Memoir on the double refraction that light rays undergo in traversing the needles of quartz in the directions parallel to the axis

by Augustin Fresnel
(read 9 December 1822)

with notes by the editors of Fresnel's *Oeuvres complètes*
1866

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English translation of "Mémoire sur la double réfraction que les rayons lumineux éprouvent en traversant les aiguilles de cristal de roche suivant les directions parallèles à l'axe" (9 December 1822), in which Fresnel coins the terms *linear polarization*, *circular polarization*, and *elliptical polarization*, and reports a direct refraction experiment showing that optical rotation is circular birefringence. There were two versions of the experiment. In the first version, one extremely obtuse isosceles quartz prism was approximately achromatized by two glass half-prisms. In the second version, for improved achromatism and a wider separation of the two images, the two glass half-prisms were replaced by quartz half-prisms whose chirality was opposite to that of the first prism.

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Translator's preface

It has been suggested that the discovery of conical refraction in 1832–3 was the first occasion on which a qualitatively new phenomenon was predicted by mathematics and confirmed by experiment.[1] As the suggestion is tentative, let us suspend acceptance. Was conical refraction really the first case of this kind? Was it even the first in optics?

Poisson's spot, even if it had not been noticed by G.F. Maraldi nearly a century before its "prediction" by Poisson, would not make the grade, because it was not a *qualitatively* new phenomenon, but merely a quantitative variation of a qualitatively familiar one: internal fringes in shadows [13]. However, it is obvious from the circular symmetry that the illumination at any internal point must be a function of the distance from the edge of the geometric shadow, and that as this distance approaches the radius of the shadow, the illumination will be more and more concentrated due to the shrinking circumference. This simple reasoning does not tell us that the illumination at the center will be precisely as bright and as white as if there were no obstruction; but it *does* tell us to expect a bright central spot, even if—yea, especially if—we cling to a corpuscular-inflection theory of the internal fringes, which all but guarantees that some corpuscles will be inflected by a distance close to the radius. I am therefore not convinced that Poisson's prediction was intended as a *reductio ad absurdum*, as seems to be universally assumed. Certainly his objections to Fresnel's theory, involving the generation and angular spread of the secondary waves, were more sophisticated than the alleged absurdity of any particular diffraction pattern.

A more eligible candidate, I submit, is the subject of the present memoir: circular birefringence. Mathematically, a plane-polarized wavetrain is a superposition of two oppositely circularly-polarized components, whose resultant direction of polarization will rotate helically if the components travel at different speeds. The inference that this difference of speed is the mechanism of optical rotation, when applied to a geometry that converts the difference of speed into a difference of refraction, predicts a new kind of double refraction, which differs *qualitatively* from the old in that the two images, instead of being polarized in perpendicular planes, are circularly polarized with opposite chiralities, so that their polarization is not apparent in a conventional analyzer unless they are viewed through a quarter-wave device, which converts the circular polarization to plane. This prediction is baldly but precisely stated at the end of Fresnel's preceding "note" [8] of September 1822, and is confirmed by experiment in the present memoir, albeit with the admission that a modification of the apparatus—which must have been inspired by a more specific prediction—gives a cleaner separation of the images.

One might try to disqualify Fresnel for his intermittent observance of the fashion, promoted by Biot, of avoiding overt reliance on any particular hypothesis on the nature of light, in consequence of which the detailed mathematics of Fresnel's prediction is not found in his preceding "note" or the present memoir, or the promised appendix that he never delivered (he had a memoir on total internal reflection to write, criticism to answer, a great age of lighthouses to launch, and less than five years to live), and receives only a partial development in his later "Second Memoir" on double refraction [11, tr. Hobson, pp. 253–7]. He is largely salvaged, however, by a footnote in his paper on chromatic polarization from mid 1821 [7, §16], which refers to aether molecules revolving about their equilibrium positions at a constant speed, and cuts through his subsequent reticence.

The present memoir, as Buchwald says [3, pp. 230–33], gave us "a new vocabulary for polarization", in which a *polarized* wavetrain is one whose perpendicular components of vibration have a fixed amplitude ratio and a fixed phase difference. The special case in which the perpendicular components are in phase or in antiphase, which was called simply "polarization" in the old terminology, is renamed *linear* polarization in the new terminology, which also recognizes other special cases with other names.

According to the *Procès-verbaux* of the Academy of Sciences [1, p. 401], confirmed by the extract thereof in *Annales de Chimie et de Physique* (Ser. 2, vol. 21, p. 435), the memoir was read to the Academy on 9 December 1822. A somewhat updated *extrait* of the memoir was published in the *Bulletin de la Société philomathique* for December 1822, pp. 191–8, reprinted in the *Annales*, Ser. 2, vol. 28 (1825), pp. 147–61, and translated into German in *Annalen der Physik und Chemie*, vol. 21 (1831), pp. 276–88

[1] See e.g. Berry & Jeffrey [2], p. 15, and Lunney & Weaire [15], p. 26.

(with an addendum by Poggendorff on pp. 288–90). But, as far as I know, the full memoir was not published until it appeared in Fresnel's *Oeuvres complètes* [12], vol. 1 (1866), pp. 731–51. In particular, and contrary to the bibliographic entry in Buchwald [3, p. 462, ref. 1822b], the full text did not appear in *Annales* 28:147–61, 263–79 (1825); the first of these page ranges is the aforesaid *extrait* and the second is a preview of the "Second Memoir" on double refraction (cf. Buchwald's ref. 1822i). Neither was any shorter *extrait* published in German in Froriep's *Notizen aus dem Gebiete der Natur- und Heilkunde*, vol. 3, cols. 321–3 (1823), or in English in the *Quarterly Journal of Science, Literature and the Arts*, vol. 15 (1823), pp. 165–6; in both places, the featured work is Fresnel's brief note on the ascent of clouds in the atmosphere [12, vol. 2, pp. 663–6]. Hence, as far as I know, neither the memoir nor any abbreviated version of it has been previously published in English.

