Equivalence Principles are Confirmed – Gravitational and Inertial Masses are Equal.

Nahum Haik, Moshe Haick

Controlnet Ltd, POB 1009 Omer, 8496500 Israel

Email: nahumh@controlnet.co.il, mosheh@conrolnet.co.il

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Abstract

The equivalence Principles of general relativity are an open question of physics – the relationship between the gravitational mass (m_g) and the inertial mass (m_i) of an object is unknown. Solving this problem is essential for testing variants of alternative theories of gravity, predicting the failure of the relationship $m_g = m_i$. Experimental tests of the relationship $m_g = m_i$, are based on the Universal Free Fall (UFF) approach: "a comparison of accelerations of two masses, having a variety of material compositions, falling into an external gravitational field". The UFF approach can only confirm the relationship $m_g \neq m_i$, but not the relationship $m_g = m_i$ because of experimental errors. We confirm the relationship $m_g = m_i$ using a new approach: "a study of a single mass at rest relative to a massive object". We also show that conservation of energy is a sufficient condition for $m_g = m_i$. We conclude that alternative theories of gravity must be constrained by the relationship $m_g = m_i$.

Keywords: Gravity; Gravitational and Inertial mass; Equivalence Principle; General Relativity; Alternative Theories.

1. Introduction

The correlation between the gravitational mass (m_g) and the inertial mass (m_i) of an object is an unsolved physics problem. A key principle of general relativity (GR) is that $m_g = m_i$[1,2], whereas many alternative theories of gravity[2-8], predict that $m_g \neq m_i$.

A key assumption of GR is the Galileo and Newton Universal Free Fall (UFF) principle[1, 2], states that any two masses fall with the same acceleration in an external gravitational field regardless of mass or material composition. It is easy to show that UFF is only true if $m_g = m_i$[2].

The relationship $m_g = m_i$ for a test mass is called the "*weak equivalence principle*" (WEP). Whereas the relationship $m_g = m_i$ for an object with a self-gravitational energy, is called the "*gravitational weak equivalence principle*" (GWEP)[2].

Based on WEP and GWEP, two important equivalence principles are defined:

1. The Einstein Equivalence Principle (EEP):

Consists of the WEP and the principles of local Lorenz invariance (LLI) and the local position invariance (LPI).

2. The strong equivalence principle (SEP):

Consists of the GWEP and the principles of local Lorenz invariance (LLI) and the local position invariance (LPI).

GR is the only theory of gravity that obey SEP[2], whereas many alternative theories of gravity[2,3,4] predict the failure of GWEP, and therefore the failure of the relationship $m_g = m_i$ and the SEP.

Obviously, proving or disproving, WEP, GWEP, and the relationship $m_g = m_i$ is extremely important for physics.

So far, the relationship $m_g = m_i$ has been investigated experimentally using many techniques, however, the basic approach used by all the experiments is based on the Galileo and Newton Universal Free Fall (UFF) approach: "a

comparison of accelerations of two masses having a variety of material compositions, falling into an external gravitational field".

Beginning with Newton's and Bessel's pendulum experiments and Eötvös torsion balance experiments[9,10], to name a few, the techniques were constantly improved. The best measured value of $| m_g - m_i |$, to test the WEP, is the US group Eöt-Wash 2012 torsion balance test with a precision of the order of $1x10^{-13}$ [11] and the recent satellite experiment[12] with a precision of the order of $9x10^{-15}$. New proposals to test the WEP by satellite[13,14] target precisions of the order of $9x10^{-18}$.

Likewise, the GWEP tests are based on the same approach of comparing the accelerations of two bodies falling into the gravity field of a third body. Current tests include:

1. Laser ranging experiments[15,16], looking at the Earth and the Moon, falling towards the Sun.

2. The recent experiments[17,18]: "a study of a triple system - PSR J0337 + 1715" in which a pulsar and a white dwarf fall into the field of a third body–a second white dwarf.

Lack of knowledge about WEP, GWEP, and the relationship $m_g = m_i$ hinders physics. We do not know whether general relativity is correct, or whether variants of alternative theories of gravity, predicting the failure of WEP and GWEP, are correct. On top of that, experimental tests cannot confirm or refute the WEP, GWEP and the relationship $m_g = m_i$.

The problem with experimental tests of WEP and GWEP and the relationship $m_g = m_i$ is the following: the tests can possibly confirm the relationship $m_g \neq m_i$. However, if $m_g = m_i$, experimental tests cannot confirm it because of experimental errors, that cannot be eliminated.

