EMUE activity A1.2.5
GUM-LPU uncertainty evaluation - importing measurement traceability from a conformity statement

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1 Summary

Measurement traceability is commonly obtained from calibration measurements that provide a result in terms of a single value and its associated uncertainty. However, there are circumstances where instead, the result may consist of a range of possible values. Such circumstances might arise when a result is provided in the form of the output from a conformity decision process, for example as a conformity statement in which a range of acceptable values rather than a specific value is reported. In terms of metrological traceability this style of result provides less information than a specific value, but it may be sufficient to obtain an acceptable target measurement uncertainty for a given application. The standard ISO/IEC 17025 [10] acknowledges this in informative annex A. This example describes how such information might be used to propagate traceability.

2 Introduction

Under typical circumstances, evaluation of measurement uncertainty following the GUM [1] law of propagation of uncertainty (LPU), involves assigning a probability density function (PDF) to the measurand that usually has a normal distribution (or sometimes a t distribution). This 'output' PDF corresponds to a combination of the PDFs for all the inputs to the measurement. It is characterised by: a location parameter — the mean value corresponding to the estimate y of the value of the measurand; and a dispersion parameter — the corresponding variance $u^2(y)$ associated with that estimate. If this result subsequently becomes an input to a further measurement process, the variance is 'imported' into the corresponding uncertainty evaluation.

However, suppose instead that whilst we still obtain information that allows us to establish the variance $u^2 = u^2(y)$, we are *not* given a specific value for y. Instead we receive only information about the interval A, e.g. $[-a, a]$, in which the estimate y is located. In other words, we know the dispersion of y, but we do not have a value for the location of y, only a range of possible values.

In this case, the information can be still be brought into a subsequent uncertainty budget, but now as *two* independent PDFs: say for example a normal distribution $\mathrm{N}(0, u)$ characterising the dispersion of values around any given value of y, and a rectangular distribution $\mathrm{R}(-a, a)$ characterising the available information about the location parameter. For ease of explanation we will usually assume here that intervals A for y are centred on zero, but this is not a necessary requirement.

This situation is of potential interest to those concerned with meeting the requirements of ISO/IEC 17025:2017 since this standard [10, Informative Annex A.2.3] accepts that metrological traceability could be provided by statements of conformity.

Ideally a statement of conformity will include (i) the specification or tolerance interval C for the measurand Y (such that $-c \leq Y \leq c$), (ii) an acceptance interval A for the estimate y (such that $-a \leq y \leq a$) defined by a Decision Rule that takes direct or indirect account of measurement uncertainty, and (iii) a conformance probability p_c, which is the basis for (or a consequence of) how the acceptance interval is defined. In fact, in many practical situations a so-called 'Simple Acceptance' criterion is used to define the limits for deciding conformity, in which case $A = C$. In this case, in order to meet the requirements for a Decision Rule appropriate for ISO/IEC 17025:2017, uncertainty is taken into account *indirectly*, usually by specifying an upper limit $u_{\max}$ that, as a *prerequisite*, must not be exceeded for the Simple Acceptance criteria to be applied.

The aim here is to provide examples with various forms for the statement of results and to show whether they allow the results to be traceably propagated. We begin by describing some likely scenarios and then provide two extended examples.

3 Examples

In all the following examples it is assumed that the estimate $y \in A$, that is, the outcome is accepted as conforming, and that intervals are centred on zero. For this (two-distribution) model to be applied it is therefore necessary to identify A and u in each case.

3.1 Information given: Acceptance interval and measurement uncertainty for any specific value

For purposes of metrological traceability, it makes no difference *how* the interval A has been established ($A \neq C$ or $A = C$), only that it *is* somehow defined. Given A and u the approach is straightforward; the information can be brought into a subsequent uncertainty budget as two distinct distributions e.g. $R(-a, a)$ and $N(0, u)$ respectively.

In this situation the information might be obtained from a statement such as

> "Measured value y has a standard uncertainty $u = 1.3$ and is within the range $-10.0 \leq y \leq 10.0$."

Note that such a statement is not a conformity statement, as no specification or tolerance interval is given, nor is there an associated Decision Rule.

3.2 Information given: Acceptance interval is the same as the tolerance interval

This scenario, in which we *only* know that $y \in A$ and $A = C$, corresponds to 'unconstrained' simple acceptance, as there is no account of measurement uncertainty either 'directly' or 'indirectly'. (For this reason alone it would *not* meet the Decision Rule requirements of ISO/IEC 17025:2017.)