In the printing of this memoir in the *Oeuvres complètes*, the editors' first footnote is attached to the title and says, when translated:

> [(a)] See, as introduction to this Memoir, Nos. XVI, XVII, and XXIII, and as supplement No. XXX.

The works cited are respectively references [4], [5], [6], and [10], the last being a "supplement" in the sense that it includes a precise calculation of the required angle of incidence for a Fresnel rhomb, the result of which is incorporated into the later *extrait* [9] of the present memoir. The angle recommended in the *extrait*, namely $54°\frac{1}{2}$, is the greater of two angles giving the desired phase difference for a refractive index of 1.51, the lesser angle being reasonably close to 50° as recommended in the memoir. The small difference between these angles indicates that both give a phase difference close to the peak, so that the phase difference is—fortunately—not very sensitive to the angle of incidence. At the smaller angle, however, the phase difference is more sensitive to the refractive index and therefore to the color, so that the larger angle is preferable.

Conventions: Footnotes to this translation are numbered sequentially. After their sequential numbers, footnotes by the editors of the *Oeuvres complètes* are further identified by their original parenthesized letters. Fresnel himself did not insert footnotes in this memoir (although he reduced some passages to footnotes in the *extrait*). Footnotes identified by sequential numbers alone, together with all items in *square brackets* (in the main text or the footnotes, and including citations such as "[12, vol. 1, p. 722]"), are mine. Section numbers are from the *Oeuvres complètes*.

— *Translator*.

1. Before the beautiful discoveries of Malus, it had long been noticed that the two beams into which light is divided by crossing a rhomb of calcite[2] receive that singular modification to which he gave the name *polarization*, after Newton's ideas on the physical cause of the phenomenon.[3] Thus Malus, strictly speaking, did not discover the polarization of light; but he was the first to show that one could impart to the rays, by simple reflection on a transparent body at a suitable incidence, or by oblique passage through a series of diaphanous plates, the same modification that they receive when they are divided into two distinct beams by crystals endowed with double refraction.

It is known that when a polarized beam falls perpendicularly on one of the natural faces of a rhomb of calcite, it is generally divided into two beams of unequal intensities, whereas unpolarized light always gives two beams sensibly equal in intensity. If the calcite rhomb is rotated about the polarized ray as axis, we notice two positions of the rhomb in which one of the two beams entirely vanishes and the incident light suffers only one mode of refraction in crossing the crystal; in one position this is the ordinary refraction, in the other the extraordinary refraction. If we define a plane passing through the polarized ray and the axis of the crystal, it will rotate with the rhomb, and for the two positions just mentioned it will take successively two directions perpendicular to each other; thus there are two perpendicular planes through the polarized ray, such that when the axis of the crystal is parallel to one of them, the ray suffers only one mode of refraction: we call the *plane of polarization* the one with which the axis of the crystal must coincide in order for the extraordinary ray to vanish.[4] By gradually turning the principal section of the rhomb (that is, the normal plane containing the axis), we see the reappearance of the image that had vanished; its intensity grows successively until it is equal to that of the other, which happens when the principal section bisects the right angle of the two planes just mentioned. If we continue turning the rhomb in the same direction, the image that had vanished becomes more luminous than the other, and the latter eventually disappears in its turn, when the principal section coincides with the second plane. Thus the properties of the polarized ray are not the same in these two planes and vary all around it.

This difference in properties of the different sides of a polarized beam is manifested not only in its passage through crystals endowed with double refraction, but in several other circumstances which Malus has made known and which we do not think necessary to recite here, the process that we have just described being always sufficient to distinguish light which is polarized from that which is not.

2. In a memoir that I had the honor of reading to the Academy towards the end of 1817,[5] I made known a new modification of light, as general, or rather as uniform, as polarization itself, in that the variously colored rays that compose white light receive it all at once and to the same degree, as happens for ordinary polarization. The process is as follows: having initially polarized the beam of light, either by its passage through a rhomb of calcite or by its reflection on an untinned glass inclined at 35°, we introduce it into a glass parallelepiped,[6] where it suffers successively, at the two opposite faces, two total internal reflections at the incidence of about 50°, and in a plane inclined at 45° to the initial plane of polarization. The angle of the entry and exit faces of the parallelepiped with the two reflecting faces must be such that the former are roughly perpendicular to the incident and emergent rays, in order not to exert any polarizing action on them.

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[2] Literally "calcareous spar".

[3, (b)] It is to Huygens that this first observation of the phenomena is due; see *Treatise on Light*, Chap. V, towards the end [14, pp. 92–4].

[4] This definition, due to Malus, has the convenient implication that when a ray is polarized by reflection off a transparent body, the plane of polarization is the plane of reflection, but the inconvenient implication that the plane of polarization is perpendicular, not parallel, to Fresnel's vibration. Fresnel modified the definition so that the plane of polarization remains perpendicular to the vibration even inside an anisotropic crystal (see e.g. [11], tr. Hobson, pp. 318–20), with the result that the plane of polarization contains the wave-normal but not necessarily the ray. In modern terms, Fresnel's "vibration" is that of the electric displacement vector **D**.

[5, (a)] Mémoire sur les modifications que la réflexion imprime à la lumière polarisée [Memoir on the modifications that reflection impresses on polarized light], and Supplement, Nos. XVI and XVII [refs. [4] and [5], respectively].

