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Neutrosophic $\mathcal{N}$-Topological Ordered Space

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Abstract. This research article presents a new concept, "Neutrosophic $\mathcal{N}$-topological ordered space". Also we define some of the separation axioms, weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space and Neutrosophic $\mathcal{N}\varsigma$-regularly ordered space in Neutrosophic $\mathcal{N}$-topological ordered space. Besides giving some of the innovative properties of these spaces.

Keywords: Neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space, Neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, Weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, Almost Neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space and Neutrosophic $\mathcal{N}\varsigma$-regularly ordered space.

1. Introduction

L.A. Zadeh introduced the concept of fuzzy sets [14]. The theory of fuzzy topological spaces was developed by Chang [3] . The study of intutionistic fuzzy set was established by Atanassov [1] in 1983. In [4], the another notion called intutionistic fuzzy topological space was found by Coker. F. Smarandache originated the concepts of neutrosophy and neutrosophic set ([12], [13]). The concept of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A.A. Salama and S.A. Alblowi [11]. Leopoldo Nachbin [9] initiated the study of topological ordered spaces in 1965. Lellis Thivagar et al. [6] have proposed the concept of N-topological space. Recently we found the new concept called $\mathcal{N}$-topological ordered spaces [5]. In this paper, we investigate the concept called Neutrosophic $\mathcal{N}$-topological Ordered Space. And also, we establish some of the Separation Axioms and its characterizations.

2. Preliminaries

Definition 2.1. [8] Let X be a non-empty set, $\tau_1, \tau_2, ..., \tau_N$ be N-arbitrary topologies defined on X and let the collection $N\tau$ be defined by

$N\tau = \{S \subseteq X : S = (\cup_{i=1}^{N} A_i) \bigcup (\cap_{i=1}^{N} B_i), A_i, B_i \in \tau_i\}$

satisfying the following axioms:

(i) $X, \emptyset \in N\tau$.

(ii) $\bigcup_{i=1}^{\infty} S_i \in N\tau$ for all $S_i \in N\tau$.

(iii) $\bigcap_{i=1}^{n} S_i \in N\tau$ for all $S_i \in N\tau$.

Then the pair $(X, N\tau)$ is called a *N-topological space* on X and the elements of the collection $N\tau$ are known as *$N\tau$-open sets* on X. A subset A of X is called *$N\tau$-closed* on X if the complement of A is $N\tau$-open on X. The set of all $N\tau$-open sets on X and the set of all $N\tau$-closed sets on X are respectively, denoted by $N\tau O(X)$ and $N\tau C(X)$.

Definition 2.2. [5] An $\mathcal{N}$-topological Space $(X, \mathcal{N}\tau)$ equipped with a partial order relation $\leq$ (that is, $Reflexive$, $Transitive$ and $Antisymmetric$) is called an $\mathcal{N}$-topological Ordered Space $(X, \mathcal{N}\tau, \leq)$.

Definition 2.3. [12] Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x), \ \gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A. Also $^{-}0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^{+}$ for all $x \in X$.

Remark 2.4. [12, 13] (1) A neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to an ordered triple set $\langle \mu_A, \sigma_A, \gamma_A\rangle$ in $]0^{-}, 1^{+}[$ on X.

(2) For the sake of simplicity, we shall use the symbol $A = \langle \mu_A, \sigma_A, \gamma_A\rangle$ for the neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$

Definition 2.5. [10] Let $\{A_i, i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then

(a) $\cap A_i = \{\langle x, \wedge\mu_{A_i}(x), \wedge\sigma_{A_i}(x), \vee\gamma_{A_i}(x)\rangle : x \in X\}$;

(b) $\cup A_i = \{\langle x, \vee\mu_{A_i}(x), \vee\sigma_{A_i}(x), \wedge\gamma_{A_i}(x)\rangle : x \in X\}$

Definition 2.6. [10]

$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$ and $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

Definition 2.7. [6] A neutrosophic N-topology on a non-empty set X is a family $N_n\tau$ of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in N_n\tau$

(ii) $\cup_{i=1}^{\infty} A_i \in N_n\tau$ for all $A_i \in N_n\tau$

(iii) $\cap_{i=1}^{n} A_i \in N_n\tau$ for all $A_i \in N_n\tau$.