For example, suppose that a result is stated as:

> "Specified tolerance interval is from –2.0 to +2.0; measured result is 'conforming' as it is within the tolerance interval"

In this case there is insufficient information to establish a PDF for the outcome. The 'unconstrained' simple acceptance conformity statement is therefore insufficient to provide metrological traceability. It could not be 'imported' into an uncertainty evaluation, nor could any statement of risk be made on the basis of this information.

To make use of such a statement it would be necessary to establish uncertainty by other (external) means, e.g. to request the value of u from the information provider.

3.3 Information given: Tolerance and acceptance intervals and a statement about limits of probability or risk of acceptance

In this case, as well as stating C and A, a statement may include the minimum conformance probability $p_{c_{\min}}$ or the related quantity, maximum probability of false acceptance $R^*_{C_{\max}}$ (in the notation of [3]), which for the usual *specific* risk scenario is given by $R^*_{C_{\max}} = 1 - p_{c_{\min}}$.

The information might be found in a statement of conformity, for example, a statement such as

> "…specified tolerance interval is from –2.0 to +2.0; measured value is conforming as it is within the acceptance interval –1.5 to 1.5. Minimum conformance probability is 0.97"

An acceptance interval A has been provided for which we see that $-1.5 \leq y \leq 1.5$, that is, $a = 1.5$. Uncertainty u is not given, but can be calculated from the information provided since $p_{c_{\min}}$ occurs when $y = \pm a$; hence, for a normal distribution, measurement uncertainty u is calculable from

$$u = (c - a)/r, \tag{1}$$

where r is the guard band multiplier [3] (sometimes called the guard band factor) by which the standard uncertainty has been scaled to obtain the particular conformance probability,

$$r = g^{-1}(p_{c_{\min}}), \tag{2}$$

and $g^{-1}(x)$ is the inverse of the cumulative standard normal distribution.

In Microsoft Excel r can be evaluated using $r = \text{NORM.S.INV}(p_{c_{\min}})$ for situations where a significant proportion of the PDF lies beyond *only one or other* of the tolerance limits. Otherwise, in situations where the PDF is broad with respect to the tolerance interval, r must be established by other means (for example, UKAS LAB-48 ed 2, appendix D [12])

In this example we find that $c = 2$, $a = 1.5$ and $r = 1.88$; hence $u = 0.266$. As above, this information can be brought into a subsequent uncertainty budget as two distinct distributions $\text{N}(0, u) = \text{N}(0, 0.266)$ and $\text{R}(-a, a) = \text{R}(-1.5, 1.5)$.

3.3.1 Special case 1: interval not centred at zero

For a tolerance interval that is not centred on zero, say $[c_1, c_2]$, with corresponding (co-centred) acceptance interval $[a_1, a_2]$, the uncertainty is instead

$$u = [(c_2 - c_1) - (a_2 - a_1)]/(2r). \tag{3}$$

3.3.2 Special case 2: Simple acceptance

Consider the special case when $A = C$ which corresponds to so-called Simple Acceptance criteria. In this scenario it is usually reported that $p_{c_{\min}} = 50\,\%$. For such a case (where $A = C$, $p_{c_{\min}} = 50\,\%$) we find that u is undefined since $(c - a)/r = 0/0$, that is, there is insufficient information to calculate u; therefore the information is not sufficient to provide metrological traceability.

Note that this simple acceptance scenario ($A = C$) is sometimes misleadingly referred to as 'shared risk', referring to the situation when an accepted value corresponds to the tolerance limit ($y = \pm a$). In fact, this equality of risk is only true for single-sided specifications, or situations where $u \ll c$. In other situations where u is sufficiently large that *both* tails of the PDF have a significant portion outside C, then $p_{c_{\min}} < 50\,\%$ and the risk is no longer 'shared' equally. Fortunately, in those cases (where $A = C$ and $p_{c_{\min}} < 50\,\%$), it is possible to calculate u for a normal PDF from

$$u = \frac{2c}{g'(p_{c_{\min}} + 0.5)}. \tag{4}$$

or, for a tolerance interval that is not centred on zero, say $[c_1, c_2]$, the uncertainty is instead

$$u = \frac{c_2 - c_1}{g'(p_{c_{\min}} + 0.5)}. \tag{5}$$

In Microsoft Excel $g'(p_{c_{\min}} + 0.5)$ is given by the cell function $\text{NORM.INV}([p_{c_{\min}} + 0.5], 0, 1)$.