[6] That is, a Fresnel rhomb.

The light emerging from the glass parallelepiped appears completely depolarized; that is, if it is analyzed with a rhomb of calcite, it always presents two white images[7] of equal intensities, in whatever azimuth we turn the principal section of the rhomb. Nevertheless it is not ordinary light; for if we make it pass through a thin plate of calcium sulfate or quartz, and then analyze it with a rhomb of calcite, then instead of the two white images that direct light would give in this case, we observe two vividly colored images, whose tints however are different from those that would have been developed in the same plates by simply polarized light [6, pp. 46–7]. Another quite remarkable characteristic further distinguishes the new modification in question, both from the polarization of Malus and from the absence of all modification: it is that the light thus modified resumes all the characteristics of perfect polarization when we make it suffer two total reflections at the incidence of 50° in the interior of a glass parallelepiped; then the plane of polarization of the emergent rays finds itself inclined at 45° to the plane of reflection, whatever its direction may be. Direct unmodified light, on the contrary, does not take on any new property after two total reflections; and these give to polarized light the appearance of complete depolarization when it is analyzed with a rhomb of calcite, if the plane of reflection makes an angle of 45° with the initial plane of polarization, as we have just said.

It was these first experiments that made me realize that the light thus modified could be considered as composed of two beams that follow the same route, but are polarized in perpendicular directions and differ in their progress by a quarter of a cycle. By introducing this definition of the new modification into the same formulae that I had used to calculate the ordinary phenomena of coloration of crystalline plates, I easily discovered the laws of the particular tints that these plates present when, instead of ordinary polarized light, a polarized beam modified by two total reflections is passed through. Thus I was led to several curious theorems, and I found that the phenomena of coloration presented by plates of quartz perpendicular to the axis, and certain homogeneous liquids such as turpentine[8], etc., were imitated by placing a thin crystalline plate, [with faces] parallel to the axis, between two glass parallelepipeds in which the incident polarized light underwent the modification that I have just defined, before its entry into the crystalline plate and after its exit; the axis of the crystalline plate must make an angle of 45° with each of the planes of incidence of the two parallelepipeds, which are perpendicular to each other. And indeed, if we rotate the principal section of the rhomb with which we analyze the emergent rays, we observe color changes similar to those given by certain liquids or by plates of quartz perpendicular to the axis; and as in these cases the nature of the tints depends only on the mutual inclinations of the initial plane of polarization and the principal section of the calcite rhomb—that is, of the two extreme planes of polarization—for if, while keeping those in the same relative directions, we rotate the small system comprising the crystalline plate between the two glass parallelepipeds, we do not perceive any variation in the nature or in the intensity of the tints [6, pp. 48–9].

It follows from the same formulae that an assembly of such systems turned in any azimuths produces the same effect as if the axes of the plates contained in them were parallel: that the rays which have suffered the ordinary refraction in the first plate suffer only the ordinary refraction in the subsequent plates, whatever be the azimuths in which the other systems are turned, so that light can pass through such an assembly with only two kinds of speeds [6, pp. 52–5].

3. These consequences, which seemed to remove all the theoretical difficulties of the coloration by turpentine, naturally led me to suppose that this liquid, in which I had demonstrated the existence of double refraction by several interference experiments [6, pp. 45–6], had its particles constituted in such a way that each of them possessed double refraction and moreover impressed on the light rays, at their entry and at their exit, the same modification that they receive by two total reflections in a glass parallelepiped [6, p. 52]. To achieve faithful representation of the phenomena, it was necessary to suppose further that in these particles the double refraction is very different for rays of different colors [6, pp. 49–52], and in inverse proportion to their wavelengths, in accordance with Mr. Biot's law on the deviations of the plane

[7] The incident light is assumed to be white.

[8] French: *essence de térébenthine*.

of polarization of all the light that has passed through a tube filled with turpentine;[9] for by supposing that the double refraction of each species of ray in the particles of this liquid is in inverse proportion to its wavelength, we find, by the interference formulae that I have used, that the deviation of the plane of polarization of the whole beam of homogeneous light, on exit from the liquid, is in inverse proportion to the square of the length of fit or of undulation [12, vol. 1, pp. 664, 681–3], as Mr. Biot had concluded from his observations. Such are the main results contained in a Memoir presented to the Academy at the beginning of 1818,[10] which I thought necessary to recall here for the understanding of the new facts.

This explanation was applicable to quartz plates perpendicular to the axis, as to oil of turpentine[11], since Mr. Biot has made sure of the identity of the coloration phenomena that they present. However, I have regarded the hypothesis just mentioned, on the modifications that light undergoes at its entry into the particles of turpentine and at its exit, not as a reality, but only as a manner of representing the facts, though all those that I have observed up to now confirm the analytical consequences of this explanation—for example, that the polarized light modified by two total reflections, which develops such vivid colors in crystalline plates, must no longer produce them in turpentine and in quartz plates perpendicular to the axis [6, p. 47]. This agreement indeed does not prove the reality of the hypothesis, but only that the results are the same as if the light underwent the said modifications in each particle of turpentine. But without probing the mechanical causes of these phenomena, I was able to deduce, from the formulae that represented them so well, some consequences which if not certain were at least extremely probable, and predict some singular phenomena that I had not yet verified by experiment.