Then the pair $(X, N_n\tau)$ is called neutrosophic N-topological space and each neutrosophic set in $N_n\tau$ is called neutrosophic $N_n\tau$-open set. The complement of neutrosophic $N_n\tau$-open set is called neutrosophic $N_n\tau$-closed set.

Definition 2.8. [6] Let $(X, N_n\tau)$ be a neutrosophic N-topological space on X and A be a neutrosophic set on X, then $N_nint(A)$ and $N_ncl(A)$ are respectively defined as

(i) $N_nint(A) = \cup\{G\colon G \subseteq A$ and G is a $N_n\tau - opensetinX\}$

(ii) $N_ncl(A) = \cap\{F\colon A \subseteq F$ and F is a $N_n\tau - closedsetinX\}$

Definition 2.9. [10] A neutrosophic set A $= \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ in a neutrosophic topological space (X,T) is said to be a neutrosophic neighbourhood of a neutrosophic point $x_{r,t,s} \in X$, if there exists a neutrosophic open set B $= \langle x, \mu_B, \sigma_B, \gamma_B\rangle$ with $x_{r,t,s} \subseteq B \subseteq A$.

Notation 1. *[10] We denote neutrosophic neighbourhood A of a in X by neutrosophic neighbourhood A of a neutrosophic point $a_{r,t,s}$ for $a \in X$*

Definition 2.10. [10] A neutrosophic set $A = \langle x, \mu_A, \sigma_A, \gamma_A\rangle$ in a partially ordered set $(X, \leq)$ is said to be

(i) an increasing neutrosophic set if $x \leq y$ implies $A(x) \subseteq A(y)$. That is, $\mu_A(x) \leq \mu_A(y), \sigma_A(x) \leq \sigma_A(y)$ and $\gamma_A(x) \geq \gamma_A(y)$.

(ii) a decreasing neutrosophic set if $x \leq y$ implies $A(x) \supseteq A(y)$. That is, $\mu_A(x) \geq \mu_A(y), \sigma_A(x) \geq \sigma_A(y)$ and $\gamma_A(x) \leq \gamma_A(y)$.

Definition 2.11. A neutrosophic set A is called neutrosophic $\mathcal{N}\varsigma$-clopen set if it is both neutrosophic $\mathcal{N}\varsigma$-open set and neutrosophic $\mathcal{N}\varsigma$-closed set.

3. Neutrosophic $\mathcal{N}$-topological Ordered Space

In this paper, we define the notation of Neutrosophic $\mathcal{N}$-Topological Space as Neutrosophic $\mathcal{N}$-TS, partial order relation as por and also Neutrosophic $\mathcal{N}$-topological Ordered Space as Neutrosophic $\mathcal{N}$-TOS. We found some results of Neutrosophic $\mathcal{N}$-topological Ordered Spaces like Neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space, Neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, weakly Neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, almost Neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space and Neutrosophic $\mathcal{N}\varsigma$-T_3-ordered space.

Definition 3.1. A neutrosophic $\mathcal{N}$-TS $(X, \mathcal{N}_n\varsigma)$ equipped with a por $\leq$ is called Neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$.

Definition 3.2. For every $u, v \in X$ such that $u \nleq v$ (i.e., u is not related to v) in X, if there exists a decreasing neutrosophic $\mathcal{N}\varsigma$-open set G containing v such that $u \notin G$, then neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$ is called *upper* neutrosophic $\mathcal{N}\varsigma$-T_1-*ordered* space.

Definition 3.3. For every $u, v \in X$ such that $u \nleq v$ (i.e., u is not related to v) in X, if there exists an increasing neutrosophic $\mathcal{N}\varsigma$-open set H containing u such that $v \notin H$, then neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$ is called *lower* neutrosophic $\mathcal{N}\varsigma$-T_1-*ordered* space.

Definition 3.4. $(X, \mathcal{N}_n\varsigma, \leq)$ is said to be neutrosophic $\mathcal{N}\varsigma$-T_1-*ordered space* if it is both lower and upper neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space.

