SINGLE VALUED NEUTROSOPHIC GRAPHS

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Abstract - The notion of single valued neutrosophic sets is a generalization of fuzzy sets, intuitionistic fuzzy sets. We apply the concept of single valued neutrosophic sets, an instance of neutrosophic sets, to graphs. We introduce certain types of single valued neutrosophic graphs (SVNG) and investigate some of their properties with proofs and examples.

1. Introduction

Neutrosophic sets (NSs) proposed by Smarandache [12, 13] is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. they are a generalization of the theory of fuzzy sets [24], intuitionistic fuzzy sets [21, 23] and interval valued intuitionistic fuzzy sets [22]. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity-membership function (f) independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. In order to practice NS in real life applications conveniently, Wang et al.[16] introduced the concept of a single-valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets. The SVNS is a generalization of intuitionistic fuzzy sets, in which three membership functions are independent and their value belong to the unit interval [0, 1]. Some more work on single valued neutrosophic sets and their extensions may be found on [2, 3, 4, 5,15, 17, 19, 20, 27, 28, 29, 30].

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. Lots of works on fuzzy graphs and intuitionistic fuzzy graphs [6, 7, 8, 25, 27] have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes(or vertices) in problems are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs are failed. For this purpose, Samarandache [9, 10, 11, 14, 34] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), which called them; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and has gained popularity among the researchers due to its applications via real world problems [1, 33, 35]. The two others graphs are based on (t, i, f) components and called them; The (t, i, f)-Edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, these concepts are not developed at all. In the literature the study of single valued neutrosophic graphs (SVN-graph) is still blank, we shall focus on the study of single valued neutrosophic graphs in this paper.

In this paper, some certain types of single valued neutrosophic graphs are developed and some interesting properties are explored.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, fuzzy graph and intuitionistic fuzzy graph relevant to the present work. See especially [6, 7, 12, 13, 16] for further details and background.

Definition 2.1 [12]. Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$$

where the functions T, I, F: $X \rightarrow]^-0,1^+[$ define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+ \tag{1}$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0,1^+[$.

Since it is difficult to apply NSs to practical problems, Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [16]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership

function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\} \qquad (2)$$

Definition 2.3 [6]. A fuzzy graph is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. i.e $\sigma : V \rightarrow [0,1]$ and $\mu: VxV \rightarrow [0,1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all u, v $\in$ V where uv denotes the edge between u and v and $\sigma(u) \wedge \sigma(v)$ denotes the minimum of $\sigma(u)$ and $\sigma(v)$. σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.
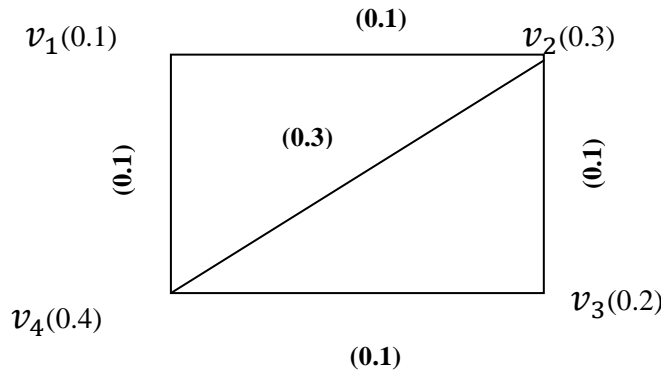

Figure 1: Fuzzy Graph

Definition 2.4 [6]. The fuzzy subgraph $H = (\tau, \rho)$ is called a fuzzy subgraph of $G = (\sigma, \mu)$ If $\tau(u) \leq \sigma(u)$ for all u $\in$ V and $\rho(u, v) \leq \mu(u, v)$ for all u, v $\in$ V.

Definition 2.5 [7]. An Intuitionistic fuzzy graph is of the form G =(V, E) where

i. V=$\{v_1, v_2,...., v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\gamma_1: V \rightarrow [0,1]$ denote the degree of membership and nonmembership of the element $v_i \in V$, respectively, and $0 \leq \mu_1(v_i) + \gamma_1(v_i)) \leq 1$ for every $v_i \in V$, (i = 1, 2, ……. n),

ii. E $\subseteq$ V x V where $\mu_2: VxV \rightarrow [0,1]$ and $\gamma_2: VxV \rightarrow [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, (i, j = 1,2, ……. n)
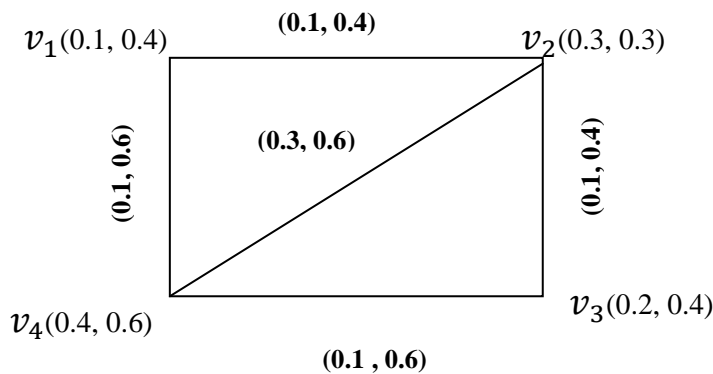

Figure 2: Intuitionistic Fuzzy Graph

Definition 2.6 [31]. Let $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X. If $A = (T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then $A = (T_A, I_A, F_A)$ is called a single valued neutrosophic relation on $B = (T_B, I_B, F_B)$ if

T$_B$(x, y) $\leq$ min(T$_A$(x), T$_A$(y))
I$_B$(x, y) $\geq$ max(I$_A$(x), I$_A$(y)) and
F$_B$(x, y) $\geq$ max(F$_A$x), F$_A$(y)) for all x, y $\in$ X.