4. This is what I did at the end of a Note on the double refraction of compressed glass, which I had the honor of reading to the Academy on 16 September and which has been published in the *Annales de Chimie et de Physique*.[12] I announced that if we made manifest the double refraction that light undergoes in quartz, in traversing it parallel to the axis of the needles, we would find that the two beams into which the light is divided would then not present any appearance of ordinary polarization when tested with a rhomb of calcite, but would still differ from direct rays in that if we made them suffer two total reflections in a glass parallelepiped at the internal incidence of about 50°, they would be polarized each in a plane inclined at 45° to the plane of reflection, one to the left and the other to the right of that plane; this does not happen to ordinary light, which these two total reflections leave as it was before. As soon as I could, I verified by experiment these curious consequences of my formulae, and I found what I had foreseen. I would have been able to announce from the same formulae the other characteristics of this double refraction; but it was enough to indicate the one just stated, because it perfectly distinguishes this double refraction from all others hitherto observed.

Indeed it had been found until now that the double refraction of crystals of two axes, like that of crystals of one axis, completely polarizes the two beams into which it divides the incident light, one beam in one direction, the other in a perpendicular direction. The double refraction produced by the compression of glass is accompanied by the same phenomena of polarization, as one may ascertain with the small device that I had the honor to set before the eyes of the Academy [8], and by means of which one obtains two distinct images. One would therefore be tempted to believe at first that this is a general rule applicable to any kind of double refraction; but it is no longer so for that which the light undergoes when it traverses the needles of quartz in the directions sensibly parallel to their axes. The two beams of light come out modified in the same manner as they would have been by the process that we have

[9, (a)] Extrait d'un Mémoire sur les rotations que certaines substances impriment aux axes de polarisation des rayons lumineux [Extract of a Memoir on the rotations that certain substances impart to the axes of polarization of light rays], *Annales de Chimie et de Physique* [Ser. 2], vol. 9 [1818], from p.372; vol. 10 [1819], from p.63. Mémoire sur les rotations que certaines substances impriment aux axes de polarisation des rayons lumineux [Memoir on the rotations that certain substances impart to the axes of polarization of light rays, read 22 Sep. 1818], *Mémoires de l'Académie des Sciences de l'Institut*, vol. 2, from p. 41, [nominally for] year 1817.

[10, (b)] Memoir on the colors developed in homogeneous fluids by polarized light, No. XXIII. [This memoir was translated into English as early as 1852 [6], but remains somewhat cryptic because it was written before Fresnel adopted the transverse-wave hypothesis. Where citations from this memoir [6] have been inserted in the text, the page numbers refer to the translation.]

[11] French: *huile de térébenthine*.

[12, (a)] Vol. 20 [Ser. 2], from p.376, issue [nominally] for August 1822; see No. XXVI [ref. [8], recently translated into English].

recalled. So now, for this new modification, there are two ways to produce it, analogous to the two principal means employed to polarize light. The one consists in the division of the beam of direct light by a particular double refraction, and the other in a certain combination of reflections: one outside the glass at an inclination of 35°, then two more in the interior of this same substance at an incidence of 50°.

To obtain separation of the light into two distinct beams by the very weak double refraction that quartz exerts along its axis, I had a crystal prism cut whose entry and exit faces were equally inclined to the axis and formed between them an angle of 152°, and at first I achromatized this prism as well as I could with two half-prisms of Saint-Gobain glass, whose refracting angles were much smaller than half of 152°, because the Saint-Gobain crown[13] is more dispersive than quartz. Although one could use this apparatus in a pinch, and although it was enough for my initial verifications, yet because it did not seem to allow a perfect achromatism, I thought I would better fulfill this condition by replacing the two crown half-prisms with two half-prisms of quartz, whose double refraction along the axis would be of the opposite type to that of the intermediate prism. For, as Mr. Biot first noted, there are some quartz plates that rotate the plane of polarization of the incident light from left to right, while others rotate it from right to left;[14] and I was able to conclude from there, according to the theoretical representation that I had found for these phenomena, that the one of the two beams that travels faster in the first type of crystal must, on the contrary, travel slower in the second, and consequently that the angular deviations produced by the two achromatizing half-prisms must add to that produced by the obtuse prism if it is of the opposite type (instead of subtracting from it, as would happen if it were of the same type, because of the opposition of the refracting angles). This indeed is what happens, and we obtain in this manner a very noticeable separation of the two images, which we could further increase by multiplying the number of prisms.

5. I believe that one would succeed, by an analogous process, in exposing the double refraction of liquids that enjoy the optical properties of quartz plates perpendicular to the axis, such as turpentine, lemon essence, etc., by employing an apparatus similar to the above. As the essences of lemon and turpentine rotate the plane of polarization of the light in contrary directions, one could combine some hollow prisms full of turpentine with some prisms containing lemon essence, which would achromatize the former while increasing the divergence of the two beams of light. I estimate that forty prisms would suffice to make the separation of the two images quite perceptible; but because of this large number of prisms and the considerable width of their refracting angles, achromatization would undoubtedly become very difficult. Perhaps it would be facilitated by mixing with one of these essential oils some other liquid, such as spirit-of-wine. These mixtures of liquids generally present so many such possibilities that I can hardly believe the experiment to be impracticable, and although it must be long in trial and error and quite expensive, I would have tried it if I had not long been assured by interference procedures [6, pp. 45–6] that light passes through turpentine with two different speeds, and that this double refraction has the same characteristics as that of quartz along the axis—an identity that one could already deduce, as least as very probable, from the perfect similarity that Mr. Biot had recognized in their coloration phenomena.

Having obtained, by the combination of the two different types of quartz, an apparatus that neatly shows the effects of double refraction along the axis of the needles, I was able to verify the formulae by which, in the Memoir [6] submitted to the Academy at the beginning of 1818, I had represented the optical properties of turpentine and the plates of quartz perpendicular to the axis.

