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Application in decision making based on fuzzy parameterized hypersoft set theory

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Abstract: Hypersoft set is the generalization of soft set as it converts single attribute function to multi-attribute function. The core purpose of this study is to make the existing literature regarding fuzzy parameterized soft set in line with the need of multi-attribute function. We first conceptualize the fuzzy parameterized hypersoft set along with some of its fundamentals. Then we propose decision making based algorithm with the help of this theory. Moreover, an illustrative example is presented which depicts its validity for successful application to the problems involving vagueness and uncertainties.

Key words: Soft set, fuzzy soft set, fuzzy parameterized soft set, hypersoft set.
AMS Subject Classification: 03E72, 03E75.

1. Introduction

In 1965, Zadeh [1] conceptualized the theory of fuzzy sets as mathematical mean to tackle many intricate problems involving various uncertainties, in different fields of mathematical sciences. But it has its own complexities which restrain it to solve these problems successfully. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. A mathematical tool is needed for dealing with uncertainties which should be free of all such impediments. In 1999, Molodtsov [2] has the honor to introduce such mathematical tool called soft sets in literature as a new parameterized family of subsets of the universe of discourse. Maji et al. [3] extended the concept and introduced some fundamental terminologies and operations of soft set. They also defined fuzzy soft set in [4] and successfully applied it in decision making. Some authors [5–11] discussed some more properties and relational features of soft set theory. Many researchers [12–24] developed certain hybrids with soft sets to get more generalized results for implementation in decision making and other related disciplines. Ça$\breve{g}$man et al. [25, 26] conceptualized fuzzy parameterized soft set which is the ordered pair of membership function of fuzzy set and approximate function of fuzzy soft. They discussed its properties and applied this theory to different fields.

In 2018, Smarandache [27] introduced the concept of hypersoft set as a generalization of soft set. Saeed et al. [28, 29] and Mujahid et al. [30] discussed some of its fundamentals and basic operations. Rahman et al. [35, 36] applied this concept in complex and convex set theories.

In many real life situations, we have disjoint attribute-valued sets corresponding to distinct attributes. The existing soft set theory doesn't deal such sets; therefore hypersoft set theory is conceptualized to tackle such situations. Motivating from the work of [25, 27, 30], fuzzy parameterized hypersoft set is characterized in

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order to adequate the literature regarding fuzzy parameterized soft set for multi attribute-valued functions and its some essential elementary properties are discussed. A decision making based algorithm is proposed with successful application for the best choice of product.

2. Preliminaries

Here some basic terms are recalled from existing literature to support the proposed work. Throughout the paper, $\mathcal{U}$, $P(\mathcal{U})$ and $I^\bullet$ will denote the universe of discourse, power set of $\mathcal{U}$ and closed unit interval respectively.

Definition 2.1. [1]
A *fuzzy set* $\mathcal{X} \subseteq \mathcal{U}$ defined as $\mathcal{X} = \{(\epsilon, \zeta_{\mathcal{X}}(\epsilon)) | \epsilon \in \mathcal{U}\}$ such that $\zeta_{\mathcal{X}} : \mathcal{U} \to I^\bullet$ where $\zeta_{\mathcal{X}}(\epsilon)$ denotes the belonging value of $\epsilon \in \mathcal{X}$.

Definition 2.2. [2]
A pair (ζ_S, Λ) is called a *soft set* over $\mathcal{U}$, where $\zeta_S : \Lambda \to P(\mathcal{U})$ and Λ be a set of attributes.

Definition 2.3. [3]
A soft set (ζ_{S_1}, Λ_1) is a *soft subset* of another soft set (ζ_{S_2}, Λ_2) if

i $\Lambda_1 \subseteq \Lambda_2$, and

ii $\forall \omega \in \Lambda_1, \zeta_{S_1}(\omega)$ and $\zeta_{S_2}(\omega)$ are identical approximations.

For more detail on soft set, see [2–11]

Definition 2.4. [25]
The pair (Ψ, G) is called a hypersoft set over $\mathcal{U}$, where G is the cartesian product of n disjoint sets $G_1, G_2, G_3, ..., G_n$ having attribute values of n distinct attributes $g_1, g_2, g_3, ..., g_n$ respectively and $\Psi : G \to P(\mathcal{U})$.