3.4 Information given: Acceptance intervals and a statement about limits of probability or risk of acceptance

This case corresponds to that described in section 3.3 but without information concerning the tolerance interval C. There is now insufficient information to establish a PDF for the outcome as u is not provided and cannot be calculated from the information given. The information is therefore not sufficient to provide metrological traceability (as it could not be 'imported' into an uncertainty evaluation).

Note that, for *accredited* conformity decisions under 17025:2017 it is a requirement to define and report the specification (or standard), which usually corresponds to providing C.

3.5 Information given: Tolerance and acceptance intervals and a statement about limits of global conformance probability or global risk of acceptance

In certain situations it is possible that the conformance probability p_c may be presented in terms of *global* risk [3]. Global risk is a measure of the risk associated with future measurements i.e. measurements that have not yet taken place. Although it is an important quantity in the evaluation of risk in general quality processes, it is arguably not consistent with the definitions of calibration [4, clause 2.39] or of a metrological traceability chain [4, clause 2.42] being a "sequence of measurement standards and calibrations that is used to relate a measurement result to a reference".

The information needed to implement the approach described in this example is therefore not generally available in such a conformity statement.

3.6 Traceability from a statement of conformance to an OIML weight classification

In this example we demonstrate how traceability might be propagated when the available information consists only of an OIMLR111-1 [11] weight classification. This example corresponds to the case in section 3.3 above and is depicted graphically in Figure 1.

From OIML R111-1, for each weight, the expanded uncertainty U of the conventional mass shall be less than or equal to one-third of the maximum permissible error: $U \leq \delta m/3$, where U relates to a coverage interval with a 95 % coverage probability.

Also, for each weight, the conventional mass, m_c shall not differ from the nominal value of the weight m_0 by more than the maximum permissible error (δm) minus the expanded uncertainty. The acceptance interval A is defined such that

$$m_0 - (\delta m - U) \leq m_c \leq m_0 + (\delta m - U) \tag{6}$$

and the tolerance interval C is defined by $[m_0 - \delta m, m_0 + \delta m]$.

Following the approach described above, the uncertainty associated with a classified weight value can be evaluated by combining the standard uncertainties for the PDFs describing the acceptance interval (information about location) and uncertainty (information about dispersion).

For example, for an E_2 class weight of nominal value 2 kg, OIML R111 defines the maximum permissible error δm as

$$\delta m = 3\,\text{mg}. \tag{7}$$

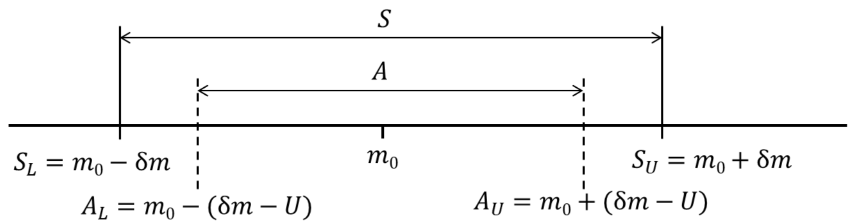

Figure 1: OIML Guard Band criteria

The corresponding maximum expanded uncertainty (95 % coverage, assumed normal distribution) is defined as

$$U = \frac{\delta m}{3} = 1\,\text{mg} \tag{8}$$

with the related standard uncertainty

$$u_1 = \frac{U}{1.96} = 0.51\,\text{mg}. \tag{9}$$

Note that the estimate of the standard uncertainty characterising dispersion in this and similar scenarios is based upon an upper limit of possible values. In situations where this is likely to represent a significant contribution to the overall uncertainty it may be appropriate to seek further information.