6. From the first I recognized that this double refraction was very different for rays of different colors, and much stronger for violet rays than for red rays,[15] as the coloration phenomena of turpentine had led me to suppose. It suffices to look at a bright line through a prism achromatized as I have just described: one will notice that the two images are bordered by a fringe of a purplish blue on the extreme

[13] Fresnel names *crown* in English.

[14, (a)] Expériences sur les plaques de cristal de roche taillées perpendiculairement à l'axe de cristallisation [Experiments on quartz plates cut perpendicular to the axis of crystallization], *Mémoires de la Classe des Sciences mathématiques et physiques de l'Institut*, for 1812, Part 1, p. 218.

[15] Corrected from the *extrait* [12, vol. 1, p. 722]. The printed text of the original memoir [12, vol. 1, p. 740] has it the other way around, inconsistent with the rest of the paragraph and §3 above.

sides, and to the contrary by a tawny red on the two sides nearest to each other; and when the bright line has even a slightly sensible width, the middle of the interval that separates the two images, instead of being entirely black, presents a dark red. We could if necessary measure this dispersion, which should be called *dispersion of double refraction*, and compare these measurements taken for the seven principal species of rays with the differences between their double refractions as deduced from Mr. Biot's law on the deviation of the planes of polarization in plates perpendicular to the axis, or even with the results that I had obtained, before the discovery of this law, by compensating for the polarizing effect of a tube full of turpentine oil by means of a plate of calcium sulfate parallel to the axis [6, pp. 48,61–5]. But this verification, which I may try later and whose result I regard as infallible, would require many precautions and a proper apparatus; for the moment I have contented myself with the crude verification offered by the simple appearance of the two images, which suffices to show that the dispersion of this double refraction is very large relative to the double refraction itself, as I had announced in my Memoir on the phenomena of coloration of turpentine [6, pp. 49–52].

It also followed from my formulae that simply polarized light, like ordinary light, should always give two images of equal intensity when subjected to this double refraction, whatever be the azimuth of its plane of polarization, whereas polarized light modified by two total reflections must then give only a single image, sometimes that which suffers the stronger refraction, and sometimes that which suffers the weaker refraction, depending on whether the plane of the two successive reflections would have been directed to the right or to the left of the initial plane of polarization, and also, from what we have said previously, on the nature of the quartz needles; for in some it is the light modified from right to left that must travel slower, and in others the light modified from left to right.

7. As the two beams produced by this double refraction must show the same characteristics as two beams of previously polarized light which have then suffered two total reflections in the azimuths of 45° relative to the initial plane of polarization, one to the right of this plane and the other to the left, it follows that if the two emergent beams pass through a second prism of quartz, parallel to its axis, each beam must there suffer the same refraction as in the first prism if the two prisms are of the same species, and the opposite refraction if they belong to needles of opposite species. But in all cases the superposition of these two prisms and even of a larger number of such prisms, always traversed by the rays in directions nearly parallel to the axes, must never give more than two images of the same object, in whatever azimuths they are turned relative to each other—whereas with the double refractions observed hitherto, one can always obtain four images by the superposition of two prisms, eight with three prisms, and so on.

All these consequences of my formulae are found to be confirmed by experiment. I must declare however that I have not combined more than two prisms and that, one of them being achromatized with crown, I have not been able to make observations as clear and as sure as if it had been achromatized like the other with quartz of the opposite species. But once it is well established by experiment that the beams emerging from the first prism are modified precisely like the light that has suffered two total reflections, and that this light gives only one image through the prism, it is evident that any number of such prisms traversed by ordinary light will never divide it into more than two beams.

If in the preceding note I announced only one of these consequences, it was because the others necessarily followed therefrom. For according to the principles of interference, any light that takes on the characteristics of ordinary polarization by the two total reflections, which entirely depolarize polarized light, must be modified in the same manner as polarized light subjected to these two total reflections; and from this follow all the other phenomena that I have just described.

But considering only the facts, we see at once that the two beams into which direct light is divided by the double refraction in question behave each like polarized light modified by two total reflections: first when we analyze them with a rhomb of calcite, since each of them gives two images of equal intensity, in whatever azimuth we turn the principal section of the rhomb; and second when we subject them to two total reflections in the interior of a glass parallelepiped at the incidence of about 50°, since they are then found to be polarized in two planes inclined at 45° to the plane of reflection, one to the left and the other to the right of this plane.

8. I wanted to reassure myself by another experiment of the identity of the modifications that light undergoes in the two cases, by comparing the colors produced in crystalline plates by the beams resulting from this double refraction, with the tints developed in the same plates by polarized light that has suffered the double total reflection: now I found that they were absolutely the same. It is thus well demonstrated that the two processes give light the same modification.

It shows the remarkable characteristic that the light ray which has received it has the same properties all around; that is, it behaves in the same way on whichever side we take it. For if we make it traverse a rhomb of calcite, it always gives two white images of the same intensity, whichever way we turn the principal section of the rhomb; and if this ray is totally reflected twice in the interior of glass at the incidence of 50°, it is always polarized in a plane inclined at 45° to the plane of incidence, whatever azimuth we have chosen for the latter, except that its new plane of polarization can be to the right or to the left of the plane of reflection, according as the ray will have received the modification from right to left or that from left to right; and finally when we make it traverse a thin crystalline plate and analyze the emergent light with a rhomb of calcite, we observe the same tints whichever way we direct the axis of the crystalline plate while leaving it perpendicular to the ray, and the absence of color, and respectively the maximum of coloration, always occur where the principal section of the rhomb is parallel or perpendicular to that of the plate, and respectively when it makes an angle of 45° therewith.

