Application of Mathematical Software in Teaching Numerical Mathematics for Students of Electrical Engineering

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Abstract

The successful education of numerical mathematics for engineering students requires usage of software tools for various applications. In this paper we present our experiences in using both open and proprietary software tools in teaching numerical mathematics for bachelor students of electrical engineering at the School of Electrical Engineering, University of Belgrade in different applications. We compare the mathematical software used for implementation of numerical solutions in class and present an illustrative example showing the application of mathematical software in teaching process.

- **Keywords**: numerical mathematics; higher education; open software; proprietary software.

Introduction

- Software tools are used for various applications in teaching numerical mathematics courses for students of electrical engineering.
- These include:
  - online educational platforms as administrative support to the educational process,
  - text processing tools
  - mathematical software tools
  - tools for spreadsheet manipulation

A. Online educational platforms

- To facilitate access to additional learning material such as examples of solved problems in various mathematical software used in the classroom
- Selection, uploading and grading of programming assignments implemented by students
- **eLearning** - The Computer Center of the University of Belgrade hosts and administrates the eLearning platform based on **Moodle**
- **Microsoft Teams** – propriety software

Moodle based platforms

- “Moodle is a learning management system and it is a free and open source software package designed to provide educators, administrators, and learners with a single robust, secure and integrated system to create personalized learning environments” [2]
- **eLearning** - The Computer Center of the University of Belgrade hosts and administrates the eLearning platform based on Moodle

  (https://elearning.rcub.bg.ac.rs/moodle/)
- The capabilities of Moodle greatly depend on the administrative settings done by the host, but it allows a large number of plug-ins that allow various capabilities, e.g. video streaming.

Microsoft Teams (MST) – propriety software

- During the spring semester 2020 the COVID-19 pandemic forced the learning process from the classroom to the online environment.
- School of Electrical Engineering, University of Belgrade, provided the Microsoft Teams (MST) platform for online teaching.
- Created primarily as collaboration platform, bat has the necessary features for course administration, such as assignment posting and grading.
- Lacks some important features such as creating groups within class and delegating specific assignments to different groups.
- Live streaming is very easily implemented through MST, together with live recording.

B. Text processing tools

- **LaTeX** is a free and open source document preparation system
- For offline installation: **MiKTeX** [4] with **TeXstudio** editor [5]
- **Overleaf** [6], an online collaborative LaTeX editor, allows fast editing without installation
- LibreOffice Writer is part of **LibreOffice**, one of the leading open source office software suite [7]
- **Microsoft Word** is part of Microsoft Office, a propriety office package [8]

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Додатак: [Програмски код]

```
import math
import matplotlib.pyplot as plt
import numpy as np

def bisekcija(f,a,b,N):

    if f(a)*f(b) >= 0:
        print("Pogresan interval")
        return None
    a_n = a
    b_n = b
    for n in range(1,N+1):
```

LaTeX code for reporting programming assignment template using the package listings for source code printing and the output on the right

Spreadsheet software

- Spreadsheet software is used for automatic calculations in class.
- This approach has shown especially useful in the teaching process of numerical mathematics, allowing students to go through the steps of the numerical method in detail while avoiding tiresome calculation by hand.
- It is suitable for solving problems of lower dimensions which are typically considered within the undergraduate numerical mathematics courses [11].
- For this purpose **LibreOffice Calc** or **Microsoft Excel** can be used with equal success.

Mathematical software tools

- Mathematical software tools developed particularly for numerical mathematical computations
- Numeric and symbolic mathematical computations
- The predominant propriety packages in the academic environment are Maple [12], Mathematica [13] and MATLAB [14], [15].
- In parallel, a number of free and open source packages were developed during the past decades, with more or less success.

Maple

- Maple is a proprietary software created and developed by Maplesoft company, primarily for numeric and symbolic mathematical computations.
- Within the project supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia in 2019 we were able to obtain the license for Maple
- It includes a sub-package of Student package specifically designed for numerical analysis topics and it contains computation commands, visualization commands and interactive routines, allowing the students to gain insight on how a specific numerical method operates [12].

