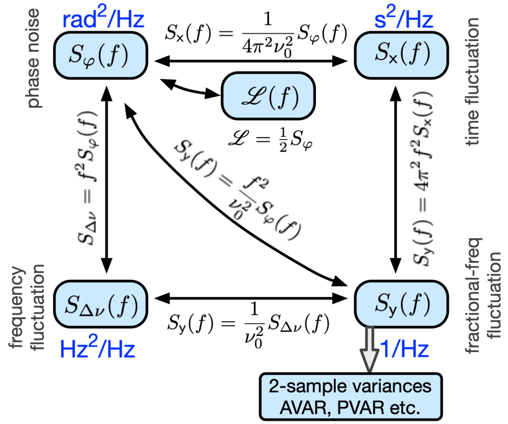
Leeson effect



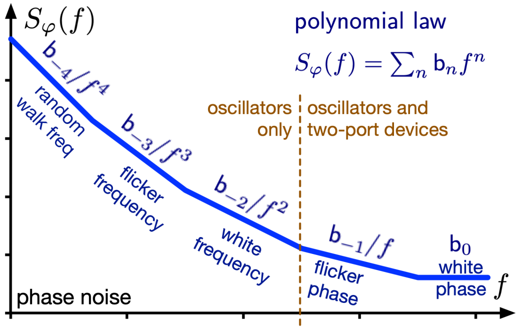
Time Domain

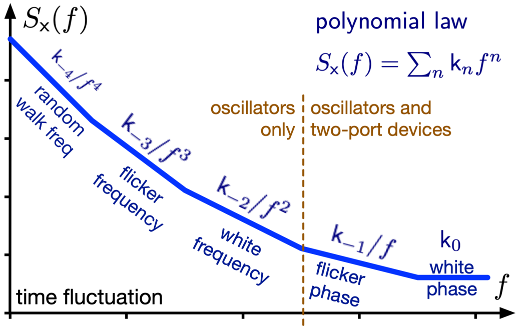


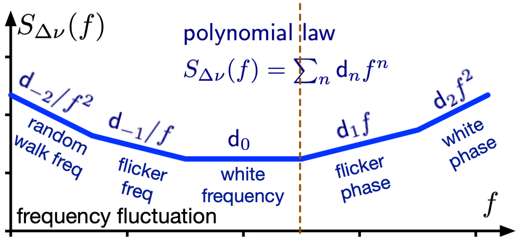
Frequency Domain

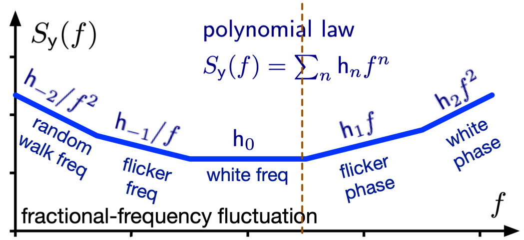


Spectra and Polynomial Law









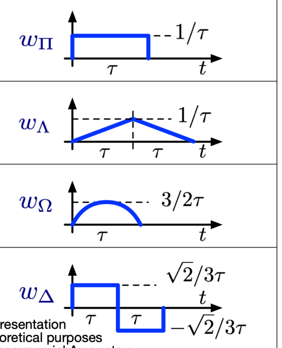
Enrico's Chart of Phase Noise and Two-Sample Variances

Enrico Rubiola – http://rubiola.org
European Frequency and Time Seminar – http://efts.eu
Oscillator Instability Measurement Platform http://oscillator-imp.com

Frequency Counter	Wavelet Variance
$\bar{y}(\tau) = \int_{\mathbb{R}} y(t)\, w(t;\tau)\, dt$	$\sigma_y^2(\tau) = \mathbb{E}\left\{\frac{1}{2}\left[\bar{y}_2 - \bar{y}_1\right]^2\right\} = \mathbb{E}\left\{\int_{\mathbb{R}}\left[y(t)\, w(t;\tau)\right]^2 dt\right\}$

type of frequency counter

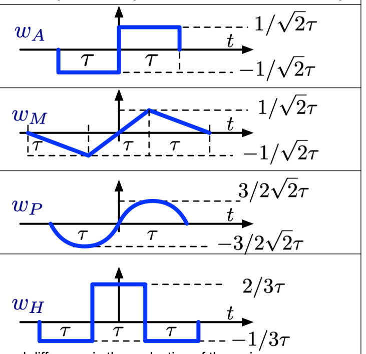
- Π — w_Π
- Λ — w_Λ
- Ω — w_Ω
- (Δ) — w_Δ