On the contrary, a ray that has received the ordinary polarization presents different properties around it in the different azimuths, and does not behave in the same way on whichever side we take it; there are above all two perpendicular directions in which it offers very different characteristics: when we make it traverse a rhomb of calcite whose principal section is parallel to the first direction, it suffers therein only the ordinary refraction, and it suffers the extraordinary refraction when this principal section is parallel to the other direction.

9. From the sole consideration of the facts, one could give the name *linear polarization*[16] to that which had long been observed in the double refraction of calcite, and which Malus was first to notice in the light reflected on transparent bodies, and the name *circular polarization* to the new modification whose characteristic properties I have just described: this is naturally divided into *circular polarization from left to right*, and *circular polarization from right to left*. These denominations, which were suggested to me by the hypothesis that I have adopted on the vibrations of light, indicate the very nature of their movements in both cases; but, fearing to abuse the Academy's time, I thought I should limit myself here to justifying the new proposed names by the simple exposition of the facts. The theoretical developments naturally find their place in a supplement, which I shall join to this Memoir.[17]

10. Between the linear polarization and the circular polarization, there exist a multitude of different intermediate orders of polarization, which partake of the characteristics of the two extremes, and which one could call *elliptical polarizations*, in accordance with the same theoretical views. One can produce various kinds of polarization, perhaps by a single total reflection or several such reflections, varying the angle of incidence, or perhaps by two total reflections always at the incidence of 50°, but varying the angle that the plane of reflection makes with the initial plane of polarization, which angle we had previously supposed to be 45°.

The laws of interference of polarized rays give a very simple means of comparing all these different kinds of polarization and of understanding them in a general formula. We have already said that a beam of circularly polarized light could be considered as composed of two beams of equal intensity polarized in perpendicular directions, and differing in their progress by a quarter of a cycle. When the beam that precedes the other in its path has its plane of polarization to the left of that of the lagging beam, the circular polarization is from left to right; it is from right to left in the contrary case, or if the planes of

[16] Literally "rectilinear polarization".

[17, (a)] This supplement was probably never composed; but the Memoir on double refraction, printed in vol. 7 of the Memoirs of the Academy, contains the sufficiently complete theory of circular polarization and elliptical polarization; see No. XLVIII [sic], §§ 10 to 15. [This posthumously published memoir [11] is actually numbered XLVII in the *Oeuvres complètes*. For an English translation of the cited paragraphs, see [11], tr. Hobson, from p. 253, "Suppose now..." to p. 257, "... left to right."]

polarization are arranged as we first supposed but the difference in progress is equal to three quarters of a cycle instead of one quarter.

When the difference in progress is a half-cycle or a whole cycle or, in general, a whole number of half-cycles, the union of the two beams constantly offers all the characteristics of linear polarization. If the two beams are of the same intensity, as we have supposed, the plane of polarization of the composite beam bisects the angle [between the planes of polarization] of the two constituent beams; if they are of unequal intensities, this plane comes closer to the plane of polarization of the more intense beam, and the cosines of the angles that it makes with the planes of polarization of the two constituent beams are proportional to the square roots of the respective intensities of these two beams.

11. When the difference in progress between the two beams (always supposed of equal intensity) is neither an even number nor an odd number of quarter-cycles, but a fractional number of quarter-cycles, then the total light possesses neither circular polarization nor linear polarization, but a polarization of an intermediate order, of the kind discussed above; it comes closer to circular polarization or to linear polarization according as the difference in progress between the two beams is closer to an odd number or to an even number of quarter-cycles. By gradually varying this path difference, one will have all the orders of intermediate modification between linear polarization and circular polarization.

One can also obtain them with a path difference equal to an odd number of quarter-cycles, by varying the relative intensities of the two constituent beams, or the angle between their planes of polarization. Very simple calculations show how these various combinations relate to each other.

12. In all that I have just said, I have always supposed that the path difference between the two beams polarized at right angles is proportional to the wavelength of the species of rays considered; so, by speaking in general of a path difference of a quarter-cycle, I mean a difference of a quarter of a red wave for the red rays, and a quarter of a violet wave for the violet rays, and so on. It is precisely because of this (at least very close) similarity of modification, which the various rays receive in the aforesaid total reflections, that white light modified in this way does not present any perceptible coloration when we analyze it with a rhomb of calcite.

It is no longer so in the beautiful phenomena that Mr. Arago has discovered by passing polarized light through thin crystalline plates and then analyzing it with a rhomb of calcite. The emergent light is indeed composed of two beams which are polarized at right angles, one parallel to the axis of the plate and the other in a perpendicular direction, and which, not having passed through the plate at the same speed, differ in their progress by a certain interval depending on its thickness and the energy of the double refraction. But this interval for the various rays is not proportional to their wavelengths: it is about the same for the rays of different colors, at least in many crystals such as calcium sulfate, mica, and plates of quartz parallel to the axis, etc.; and when it differs notably from one ray to another, far from approaching proportionality to the wavelengths, it seems to be always in a contrary sense. Hence it follows that if the path difference arising from the double refraction of the crystalline plate corresponds to (e.g.) three quarters of a cycle for the red rays, it will not correspond to three quarters of a cycle for the green rays, whose wavelength is smaller, and that therefore the rays of diverse colors will have been diversely modified. The phenomena of coloration presented by white light on exit from a crystalline plate, when analyzed with a rhomb of calcite, follow from precisely this diversity.

