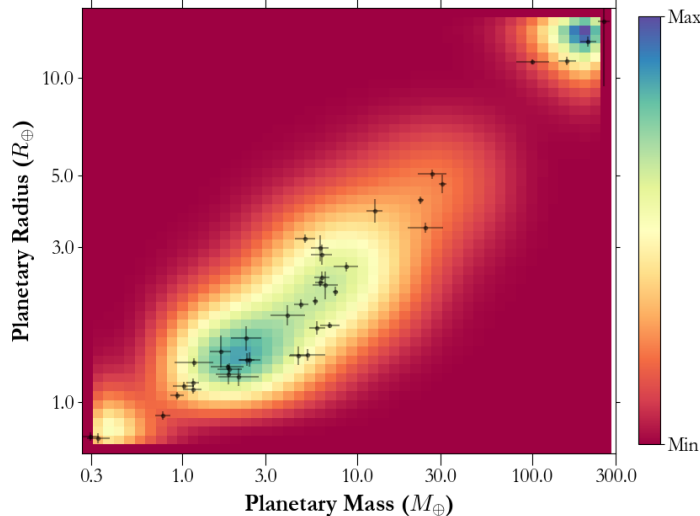


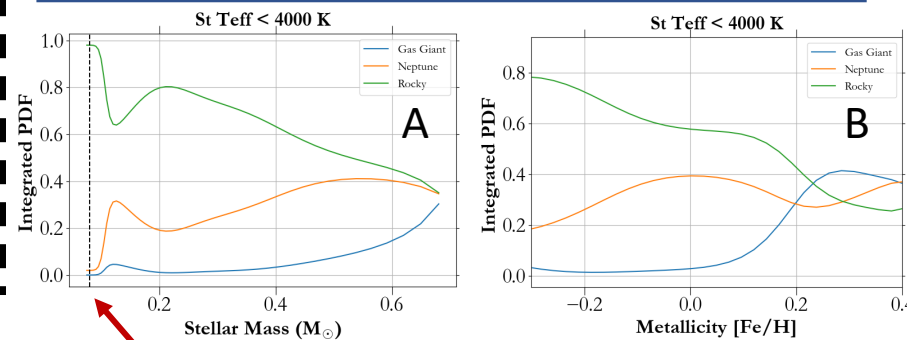
- With larger exoplanet sample sets, we can move beyond simple power law scaling relations.
  - We use nonparametric basis sets (Beta density functions) to do so, which –
    1. Can model conditional distributions ( $f(x|y)$ ) that are not Gaussian.
    2. Are intrinsically more flexible than power laws.
    3. Easier to expand to > 2 dimensions.
- Ning et al. (2018), and Kanodia et al. (2019), used these to model the Mass-Radius relationship for FGK and M dwarf planet samples, respectively.



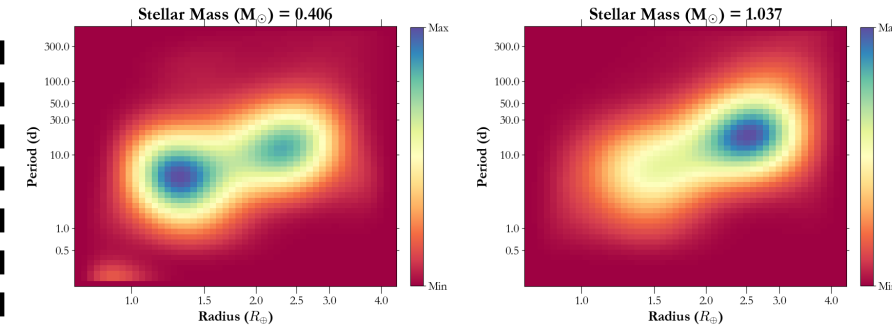
**Figure 1:** Joint mass-radius (M-R) distribution fit to a sample of M-dwarf planets. The color-scale represents the probability density function (PDF) of the model.

## Expanding from Mass-Radius to higher dimensions (Ongoing)

- Expand from Mass-Radius relationships to higher dimensions (3, 4, even 5 dimensions). Such as planetary mass, radius, period, eccentricity, stellar mass, metallicity
- Some example applications –
  1. Model the Mass-Radius relationship as a function of Period (**M-R-P**) to include the period dependence of density (especially important for photoevaporation).
  2. Then include stellar mass, to probe how the M-R-P space evolves across spectral types (**M-R-P-StM**).
  3. Evolution of Radius-Insolation space features for features such as the Radius-gap or Neptune desert with stellar parameters such as metallicity, stellar mass (**R-P-StM-Fe/H**). Especially important for M dwarfs, where Stellar Mass can change 6-7x between late and early Ms.
  4. Correlations between Gas Giant occurrence with Period, Metallicity, Stellar Mass (**R-P-StM-Fe/H**).



**TRAPPIST-1**



**Figure 3:** Projections of the joint R-P-StM distribution for a sample of FGKM planets. Conditioning on stellar mass allows us to probe how the planet radius-valley (in R-P) changes as a function of the stellar mass.

## FUTURE WORK

- Use a combination of TESS + precision RV instruments to find and characterize planets with mass measurements, especially for M dwarfs (currently only about 40 planets.). Example – TOI-1728b (Kanodia et al 2020), TOI-1899b (Canas et al. 2020), TOI-1266b,c (Stefansson et al. 2020).
- Consider science cases which are less sensitive to biases due to selection effects (esp. for non-Kepler samples).
- Account for selection effects to transition from exoplanet samples to populations.
- Include asymmetric measurement errorbars.
- Plot 4+ dimensions in 3 dimensions...

**Figure 2:** M dwarf sample set. Integrated probability density function (PDF) for the relative occurrence of planets in different size regimes (Rocky, < 2 R\_earth; Neptune, 2-4 R\_earth; Gas giant, >4 R\_earth), as a function of stellar mass (**R-P-StM**) in (A) and host star metallicity (**R-P-Fe/H**) (B).