For more definitions and operations of hypersoft set, see [25–28, 30–36].

3. Fuzzy Parameterized Hypersoft Set ($fphs$-set)with Application

In this section, fuzzy parameterized hypersoft set is conceptualized and some of its fundamentals are discussed.

Definition 3.1. Let $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ be a collection of disjoint attribute-valued sets corresponding to n distinct attributes $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ respectively. An FP-hypersoft set ($fphs$-set) $\Psi_{\mathcal{F}}$ over $\mathcal{U}$ is defined as

$$\Psi_{\mathcal{F}} = \{(\zeta_{\mathcal{F}}(g)/g, \psi_{\mathcal{F}}(g)) : g \in G,\ \psi_{\mathcal{F}}(g) \in P(\mathcal{U}),\ \zeta_{\mathcal{F}}(g) \in I^\bullet\}$$

where

(i) $G = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times .... \times \mathcal{A}_n$

(ii) $\mathcal{F}$ is a fuzzy set over G with $\zeta_{\mathcal{F}} : G \to I^\bullet$ as membership function of $fphs$-set.

(iii) $\psi_{\mathcal{F}} : G \to P(\mathcal{U})$ is called approximate function of $fphs$-set.

Now throughout the remaining part of the paper, $\Omega_{FPHS}(\mathcal{U})$, $\zeta_{\mathcal{F}}$ and $\psi_{\mathcal{F}}$ will represent the collection of all $fphs$-sets over $\mathcal{U}$, membership function and approximate function respectively.

Definition 3.2. Let $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$. If $\psi_{\mathcal{F}}(g) = \phi$ for all $g \in G$, then $\Psi_{\mathcal{F}}$ is called an $\mathcal{F}$-empty $fphs$-set, denoted by $\Psi_{\Phi_{\mathcal{F}}}$. If $\mathcal{F} = \phi$, then $\Psi_{\mathcal{F}}$ is called an empty $fphs$-set, denoted by Ψ_{Φ}.

Definition 3.3. Let $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$. If $\mathcal{F}$ is a crisp subset of G and $\psi_{\mathcal{F}}(g) = \mathcal{U}$ for all $g \in \mathcal{F}$, then $\Psi_{\mathcal{F}}$ is called $\mathcal{F}$-universal $fphs$-set, denoted by $\Psi_{\tilde{\mathcal{F}}}$. If $\mathcal{F} = G$, then the $\mathcal{F}$-universal $fphs$-set is called universal $fphs$-set, denoted by $\Psi_{\tilde{G}}$.

Example 3.1. *Consider* $\mathcal{U} = \{u_1, u_2, u_3, u_4, u_5\}$ *and* $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ *with* $\mathcal{A}_1 = \{a_{11}, a_{12}\}$, $\mathcal{A}_2 = \{a_{21}, a_{22}\}, \mathcal{A}_3 = \{a_{31}\}$, *then*
$G = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$
$G = \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{22}, a_{31}), (a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{31})\} = \{g_1, g_2, g_3, g_4\}$.
Case 1.
If $\mathcal{F}_1 = \{0.2/g_2, 0.5/g_3, 1.0/g_4\}$ *and* $\psi_{\mathcal{F}_1}(g_2) = \{u_2, u_4\}$, $\psi_{\mathcal{F}_1}(g_3) = \phi$, *and* $\psi_{\mathcal{F}_1}(g_4) = \mathcal{U}$, *then* $\Psi_{\mathcal{F}_1} = \{(0.2/g_2, \{u_2, u_4\}), (0.5/g_3, \phi), (1.0/g_4, U)\}$.
Case 2.
If $\mathcal{F}_2 = \{0.3/g_2, 0.7/g_3\}, \psi_{\mathcal{F}_2}(g_2) = \phi$ *and* $\psi_{\mathcal{F}_2}(g_3) = \phi$, *then* $\Psi_{\mathcal{F}_2} = \Psi_{\phi\mathcal{F}_2}$.
Case 3.
If $\mathcal{F}_3 = \phi$, *then* $\Psi_{\mathcal{F}_3} = \Psi_{\phi}$.
Case 4.
If $\mathcal{F}_4 = \{1.0/g_1, 1.0/g_2\}, \psi_{\mathcal{F}_4}(g_1) = \mathcal{U}$, *and* $\psi_{\mathcal{F}_4}(g_2) = \mathcal{U}$, *then* $\Psi_{\mathcal{F}_4} = \Psi_{\tilde{\mathcal{F}}_4}$.