Open mathematical software tools

- **GNU Octave** is one of the most successful alternatives to MATLAB. It is compatible with many MATLAB scripts and has available online version, which is an advantage for students with modest technical capacities [16].
- **SageMath** is an open source mathematical software built on top of many existing open source packages with a unified Python interface. SageMath can be used on a local computer, in a local network and online. The online version is called CoCalc and it can operate as a collaborative and educational platform as well [17,18].

Python programming language

- Python is a high-level general purpose programming language [19].
- It is highly extensible, fast-growing environment, allowing creation of large number of libraries that support numerical and symbolic computation.
- Libraries that support numerical methods include:
  - *NumPy*, defines a multi-dimensional array object and associated fast math functions that operate on it [20]
  - *SymPy*, for symbolic mathematics, aiming to become a full-featured computer algebra system [21]
  - *SciPy*, build on NumPy, used for solving scientific and mathematical problems
  - *matplotlib*, for creating static, animated, and interactive visualizations in Python [23]

Python with libraries

- SciPy with matplotlib library allows creation of interactive routines and animations of various numerical methods.
- There is a large number of routines available online that are created with the aim to improve the learning process and to provide visualization of various numerical methods.
- It is not hard to create such routines, even for inexperienced programmers.

Classroom example

- We present an example with the aim to illustrate the application of mathematical software tools in teaching numerical mathematics methods for students of electrical engineering.
- Implementation of the solution of nonlinear equation with the Newton-Raphson iterative method using Maple Student package, Python and LibreOffice Calc.
- As an illustrative example we solve the equation $(x-1)e^{3x}=0$
- Knowing the correct solution allows us to compare the exact error in each iteration

Maple Student package

```
> with(Student[NumericalAnalysis]):
> f:=(x-1)*exp(3*x):
```

Finding zero of function f using Newton iterative method with initial point $x = 2$ and tolerance 10^{-4}.

```
> Newton(f,x=2.0,tolerance = 10^(-4))
```

1.000000000 (1)

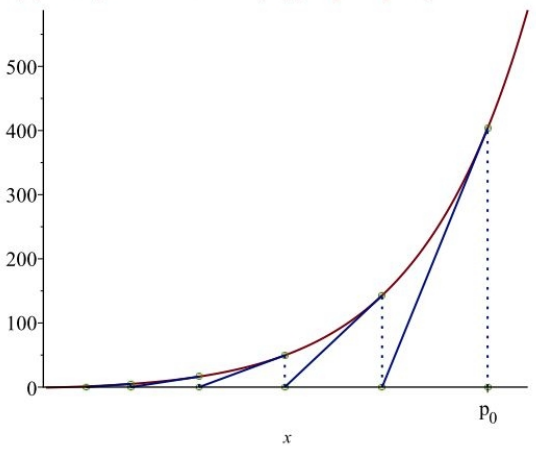
Show sequence of all iterations.

```
> Newton(f,x=2.0,tolerance = 10^(-4),output = sequence)
```

2.0, 1.750000000, 1.519230769, 1.316223250, 1.153946779, 1.048636524, 1.006192927, 1.000112958, 1.000000038, 1.000000000 (2)

