The Klein-Gordon Equation, exp(i Action) [Propagator] and Fluctuations Part II

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In a previous note (Part 1), we argued that for both a relativistic and nonrelativistic free particle Lagrangian, id/dT exp(i Action) = E exp(i Action) and -id/dX exp(i Action) = p exp(i Action) where velocity= X/T. We further argued that these results allow one to convert an energy conservation equation (or even a Dirac type equation) in E and p into a differential equation in terms of d/dT and d/dX. Thus, d/dT and d/dX seem to represent fluctuations because velocityt=X/=constant is not imposed.For the case of a free particle with no velocity related potential such as vA(x), we argued Action= L(v) T as v is constant. Furthermore, L(v) so d/dt and d/dx both make use of dL/dv. In this note, we consider the presence of a magnetic vector potential term vA(x), but no electric field so the particle does not accelerate.

Action in terms of T and X

In a previous note, we argued that the relativistic action Integral dt L(v) = T sqrt(1-vv) mo and the nonrelativistic T .5m vv could be written with v=X/T. As a next step, we argued that one may introduce fluctuations into exp(i Action) (introduced by P. Dirac) through d/dX and d/dT. These are related to p and E i.e.

i d/dT exp(i Action) = E exp(i Action) and -i d/dX exp(i Action) = p exp(i Action) ((1))

For the relativistic case, E=mo/sqrt(1-vv) and p=mov/sqrt(1-vv) and for the nonrelativistic, E=.5movv and p=mo v.

Thus, one may take a classical equation containing p and E and replace these with -id/dx exp(i Action) and id/dt exp(i Action), thus obtaining the Klein-Gordon and Schrodinger equations i.e fluctuation differential equations which conserve classical energy on average.

In this note, we consider the case of an extra v A(x) potential due to a magnetic vector potential which does not accelerate the particle.

Case of a Magnetic Vector Potential (Non-relativistic case)

For the addition of a magnetic vector potential, the nonrelativistic Lagrangian is:

L= .5mvv + vA(x) and Action = T .5mvv + Integral dx1/dt A(x1) dt ((2))

The integral on the RHS of ((2)) may be converted into:

Integral (0,X) dx1 A(x1) ((3))

Thus, a fluctuation in X independent of T changes the integration bound of the integral.

The result is:

exp(i Action) = exp{i [.5mvvT + Integral (0,X) dx1 A(x1)]} ((4))

This may be varied with respect to T and X independently which suggests a fluctuation because v = X/T =constant is not imposed. For the case of i d/dT, one has again:

id/dt exp(i Action) = .5mvv exp(i Action) ((5a)) as in the absence of a magnetic vector potential

-id/dX exp(i Action) = exp(i Action) { mv + A(x) } ((5b))

In order to link ((5a)) and ((5b)) one may use the classical conservation of energy equation because the particle is not accelerated:

E = mvv/2 or id/dt exp(i Action) = 1/2m { p-A(x) } {p-A(x)} where p=-id/dX ((6))

This is the standard result, although reached using d/dX and d/dT variation.

One may note that ((5a)) and ((5b)) are equivalent to exp(i action) being replaced by:

exp(ipx - iEt) ((7))

Here p (which is a number) is not mv, but rather mv+Integral (0,X) dx1 A(x1)

In the case of an accelerating potential, one may consider a momentum distribution, i.e.

W(x,t)=exp(-iEt) Sum over p a(p) exp(ipx) ((8))

Energy conservation is still retained in ((6) i.e.

id/dt exp(-iEt) W(x) = 1/2m { p-A(x) } {p-A(x)} W(x) + V(x) W(x) where V(x) may be e Phi(x) the electric potential. ((9))

Conclusion

In conclusion, we extend the ideas of replacing v=velocity with X/T in a free particle relativistic or nonrelativistic Lagrangian, and computing id/dT exp(i Action) and -id/dX exp(i Action) for the case of a magnetic vector potential vA(x). We find that the approach yields the usual p=mv+ A(x) and: id/dt exp(i Action) = .5mvv exp(i Action) and -id/dx exp(i Action) = p exp(i Action). Using a conservation of energy equation for the nonaccelerating particle gives:

id/dt exp(i Action) = 1/2m [p-A][p-A] exp(i Action) where p=-id/dX. exp(i Action) is equivalent in this case to exp(ipx -iEt) where p1=mv+ Integral (0,X) A(x1)dx1. For the case of a potential which accelerates the particle, we argue it is stochastic i.e. V(x)=Sum over k Vk exp(ikx) . This leads to a momentum distribution W(x)=Sum over p a(p)exp(ipx). Thus, one has:

id/dt W(x)exp(-iEt) = 1/2m [p-A][p-A] W(x) + V(x) W(x)