A single valued neutrosophic relation A on X is called symmetric if

T_A(x, y) = T_A(y, x),
I_A(x, y) = I_A(y, x), F_A(x, y) = F_A(y, x) and
T_B(x, y) = T_B(y, x), I_B(x, y) = I_B(y, x) and
F_B(x, y) = F_B(y, x) for all $x, y \in X$.

3. Single Valued Neutrosophic Graphs

Throught this paper, we denote $G^* = $ (V, E) a crisp graph, and G =(A, B) a single valued neutrosophic graph.

Definition 3.1. A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B) where

1.The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in$ V, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in V \text{ (i=1, 2, ...,n)}$$

2. The functions T_B: E $\subseteq$ V x V →[0, 1], I_B:E $\subseteq$ V x V →[0, 1] and F_B: E $\subseteq$ V x V →[0, 1] are defined by

$T_B(\{v_i, v_j\}) \leq$ min [$T_A(v_i), T_A(v_j)$],
$I_B(\{v_i, v_j\}) \geq$ max [$I_A(v_i), I_A(v_j)$] and
$F_B(\{v_i, v_j\}) \geq$ max [$F_A(v_i), F_A(v_j)$]

Denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3 \text{ for all } \{v_i, v_j\} \in E \text{ (i, j = 1, 2,..., n)}$$

We call A the single valued neutrosophic vertex set of V, B the single valued neutrosophic edge set of E, respectively, Note that B is a symmetric single valued neutrosophic relation on A. We use the notation (v_i, v_j) for an element of E Thus, G = (A, B) is a single valued neutrosophic graph of G*= (V, E) if
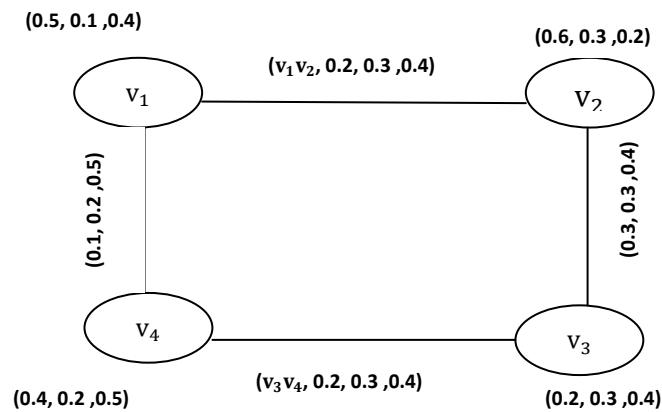
$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)]$,
$I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)]$ and
$F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)]$ for all $(v_i, v_j) \in E$

Example 3.2. Consider   a graph G^* such that V= $\{v_1, v_2, v_3, v_4\}$, E=$\{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be a single valued neutrosophic subset of V and let B a single valued neutrosophic subset of E denoted by

	v_1	v_2	v_3	v_4
T_A	0.5	0.6	0.2	0.4
I_A	0.1	0.3	0.3	0.2
F_A	0.4	0.2	0.4	0.5

	v_1v_2	v_2v_3	v_3v_4	v_4v_1
T_B	0.2	0.3	0.2	0.1
I_B	0.3	0.3	0.3	0.2
F_B	0.4	0.4	0.4	0.5

Figure 3: G: Single valued neutrosophic graph

In figure 3, (i)   (v$_1$,0.5, 0.1,0.4) is a single valued neutrosophic vertex or SVN-vertex

(ii) (v$_1$v$_2$, 0.2, 0.3, 0.4) is a single valued neutrosophic edge or SVN-edge

(iii) (v$_1$, 0.5, 0.1, 0.4) and (v$_2$, 0.6, 0.3, 0.2) are single valued neutrosophic adjacent vertices.

(iv) (v$_1$v$_2$, 0.2, 0.3, 0.4) and (v$_1$v$_4$, 0.1, 0.2, 0.5)   are a single valued neutrosophic adjacent edge.

Note 1. (i) When $T_{Bij} = I_{Bij} = F_{Bij}$ for some i and j, then there is no edge between v$_i$ and v$_j$.
Otherwise there exists an edge between v$_i$ and v$_j$.
(ii)If one of the inequalities is not satisfied in (1) and (2), then G is not an SVNG

The single valued neutrosophic graph G depicted in figure 3 is represented by the following adjacency matrix M_G

$$M_G = \begin{bmatrix} (0.5, 0.1, 0.4) & (0.2, 0.3, 0.4) & (0, 0, 0) & (0.1, 0.2, 0.5) \\ (0.2, 0.3, 0.4) & (0.6, 0.3, 0.2) & (0.3, 0.3, 0.4) & (0, 0, 0) \\ (0, 0, 0) & (0.3, 0.3, 0.4) & (0.2, 0.3, 0.4) & (0.2, 0.3, 0.4) \\ (0.1, 0.2, 0.5) & (0, 0, 0) & (0.2, 0.3, 0.4) & (0.4, 0.2, 0.5) \end{bmatrix}$$
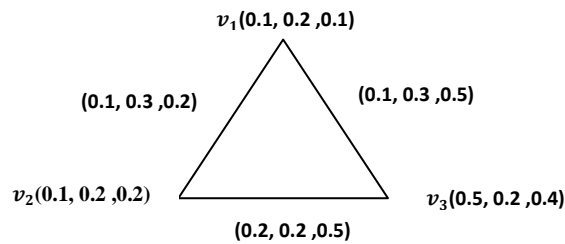
Definition 3.3. A partial SVN-subgraph of SVN-graph G= (A, B) is a SVN-graph H = (V', E') such that