Example 3.5. Let $X = \{a, b, c\}$ with a por $\leq$. For $\mathcal{N} = 2$, let the neutrosophic sets be $U = \{x, (0.2, 0.2, 0.4), \quad (0.3, 0.3, 0.1), (0.6, 0.6, 0.2)\}$ and $V = \{x, (0.4, 0.4, 0.4), (0.4, 0.4, 0.3), (0.4, 0.4, 0.3)\}$. Then $U \cup V = \{(x, (0.4, 0.4, 0.4), (0.4, 0.4, 0.3), (0.4, 0.4, 0.3))\}$ and $U \cap V = \{(x, (0.2, 0.2, 0.4), (0.3, 0.3, 0.1), (0.6, 0.6, 0.2))\}$. Considering $\varsigma_1 = \{0_N, 1_N, U\}$ and $\varsigma_2 = \{0_N, 1_N, V\}$, then $2\varsigma O(X) = \{0_N, 1_N, U, V, U \cap V, U \cup V\}$ which is a neutrosophic bitopology on X. Then $(X, 2_n\varsigma, \leq)$ is a neutrosophic bi-topological ordered space. Let $a_{(0.15,0.2,0.4)}$ and $b_{(0.15,0.15,0.25)}$ be any two neutrosophic points on X. For $a_{(0.15,0.2,0.4)} \nleq b_{(0.15,0.15,0.25)}$, there exists an increasing neutrosophic 2ς-neighbourhood U of $a_{(0.15,0.2,0.4)}$ such that U is not a neutrosophic 2ς-neighbourhood of $b_{(0.15,0.15,0.25)}$. Therefore, $(X, 2_n\varsigma, \leq)$ is a lower neutrosophic 2ς-T_1-ordered space. Similarly, we can do for upper neutrosophic 2ς-T_1-ordered space. For $\mathcal{N} = 3$, define the neutrosophic sets $U = \{x, (0.3, 0.3, 0.5), (0.5, 0.5, 0.3), (0.7, 0.7, 0.2)\}$, $V = \{x, (0.6, 0.6, 0.5), (0.6, 0.6, 0.5), (0.6, 0.6, 0.5)\}$. Then $U \cup V = \{(x, (0.6, 0.6, 0.5), (0.6, 0.6, 0.5), (0.6, 0.6, 0.5))\}$ and $U \cap V = \{(x, (0.3, 0.3, 0.5), (0.5, 0.5, 0.3), (0.7, 0.7, 0.2))\}$. Considering $\varsigma_1 = \{0_N, 1_N, U\}$, $\varsigma_2 = \{0_N, 1_N, V\}$ and $\varsigma_3 = \{0_N, 1_N\}$, then $3\varsigma O(X) = \{0_N, 1_N, U, V, U \cap V, U \cup V\}$ which is a neutrosophic tritopology on X. Then $(X, 3_n\varsigma, \leq)$ is neutrosophic tri-topological ordered space. Let $a_{(0.25,0.3,0.5)}, b_{(0.25,0.25,0.35)} \in X$ such that $a_{(0.25,0.3,0.5)} \nleq b_{(0.25,0.25,0.35)}$. Then there exists an increasing neutrosophic 3ς-neighbourhood U of $a_{(0.25,0.3,0.5)}$ such that U is not a neutrosophic 3ς-neighbourhood of $b_{(0.25,0.25,0.35)}$. Therefore, $(X, 3_n\varsigma, \leq)$ is a lower neutrosophic 3ς-T_1-ordered space. Similarly, we can do for upper neutrosophic 3ς-T_1-ordered space.

Theorem 3.6. *For a neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$, the following are equivalent:*

(i) X is a lower(respectively upper) neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space.

(ii) For each $u, v \in X$ such that $u \nleq v$, there exists an increasing(respectively decreasing) neutrosophic $\mathcal{N}\varsigma$-open set $G = \langle x, \mu_G, \sigma_G, \gamma_G \rangle$ containing u(respectively v) such that $r \nleq v$(respectively $u \nleq r$) for all $r \in G$.

Proof. Now we prove the theorem only for lower neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space.

(i) $\Rightarrow$ (ii): Let $u \nleq v$. By hypothesis, there exists an increasing neutrosophic $\mathcal{N}\varsigma$-open set G containing u such that $v \notin G$. If $r \in G$ and $r \leq v$, then $v \in G$, a contradiction. Therefore, $r \nleq v$ for all $r \in G$.

