Online Trajectory Generation for Step Recovery in Bipedal Robot Locomotion

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Abstract—We present a computationally efficient technique aimed at planning bipedal walking trajectories for push recovery. By modifying a layered walking controller, in case of pushes, we are able to plan a new position and timing for the forthcoming step. Tests have been performed in the Gazebo simulation environment using the one-meter-tall iCub humanoid robot model. Results show the effectiveness of the approach in stabilizing walking motions in case of strong external disturbances.

Index Terms—step adjustment, push recovery, divergent component of motion, bipedal walking

I. INTRODUCTION

Robust bipedal locomotion remains an open problem despite decades of research in the subject. In case of unforeseen disturbances acting on the robot, it would be necessary to modify the generated patterns such that fall states are avoided. In recent years, several attempts have been made for considering the state of the robot in order to compensate the external disturbances. In particular, in the presence of highly external disturbances the principal strategy for balance recovery is step adjustment [1].

This paper extends the controller architecture presented in [2] by developing a step-timing and step-position adjustment algorithm for generating the desired footprints. The proposed approach is general enough to be applied with different kind of walking controllers, while it considers a time-varying Zero Moment Point (ZMP) during the single support phase, allowing heel to toe motion. Indeed the step adjustment algorithm is able to define contemporaneously modifications for the step position, step timing and Divergent Component of Motion (DCM) trajectory, with a single QP problem.

II. CONTROL ARCHITECTURE

This section reviews the components of the control architecture presented in [2]. It is composed of three main layers. The first layer is represented by the *trajectory optimization*, whose main purpose is to generate desired footstep positions and orientation and also the desired DCM trajectory using the robot state. The second layer employs *simplified robot models*

Fig. 1: iCub reacts to the application of an external force, indicated with a red arrow.

to track the desired DCM trajectory. Finally, the third control layer is given by the *whole-body QP* block. It has the main purpose of ensuring the tracking of the desired feet positions and orientations and also the CoM acceleration by generating desired joint torques.

In this work the above mentioned control architecture is improved by adding the step position and timing adaptation module to the *trajectory optimization* block.

III. STEP TIMING & POSITION ADAPTATION

Once the nominal trajectory is designed, the step adjustment module optimizes the next step position and step timing based on the measured DCM. The DCM ξ and its dynamics are defined as [3]:

$$
\xi = x + b\dot{x}, \qquad \dot{\xi} = \frac{1}{b}(\xi - r^{zmp}),
$$
\n(1)

where $x \in \mathbb{R}^2$ represents the position of the CoM, z_0 is the constant CoM height, $b = \sqrt{z_0/g}$ and $r^{zmp} \in \mathbb{R}^2$ is the position of the ZMP.

Unlike previous researches [4], the proposed algorithm allows considering a desired ZMP trajectory also during single support phase. In particular, we define an exponential interpolation for the ZMP trajectory as:

$$
r^{zmp}(t) = Ae^{-\frac{t}{b}} + B,
$$
 (2)

Fig. 2: The trajectory optimization block.

where $A, B \in \mathbb{R}^2$ are chosen to satisfy the ZMP boundary conditions, i.e. $r^{zmp}(0) = r_1^{zmp} r^{zmp}(T) = r_2^{zmp}$ as:

$$
A = \frac{(r_2^{zmp} - r_1^{zmp})\sigma}{1 - \sigma} , \quad B = \frac{r_2^{zmp} - r_2^{zmp}\sigma}{1 - \sigma}, \quad (3)
$$

where T represents the duration of the single support phase and $\sigma = e^{\frac{T}{b}}$. By substituting (2) into the DCM dynamics (1) the following first order in-homogeneous Ordinary Differential Equation(ODE) holds:

$$
\dot{\xi} - \frac{\xi}{b} = -\frac{1}{b}r^{zmp}(t) = -\frac{A}{b}e^{-\frac{t}{b}} - \frac{B}{b}.
$$
 (4)