(i) $V' \subseteq V$, where $T'_{Ai} \leq T_{Ai}$, $I'_{Ai} \geq I_{Ai}$, $F'_{Ai} \geq F_{Ai}$ for all $v_i \in V$.
(ii) $E' \subseteq E$, where $T'_{Bij} \leq T_{Bij}$, $I'_{Bij} \geq I_{Bij}$, $F'_{Bij} \geq F_{Bij}$ for all $(v_i v_j) \in E$.

Definition 3.4. A SVN-subgraph of SVN-graph G= (V, E) is a SVN-graph H = (V', E') such that

(i) $V' = V$, where $T'_{Ai} = T_{Ai}$, $I'_{Ai} = I_{Ai}$, $F'_{Ai} = F_{Ai}$ for all v_i in the vertex set of V'.
(ii) $E' = E$, where $T'_{Bij} = T_{Bij}$, $I'_{Bij} = I_{Bij}$, $F'_{Bij} = F_{Bij}$ for every $(v_i v_j) \in E$ in the edge set of E'.
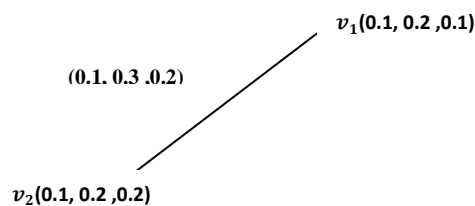
Example 3.5. G_1 in Figure 4 is a SVN-graph . H_1 in Figure 5 is a partial SVN-subgraph and

H_2 in Figure 6 is a SVN-subgraph of G_1

Figure 4: G_1, a single valued neutrosophic graph
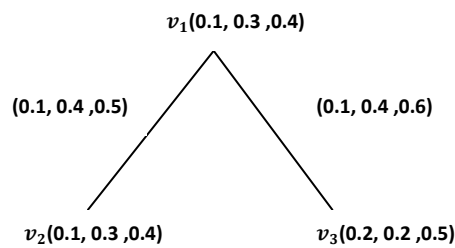
Figure 5: H_1, a partial SVN-subgraph of G_1

Figure 6: H_2, a SVN-subgraph of G_1.

Definition 3.6. The two vertices are said to be adjacent in a single valued neutrosophic graph G= (**A**, **B**) if $T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)]$, $I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)]$ and $F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)]$. In this case, v_i and v_j are said to be neighbours and (v_i, v_j) is incident at v_i and v_j also.

Definition 3.7. A path P in a single valued neutrosophic graph G= (**A**, **B**) is a sequence of distinct vertices v_0, v_1, v_3,... v_n such that $T_B(v_{i-1}, v_i) > 0$, $I_B(v_{i-1}, v_i) > 0$ and $F_B(v_{i-1}, v_i) > 0$ for $0 \leq i \leq 1$. Here n ≥ 1 is called the length of the path P. A single node or vertex v_i may also be considered as a path. In this case the path is of the length (0, 0, 0). The consecutive pairs (v_{i-1}, v_i) are called edges of the path. We call P a cycle if $v_0 = v_n$ and n≥3.

Definition 3.8. A single valued neutrosophic graph G= (A, B) is said to be connected if every pair of vertices has at least one single valued neutrosophic path between them, otherwise it is disconnected.

Definition 3.9. A vertex v_j ∈ V of single valued neutrosophic graph G= (**A**, **B**) is said to be an isolated vertex if there is no effective edge incident at v_j.

v_1(0.1, 0.2 ,0.1) v_4(0.1, 0.3 ,0.5)

(0.1, 0.3 ,0.2)   (0.1, 0.3 ,0.5)

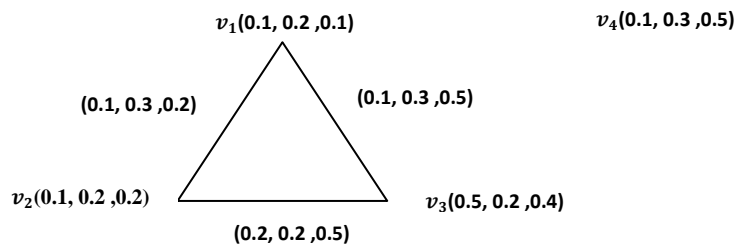v_2(0.1, 0.2 ,0.2) v_3(0.5, 0.2 ,0.4)

(0.2, 0.2 ,0.5)

Figure 7: Example of single valued neutrosophic graph

In figure 7, the single valued neutrosophic vertex v_4 is an isolated vertex.