(ii) $\Rightarrow$ (i): Let $u, v \in X$ such that $u \nleq v$. Therefore there exists an increasing neutrosophic $\mathcal{N}\varsigma$-open set G containing u such that $r \nleq v$ for all $r \in G$. Then $i(G)$ is an increasing neutrosophic $\mathcal{N}\varsigma$-open set of u such that $v \notin i(G)$. This implies that X is a lower neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space. Similar proof holds for upper neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space. □

Theorem 3.7. *If $(X, \mathcal{N}_n\varsigma, \leq)$ is a lower(respectively upper) neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space and $\mathcal{N}_n\varsigma \subseteq \mathcal{N}_n\varsigma^*$, then $(X, \mathcal{N}_n\varsigma^*, \leq)$ is a lower(respectively upper) neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space.*

Proof. Let $(X, \mathcal{N}_n\varsigma, \leq)$ be a lower neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space. Then if $u, v \in X$ such that $u \nleq v$, there exists an increasing neutrosophic $\mathcal{N}\varsigma$-open set $U = \langle x, \mu_U, \sigma_U, \gamma_U \rangle$ of u such that U is not a neutropsophic $\mathcal{N}\varsigma$-open set of v. Since $\mathcal{N}_n\varsigma \subseteq \mathcal{N}_n\varsigma^*$, therefore if $u, v \in X$ such that $u \nleq v$, there exists an increasing neutrosophic $\mathcal{N}\varsigma^*$-open set U^* of u such that U^* is not a neutrosophic $\mathcal{N}\varsigma^*$-open set of v. Thus $(X, \mathcal{N}_n\varsigma^*, \leq)$ is a lower neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space. Similarly, we can prove for upper neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space. □

Definition 3.8. For each pair of elements $u \nleq v$ in X, there exists neutrosophic $\mathcal{N}\varsigma$-open sets $G = \langle x, \mu_G, \sigma_G, \gamma_G \rangle$ and $H = \langle x, \mu_H, \sigma_H, \gamma_H \rangle$ such that G is an increasing neutrosophic $\mathcal{N}\varsigma$-neighbourhood of u, H is a decreasing neutrosophic $\mathcal{N}\varsigma$-neighbourhood of v and $G \cap H = 0_N$, then $(X, \mathcal{N}_n\varsigma, \leq)$ is defined to be neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.

Theorem 3.9. *For a neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$, the following are equivalent:*

(i) X is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.

(ii) For each pair $u, v \in X$ such that $u \nleq v$, there exists neutrosophic $\mathcal{N}\varsigma$-open sets $G = \langle x, \mu_G, \sigma_G, \gamma_G \rangle$ and $H = \langle x, \mu_H, \sigma_H, \gamma_H \rangle$ such that $u \in G$, $v \in H$ and $s \in G$, $t \in H$ together imply that $s \nleq t$.

(iii) The graph of the partial order of X is a neutrosophic $\mathcal{N}\varsigma^*$-closed where $\mathcal{N}\varsigma^*$ is the product topology for $X \times X$.

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (i): Let $u, v \in X$ with $u \nleq v$, there exists neutrosophic $\mathcal{N}\varsigma$-open sets G and H satisfying the properties in (ii). Since $i(G)$ is an increasing neutrosophic $\mathcal{N}\varsigma$-open set and $d(H)$ is a decreasing neutrosophic $\mathcal{N}\varsigma$-open set, we have $i(G) \cap d(H) = 0_N$. Suppose if $w \in i(G) \cap d(H)$, there exists $s \in G$ such that $s \leq w$ and there exists $t \in H$ such that $w \leq t$. Then $s \leq t$, a contradiction. Therefore $i(G) \cap d(H) = 0_N$. Hence X is neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.

