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The Pressure Problem of The Incompressible Flow Equations

Editorial

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Numerical solutions for the incompressible flow equations encounter a numerical problem because of the lack of an equation for the pressure. To resolve this problem, the continuity equation is modified or replaced to derive pressure equations. These equations are derived from: (1) analogy with the compressible flow equations by introducing a time dependent pressure term into the continuity equation, and (2) divergence of the momentum equation and the enforcement of the continuity equation. These techniques are known as the artificial compressibility (AC) method, and the pressure Poisson (PP) method respectively. The pressure equation in the AC method is first order partial differential equation, and in the PP method is second order partial differential equation of the Poisson type. Although the pressure Poisson equation is consistent with the elliptic nature of the pressure field, it imposes an additional integral compatibility constraint that is cumbersome to satisfy numerically on collocated grids. Using these techniques, the continuity equation is satisfied to non zero dilatation [5].

I developed a consistent Finite Difference approximation for the pressure Poisson equation and its Neumann boundary conditions that automatically satisfy the compatibility constraint on non-staggered (collocated) grids.[1] and [2]. Numerical solutions are obtained using our scheme for two- and three-dimensional problems [3-7]. The computed results show that the compatibility constraint is satisfied, and the pressure Poisson equation with Neumann boundary conditions is converged. The continuity equation is satisfied upon convergence of the numerical solutions.

Another approach to resolve the pressure problem is the elimination of the pressure as a dependent variable from the governing equations. The pressure can be eliminated using the Curl of the momentum equation, and the Curl Gradient identity. The Curl of the momentum equation, however, is a third order partial differential equation in terms of the velocity as a dependent variable. To maintain second order formulation of the curl of the momentum equation, the vorticity is introduced as a dependent variable. Methods such as stream function-vorticity, vector potential-vorticity and velocity-vorticity are known as the non-primitive variables approaches. Additional differential constraints, and difficulties in deriving boundary conditions for the new dependent variables arise in these techniques.

Unlike the compressible flow equations, solutions of the incompressible flow equations do not depend on the pressure. They instead depend on the pressure derivatives. Consistent with the physics of incompressible flows, we derived a new approach that employ the velocity and pressure derivatives as dependent variables [8]. In this case the number of dependent variables, as compared with the velocity and pressure as dependent variables, is increased from three to four in two-dimensions and four to six in three dimensions. Additional governing equations are obtained from the Curl Gradient identity for the pressure derivatives. This scheme has primitive variables as the primitive variables approach, and employs the Curl Gradient identity as the non-primitive variables approach. However, the non-primitive variables approach employs the Curl Gradient identity to eliminate the pressure as dependent variable, while our approach employs the identity as differential equations to calculate the pressure derivatives. The main advantages of this approach can be summarized as follows: (1) boundary conditions for the pressure derivatives are of the Dirichlet type, which eliminates the additional constraints imposed on the primitive- and non-primitive formulations. (2) the continuity equation is satisfied numerically to machine zero by solving it for one of the velocity components, and (3) the numerical scheme is robust because of the Dirichlet boundary condition on the dependent variables.

In conclusion, the use of the pressure derivatives as dependent variables for solutions of the incompressible flow equations solves the pressure problem.

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