The limits of the acceptance interval for δm are $\pm a$, where

$$a = \delta m - U = 2\,\text{mg}. \tag{10}$$

If A is represented by a rectangular PDF, then the corresponding standard uncertainty is

$$u_2 = \frac{\delta m - U}{\sqrt{3}} = 1.15\,\text{mg}. \tag{11}$$

The standard uncertainty u_c associated with the nominal mass value can therefore be evaluated by combining these uncertainties:

$$u_c = \sqrt{u_1^2 + u_2^2} = 1.3\,\text{mg}. \tag{12}$$

More generally, if the expanded uncertainty U is required to be some factor D less than a maximum permissible error δm ($D = 3$ for the example above), and if the coverage probability p is obtained using a coverage factor k_p, then

$$u_1 = \frac{\delta m}{k_p D} \tag{13}$$

and

$$u_2 = \frac{\delta m(1 - 1/D)}{\sqrt{3}}. \tag{14}$$

Figure 2 shows how the standard uncertainty varies with factor D for a maximum permissible error$\delta m =$ 3 mg.

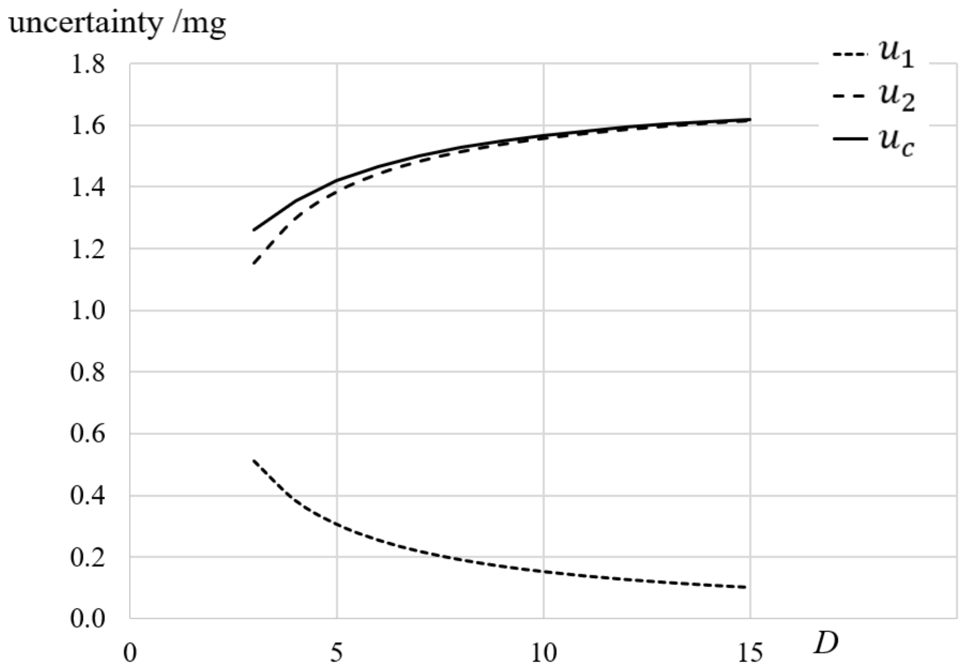

Figure 2: Maximum permitted standard uncertainty u_1, the semi-range of the acceptance interval a, and the combined standard uncertainty u for a range of values of $U = \delta m/D$

Note that as the standard uncertainty u_1 *decreases* with increasing D, the overall uncertainty u_c *increases* (due to the proportionately greater contribution corresponding to a). In this situation (where, in use, the value assigned to a weight will be the nominal value) we might perhaps conclude that it is not in the interest of a purchaser for U to be low when the weight is classified, whereas it *is* in the interest of a supplier of weights, as fewer potentially conforming products will be rejected.

Note also that the PDF associated with u_c is not normal. However, provided that u_2 is not a dominant quantity in the budget into which it is subsequently imported, the shape of the corresponding output PDF should not be significantly affected.

3.6.1 Comment on ISO/IEC 17025:2017 Annex A.2.3

Those readers familiar with ISO/IEC 17025:2017 [10] and in particular Annex A.2.3 might perhaps interpret that (informative) Annex to suggest that metrological traceability can be obtained from a rectangular PDF with limits corresponding to the tolerance interval $C = [-c, c]$. Annex A.2.3 cites "The use of OIML R 111 class weights to calibrate a balance", which might be interpreted as an example of that practice.