Definition 3.10. A vertex in a single valued neutrosophic G= (**A**, **B**) having exactly one neighbor is called *a pendent vertex*. Otherwise, it is called *non-pendent vertex*. An edge in a single valued neutrosophic graph incident with a pendent vertex is called a *pendent edge*. Otherwise it is called *non-pendent edge*. A vertex in a single valued neutrosophic graph adjacent to the pendent vertex is called **a support** of the pendent edge

(0.5, 0.1 ,0.4)

(0.6, 0.3 ,0.2)   (0.2, 0.2 ,0.3)

(0.2, 0.3 ,0.4)

v_1 v_2 v_5

(0.1, 0.2 ,0.5)   (0.3, 0.3 ,0.4)

(0.2, 0.4 ,0.5)

v_4 v_3

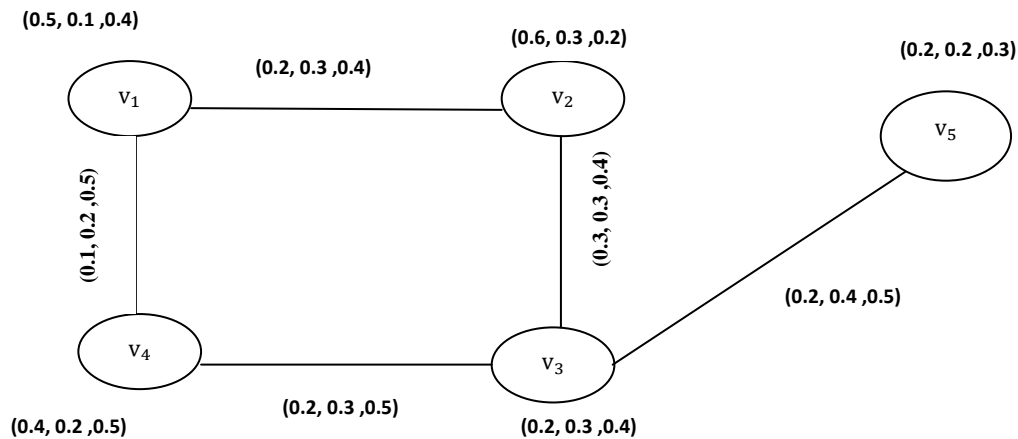(0.2, 0.3 ,0.5)

(0.4, 0.2 ,0.5)   (0.2, 0.3 ,0.4)

Figure 8 : Incident SVN-graph.

Definition 3.11. A single valued neutrosophic graph G= (A, B) that has neither self loops nor parallel edge is called **simple single valued neutrosophic graph.**

Definition 3.12. When a vertex $\mathbf{v_i}$ is end vertex of some edges $(\mathbf{v_i}, \mathbf{v_j})$ of any SVN-graph G= (A, B). Then $\mathbf{v_i}$ and $(\mathbf{v_i}, \mathbf{v_j})$ are said to be **incident** to each other. **In Figure 8, $\mathbf{v_2 v_3}$, $\mathbf{v_3 v_4}$ and $\mathbf{v_3 v_5}$ are incident on $\mathbf{v_3}$.**
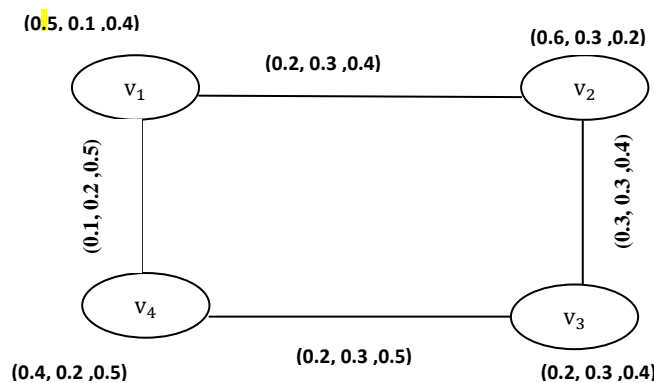
Definition 3.13. Let G= (**A**, **B**) be a single valued neutrosophic graph. Then the degree of any vertex **v** is sum of degree of truth-membership, sum of degree of indeterminacy-membership and sum of degree of falsity-membership of all those edges which are incident on vertex **v** denoted by d(v)= $(d_T(v), d_I(v), d_F(v))$ where

$d_T(v)=\sum_{u\neq v} T_B(u,v)$ denotes degree of truth-membership vertex.
$d_I(v)=\sum_{u\neq v} I_B(u,v)$ denotes degree of indeterminacy-membership vertex.
$d_F(v)=\sum_{u\neq v} F_B(u,v)$ denotes degree of falsity-membership vertex.

Example 3.14. Let us consider a single valued neutrosophic graph G= (A, B) of $G^* = $ (V, E) where V ={v_1, v_2, v_3, v_4} and E={$v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_1$}.

Figure 9: Degree of vertex of single valued neutrosophic graph

We have, the degree of each vertex as follows:

$d(v_1)= (0.3, 0.5, 0.9), d(v_2)= (0.5, 0.6, 0.8), d(v_3)= (0.5, 0.6, 0.9), d(v_4)= (0.3, 0.5, 1)$

Definition 3.15 . A single valued neutrosophic graph G= (A, B) is called constant if degree of each vertex is k =(k_1, k_2, k_3). That is, d$(v) = (k_1, k_2, k_3)$ for all $v \in$ V.
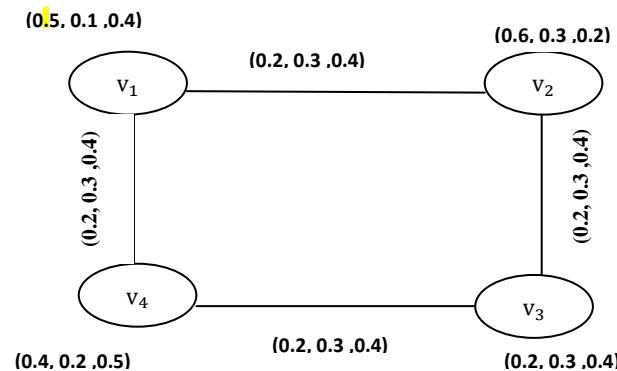
Figure 10: Constant SVN-graph.