Physics needs solutions to these problems.

The aim of the current research is to solve these problems by using a new approach, based on the law of conservation of energy to prove that WEP, GWEP, and the relationship $m_g = m_i$, are correct.

Rather than using the 400 years UFF approach: "comparing the accelerations of two masses falling in an external gravitational field", our new approach

investigates "*a single mass at rest relative to a massive object*", to compare m_g and m_i directly.

Moreover, unlike the UFF approach that uses a variety of material compositions, our approach is independent of material composition. In-addition, the new approach has an advantage in that testing a mass at rest enables us to use Newtonian reasoning without the complications of moving bodies.

In this article we achieve the following:

- Present a new approach to directly compare m_g and m_i of a single mass at rest, and based on the law of conservation of energy we prove that $m_g = m_i$. This means that the GR Equivalence principles: WEP and GWEP and the relationship $m_g = m_i$ are correct.

- Prove that the law of conservation of energy is a "sufficient" condition for the relationship $m_g = m_i$.

- We conclude that WEP and GWEP and the relationship $m_g = m_i$ must be used as constraints on alternative gravity theories and on any other theory that has predictions about WEP, GWEP and the relationship $m_g = m_i$.

2. Methodology

The relationship $m_g = m_i$ is at the heart of Newton's theory of gravity and Einstein's GR. The GR principles, WEP and GWEP are equivalent to the relationship $m_g = m_i$. Assuming the Uniqueness of Free Fall (UFF) and using Newtonian reasoning, it is easy to show that $m_g = m_i$, and vice versa.

All the last 400 years of experimental testing of the relationship $m_g = m_i$, are based on the UFF and Newtonian reasoning, starting with the Galileo free fall experiments and Newton and Bessel pendulum experiments, Eötvös torsion balance experiments and the recent satellite experiment[12], to name a few.

The experimental approach can succeed only if $m_g \neq m_i$, however, if $m_g = m_i$, the experimental approach cannot confirm it even if an infinite number of experiments are carried out, because the experimental errors cannot be eliminated.

In the present article we use a new approach to study the relationship $m_g = m_i$, in which, "*a single mass at rest relative to a second object*" is studied, to directly compare m_g and m_i of the mass and to prove that $m_g = m_i$.

Note that in this article we are not competing with any theory. We only study the basic principles of GR: the WEP and GWEP, and the relationship $m_g = m_i$, using our new approach and the law of conservation of energy.

Newtonian reasoning is the only option to investigate the WEP, GWEP and the relationship $m_g = m_i$ because:

We cannot use special relativity (SR) or general relativity for this task because:

1. SR cannot handle gravity.

Moreover, SR is not needed in the test of the relationship $m_g = m_i$ because the studied objects are at rest.

2. We cannot use GR to test the relationship $m_g = m_i$ because GR is based on the $m_g = m_i$ principle.

In the next section, we study a system of two objects at rest interacting gravitationally. Without losing generality, we consider a test mass or an object at rest on the surface of Earth to confirm the relationship $m_g = m_i$.

3. Gravitational and inertial masses are Equal.

In this section, based on the law of conservation of energy, we prove that the gravitational mass m_g, and the inertial mass m_i, of an object are equal. This means that WEP and GWEP are correct.

To begin with, note that only two types of force can act on any mass: gravitational, and inertial.

Without losing generality, consider *a test mass at rest relative to earth*, having any type of material composition.

The downward force is gravitational, whereas the upward force is not gravitational → the upward force is inertial.

We get:

$$\mathbf{F_g} + \mathbf{F_i} = 0 \quad \rightarrow \quad m_g / m_{i\,=} \, |\mathbf{a_i}| / |\mathbf{a_g}| \qquad (1)$$

where $\mathbf{F_g}$, m_g, and $\mathbf{a_g}$ are the Gravitational force, mass, and acceleration respectively, whereas $\mathbf{F_i}$, m_i, and $\mathbf{a_i}$ are the corresponding Inertial quantities.