(i) $\Rightarrow$ (iii): Let G be the graph of the partial order of X and $(s, t) \in \mathcal{N}_n\varsigma^*\text{-}cl(G)$ and $(s, t) \notin G$. Then $s \nleq t$ and therefore there exists an increasing neutrosophic $\mathcal{N}\varsigma$-open set A of

s and a decreasing neutrosophic $\mathcal{N}\varsigma$-open set B of t such that $A \cap B = 0_N$. $A \times B$ being a neutrosophic $\mathcal{N}\varsigma^*$-open set of (s,t), $(A \times B) \cap G = 0_N$. Thus $(s,t) \in A \times B$. It follows that $(s,s) \in A$ which implies $s \leq t$. Since A is an increasing neutrosophic $\mathcal{N}\varsigma$-open set, $t \in A$. Then $A \cap B \neq 0_N$, a contradiction. Therefore, $(s,t) \notin \mathcal{N}_n\varsigma^*\text{-}cl(G)$ and consequently, G is neutrosophic $\mathcal{N}\varsigma^*$-closed.

(iii) $\Rightarrow$ (i): Suppose $s \nleq t$. Then $(s,s) \notin G$ where G is the graph of the partial order of X. Since G is neutrosophic $\mathcal{N}\varsigma^*$-closed, there exists neutrosophic $\mathcal{N}\varsigma^*$-open sets S and T such that $(s,t) \in S \times T$ and $(S \times T) \cap G = 0_N$. Let $S^* = i(S)$ and $T^* = d(T)$. Then S^* is an increasing neutrosophic $\mathcal{N}\varsigma$-open set of s, T^* is a decreasing neutrosophic $\mathcal{N}\varsigma$-open set of t. Also $S^* \cap T^* = 0_N$, because suppose if $r \in S^* \cap T^*$, then there exists $p \in S, q \in T$ such that $p \leq r \leq q$ which implies $p \leq q$. So $(p,q) \in (S \times T) \cap G$, a contradiction. Therefore, $S^* \cap T^*$ must be empty. Hence X is neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space. □

Theorem 3.10. *A neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$ is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space if and only if for each $r \in X$, there exists an increasing(respectively decreasing) neutrosophic $\mathcal{N}\varsigma$-clopen subset of X containing r.*

Proof. If X is neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space and let $H \subseteq X$, then H is the required increasing (respectively decreasing) neutrosophic $\mathcal{N}\varsigma$-clopen subset of X for all $r \in X$. Conversely, let us assume $r \nleq s$ in X. By hypothesis, there exists an increasing(respectively decreasing) neutrosophic $\mathcal{N}\varsigma$-clopen subset H in X containing r. If $s \in H$, then there is nothing to prove. If $s \notin H$, then $X \setminus H$ is a decreasing neutrosophic $\mathcal{N}\varsigma$-clopen subset of X containing s. Also $H \cap X \setminus H = \emptyset$. Hence $(X, \mathcal{N}_n\varsigma, \leq)$ is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.

□

4. Weakly Neutrosophic $\mathcal{N}\varsigma$-T_2-Ordered and Almost Neutrosophic $\mathcal{N}\varsigma$-T_2-Ordered Space

Definition 4.1. A neutrosophic $\mathcal{N}$-TOS is said to be weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space if for given $v < u$(that is $v \leq u$ and $v \neq u$), there exists neutrosophic $\mathcal{N}\varsigma$-open sets $G = \langle x, \mu_G, \sigma_G, \gamma_G \rangle$ and $H = \langle x, \mu_H, \sigma_H, \gamma_H \rangle$ containing u and v respectively such that $r \in G$ and $s \in H$ together imply that $s < r$.

Definition 4.2. A neutrosophic $\mathcal{N}$-TOS is said to be an almost neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space if for given $u \parallel v$, there exists neutrosophic $\mathcal{N}\varsigma$-open sets $G = \langle x, \mu_G, \sigma_G, \gamma_G \rangle$ and $H = \langle x, \mu_H, \sigma_H, \gamma_H \rangle$ containing u and v respectively such that $r \in G$ and $s \in H$ together imply that $r \parallel s$.