Example 3.16. Consider a single valued neutrosophic graph G such that V ={v_1, v_2, v_3, v_4} and E={v_1v_2, v_2v_3, v_3v_4, v_4v_1}.

Clearly, as it is seen in Figure 10, G is constant SVN-graph since the degree of **v_1, v_2, v_3** and **v_4** is (0.4, 0.6, 0.8).

Definition 3.17. A single valued neutrosophic graph G=(A, B) of G^*= (V, E) is called strong single valued neutrosophic graph if

$T_B(v_i, v_j) = \min [T_A(v_i),\ T_A(v_j)]$
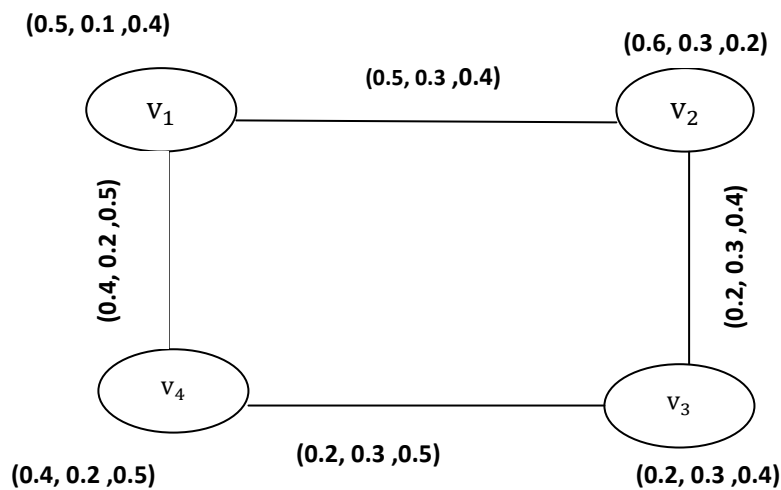$I_B(v_i, v_j) = \max [I_A(v_i),\ I_A(v_j)]$
$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)]$
For all $(v_i, v_j) \in$ E.

Example 3.18. Consider a graph G^* such that V= {v_1, v_2, v_3, v_4}, E={v_1v_2, v_2v_3, v_3v_4, v_4v_1}. Let A be a single valued neutrosophic subset of V and let B a single valued neutrosophic subset of E denoted by

	v_1	v_2	v_3	v_4
T_A	0.5	0.6	0.2	0.4
I_A	0.1	0.3	0.3	0.2
F_A	0.4	0.2	0.4	0.5

	v_1v_2	v_2v_3	v_3v_4	v_4v_1
T_B	0.5	0.2	0.2	0.4
I_B	0.3	0.3	0.3	0.2
F_B	0.4	0.4	0.5	0.5

Figure 11: Strong SVN-graph

By routing computations, it is easy to see that G is a strong single valued neutrosophic of G^*.

Proposition 3.19. A single valued neutrosophic graph is the generalization of fuzzy graph

Proof: Suppose G= (V, E) be a single valued neutrosophic graph. Then by setting the indeterminacy- membership and falsity- membership values of vertex set and edge set equals to zero reduces the single valued neutrosophic graph to fuzzy graph.

Proposition 3.20. A single valued neutrosophic graph is the generalization of intuitionistic fuzzy graph.

Proof: Suppose G=(V, E) be a single valued neutrosophic graph. Then by setting the indeterminacy- membership value of vertex set and edge set equals to zero reduces the single valued neutrosophic graph to intuitionistic fuzzy graph.

Definition 3.21. The complement of a single valued neutrosophic graph G (A, B) on G^* is a single valued neutrosophic graph $\bar{G}$ on G^* where:

1. $\bar{A}$ =A
2. $\overline{T_A}(v_i)$= $T_A(v_i)$, $\overline{I_A}(v_i)$= $I_A(v_i)$, $\overline{F_A}(v_i)$ = $F_A(v_i)$, for all $v_j \in$ V.
3. $\overline{T_B}(v_i, v_j)$= min $\left[T_A(v_i), T_A(v_j)\right]$ -$T_B(v_i, v_j)$

$\overline{I_B}(v_i, v_j)$= max $\left[I_A(v_i), I_A(v_j)\right]$ -$I_B(v_i, v_j)$ and

$\overline{F_B}(v_i, v_j)$= max $\left[F_A(v_i), F_A(v_j)\right]$ -$F_B(v_i, v_j)$, For all $(v_i, v_j) \in$ E

Remark 3.22. if G= (V, E) is a single valued neutrosophic graph on G^*. Then from above definition, it follow that $\bar{\bar{G}}$ is given by the single valued neutrosophic graph $\bar{\bar{G}}$ =($\bar{\bar{V}}$, $\bar{\bar{E}}$) on G^* where

$\bar{\bar{V}}$ =V and $\overline{\overline{T_B}}(v_i, v_j)$= min $\left[T_A(v_i), T_A(v_j)\right]$-$T_B(v_i, v_j)$,

$\overline{\overline{I_B}}(v_i, v_j)$= min $\left[I_A(v_i), I_A(v_j)\right]$-$I_B(v_i, v_j)$,and

$\overline{\overline{F_B}}(v_i, v_j)$ = min $\left[F_A(v_i), F_A(v_j)\right]$-$F_B(v_i, v_j)$ For all $(v_i, v_j) \in$ E.