If we wanted, by means of such a plate, to impart to the rays a *single* mode of polarization, it would be necessary to employ a light as homogeneous as possible, and to thin the plate or incline it somewhat, until the path difference between the two beams was equal to an odd multiple of the quarter-wavelength of the rays employed, if it is (e.g.) the circular polarization that we want to impart to them. So I suppose that we use red light and that having previously polarized it, we make it traverse a crystalline plate whose axis is turned in an azimuth of 45°, and whose thickness is such that the difference in progress between the ordinary and extraordinary rays is equal to 3/4 of a red wavelength; then the emergent light—being composed of two beams equal in intensity, polarized at right angles, and differing in their progress by a quarter of a cycle—must present all the characteristics of circular polarization. If we make it traverse a rhomb of calcite[18], it will always give two images of the same intensity, in whatever azimuth we turn the

[18] Literally "spar of Iceland" in this instance.

principal section of the rhomb; this is what I had long verified by experiment. And if we subject it to two total reflections in the interior of a glass parallelepiped at the incidence of 50°, it finds itself polarized linearly in an azimuth of 45° relative to the plane of reflection. And finally if we make it traverse a quartz prism in a direction parallel to the axis of crystallization, instead of dividing into two distinct beams it gives only a single image. I have not yet done these last two experiments, but they cannot fail to confirm what I have just said.

13. Having set out the main characteristics of the peculiar double refraction that manifests itself in quartz parallel to the axis of the needles, and of the circular polarization that this imparts to the light in dividing it into two distinct beams, I still need to explain the phenomena of coloration of quartz plates perpendicular to the axis.

It follows from this double refraction, as we have seen, that the rays circularly polarized from right to left do not travel through the quartz with the same speed as the rays circularly polarized from left to right; for, whether we adopt the theory of waves or that of emission, a difference of refraction between two beams always supposes a difference of speed; if they had the same speed, it would be impossible to separate them, whichever way we cut the prism. I had already long recognized the existence of these two speeds of the light in turpentine, by interference experiments reported in the Memoir already cited [6, pp. 45, 46, 47].

It follows from the laws of interference of polarized rays that a beam of light which has received linear polarization can always be replaced by two beams equal in intensity and circularly polarized, one from right to left and the other from left to right, the combination of these two beams being equivalent to the incident beam. But these two constituent beams, as they do not travel through the quartz with the same speed, will differ in their progress, in proportion as the path is longer, or even in their directions if the refracting faces are not perpendicular to them. It is this divergence that we have made visible by cutting the crystal into a prism.

That being said, let us consider what happens when a polarized beam travels parallel to the axis of a quartz plate [with faces] perpendicular to its axis. Then the two circularly polarized beams, into which the incident beam can be considered divided, pass through the crystal with different speeds, but will not separate in their directions; one of them merely falls behind the other by a margin that grows in proportion to the length of the path. Now if we calculate, by the same rules of interference, the result of this difference in progress for any species of rays, we find that the ensemble of the two beams must always offer the characteristics of linear polarization, but that its plane of polarization, instead of coinciding with that of the incident beam, will have deviated from it by a certain angle, and that this angle must be proportional to the difference in progress divided by the wavelength, and hence to the length of the path for a given species of rays, as Mr. Biot had concluded from his observations. The initial plane of polarization is tilted from right to left or from left to right, according as the beam circularly polarized from right to left traverses the crystal faster or slower than the beam circularly polarized from left to right. There are some needles of quartz in which the first travels faster than the second, and some, on the contrary, in which it travels slower; this is the cause of the opposition of their optical properties. In these needles, where Mr. Biot has discovered two sorts of rotation of the plane of polarization, one from right to left and the other from left to right, I always see, for each, the incident light dividing into two beams circularly polarized, one from right to left and the other from left to right; but the one of the two that travels faster in the former needles is the one that falls behind in the latter. This difference in ways of stating the facts arises because Mr. Biot has always considered the total light emerging from the crystal, whereas I have got down to the elements that compose it; but as soon as we apply the formulae of interference to the ensemble of these elements, we fall back upon the law discovered by Mr. Biot.

The angular deviation of the plane of polarization of the total light being proportional to the aforesaid difference in progress, divided by the wavelength, it follows that if this difference in progress over the same path were equal in length for the various light rays—that is, if they suffered double refraction to the same degree—the angles of deviation of their planes of polarization would be to each other in inverse proportion to their wavelengths. But the double refraction is very different for each of them, being (e.g.) much greater for violet rays than for red rays, and this increases in the same proportion the separation of their planes of polarization. If we admit that this double refraction is inversely proportional to the wavelength of the ray—or, in other words, that the difference in progress between the beam circularly polarized from right to left and the beam circularly polarized from left to right is always the same for the same number of luminous undulations, whatever be their length—then we are brought back to the law that Mr. Biot deduced from direct experiments made with various species of homogeneous light: that the deviation of the plane of polarization of the total emergent light is, for the same plate, inversely proportional to the square of the length of the fits.

It is these unequal deviations of the planes of polarization of rays of various colors that cause the phenomena of coloration presented by polarized white light when it is passed through a plate of quartz [with faces] perpendicular to the axis, or a tube filled with turpentine, and then analyzed with a rhomb of calcite. But it is understandable that if we use circularly polarized light, as it can take only a single speed in the quartz plate or in turpentine oil, it will not suffer any division therein, and will come out as it went in, with all the characteristics that it had before; hence, being analyzed by means of a rhomb of calcite, it will always give two images of equal intensity, which will be white if the incident light was white. If, on exit from the plate or from the turpentine, it is subjected to two total reflections at the incidence of 50°, it will be polarized in a plane inclined at 45° to the plane of reflection; or if we pass it through a thin crystalline plate and then analyze it with a rhomb of calcite, it will give colors absolutely similar to those that it developed in the same plate before having traversed the plate of quartz or the tube filled with turpentine. This is what I had long ago verified by experiment.