Note: this representation is only for theoretical purposes
There are no commercial Δ counters

associated variance

- AVAR ${}^A\sigma_y^2(\tau)$ — w_A
- MVAR ${}^H\sigma_y^2(\tau)$ — w_M
- PVAR ${}^P\sigma_y^2(\tau)$ — w_P
- HVAR ${}^H\sigma_y^2(\tau)$ — w_H

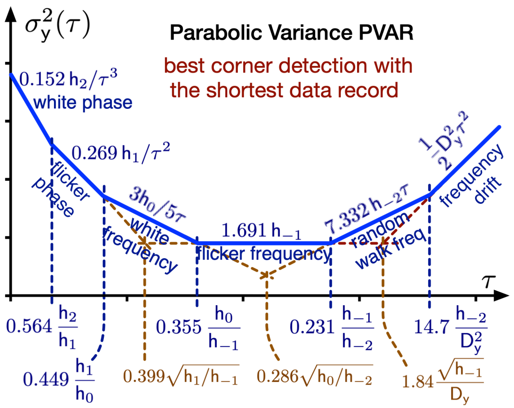


Note: the wavelet representation hides the second difference in the evaluation of the variance

Other Two-Sample Variances

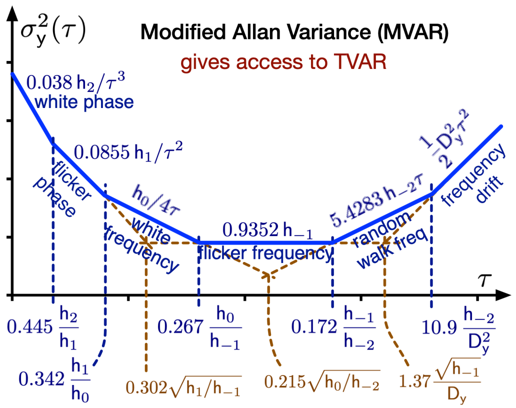
Parabolic Variance PVAR

best corner detection with the shortest data record



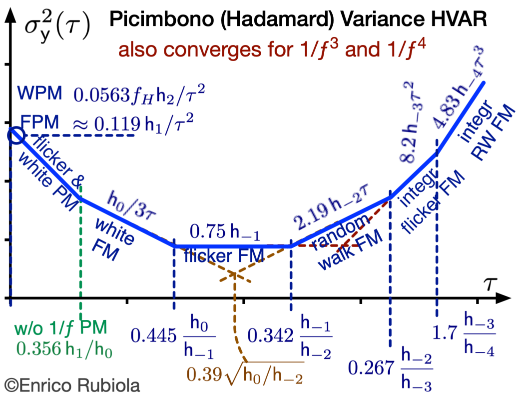
Modified Allan Variance (MVAR)