Using eq. (1) there are three options:

$$m_g > m_i \rightarrow |\mathbf{a_g}| < |\mathbf{a_i}| \rightarrow \qquad (2)$$

$\mathbf{F_g} + \mathbf{F_i} = 0$, whereas the acceleration $|\mathbf{a_g} + \mathbf{a_i}| > 0$.

$$m_g < m_i \rightarrow |\mathbf{a_g}| > |\mathbf{a_i}| \rightarrow \qquad (3)$$

$\mathbf{F_g} + \mathbf{F_i} = 0$, whereas the acceleration $|\mathbf{a_g} + \mathbf{a_i}| > 0$.

$$m_g = m_i \rightarrow |\mathbf{a_g} + \mathbf{a_i}| = 0 \rightarrow \qquad (4)$$

Gravitational mass is equal to Inertial mass.

In eq. 2 and 3, the vector sum of forces on the mass is 0, whereas the value of the vector sum of accelerations $|\mathbf{a_g} + \mathbf{a_i}| > 0$. These results contradict the

law of conservation of energy. We get that eq. 4 is the only option that is consistent with the law of conservation of energy.

This proves the WEP:

The gravitational mass and the inertial mass are equal, $m_g = m_i$.

Next, we prove that the GWEP is correct:

Let us consider a mass, having any type of material composition, at rest relative to earth.

Looking at the relationship between the Earth's gravitational mass (M_g), and the Earth's inertial mass (M_i), and using the same considerations leading to eq. 1, we get that the vector sum of forces acting on Earth is 0.

Here again, we have three options leading to equations similar- to eq. 2 - 4 in which $\mathbf{a_g}$ and $\mathbf{a_i}$ are Earth's gravitational and inertial accelerations, respectively.

The only option that is consistent with the law of conservation of energy is:

$$M_g = M_i \rightarrow |\mathbf{a_g} + \mathbf{a_i}| = 0 \rightarrow \quad (5)$$

The gravitational mass M_g is equal to the inertial mass M_i.

The equality of the gravitational and the inertial masses, $M_g = M_i$, leads to the conclusion that the contribution of the Earth's self-gravitational energy to M_g and to M_i is the same.

This proves the GWEP:

*In a field of a gravitating body, including its self-*gravitational *energy,*

the *gravitational mass and the inertial mass are equal, $M_g = M_i$.*

In eq. 2 - 5, the only assumption that we made is the law of conservation of energy. This shows that the law of conservation of energy is a **"**sufficient*"* condition for $m_g = m_i$ independent of the material composition and the self-energy of the object.

These equations also prove a second relationship: $m_g = m_i$ is a **"**necessary*"* condition for the law of conservation of energy.

4**. Discussion**

The relationship between gravitational mass (m_g) and inertial mass (m_i) of an object is an open question of physics. Historically[10] and recently[12,17], the relationship $m_g = m_i$ and WEP and GWEP have been investigated experimentally, using an approach based on UFF, in which: "the accelerations of two masses falling in an external gravitational field are compared".

Note that, if $m_g \neq m_i$, it is possible by using UFF to confirm this inequality experimentally by conducting a finite number of experiments, using improved technologies and accuracies.

However, if $m_g = m_i$, it is not possible to confirm it experimentally even if an infinite number of experiments are conducted, because of experimental errors.

The fact that GR is based on the WEP and GWEP principles is a weakness that has made it possible to create alternative theories of gravity based on a possible failure of WEP or GWEP and therefore a possible failure of GR.

In this article we use a new approach, based on the law of conservation of energy, in which "*a single object at rest relative to a massive object*" is studied to prove that WEP, GWEP, and the relationship $m_g = m_i$, are correct.

The confirmation of the relationship $m_g = m_i$ in this article can explain the fact that experimental tests conducted since Galileo and Newton could not confirm or refute the relationship $m_g = m_i$. As a result, theories predicting the failure of the WEP, the GWEP and the relationship $m_g = m_i$, could not test their predictions.

5. **Conclusions**

Using a new approach: "*a study of a single object at rest relative to a massive object*" and the law of conservation of energy, we prove that the founding principles of GR: the weak equivalence principle (WEP), and the gravitational weak equivalence principle (GWEP), and the relationship $m_g = m_i$ are correct.

We conclude that WEP and GWEP and the relationship $m_g = m_i$ must be used as constraints on alternative gravity theories and on any other theory that has predictions about WEP, GWEP and the relationship $m_g = m_i$.

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Author Information:

Affiliations:

Controlnet Ltd, POB 1009 Omer, 8496500 Israel

Nahum Haik, Moshe Haick

Contributions:

N.H and M.H contributed equally to the research and to the writing of this paper.

Corresponding author:

Correspondence to Nahum Haik.

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The authors declare no competing interests.