Definition 3.4. Let $\Psi_{\mathcal{F}_1}$, $\Psi_{\mathcal{F}_2} \in \Omega_{FPHS}(\mathcal{U})$ then $\Psi_{\mathcal{F}_1}$ is a $fphs$-subset of $\Psi_{\mathcal{F}_2}$, denoted by $\Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\mathcal{F}_2}$ if $\zeta_{\mathcal{F}_1}(g) \leq \zeta_{\mathcal{F}_2}(g)$ and $\psi_{\mathcal{F}_1}(g) \subseteq \psi_{\mathcal{F}_2}(g)$ for all $g \in G$.

Proposition 3.1. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2}, \Psi_{\mathcal{F}_3} \in \Omega_{FPHS}(\mathcal{U})$ *then*

1. $\Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\tilde{G}}$.
2. $\Psi_{\phi} \tilde{\subseteq} \Psi_{\mathcal{F}_1}$.
3. $\Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\mathcal{F}_1}$.
4. $\Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\mathcal{F}_2}$ *and* $\Psi_{\mathcal{F}_2} \tilde{\subseteq} \Psi_{\mathcal{F}_3} \Rightarrow \Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\mathcal{F}_3}$.

Definition 3.5. Let $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2} \in \Omega_{FPHS}(\mathcal{U})$ then, $\Psi_{\mathcal{F}_1}$ and $\Psi_{\mathcal{F}_2}$ are $fphs$-equal, represented as $\Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_2}$, iff $\zeta_{\mathcal{F}_1}(x) = \zeta_{\mathcal{F}_2}(x)$ and $\psi_{\mathcal{F}_1}(g) = \psi_{\mathcal{F}_2}(g)$ for all $g \in G$.

Proposition 3.2. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2}, \Psi_{\mathcal{F}_3} \in \Omega_{FPHS}(\mathcal{U})$ *then,*

1. *If* $\Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_2}$ *and* $\Psi_{\mathcal{F}_2} = \Psi_{\mathcal{F}_3} \Rightarrow \Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_3}$.
2. $\Psi_{\mathcal{F}_1} \tilde{\subseteq} \Psi_{\mathcal{F}_2}$ *and* $\Psi_{\mathcal{F}_2} \tilde{\subseteq} \Psi_{\mathcal{F}_1} \Leftrightarrow \Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_2}$.

Definition 3.6. Let $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$ then, complement of $\Psi_{\mathcal{F}}$ (i.e. $\Psi^{\tilde{c}}_{\mathcal{F}}$) is an $fphs$-set given as $\zeta^{\tilde{c}}_{\mathcal{F}}(g) = 1 - \zeta_{\mathcal{F}}(g)$ and $\psi^{\tilde{c}}_{\mathcal{F}}(g) = \mathcal{U} \setminus \psi_{\mathcal{F}}(g)$

Proposition 3.3. *Let* $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$ *then,*

1. $(\Psi^{\tilde{c}}_{\mathcal{F}})^{\tilde{c}} = \Psi_{\mathcal{F}}$.

2. $\Psi_{\Phi}^{\tilde{c}} = \Psi_{\tilde{G}}$.

Definition 3.7. Let $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2} \in \Omega_{FPHS}(\mathcal{U})$ then, union of $\Psi_{\mathcal{F}_1}$ and $\Psi_{\mathcal{F}_2}$, denoted by $\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_2}$, is defined by

1. $\zeta_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g) = max\{\zeta_{\mathcal{F}_1}(x), \zeta_{\mathcal{F}_2}(g)\}$ and
2. $\psi_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g) = \psi_{\mathcal{F}_1}(g) \cup \psi_{\mathcal{F}_2}(g)$, for all $g \in G$.