However, such an approach does not make best use of the information available: whereas the approach described in this example correctly converges on the appropriate 'combined' PDF in the limits of small and large u_1/c , the A.2.3 interpretation as stated above employs a rectangular PDF throughout, even though the normal distribution becomes proportionately more significant as u_1/c increases. As a consequence, that particular interpretation of A.2.3. can significantly overestimate the uncertainty. This is demonstrated in Table **??** where, for the particular scenario given, we see that the difference between estimates can exceed 50 %.

c/mg	a/mg	$u(c)$/mg	$u(a, u_m)$/mg
1 000	998	577	576
10	8	5.8	4.7
9	7	5.2	4.2
8	6	4.6	3.6
7	5	4.0	3.1
6	4	3.5	2.5
5	3	2.9	2.0
4	2	2.3	1.5
3	1	1.7	1.2
2	0	1.2	1.0

Table 1: Comparison between standard uncertainty estimates obtained for: the interpretation of ISO/IEc 17025 A.2.3 described above, identified as "$u(c)$"; and estimates based upon the two PDF approach described in this example, identified as "$u(a, u_1)$". PDFs are centred at zero. Estimates are for model data over a range of tolerance intervals $[-c, c]$ and corresponding acceptance intervals $[-a, a]$ with $u_1 = 1$ and guard band $w = c - a = 2u_1$.

For the OIML E2 – 2kg weight discussed in this example, $c = \delta m = 3$ mg and $w = U = 1$ mg hence $u_1 = 0.51$ mg, yielding $u(c) = 1.7$ mg and $u(a, u_1) = 1.3$ mg, a difference of nearly 40 %.

3.7 Calibration and verification of a caliper according to Geometrical Product Specification (GPS) standard ISO 13385-1:2019

In this example we demonstrate how traceability can be obtained from a verification statement for an instrument certified under a Geometrical Product Specifications (GPS) standard.

In general, laboratories accredited for the calibration of calipers adopt the GPS standard ISO 13385-1 [9]. According to this standard — which includes requirements for test methods, default values for maximum permissible errors (MPEs) and related decision rules — laboratories are variously required to provide two different uncertainty evaluations: one for the measured calibration values of the instrument, u_{cal}, and the other for its verification 'test uncertainty' u_{test} as defined in ISO 14253-5:2015) [6].

Certificates that meet the requirements of ISO/IEC 17025:2017 [10] concerning the reporting of calibration results (variously described in clauses 7.8.4, 7.8.6 and A.2.3) could present the information in various forms, as considered in the following scenarios:

1. Calibration certificate containing indication errors with the associated calibration standard uncertainty u_{cal},
2. Calibration certificate containing statement of conformity with a specification (MPE), associated test standard uncertainty u_{test} and decision rule, without indication errors (consistent with paragraph 3.3);
3. Calibration certificate containing only a statement of conformity with a specification (expressed in terms of MPE) with no reported indication errors, calibration measurement uncertainty or test verification uncertainty, but with a GPS decision rule that has somehow accounted for these quantities (consistent with paragraph 3.3).

Note that the purpose of *calibration* is to establish a traceable link to the SI, whereas the purpose of *verification* is only to decide conformity with a specification. In the case of specifications such as those represented by the GPS standards, the calibration standard measurement uncertainty u_{cal} (scenario 1) is therefore different from the test verification uncertainty u_{test}.

In the case of GPS, a specification is defined in terms of limits (MPE) that somehow already account for various influence quantities such as repeatability and resolution that would normally be incorporated into a calibration uncertainty evaluation. The evaluation of test verification uncertainty therefore does not include these quantities and is therefore less than the calibration measurement uncertainty (for further details see ISO 14978:2018 [8, annex D]). In principle however, all relevant influence quantities are present and, if combined correctly, the test verification uncertainty and information represented by the specification can be used to provide an evaluation of calibration measurement uncertainty required for dissemination of measurement traceability.

Scenario 1

In this straightforward scenario, the calibration of the caliper produces indication errors with associated calibration measurement uncertainty. Following best practice the errors can be corrected and the calibration measurement uncertainty can be propagated into the measurement chain.

Note that it is however common practice that error correction is not performed, and the associated uncertainty is enlarged by one of a variety of methods. Notwithstanding the potential issues associated with this poor practice [5], the GUM suggests a method to achieve this [1, clause F.2.4.5].

Scenario 2

In this case no quantitative information on the indication errors is available other than their values being within specification limits. In the absence of any other information, the best estimate of the error is therefore zero.