Thus $\overline{\overline{T_B}}$ =T_B, $\overline{\overline{I_B}}$ =I_B, and $\overline{\overline{F_B}}$ =F_B on V, where E =(T_B, I_B, F_B) is the single valued neutrosophic relation on V. For any single valued neutrosophic graph G, $\bar{G}$ is strong single valued neutrosophic graph and G ⊆ $\bar{G}$.

Proposition 3.23. G= $\bar{\bar{G}}$ if and only if G is a strong single valued neutrosophic graph.
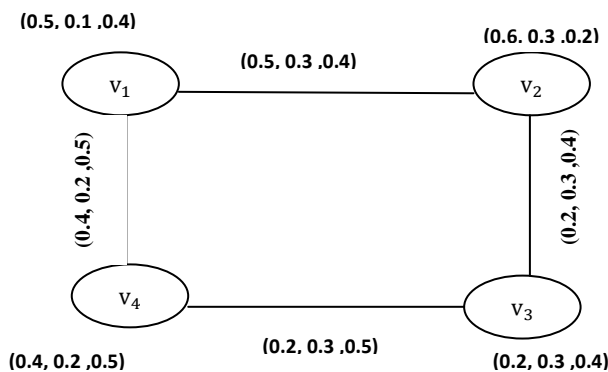
Proof. it is obvious.

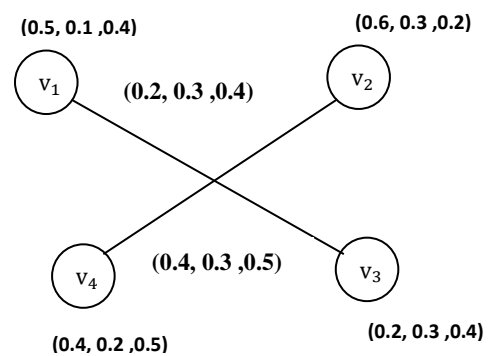**Figure 12:** G: Strong SVN- graph   **Figure 13:** $\bar{G}$ Strong SVN- graph
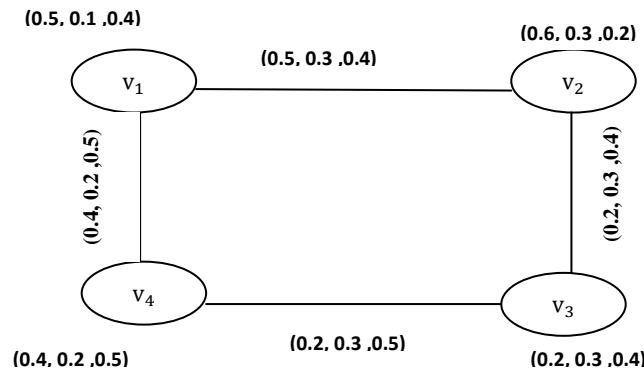
(0.5, 0.1 ,0.4)   (0.6, 0.3 ,0.2)

v_1 —— (0.5, 0.3 ,0.4) —— v_2

(0.4, 0.2 ,0.5)   (0.2, 0.3 ,0.4)

v_4 v_3

(0.4, 0.2 ,0.5)   (0.2, 0.3 ,0.5)   (0.2, 0.3 ,0.4)

Figure 14: $\overline{\overline{G}}$ Strong SVN- graph

Definition 3.24. A strong single valued neutrosophic graph G is called self complementary if G≅ $\overline{G}$. Where $\overline{G}$ is the complement of single valued neutrosophic graph G.

Example 3.25. Consider a graph $G^* = (V, E)$ such that V ={v_1, v_2, v_3, v_4}, E={v_1v_2, v_2v_3, v_3v_4, v_1v_4}. Consider a single valued neutrosophic graph G as in Figure 12 and 13,

Clearly, as it is seen in Figure 14, G≅ $\overline{\overline{G}}$. Hence G is self complementary.

Proposition 3.26. Let G=(A, B) be a ***strong*** single valued neutrosophic graph. If

$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)]$,
$I_B(v_i, v_j) = \max[I_A(v_i), I_A(v_j)]$ and
$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)]$ for all $v_i, v_j \in$ V.

Then G is self complementary.

Proof. Let G=(A, B) be a strong single valued neutrosophic graph such that

$T_B(v_i, v_j) = \min [T_A(v_i), T_A(v_j)]$
$I_B(v_i, v_j) = \max [I_A(v_i), I_A(v_j)]$
$F_B(v_i, v_j) = \max [F_A(v_i), F_A(v_j)]$

For all $v_i, v_j \in$ V. Then G $\approx$ $\overline{\overline{G}}$ under the identity map *I*: V $\rightarrow$ V. Hence G is self complementary .