Proposition 3.4. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2}, \Psi_{\mathcal{F}_3} \in \Omega_{FPHS}(\mathcal{U})$ *then,*

1. $\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_1}$.
2. $\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\Phi} = \Psi_{\mathcal{F}_1}$.
3. $\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\tilde{G}} = \Psi_{\tilde{G}}$.
4. $\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_2} = \Psi_{\mathcal{F}_2} \tilde{\cup} \Psi_{\mathcal{F}_1}$.
5. $(\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_2}) \tilde{\cup} \Psi_{\mathcal{F}_3} = \Psi_{\mathcal{F}_1} \tilde{\cup} (\Psi_{\mathcal{F}_2} \tilde{\cup} \Psi_{\mathcal{F}_3})$.

Definition 3.8. Let $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2} \in \Omega_{FPHS}(\mathcal{U})$ then intersection of $\Psi_{\mathcal{F}_1}$ and $\Psi_{\mathcal{F}_2}$, denoted by $\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_2}$, is an $fphs$-set defined by $\zeta_{\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2}(g) = min\{\zeta_{\mathcal{F}_1}(g), \zeta_{\mathcal{F}_2}(g)\}$ and $\psi_{\mathcal{F}_1 \tilde{\cap} \mathcal{F}_2}(g) = \psi_{\mathcal{F}_1}(g) \cap \psi_{\mathcal{F}_2}(g)$

Proposition 3.5. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2}, \Psi_{\mathcal{F}_3} \in \Omega_{FPHS}(\mathcal{U})$ *then*

1. $\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_1} = \Psi_{\mathcal{F}_1}$.
2. $\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\Phi} = \Psi_{\Phi}$.
3. $\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\tilde{G}} = \Psi_{\tilde{\mathcal{F}}_1}$.
4. $\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_2} = \Psi_{\mathcal{F}_2} \tilde{\cap} \Psi_{\mathcal{F}_1}$.
5. $(\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_2}) \tilde{\cap} \Psi_{\Psi_{\mathcal{F}_3}} = \Psi_X \tilde{\cap} (\Psi_{\mathcal{F}_2} \tilde{\cap} \Psi_{\Psi_{\mathcal{F}_3}})$.

Remark 3.1. *Let* $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$. *If* $\Psi_{\mathcal{F}} \neq \Psi_{\tilde{G}}$, *then* $\Psi_{\mathcal{F}} \tilde{\cup} \Psi_{\mathcal{F}}^{\tilde{c}} \neq \Psi_{\tilde{G}}$ *and* $\Psi_{\mathcal{F}} \tilde{\cap} \Psi_{\mathcal{F}}^{\tilde{c}} \neq \Psi_{\Phi}$

Proposition 3.6. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2} \in \Omega_{FPHS}(\mathcal{U})$ *D'Morgan's laws are valid*

1. $(\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_2})^{\tilde{c}} = \Psi_{\mathcal{F}_1}^{\tilde{c}} \tilde{\cap} \Psi_{\mathcal{F}_2}^{\tilde{c}}$.
2. $(\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_2})^{\tilde{c}} = \Psi_{\mathcal{F}_1}^{\tilde{c}} \tilde{\cup} \Psi_{\mathcal{F}_2}^{\tilde{c}}$.

Proof. For all $g \in G$,
(1). Since $(\zeta_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2})^{\tilde{c}}(g) = 1 - \zeta_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g)$
$= 1 - max\{\zeta_{\mathcal{F}_1}(g), \zeta_{\mathcal{F}_2}(g)\}$
$= min\{1 - \zeta_{\mathcal{F}_1}(g), 1 - \zeta_{\mathcal{F}_2}(g)\}$
$= min\{\zeta_{\mathcal{F}_1}\tilde{c}(g), \zeta_{\mathcal{F}_2}\tilde{c}(g)\}$
$= \zeta_{\mathcal{F}_1^{\tilde{c}} \tilde{\cap} \mathcal{F}_2^{\tilde{c}}}(g)$
and
$(\psi_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2})^{\tilde{c}}(g) = \mathcal{U} \setminus \psi_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g)$