Graphical interpretation of the Newton method

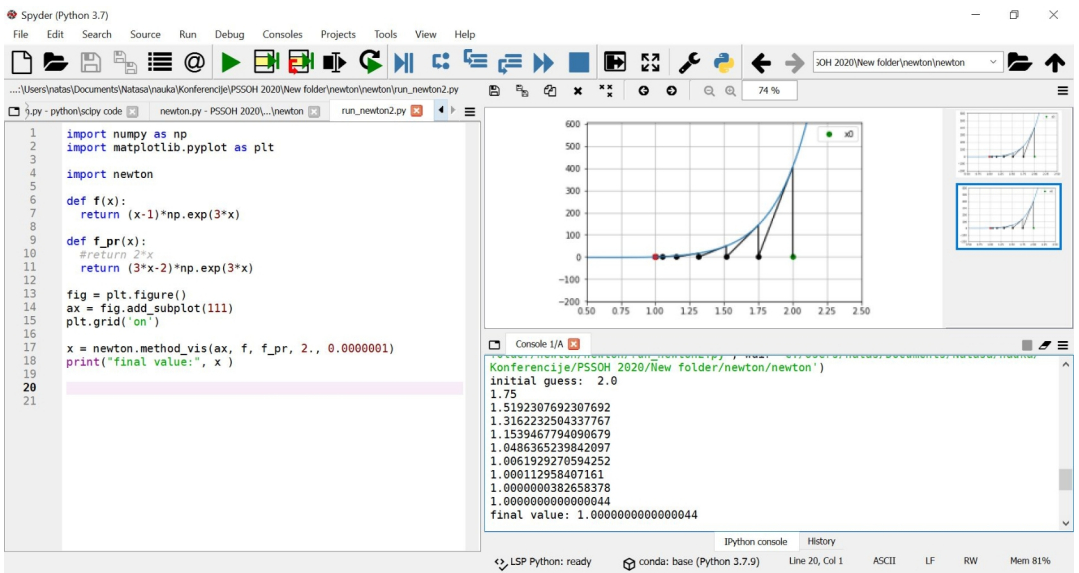
```
> Newton(f,x=2.0,tolerance = 10^(-4),output =plot)
```



5 iterations of Newton's method applied to $f(x) = (x - 1)\, e^{3x}$ with initial point $p_0 = 2.0$. The stopping criterion is not met.

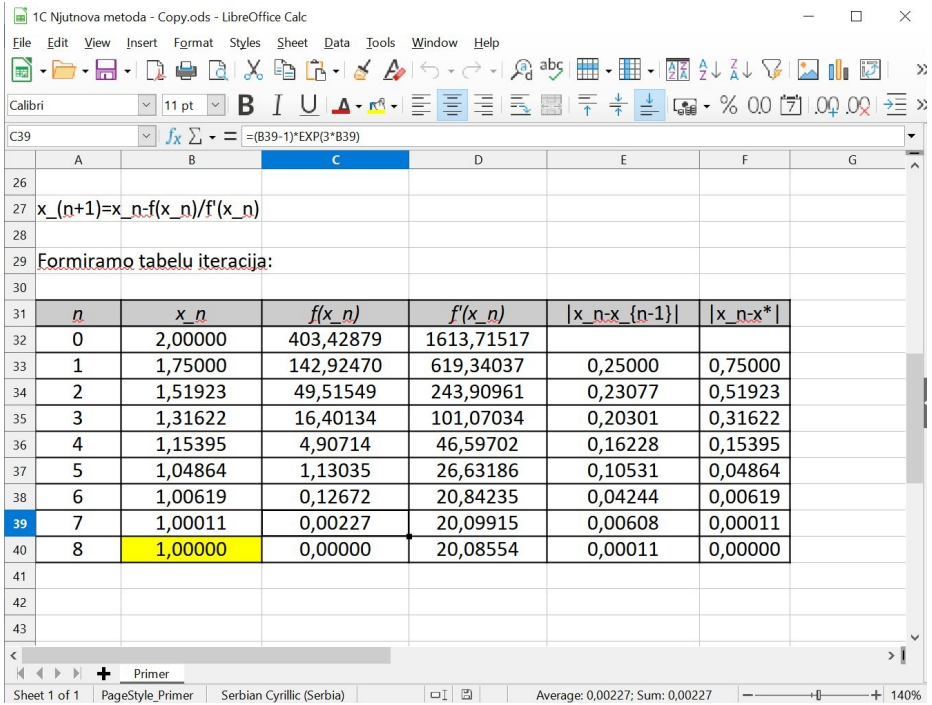
- The Maple code and graphical interpretation
- Command *Newton* within the *Student [NumericalAnalysis]* subpackage
- Option *output* displays the sequence of iteration values
- Option *plot* provides geometric interpretation of the method
- Option *animation* enables animated display of the iterative process.