Proposition 3.27. Let G be a self complementary single valued neutrosophic graph. Then

$\sum_{v_i \neq v_j} T_B(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \min [T_A(v_i), T_A(v_j)]$
$\sum_{v_i \neq v_j} I_B(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max [I_A(v_i), I_A(v_j)]$
$\sum_{v_i \neq v_j} F_B(v_i, v_j) = \frac{1}{2} \sum_{v_i \neq v_j} \max [F_A(v_i), F_A(v_j)]$

Proof. If G be a self complementary single valued neutrosophic graph. Then there exist an isomorphism $f: V_1 \rightarrow V_1$ satisfying

$$\overline{T_{V_1}}(f(v_i)) = T_{V_1}(f(v_i)) = T_{V_1}(v_i)$$
$$\overline{I_{V_1}}(f(v_i)) = I_{V_1}(f(v_i)) = I_{V_1}(v_i)$$
$$\overline{\overline{F_{V_1}}}(f(v_i)) = F_{V_1}(f(v_i)) = F_{V_1}(v_i) \quad \text{for all } v_i \in V_1.$$

And

$$\overline{T_{E_1}}(f(v_i), f(v_j)) = T_{E_1}(f(v_i), f(v_j)) = T_{E_1}(v_i, v_j)$$
$$\overline{I_{E_1}}(f(v_i), f(v_j)) = I_{E_1}(f(v_i), f(v_j)) = I_{E_1}(v_i, v_j)$$
$$\overline{F_{E_1}}(f(v_i), f(v_j)) = F_{E_1}(f(v_i), f(v_j)) = F_{E_1}(v_i, v_j) \quad \text{for all } (v_i, v_j) \in E_1$$

We have

$$\overline{T_{E_1}}(f(v_i), f(v_j)) = \min[\overline{T_{V_1}}(f(v_i)), \overline{T_{V_1}}(f(v_j))] - T_{E_1}(f(v_i), f(v_j))$$
i.e, $T_{E_1}(v_i, v_j) = \min[T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(f(v_i), f(v_j))$
$$T_{E_1}(v_i, v_j) = \min[T_{V_1}(v_i), T_{V_1}(v_j)] - T_{E_1}(v_i, v_j)$$

That is

$$\sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \min[T_{V_1}(v_i), T_{V_1}(v_j)]$$
$$\sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max[I_{V_1}(v_i), I_{V_1}(v_j)]$$
$$\sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) + \sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max[F_{V_1}(v_i), F_{V_1}(v_j)]$$
$$2\sum_{v_i \neq v_j} T_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \min[T_{V_1}(v_i), T_{V_1}(v_j)]$$
$$2\sum_{v_i \neq v_j} I_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max[I_{V_1}(v_i), I_{V_1}(v_j)]$$
$$2\sum_{v_i \neq v_j} F_{E_1}(v_i, v_j) = \sum_{v_i \neq v_j} \max[F_{V_1}(v_i), F_{V_1}(v_j)]$$

From these equations, Proposition 3.27 holds

Proposition 3.28. Let G_1 and G_2 be strong single valued neutrosophic graph, $\overline{G_1} \approx \overline{G_2}$ (isomorphism)

Proof. Assume that G_1 and G_2 are isomorphic, there exist a bijective map $f: V_1 \rightarrow V_2$ satisfying

$$T_{V_1}(v_i) = T_{V_2}(f(v_i)),$$
$$I_{V_1}(v_i) = I_{V_2}(f(v_i)),$$
$$F_{V_1}(v_i) = F_{V_2}(f(v_i)) \quad \text{for all } v_i \in V_1.$$

And

$$T_{E_1}(v_i, v_j) = T_{E_2}(f(v_i), f(v_j)),$$
$$I_{E_1}(v_i, v_j) = I_{E_2}(f(v_i), f(v_j)),$$
$$F_{E_1}(v_i, v_j) = F_{E_2}(f(v_i), f(v_j)) \quad \text{for all } (v_i, v_j) \in E_1$$

By definition 3.21, we have

$$\overline{T_{E_1}}(v_i, v_j) = \min \left[T_{V_1}(v_i), T_{V_1}(v_j) \right] - T_{E_1}(v_i, v_j)$$
$$= \min \left[T_{V_2}(f(v_i)), T_{V_2}(f(v_j)) \right] - T_{E_2}(f(v_i), f(v_j)),$$
$$= \overline{T_{E_2}}(f(v_i), f(v_j)),$$

$$\overline{I_{E_1}}(v_i, v_j) = \max \left[I_{V_1}(v_i), I_{V_1}(v_j) \right] - I_{E_1}(v_i, v_j)$$
$$= \max[I_{V_2}(f(v_i)), I_{V_2}(f(v_j))] - I_{E_2}(f(v_i), f(v_j)),$$
$$= \overline{I_{E_2}}(f(v_i), f(v_j)),$$

$$\overline{F_{E_1}}(v_i, v_j) = \min \left[F_{V_1}(v_i), F_{V_1}(v_j) \right] - F_{E_1}(v_i, v_j)$$
$$= \min \left[F_{V_2}(f(v_i)), F_{V_2}(f(v_j)) \right] - F_{E_2}(f(v_i), f(v_j)),$$
$$= \overline{F_{E_2}}(f(v_i), f(v_j)),$$

For all $(v_i, v_j) \in E_1$. Hence $\overline{G_1} \approx \overline{G_2}$. The converse is straightforward.

4. Complete Single Valued Neutrosophic Graphs

For the sake of simplicity we denote $T_A(v_i)$ by T_{Ai}, $I_A(v_i)$ by I_{Ai}, and $I_A(v_i)$ by I_{Ai}. Also $T_B(v_i, v_j)$ by T_{Bij}, $I_B(v_i, v_j)$ by I_{Bij} and $F_B(v_i, v_j)$ by F_{Bij}.