The uncertainty when in use by the customer, can be evaluated from the PDF resulting from the convolution of a normal probability distribution $\mathrm{N}(0, u_{\text{test}})$ and a rectangular distribution $\mathrm{R}(-a, a)$, where

$$a = \mathrm{MPE} - k u_{\text{test}} \tag{15}$$

and, for a two-sided specification, k can be calculated iteratively by applying equation (11) of JCGM 106:2012 [3], given the values of p_c and u_{test}.

For example, figures 3 and 4 present the PDFs for conformance probability values p_c equal to 50 % and 95 % respectively, for a range of values for measurement capability index C_m, where

$$C_m = \frac{2 \times \text{MPE}}{4u_{test}}. \tag{16}$$

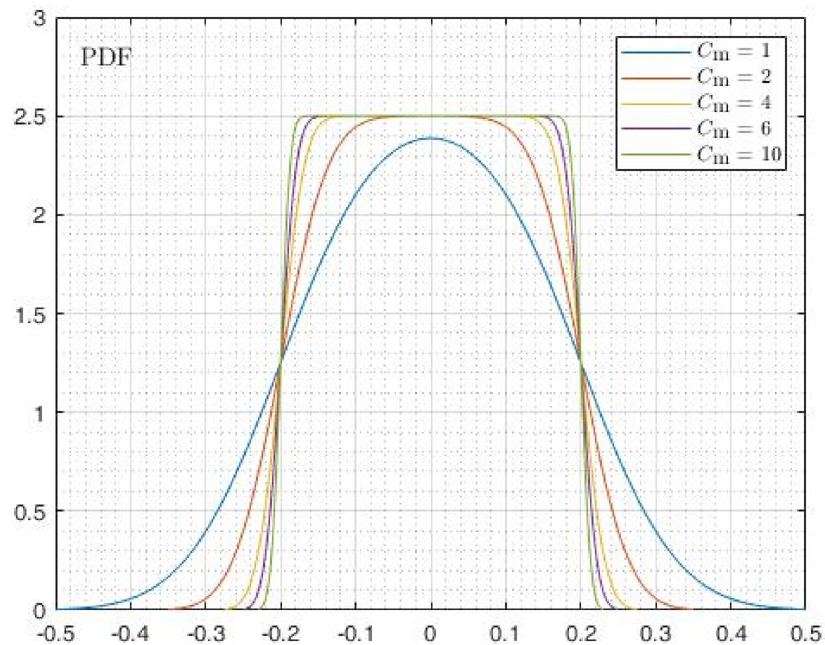

Figure 3: PDFs for $p_c = 50\%$, $MPE = 0.2$ and various C_m values

Once the PDF is established the associated standard deviation u can be found, for example by combining variances:

$$u^2 = \frac{(\text{MPE} - ku_{test})^2}{3} + u_{test}^2. \tag{17}$$

Assuming that the specification limits and u_{test} account correctly for all influence quantities that contribute to the calibration of the caliper (as is the premise of the GPS standard) then u corresponds to the calibration standard measurement uncertainty, (that is, $u_{cal} = u$).

Note that in general the standard uncertainty alone provides insufficient information for propagation of measurement results and knowledge of the PDF is needed, for example, whether it can be described by a known distribution such as a normal distribution. In cases where the shape of the PDF is dominated by the specification it will be more ‘rectangular’ than normal. In that case some other means of conveying information about the PDF is needed, such as in figures 3 and 4 or as might be provided by using a numerical approach for evaluating the uncertainty [2].