By the same principles one can also explain all the other optical properties of quartz plates perpendicular to the axis and of homogeneous fluids that color polarized light.

Paris, 9 December 1822.

References

[1] Académie des Sciences, *Procès-verbaux des séances de l'Académie tenues depuis la fondation de l'Institut jusqu'au mois d'août 1835*, vol. 7 (for 1820–23), Hendaye, Basses Pyrénées: Imprimerie de l'Observatoire d'Abbadia, 1916.

[2] M.V. Berry and M.R. Jeffrey, "Conical diffraction: Hamilton's diabolical point at the heart of crystal optics", in E. Wolf (ed.), *Progress in Optics*, vol. 50, Amsterdam: Elsevier, 2007, pp. 13–50; michaelberryphysics.files.wordpress.com/2013/07/berry400.pdf.

[3] J. Z. Buchwald, *The Rise of the Wave Theory of Light: Optical Theory and Experiment in the Early Nineteenth Century*, University of Chicago Press, 1989.

[4] A. Fresnel, "Mémoire sur les modifications que la réflexion imprime à la lumière polarisée" ("Memoir on the modifications that reflection impresses on polarized light"), signed & submitted 10 Nov. 1817, read 24 Nov. 1817, printed in [12], vol. 1, pp. 441–85.

[5] A. Fresnel, "Supplément au Mémoire sur les modifications que la réflexion imprime à la lumière polarisée" ("Supplement to the Memoir on the modifications that reflection impresses on polarized light"), signed 15 Jan. 1818, submitted 19 Jan. 1818, printed in [12], vol. 1, pp. 487–508.

[6] A. Fresnel, "Mémoire sur les couleurs développées dans les fluides homogènes par la lumière polarisée", read 30 March 1818, subsequently mislaid, published in *Mémoires de l'Académie des Sciences de l'Institut de France*, vol. 20 (1849), pp. 163–94; reprinted in *OC* [12], vol. 1, pp. 655–83; translated by E. Ronalds & H. Lloyd as "Memoir upon the colours produced in homogeneous fluids by polarized light" in Taylor [16], 1852, pp. 44–65. Cited page numbers are from the translation.

[7] A. Fresnel, "Note sur le calcul des teintes que la polarisation développe dans les lames cristallisées" et seq., *Annales de Chimie et de Physique*, Ser. 2, vol. 17, pp. 102–11 (May 1821), 167–96 (June 1821), 312–15 ("Postscript", July 1821); reprinted (adding section nos.) in [12], vol. 1, pp. 609–48; translated by G.R. Putland as "On the calculation of the tints that polarization develops in crystalline plates, & postscript", doi.org/10.5281/zenodo.4058004 (2021).

[8] A. Fresnel, "Note sur la double réfraction du verre comprimé", read 16 Sep. 1822; reprinted in *OC* [12], vol. 1, pp. 713–18; translated by G.R. Putland as "Note on the double refraction of compressed glass", doi.org/10.5281/zenodo.4706834 (2021).

[9] A. Fresnel, "Extrait d'un Mémoire sur la double réfraction particulière que présente le cristal de roche dans la direction de son axe" ("*Extrait* of a Memoir on the peculiar double refraction presented by quartz in the direction of its axis"), *Bulletin de la Société philomathique*, 1822, pp. 191–8; repr. in [12], vol. 1, pp. 719–29.

[10] A. Fresnel, "Mémoire sur la loi des modifications que la réflexion imprime à la lumière polarisée" ("Memoir on the law of the modifications that reflection impresses on polarized light"), Jan. 1823; reprinted in *OC* [12], vol. 1, pp. 767–99.

[11] A. Fresnel, "Mémoire sur la double réfraction", *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, vol. 7 (nominally for 1824, printed 1827), pp. 45–176; reprinted as "Second mémoire..." in [12], vol. 2, pp. 479–596; tr. by A.W. Hobson as "Memoir on double refraction" in Taylor [16], 1852, pp. 238–333. For notable errata in the original printing, and consequently in the translation, see [12], vol. 2, p. 596*n*.

[12] A. Fresnel (ed. H. de Sénarmont, E. Verdet, & L. Fresnel), *Oeuvres complètes d'Augustin Fresnel* (3 vols.), Paris: Imprimerie Impériale, 1866, 1868, 1870.

[13] S. Ganci, "Was Poisson's spot an intentional discovery?" *Optik*, vol. 124, no. 19 (Oct. 2013), pp. 3906–8.

[14] C. Huygens, *Traité de la Lumière* (Leiden: Van der Aa, 1690), tr. by S.P. Thompson as *Treatise on Light*, University of Chicago Press, 1912; Project Gutenberg, 2005, gutenberg.org/ebooks/14725. Cited page numbers match the Chicago edition and the Gutenberg HTML edition. See also "Errata in various editions of Huygens' *Treatise on Light*" at *www.grputland.com* or *grputland.blogspot.com*, June 2016.

[15] J. G. Lunney and D. Weaire, "The ins and outs of conical refraction", *Europhysics News*, vol. 37, no. 3 (May–June 2006), pp. 26–29; doi.org/10.1051/epn:2006305.

[16] R. Taylor (ed.), *Scientific Memoirs, selected from the Transactions of Foreign Academies of Science and Learned Societies, and from Foreign Journals*, vol. 5, London: Taylor & Francis, 1852; archive.org/details/scientificmemoir05memo.