Theorem 4.3. *A neutrosophic $\mathcal{N}$-TOS $(X, \mathcal{N}_n\varsigma, \leq)$ is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space if and only if it is weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered and almost neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.*

Proof. Let $(X, \mathcal{N}_n\varsigma, \leq)$ be a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space. Then it is weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space. Let $u \parallel v$. Then $u \nleq v$ and $v \nleq u$. Since X is neutrosophic $\mathcal{N}\varsigma$-T_2-ordered and $u \nleq v$, then there exists neutrosophic $\mathcal{N}\varsigma$-open sets G and H containing u and v respectively such that $r \in G$ and $s \in H$ together imply that $r \nleq s$. Since $v \nleq u$, there exists neutrosophic $\mathcal{N}\varsigma$-open sets H^* of $v$and G^* of u such that $s \in H^*$ and $r \in G^*$ together imply that $s \nleq r$. Thus $G \cap G^*$ is a neutrosophic $\mathcal{N}\varsigma$-open set containing u and $H \cap H^*$ is a neutrosophic $\mathcal{N}\varsigma$-open set containing v such that $r \in G \cap G^*$, $s \in H \cap H^*$ together imply that $r \parallel s$. Hence X is almost neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.

Conversely, if $u \nleq v$, then either $v < u$ or $v \nleq u$. If $v < u$ and since X is weakly neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, then there exists neutrosophic $\mathcal{N}\varsigma$-open sets G and H containing u and v respectively such that $r \in G$, $s \in H$ implies that $s < r$, that is $r \nleq s$. If $v \nleq u$, then obviously $u \parallel v$. And since X is almost neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space, for given $u \parallel v$, there exists neutrosophic $\mathcal{N}\varsigma$-open sets G^* and H^* containing u and v respectively such that $r \in G^*$ and $s \in H^*$ together imply that $r \parallel s$. Therefore $(X, \mathcal{N}_n\varsigma, \leq)$ is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space. □

5. **Neutrosophic $\mathcal{N}\varsigma$-Regularly Ordered Space**

Definition 5.1. Let $(X, \mathcal{N}_n\varsigma, \leq)$ be a neutrosophic $\mathcal{N}$-TOS. If for each decreasing(respectively increasing) neutrosophic $\mathcal{N}\varsigma$-closed subset W in X and for each $s \notin W$, there exists a neutrosophic $\mathcal{N}\varsigma$-neighbourhood G of s and a neutrosophic $\mathcal{N}\varsigma$-neighbourhood H of W such that G is increasing(respectively decreasing), H is decreasing(respectively increasing) and $G \cap H = 0_N$, then $(X, \mathcal{N}_n\varsigma, \leq)$ is said to be lower(respectively upper) neutrosophic $\mathcal{N}\varsigma$-regularly ordered space.

Definition 5.2. $(X, \mathcal{N}_n\varsigma, \leq)$ is said to be neutrosophic $\mathcal{N}\varsigma$-regularly ordered space if it is both lower and upper neutrosophic $\mathcal{N}\varsigma$-regularly ordered space.

Definition 5.3. A neutrosophic $\mathcal{N}\varsigma$-T_1-ordered neutrosophic $\mathcal{N}\varsigma$-regularly ordered space is called $\mathcal{N}\varsigma$-T_3-ordered space.

Theorem 5.4. *Every neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space, lower or upper neutrosophic $\mathcal{N}\varsigma$-regularly ordered space is neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space.*

Proof. Let X be a neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space, lower neutrosophic $\mathcal{N}\varsigma$-regularly ordered space and let $u \nleq v$. Since X is neutrosophic $\mathcal{N}\varsigma$-T_1-ordered space, $[\leftarrow, v]$ is neutrosophic $\mathcal{N}\varsigma$-closed. Also $[\leftarrow, v]$ is a decreasing neutrosophic set. Since $u \notin [\leftarrow, v]$, there exists an increasing neutrosophic $\mathcal{N}\varsigma$-neighbourhood G of u and a decreasing neutrosophic $\mathcal{N}\varsigma$-neighbourhood H of $[\leftarrow, v]$ such that $G \cap H = 0_N$. Since $v \in [\leftarrow, v] \subseteq H$, X is a neutrosophic $\mathcal{N}\varsigma$-T_2-ordered space. □

6. Conclusions

In this paper, we defined a new concept "Neutrosophic $\mathcal{N}$-Topological Ordered Spaces". some characterisitics of separation axioms $\mathcal{N}\varsigma$-T_i-ordered space $(i = 0, 1, 2, 3)$ dealing with neutrosophic were studied here. In our future work, we deal with neutrosophic $\mathcal{N}\varsigma$-T_i-ordered space (i=4,5) and its characteristics in Neutrosophic $\mathcal{N}$-Topological Ordered Spaces.

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