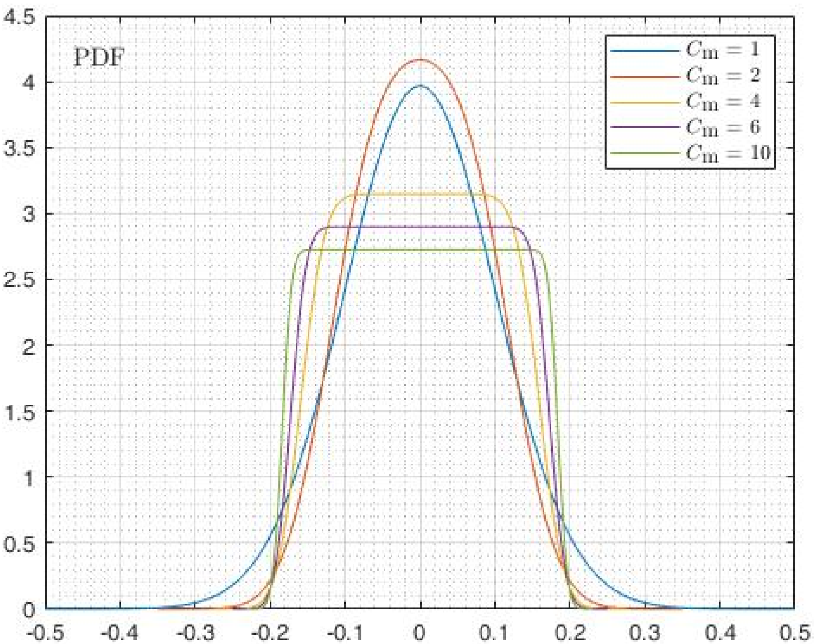

Figure 4: PDFs for $p_c = 95\,\%$, $MPE = 0.2$ and various C_m values

A standard uncertainty u_{MPE} associated with the specification can also be established by taking it to be the standard deviation of a rectangular PDF with limits ±MPE:

$$u_{MPE} = \mathrm{MPE}/\sqrt{3}. \tag{18}$$

In some situations u_{MPE} is used as an estimate of the calibration measurement uncertainty.

Figure 5 shows in the ordinate the standard deviation u of the PDF divided by u_{MPE} and in the abscissa C_m for various p_c values.

Note that for p_c values greater than 85 %, u_{MPE} provides a conservative overestimate of u_{cal} for all C_m values considered.

It is also clear that it is not possible to ensure values of p_c higher than 95 % for low values of C_m. As an example, considering the curve corresponding to the probability of 99 %, the first value of C_m that allows this probability is about 1.3.

For p_c values lower than 85 %, the use of the u_{MPE} can lead to an underestimation of measurement uncertainty for low C_m values, which happens for example in the case of p_c equal to 70 % and for C_m values lower than about 1.6.

For p_c values of 50 %, in order not to underestimate uncertainty, it is possible to multiply u_{MPE} by a 'safety factor' which can be determined by the graph for C_m less than 4. For higher C_m values the underestimation of the uncertainty is less than 1 % and therefore may not be significant.

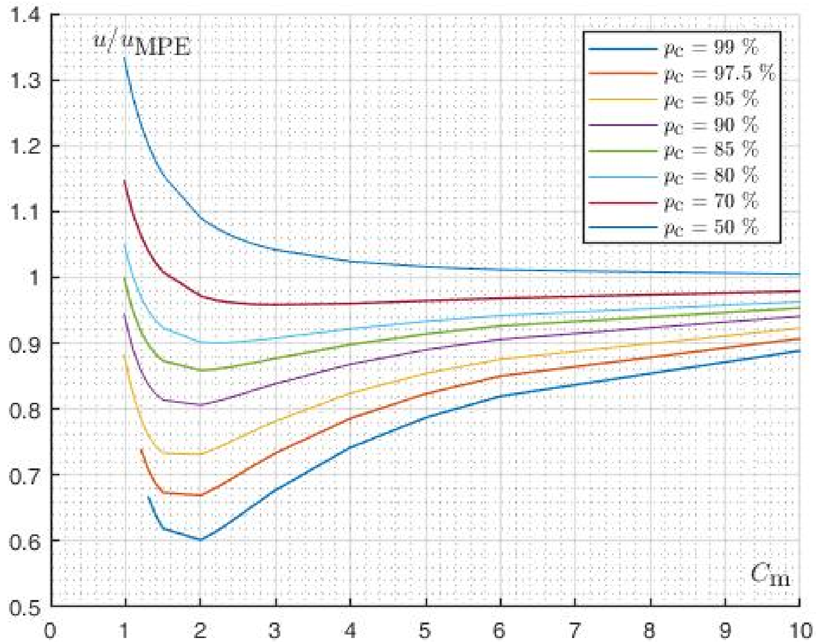

Figure 5: u/u_{MPE} as a function of C_{m} for various conformance probabilities p_{c}.

