EMUE A1.2.2 Additive or multiplicative corrections: correlation example

Observed UUC value $p'_{uuc} := 255.9$ $u_{p'uuc} := 0.115$ ($u_{p'uuc}$ is the uncertainty for observed uuc indication)

Observed std value $p'_{std} := 256.4$ $u_{p'std} := 0.75$ ($u_{p'std}$ is the uncertainty for observed reference pressure)

calibration pressure correction $\Delta p_m := \frac{0.05}{100} \cdot p'_{std}$ $u_{\Delta pm} := 0.5$ ($u_{\Delta p}$ is the uncertainty in reference pressure correction)

UUC temperature coefficient $k_{uuc} := \frac{1}{300}$ $u_{kuuc} := 0.1\ k_{uuc}$ NOTE that EMUE example uses α rather than k

std temperature coefficient $k_{std} := \frac{1}{300}$ $u_{kstd} := 0.1\ k_{std}$

estimated temperature error $\Delta T := 0$ $u_{\Delta T} := \frac{1}{\sqrt{3}}$ (we set dT =0 to represent a poorly understood measurement effect.)

'true' std pressure ...

$$p_{std} := p'_{std} \cdot \left(1 + k_{std} \cdot \Delta T\right) = 256.4$$

$$C_{pstd} := \left[\left(1 + k_{std} \cdot \Delta T\right) \quad \left(p'_{std} \cdot \Delta T\right) \quad \left(p'_{std} \cdot k_{std}\right) \right] = \left[1 \quad 0 \quad 0.855 \right]$$

$$V_{in\_pstd} := \begin{bmatrix} u_{p'std}^{\,2} & 0 & 0 \\ 0 & u_{kstd}^{\,2} & 0 \\ 0 & 0 & u_{\Delta T}^{\,2} \end{bmatrix} = \begin{bmatrix} 0.563 & 0 & 0 \\ 0 & 1.111 \cdot 10^{-7} & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

$$V_{pstd} := C_{pstd} \cdot V_{in\_pstd} \cdot C_{pstd}^{\,\mathrm{T}} = 0.806$$

$$u_{pstd} := \sqrt{V_{pstd}} = 0.898$$

'true' uuc pressure ...

$$p_{uuc} := p'_{uuc} \cdot \left(1 + k_{uuc} \cdot \Delta T\right) = 255.9$$

$$C_{puuc} := \left[\left(1 + k_{uuc} \cdot \Delta T\right) \quad \left(p'_{uuc} \cdot \Delta T\right) \quad \left(p'_{uuc} \cdot k_{uuc}\right) \right] = \left[1 \quad 0 \quad 0.853 \right]$$

$$V_{in\_puuc} := \begin{bmatrix} u_{p'uuc}^{\,2} & 0 & 0 \\ 0 & u_{kuuc}^{\,2} & 0 \\ 0 & 0 & u_{\Delta T}^{\,2} \end{bmatrix} = \begin{bmatrix} 0.013 & 0 & 0 \\ 0 & 1.111 \cdot 10^{-7} & 0 \\ 0 & 0 & 0.333 \end{bmatrix}$$

$$V_{puuc} := C_{puuc} \cdot V_{in\_puuc} \cdot C_{puuc}^{\,\mathrm{T}} = 0.256$$

$$u_{puuc} := \sqrt{V_{puuc}} = 0.506$$

'true' pressure error, Δp ...

$$\Delta p := p_{uuc} - \left(p_{std} + \Delta p_m\right) = -0.6282$$

Input covariance, $V_{in\Delta p}$ for $\Delta p = f(X)$

$$X_1 = p_{std} = f_1\left(p'_{std}, k_{std}, p'_{uuc}, k_{uuc}, \Delta T, \Delta p_m\right) = p'_{std} \cdot \left(1 + k_{std} \cdot \Delta T\right)$$

$$X_2 = p_{uuc} = f_2\left(p'_{std}, k_{std}, p'_{uuc}, k_{uuc}, \Delta T, \Delta p_m\right) = p'_{uuc} \cdot \left(1 + k_{uuc} \cdot \Delta T\right)$$

$$X_3 = \Delta p_m = f_1\left(p'_{std}, k_{std}, p'_{uuc}, k_{uuc}, \Delta T, \Delta p_m\right) = \Delta p_m$$

$$C_X := \begin{bmatrix} (1 + k_{std} \cdot \Delta T) & (p'_{std} \cdot \Delta T) & 0 & 0 & (p'_{std} \cdot k_{std}) & 0 \\ 0 & 0 & (1 + k_{uuc} \cdot \Delta T) & (p'_{uuc} \cdot \Delta T) & (p'_{uuc} \cdot k_{uuc}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$V_X := \begin{bmatrix} u_{p'std}{}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & u_{kstd}{}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & u_{p'uuc}{}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & u_{kuuc}{}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_{\Delta T}{}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & u_{\Delta pm}{}^2 \end{bmatrix}$$

$$V_{in} := C_X \cdot V_X \cdot C_X{}^{\mathrm{T}} = \begin{bmatrix} 0.806 & 0.243 & 0 \\ 0.243 & 0.256 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$$

Output covariance, $V_{\Delta p}$...

$$C_{\Delta p} := [-1 \quad 1 \quad -1]$$

$$V_{\Delta p} := C_{\Delta p} \cdot V_{in} \cdot C_{\Delta p}{}^{\mathrm{T}} = 0.826$$

$$u_{\Delta p} := \sqrt{V_{\Delta p}} = 0.909$$

Relative 'non-correlated' to 'correlated' uncertainty estimate...

$$\sqrt{V_{puuc} + V_{pstd} + u_{\Delta pm}{}^2} = 1.145$$

$$\frac{\sqrt{V_{puuc} + V_{pstd} + u_{\Delta pm}{}^2}}{u_{\Delta p}} = 1.26$$

conformance probability with covariance...

$$spec := 0.005 \cdot p_{std} = 1.282$$

$$T_L := -spec \qquad T_U := spec$$

$$m := \Delta p$$

$$\sigma := u_{\Delta p}$$

$$p_U := 0.5 \cdot \mathrm{erfc}\left(\frac{T_U - m}{\sqrt{2} \cdot \sigma}\right) = 0.017771$$

$$p_L := 0.5 \cdot \mathrm{erfc}\left(\frac{m - T_L}{\sqrt{2} \cdot \sigma}\right) = 0.236$$

$$p_c := 1 - p_U - p_L = 0.746$$

conformance probability without covariance...

$spec := 0.005 \cdot p_{std} = 1.282$

$T_L := -spec \qquad T_U := spec$

$m := \Delta p$

$\sigma := \sqrt{V_{puuc} + V_{pstd} + u_{\Delta pm}{}^2}$

$p_U := 0.5 \cdot \operatorname{erfc}\left(\frac{T_U - m}{\sqrt{2} \cdot \sigma}\right) = 0.047674$

$p_L := 0.5 \cdot \operatorname{erfc}\left(\frac{m - T_L}{\sqrt{2} \cdot \sigma}\right) = 0.284$

$p_c := 1 - p_U - p_L = 0.668$