Definition 4.1. A single valued neutrosophic graph G= (A, B) is called complete if $T_{Bij} = \min(T_{Ai}, T_{Aj})$, $I_{Bij} = \max(I_{Ai}, I_{Aj})$ and $F_{Bij} = \max(F_{Ai}, F_{Aj})$ for all $v_i, v_j \in$ V.

Example 4.2. Consider a graph $G^* =$ (V, E) such that V $=\{ v_1, v_2, v_3, v_4 \}$, E$=\{ v_1 v_2, v_1 v_3, v_2 v_3, v_1 v_4, v_3 v_4, v_2 v_4 \}$. Then G= (A, B) is a complete single valued neutrosophic graph of G^*.
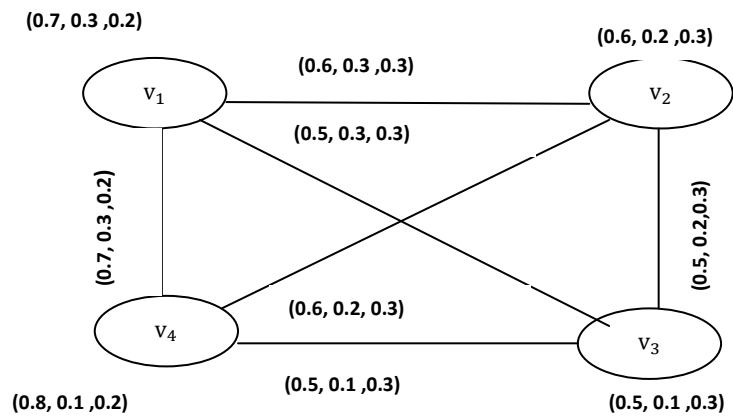
Figure 13: Complete single valued neutrosophic graph

Definition 4.3. The complement of a complete single valued neutrosophic graph G =(A, B) of $G^* =$ (V, E) is a single valued neutrosophic complete graph $\bar{G} = (\bar{A}, \bar{B})$ on $G^* = (V, \bar{E})$ where

1. $\bar{V} =$ V
2. $\overline{T_A}(v_i) = T_A(v_i)$, $\overline{I_A}(v_i) = I_A(v_i)$, $\overline{F_A}(v_i) = F_A(v_i)$, for all $v_j \in$ V.
3. $\overline{T_B}(v_i, v_j) = \min \left[T_A(v_i), T_A(v_j) \right] - T_B(v_i, v_j)$

$\overline{I_B}(v_i, v_j) = \max\left[I_A(v_i), I_A(v_j)\right] - I_B(v_i, v_j)$ and
$\overline{F_B}(v_i, v_j) = \max\left[F_A(v_i), F_A(v_j)\right] - F_B(v_i, v_j)$ for all $(v_i, v_j) \in E$

Proposition 4.4. The complement of complete SVN-graph is a SVN-graph with no edge. Or if G is a complete then in $\bar{G}$ the edge is empty.

Proof. Let G= (A, B) be a complete SVN-graph. So

$T_{Bij} = \min(T_{Ai}, T_{Aj})$, $I_{Bij} = \max(I_{Ai}, I_{Aj})$ and $F_{Bij} = \max(F_{Ai}, F_{Aj})$ for all $v_i, v_j \in V$

Hence in $\bar{G}$,

$\begin{aligned}
\bar{T}_{Bij} &= \min\left[T_{Ai}, T_{Aj}\right] - T_{Aij} \text{ for all i, j,.....,n} \\
&= \min\left[T_{Ai}, T_{Aj}\right] - \min\left[T_{Ai}, T_{Aj}\right] \text{ for all i, j,.....,n} \\
&= 0 \quad \text{for all i, j,.....,n}
\end{aligned}$

and

$\begin{aligned}
\bar{I}_{Bij} &= \max\left[I_{Ai}, I_{Aj}\right] - I_{Bij} \text{ for all i, j,.....,n} \\
&= \max\left[I_{Ai}, I_{Aj}\right] - \max\left[I_{Ai}, I_{Aj}\right] \text{ for all i, j,.....,n} \\
&= 0 \quad \text{for all i, j,.....,n}
\end{aligned}$

Also

$\begin{aligned}
\bar{F}_{Bij} &= \max\left[F_{Ai}, F_{Aj}\right] - F_{Bij} \text{ for all i, j,.....,n} \\
&= \max\left[F_{Ai}, F_{Aj}\right] - \max\left[F_{Ai}, F_{Aj}\right] \text{ for all i, j,.....,n} \\
&= 0 \quad \text{for all i, j,.....,n}
\end{aligned}$

Thus $(\bar{T}_{Bij}, \bar{I}_{Bij}, \bar{F}_{Bij}) = (0, 0, 0)$

Hence, the edge set of $\bar{G}$ is empty if G is a complete SVN-graph.

4. Conclusion

Neutrosophic sets is a generalization of the notion of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic models gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy and/or intuitionistic fuzzy models. In this paper, we have introduced certain types of single valued neutrosophic graphs, such as strong single valued neutrosophic graph, constant single valued neutrosophic graph and complete single valued neutrosophic graphs. In future study, we plan to extend our research to regular and irregular single valued neutrosophic graphs, bipolar single valued neutrosophic graphs, interval valued neutrosophic graphs, strong interval valued neutrosophic, regular and irregular interval valued neutrosophic.

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