The solution of (4) can be expressed by the following integral:

$$
\xi(t) = e^{\int \frac{1}{b}dt} \left[\int \left(-\frac{A}{b} e^{-\frac{t}{b}} - \frac{B}{b} \right) e^{\int -\frac{1}{b}dt} dt + C \right], \quad (5)
$$

in which $C \in \mathbb{R}^2$ is the vector of unknown coefficients that can be found by imposing the boundary condition. Therefore, we can find the coefficient by solving the problem as an initial value problem:

$$
\xi(0) = \xi_0 = \frac{A}{2} + B + C_0,\tag{6}
$$

or by solving (5) as a final value problem:

$$
\xi(T) = \xi_T = \frac{A}{2}e^{-\frac{T}{b}} + B + C_f e^{\frac{T}{b}}.
$$
 (7)

In order to find a DCM trajectory that satisfies both the initial and the final condition problems C_0 has to be equal to C_f , thus, combining (6) and (7) the following relation holds:

$$
\xi_0 - \frac{A}{2} - B = \left(\xi_T - \frac{A}{2}e^{-\frac{T}{b}} - B\right)e^{-\frac{T}{b}}.\tag{8}
$$

By defining the $\delta = r_2^{zmp} - r_1^{zmp}$, using $\sigma = e^{\frac{T}{b}}$ and enhanced the introduction of r_2^{zmp} as position substituting (3) to (8) and by introduction of r_T^{zmp} as position of the ZMP at the beginning of the next step, and the DCM offset as $\gamma_T = \xi_T - r_T^{zmp}$, we will have:

$$
\gamma_T + r_T^{zmp} + \left(r_2^{zmp} - \xi_0 - \frac{\delta}{2}\right)\sigma = r_1^{zmp} + \frac{\delta}{2}.\tag{9}
$$

The step adjustment problem can be now formalized as a constrained optimization problem where the unknowns variables are γ_T , \overline{r}_T^{zmp} and σ . Meanwhile, r_T^{zmp} is assumed to be at the center of the foot at the beginning of next step. Thus, we can assume this position to be considered as a target for the next footstep position.

The following cost function is chosen to yield the desired gait values as close as possible to the nominal values:

$$
J = \alpha_1 \left\| r_T^{zmp} - r_{T,nom}^{zmp} \right\|^2 + \alpha_2 \left\| \gamma_T - \gamma_{nom} \right\|^2
$$

+
$$
\alpha_3 \left\| \sigma - e^{\frac{T_{nom}}{b}} \right\|^2,
$$
 (10)

where α_1 , α_2 , α_3 are positive numbers and the next zmp position $r_{T,nom}^{zmp}$, step duration T_{nom} and next DCM offset γ_{nom} are obtained from the nominal trajectory generator.

Furthermore we introduce the following set of inequality constraints:

$$
\begin{bmatrix} I_2 & 0_{2\times 1} & 0_2 \\ -I_2 & 0_{2\times 1} & 0_2 \\ 0_{1\times 2} & I_1 & 0_1 \\ 0_{1\times 2} & -I_1 & 0_1 \end{bmatrix} \begin{bmatrix} r_2^{zmp} \\ \sigma \\ \gamma_T \end{bmatrix} \leq \begin{bmatrix} r_{T,mx}^{zmp} \\ -r_{T,min}^{zmp} \\ \sigma_{max} \\ -\sigma_{min} \end{bmatrix}, \quad (11)
$$

where $r_{T,max}^{zmp}$, $r_{T,min}^{zmp} \in \mathbb{R}^2$ and $\sigma_{max}, \sigma_{min} \in \mathbb{R}$.

The aforementioned optimization problem is expressed as a QP problem and it is solved at each control cycle by substituting ξ_0 with the current DCM position. The single support maximum duration is progressively shrunk as the robot completes the step.

Fig. 2 describes the communication of the step adaptation module with the other components inside of the trajectory optimizer block and the pipeline of data exchanged with the other layers of architecture.

IV. RESULTS

In order to validate the capability of the proposed controller architecture, we present two main step recovery experiments. In the former the robot is walking straight and an external disturbance acts on the pelvis to the lateral direction. In the latter the robot follows a circular path and an external push is exerted on the pelvis. In both scenarios, the external disturbance has a magnitude of $50 N$ and lasts $0.1 s$. The presented algorithm determines suitable modifications to the pre-planned trajectories allowing to maintain balance by both step timing and position adaptation. The simulation results is shown in the accompanying video.

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