Python with libraries



- Routine *scipy.optimize.newton*
- We created the routine that shows the graphical interpretation of Newton-Raphson method using the SciPy and matplotib libraries.
- Part of the python code with iteration values and graphical interpretation of Newton-Raphson method

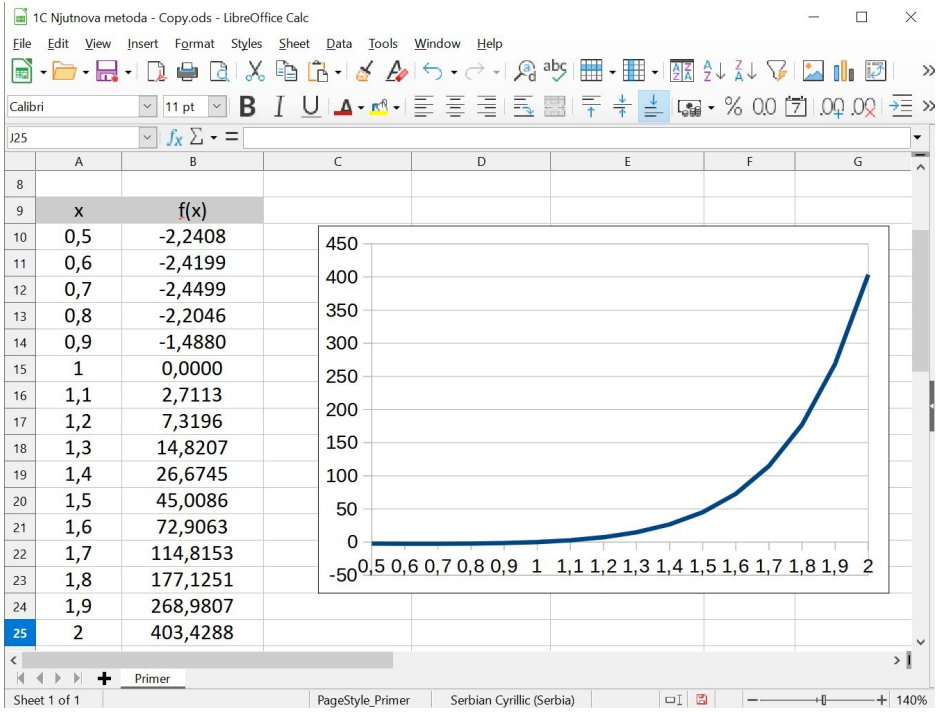
LibreOffice Calc



n	x_n	f(x_n)	f'(x_n)	|x_n-x_{n-1}|	|x_n-x*|
0	2,00000	403,42879	1613,71517		
1	1,75000	142,92470	619,34037	0,25000	0,75000
2	1,51923	49,51549	243,90961	0,23077	0,51923
3	1,31622	16,40134	101,07034	0,20301	0,31622
4	1,15395	4,90714	46,59702	0,16228	0,15395
5	1,04864	1,13035	26,63186	0,10531	0,04864
6	1,00619	0,12672	20,84235	0,04244	0,00619
7	1,00011	0,00227	20,09915	0,00608	0,00011
8	1,00000	0,00000	20,08554	0,00011	0,00000

- Table obtained during implementation of the Newton-Raphson method
- Worksheet formulas
- We compare two adjacent iteration values and the actual error

LibreOffice Calc – graphic draft



- Graphing of functions in LibreOffice Calc using the chart of linear type on function values
- Linear interpolation of data
- This approach does not handle function discontinuities!

Conclusion

- Software tools are used in bachelor numerical mathematics courses for 3 main applications: educational platforms, text processing and mathematical software tools.
- While educational platforms and text processing tools are used irrelevant of topic, mathematical software is crucial in numerical mathematics courses for understanding and implementing numerical methods.
- Spreadsheet software tools have shown very useful when a numerical method is introduces in class. Both open source LibreOffice Calc and proprietary Microsoft Excel can be successfully used.

Conclusion 2

- Numerical software allows more detailed analysis of numerical methods.
- For this purpose the Maple Student package, a proprietary software, and Python with *SciPy* and *mathplotlib* are used.
- Python may require creation of new routines and many are freely available.
- Such routines can be easily created in Python, while Maple Student package covers only the most common numerical methods.
- Other relevant software can be used with the same purpose and we plan to include SageMath in teaching process.
- For implementation of programming assignments students tend to use tools that they are already familiar with and that are available.

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Thank you for your attention!

Questions?