$= \mathcal{U} \setminus (\psi_{\mathcal{F}_1}(g) \cup \psi_{\mathcal{F}_2}(g))$
$= (\mathcal{U} \setminus \psi_{\mathcal{F}_1}(g)) \cap (\mathcal{U} \setminus \psi_{\mathcal{F}_2}(g))$
$= \psi_{\mathcal{F}_1^{\tilde{c}}}(g) \tilde{\cap} \psi_{\mathcal{F}_2^{\tilde{c}}}(g)$
$= \psi_{\mathcal{F}_1^{\tilde{c}} \tilde{\cap} \mathcal{F}_2^{\tilde{c}}}(g)$.
similarly (2) can be proved easily. □

Proposition 3.7. *Let* $\Psi_{\mathcal{F}_1}, \Psi_{\mathcal{F}_2}, \Psi_{\mathcal{F}_3} \in \Omega_{FPHS}(\mathcal{U})$ *then*

1. $\Psi_{\mathcal{F}_1} \tilde{\cup} (\Psi_{\mathcal{F}_2} \tilde{\cap} \Psi_{\mathcal{F}_3}) = (\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_2}) \tilde{\cap} (\Psi_{\mathcal{F}_1} \tilde{\cup} \Psi_{\mathcal{F}_3})$.
2. $\Psi_{\mathcal{F}_1} \tilde{\cap} (\Psi_{\mathcal{F}_2} \tilde{\cup} \Psi_{\mathcal{F}_3}) = (\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_2}) \tilde{\cup} (\Psi_{\mathcal{F}_1} \tilde{\cap} \Psi_{\mathcal{F}_3})$.

Proof. For all $g \in G$,
(1). Since $\zeta_{\mathcal{F}_1 \tilde{\cup} (\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3)}(g) = max\{\zeta_{\mathcal{F}_1}(g), \zeta_{\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3}(g)\}$
$= max\{\zeta_{\mathcal{F}_1}(g), min\{\zeta_{\mathcal{F}_2}(g), \zeta_{\mathcal{F}_3}(g)\}\}$
$= min\{max\{\zeta_{\mathcal{F}_1}(g), \zeta_{\mathcal{F}_2}(g)\}, max\{\zeta_{\mathcal{F}_1}(g), \zeta_{\mathcal{F}_3}(g)\}\}$
$= min\{\zeta_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g), \zeta_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_3}(g)\}$
$= \zeta_{(\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2) \tilde{\cap} (\mathcal{F}_1 \tilde{\cup} \mathcal{F}_3)}(g)$
and
$\psi_{\mathcal{F}_1 \tilde{\cup} (\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3)}(g) = \psi_{\mathcal{F}_1}(g) \cup \psi_{\mathcal{F}_2 \tilde{\cap} \mathcal{F}_3}(g)$
$= \psi_{\mathcal{F}_1}(g) \cup (\psi_{\mathcal{F}_2}(g) \cap \psi_{\mathcal{F}_3}(g))$
$= (\psi_{\mathcal{F}_1}(g) \cup \psi_{\mathcal{F}_2}(g)) \cap (\psi_{\mathcal{F}_1}(g) \cup \psi_{\mathcal{F}_3}(g))$
$= \psi_{\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2}(g) \cap \psi_{\mathcal{F}_1 \tilde{U} \mathcal{F}_3}(g)$
$= \psi_{(\mathcal{F}_1 \tilde{\cup} \mathcal{F}_2) \tilde{\cap} (\mathcal{F}_1 \tilde{\cup} \mathcal{F}_3)}(g)$
In the same way, (2) can be proved. □

4. Fuzzy Decision Set of $fphs$-set

Here novel algorithm is proposed with the help of characterization of fuzzy decision set on $fphs$-set which based on decision making technique and is explained with example.

Definition 4.1. Let $\Psi_{\mathcal{F}} \in \Omega_{FPHS}(\mathcal{U})$ then a fuzzy decision set of $\Psi_{\mathcal{F}}$ (i.e. $\Psi_{\mathcal{F}}^D$) is represented as

$$\Psi_{\mathcal{F}}^D = \left\{ \zeta_{\mathcal{F}}^D(u)/u : u \in \mathcal{U} \right\}$$

where $\zeta_{\Psi_{\mathcal{F}}^D} : \mathcal{U} \to I^{\bullet}$ and

$$\zeta_{\Psi_{\mathcal{F}}^D}(u) = \frac{1}{|S(\mathcal{F})|} \sum_{v \in S(\mathcal{F})} \zeta_{\Psi}(v) \Gamma_{\psi_{\mathcal{F}}(v)}(u)$$

where $S(\mathcal{F})$ is the support set of $\mathcal{F}$ with

$$\Gamma_{\psi_{\mathcal{F}}(v)}(u) = \begin{cases} 1 & ; \quad u \in \Gamma_{\psi_{\mathcal{F}}}(v) \\ 0 & ; \quad u \notin \Gamma_{\psi_{\mathcal{F}}}(v) \end{cases}$$