Scenario 3

In this case, no quantitative information on the indication errors and test uncertainty is available. The decision rule comes from ISO 13385-1:2019 [9]. However, the standard provides two different rules depending on the agreement with the customer:

Decision rule A If no decision rule is stated with the specifications, and no special agreement is made between supplier and customer, then the default rule of ISO 14253-1 [7] applies (ref. ISO 13385-1 [9, clause 6.3]). In this case the default conformance probability limit is $p_{\mathrm{c}} = 95\,\%$, which corresponds to a false acceptance probability less than or equal to 5 %. This information, combined with the MPE, can be used to evaluate the uncertainty to be attributed to the instrument when used by the customer, ensuring traceability. From this information (MPE and p_{c}), assuming that the distribution associated with the test uncertainty is normal, it is possible to provide an upper limit of standard uncertainty u_{max}. This value can be calculated from ISO 14253-1 [7, annex A, figure A.3], considering the most conservative condition with the ratio of the specification and the test uncertainty equal to 3.92, which corresponds to $C_{\mathrm{m}} = 0.98$:

$$\frac{2 \times \mathrm{MPE}}{u_{\mathrm{max}}} = 3.92. \tag{19}$$

This case is equivalent to scenario 2 with $p_{\mathrm{c}} = 95\,\%$ and $C_{\mathrm{m}} = 0.98$ (see figure 5).

Decision rule B If there is an agreement with the customer to verify the caliper with respect to the MPE values reported in [9, table B.1], the decision rule that applies shall be simple acceptance, with the measurement capability index C_m being four or larger (ref. annex B of ISO 13385-1) [9]. Although the test uncertainty is not reported in the Calibration certificate, it is possible to calculate a limit value of uncertainty u_max from the limit value of $C_\mathrm{m} = 4$:

$$u_\mathrm{max} = \frac{\mathrm{MPE}}{2C_\mathrm{m}} = \frac{\mathrm{MPE}}{8} \tag{20}$$

This case is equivalent to scenario 2 with $p_\mathrm{c} = 95\,\%$ and $C_\mathrm{m} = 4$ (see figure 5).

4 Additional notes and comments

4.1 Notes on risk in relation to uncertainty

The GUM is concerned with the propagation of PDFs (or in the case of GUM-LPU their variances).

Estimates of risk are given by integrals over certain ranges of a PDF (or a joint PDF) as described in [3]. They are not in a form that is directly propagated using GUM methodology.

Risk is usefully evaluated at times when a decision is needed concerning the acceptability of a result, normally at the *end* of a measurement chain. It may be of *passing* interest at intermediate points in the chain, but for propagation of traceability it is the underlying PDF that is of interest.

Therefore a statement of conformity and risk is generally *not* a useful alternative to a description (or summary) of the PDF for the measurand. An accredited laboratory, for example, would be expected to ensure that customers are aware of this lack of utility when their customers request a statement of conformity.

4.2 Notes on Simple Acceptance

It is worthwhile re-iterating the point that assertions of conformity based upon Simple Acceptance criteria *on their own*, with no account for measurement uncertainty whether it be direct or indirect, are *not* sufficient to provide traceability (or to define a meaningful Decision Rule, as the associated risk is *undefinable*).

Further, it is not possible to take indirect account of uncertainty by simply stating the value of the uncertainty *after* the decision is made, which corresponds to a situation in which the decision is made regardless of uncertainty or risk at the time the decision is being made.

4.3 Single sided specifications

The examples presented here have all been presented in terms of two-sided specifications which define an upper and a lower limit for the measurand. A key point is that such specifications allow a rectangular PDF to be established to describe the location of the quantity of interest, which would not be possible with a truly single-sided specification for which no such PDF can be established, there being only one defined limit.

5 Conclusions

The examples presented here have demonstrated various situations in which there is no explicit statement of a measurement result (in terms of a specific value and associated uncertainty), yet metrological traceability can be obtained from a statement of conformity together with a suitable specification and decision rule. (Such a situation is anticipated in [10]). Making optimum use of available information to establish metrological traceability is demonstrated for several general scenarios and is illustrated with two extended examples. The process recommended involves identifying two or more independent PDFs to represent the information that has been provided. Typically this approach results in a PDF that characterises the location of possible quantity values for the measurand, and a PDF that characterises the dispersion of possible values around any given value of the measurand. In practice, these PDFs are likely to have the well-known rectangular and normal distributions respectively and can be individually 'imported' into uncertainty evaluations based upon the GUM law of propagation of uncertainty or Monte Carlo Simulation as independent input quantities.

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