gives access to TVAR



Picimbono (Hadamard) Variance HVAR

also converges for $1/f^3$ and $1/f^4$

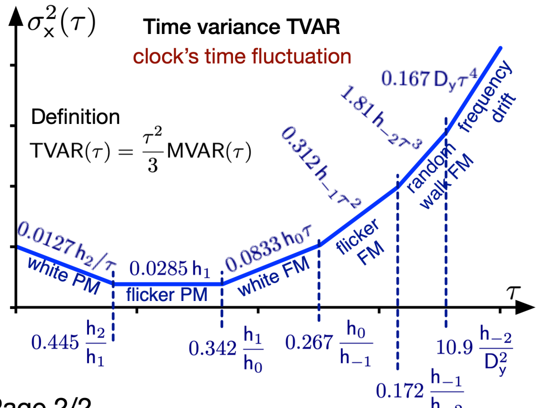


Time variance TVAR

clock's time fluctuation

Definition

$$\text{TVAR}(\tau) = \frac{\tau^2}{3}\text{MVAR}(\tau)$$



Spectra to Variances Conversion

noise type	$S_y(f)$	AVAR ${}^A\sigma_y^2(\tau)$	MVAR ${}^M\sigma_y^2(\tau)$	HVAR ${}^H\sigma_y^2(\tau)$	PVAR ${}^P\sigma_y^2(\tau)$	TVAR ${}^T\sigma_x^2(\tau)$
white PM	$h_2 f^2$	$\frac{3f_H}{4\pi^2}\frac{h_2}{\tau^2}$ $0.0760\, f_H h_2/\tau^2$	$\frac{3}{8\pi^2}\frac{h_2}{\tau^3}$ $0.0380\, h_2/\tau^3$	$\frac{5f_H}{9\pi^2}\frac{h_2}{\tau^2}$ $0.0563\, f_H h_2/\tau^2$	$\frac{3}{2\pi^2}\frac{h_2}{\tau^3}$ $0.1520\, h_2/\tau^3$	$\frac{1}{8\pi^2}\frac{h_2}{\tau}$ $0.0127\, h_2/\tau$
flicker PM	$h_1 f$	$\frac{3\gamma - \ln 2 + 3\ln(2\pi f_H\tau)}{4\pi^2}\frac{h_1}{\tau^2}$ $[3\gamma - \ln 2 + 3\ln 2\pi]/4\pi^2 = 0.166$	$\frac{(24\ln 2 - 9\ln 3)}{8\pi^2}\frac{h_1}{\tau^2}$ $0.0855\, h_1/\tau^2$	$\simeq \frac{5[\gamma + \ln(\sqrt[10]{48}\,\pi f_H\tau)]}{9\pi^2}\frac{h_1}{\tau^2}$ $5[\gamma + \ln(\sqrt[10]{48}\,\pi)]/9\pi^2 = 0.1187$	$\frac{3[\ln(16) - 1]}{2\pi^2}\frac{h_1}{\tau^2}$ $0.2694\, h_1/\tau^2$	$\frac{(8\ln 2 - 3\ln 3)}{8\pi^2} h_1$ $0.0285\, h_1$
white FM	h_0	$\frac{1}{2}\frac{h_0}{\tau}$	$\frac{1}{4}\frac{h_0}{\tau}$	$\frac{1}{3}\frac{h_0}{\tau}$	$\frac{3}{5}\frac{h_0}{\tau}$	$\frac{1}{12}h_0\tau$
flicker FM	$h_{-1}f^{-1}$	$2\ln(2)\, h_{-1}$ $1.3863\, h_{-1}$	$\frac{27\ln 3 - 32\ln 2}{8} h_{-1}$ $0.9352\, h_{-1}$	$\frac{8\ln 2 - 3\ln 3}{3} h_{-1}$ $0.7498\, h_{-1}$	$\frac{2[7 - \ln(16)]}{5} h_{-1}$ $1.691\, h_{-1}$	$\frac{27\ln 3 - 32\ln 2}{24} h_{-1}\tau^2$ $0.312\, h_{-1}\tau^2$
random walk FM	$h_{-2}f^{-2}$	$\frac{2\pi^2}{3} h_{-2}\tau$ $6.5797\, h_{-2}\tau$	$\frac{11\pi^2}{20} h_{-2}\tau$ $5.4283\, h_{-2}\tau$	$\frac{2\pi^2}{9} h_{-2}\tau$ $2.1933\, h_{-2}\tau$	$\frac{26\pi^2}{35} h_{-2}\tau$ $7.3317\, h_{-2}\tau$	$\frac{11\pi^2}{60} h_{-2}\tau^3$ $1.81\, h_{-2}\tau^3$
integrated flicker FM	$h_{-3}f^{-3}$	not converging	not converging	$\frac{\pi^2[27\ln(3) - 32\ln(2)]}{9} h_{-3}\tau^2$ $8.205\, h_{-3}\tau^2$	not converging	not converging
integrated RW FM	$h_{-4}f^{-4}$	not converging	not converging	$\frac{44\pi^2}{90} h_{-4}\tau^3$ $4.825\, h_{-4}\tau^3$	not converging	not converging
linear drift D_y		$\frac{1}{2}D_y^2\tau^2$	$\frac{1}{2}D_y^2\tau^2$	0	$\frac{1}{2}D_y^2\tau^2$	$\frac{1}{6}D_y^2\tau^2$
spectral response $\lvert H(\theta)\rvert^2$, $\theta = \pi f\tau$		$\frac{2\sin^4(\theta)}{\theta^2}$	$\frac{2\sin^6(\theta)}{\theta^4}$	$\frac{16\sin^6(\theta)}{9\theta^2}$	$\frac{9\left[2\sin^2(\theta) - \theta\sin(2\theta)\right]^2}{2\theta^6}$	$\frac{\tau^2}{3}\frac{2\sin^6(\pi f\tau)}{(\pi f\tau)^4}$
MVAR, PVAR and TVAR formulas need $\tau > 1/f_H$, where $f_H < 1/2\tau_0$ is the cutoff frequency, and τ_0 is the sampling interval.						$\sigma_x(\tau) = \frac{\tau^2}{3}\, {}^M\sigma_y(\tau)$