4.1. Proposed Algorithm

Once $\Psi_{\mathcal{F}}^{D}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm.

Step 1 Determine $\mathcal{F} = \{\zeta_{\mathcal{F}}(g)/g : \zeta_{\mathcal{F}}(g) \in I^{\bullet}, g \in G\}$,

Step 2 Find $\psi_{\mathcal{F}}(g)$

Step 3 Construct $\Psi_{\mathcal{F}}$ over $\mathcal{U}$,

Step 4 Compute $\Psi_{\mathcal{F}}^{D}$,

Step 5 Choose the maximum of $\zeta_{\Psi_{\mathcal{F}}^{D}}(u)$.

Example 4.1. *Suppose that Mr. James Peter wants to buy a mobile from a mobile market. There are eight kinds of mobiles (options) which form the set of discourse* $\mathcal{U} = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$. *The best selection may be evaluated by observing the attributes i.e.* a_1 *= Company,* a_2 *= Camera Resolution,* a_3 *= Size,* a_4 *= RAM, and* a_5 *= Battery power. The attribute-valued sets corresponding to these attributes are:*
$B_1 = \{b_{11}, b_{12}\}$
$B_2 = \{b_{21}, b_{22}\}$
$B_3 = \{b_{31}, b_{32}\}$
$B_4 = \{b_{41}, b_{42}\}$
$B_5 = \{b_{51}\}$
then $G = B_1 \times B_2 \times B_3 \times B_4 \times B_5$
$G = \{g_1, g_2, g_3, g_4, ..., g_{16}\}$ *where each* $g_i, i = 1, 2, ..., 16$, *is a 5-tuples element.*
Step 1 :
From table 1, we can construct $\mathcal{F}$ *as*

Table 1. Degrees of Membership $\zeta_{\mathcal{F}}(g_i)$

$\zeta_{\mathcal{F}}(g_i)$	Degree	$\zeta_{\mathcal{F}}(g_i)$	Degree
$\zeta_{\mathcal{F}}(g_1)$	0.1	$\zeta_{\mathcal{F}}(g_9)$	0.9
$\zeta_{\mathcal{F}}(g_2)$	0.2	$\zeta_{\mathcal{F}}(g_{10})$	0.16
$\zeta_{\mathcal{F}}(g_3)$	0.3	$\zeta_{\mathcal{F}}(g_{11})$	0.25
$\zeta_{\mathcal{F}}(g_4)$	0.4	$\zeta_{\mathcal{F}}(g_{12})$	0.45
$\zeta_{\mathcal{F}}(g_5)$	0.5	$\zeta_{\mathcal{F}}(g_{13})$	0.35
$\zeta_{\mathcal{F}}(g_6)$	0.6	$\zeta_{\mathcal{F}}(g_{14})$	0.75
$\zeta_{\mathcal{F}}(g_7)$	0.7	$\zeta_{\mathcal{F}}(g_{15})$	0.65
$\zeta_{\mathcal{F}}(g_8)$	0.8	$\zeta_{\mathcal{F}}(g_{16})$	0.85

$$\mathcal{F} = \left\{ \begin{array}{l} 0.1/g_1, 0.2/g_2, 0.3/g_3, 0.4/g_4, 0.5/g_5, 0.6/g_6, \\ 0.7/g_7, 0.8/g_8, 0.9/g_9, 0.16/g_{10}, 0.25/g_{11}, \\ 0.45/g_{12}, 0.35/g_{13}, 0.75/g_{14}, 0.65/g_{15}, 0.85/g_{16} \end{array} \right\}$$

Step 2 :
Table 2 presents $\psi_{\mathcal{F}}(g_i)$ *corresponding to each element of* G.
Step 3 :

With the help of tables 1 and 2, we can construct $\Psi_{\mathcal{F}}$ *as*

$$\Psi_{\mathcal{F}} = \left\{ \begin{array}{l} (0.1/g_1, \{m_1, m_2\}), (0.2/g_2, \{m_1, m_2, m_3\}), (0.3/g_3, \{m_2, m_3, m_4\}), (0.4/g_4, \{m_4, m_5, m_6\}), \\ (0.5/g_5, \{m_6, m_7, m_8\}), (0.6/g_6, \{m_2, m_3, m_4, m_7\}), (0.7/g_7, \{m_1, m_3, m_5, m_6\}), \\ (0.8/g_8, \{m_2, m_3, m_6, m_7\}), (0.9/g_9, \{m_2, m_3, m_6, m_7, m_8\}), (0.16/g_{10}, \{m_1, m_3, m_6, m_7, m_8\}), \\ (0.25/g_{11}, \{m_2, m_4, m_6, m_7, m_8\}), (0.45/g_{12}, \{m_1, m_2, m_3, m_6, m_7, m_8\}), \\ (0.35/g_{13}, \{m_2, m_3, m_5, m_7, m_8\}), (0.75/g_{14}, \{m_1, m_3, m_5, m_7, m_8\}), \\ (0.65/g_{15}, \{m_1, m_2, m_3, m_5, m_7, m_8\}), (0.85/g_{16}, \{m_4, m_5, m_6, m_7, m_8\}) \end{array} \right\}$$

Step 4 :

Table 2. Approximate functions $\psi_{\mathcal{F}}(g_i)$

g_i	$\psi_{\mathcal{F}}(g_i)$	g_i	$\psi_{\mathcal{F}}(g_i)$
g_1	$\{m_1, m_2\}$	g_9	$\{m_2, m_3, m_6, m_7, m_8\}$
g_2	$\{m_1, m_2, m_3\}$	g_{10}	$\{m_1, m_3, m_6, m_7, m_8\}$
g_3	$\{m_2, m_3, m_4\}$	g_{11}	$\{m_2, m_4, m_6, m_7, m_8\}$
g_4	$\{m_4, m_5, m_6\}$	g_{12}	$\{m_1, m_2, m_3, m_6, m_7, m_8\}$
g_5	$\{m_6, m_7, m_8\}$	g_{13}	$\{m_2, m_3, m_5, m_7, m_8\}$
g_6	$\{m_2, m_3, m_4, m_7\}$	g_{14}	$\{m_1, m_3, m_5, m_7, m_8\}$
g_7	$\{m_1, m_3, m_5, m_6\}$	g_{15}	$\{m_1, m_2, m_3, m_5, m_7, m_8\}$
g_8	$\{m_2, m_3, m_6, m_7\}$	g_{16}	$\{m_4, m_5, m_6, m_7, m_8\}$

Table 3. Membership values $\zeta_{\Psi^D_{\mathcal{F}}}(m_i)$

m_i	$\zeta_{\Psi^D_{\mathcal{F}}}(m_i)$	m_i	$\zeta_{\Psi^D_{\mathcal{F}}}(m_i)$
m_1	0.19	m_5	0.23
m_2	0.29	m_6	0.31
m_3	0.37	m_7	0.39
m_4	0.15	m_8	0.30

From table 3, we can construct $\Psi^D_{\mathcal{F}}$ *as*

$\Psi^D_{\mathcal{F}} = \{0.19/m_1, 0.29/m_2, 0.37/m_3, 0.15/m_4, 0.23/m_5, 0.31/m_6, 0.39/m_7, 0.30/m_8\}$

Step 5 :

Since maximum of $\zeta_{\Psi^D_{\mathcal{F}}}(m_i)$ *is 0.39 so the mobile* m_7 *is selected.*

5. Conclusion

In this study, fuzzy parameterized hypersoft set is conceptualized along with some of elementary properties and theoretic operations. A novel algorithm is proposed for decision making and is validated with the help of an illustrative example for best purchasing of mobile from mobile market. Future work may include the extension of this work for other fuzzy-like environments and the implementation for solving more real life problems in decision making.

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