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ANALYSIS OF AXIAL RIGIDITY IN BEARINGS: SPINDLES FROM HIGH ROTATION SPEED TECHNOLOGICAL SYSTEMS

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ABSTRACT

This paper presents a mathematical model for the variation of the axial rigidity of the main shaft in a CNC machine tool. It analyzes the main parameters on which the axial rigidity of the shaft depend and the way they influence the stability of functioning.

KEYWORDS: Principal shaft; axial rigidity; parameters; stability.

INTRODUCTION

The modern technological working conditions impose a series of structural changes of the main components of a CNC machine tool. One of these is referring to the construction and functioning of the main axle, which must possess an appropriate technological rigidity, in order to obtain dimensional precision and quality of the superior surface. [1]

In this regards, one must know the initial working conditions in stationary mode. Thus, for a fixed shaft, we have [2, 3]:

$$\begin{cases} F_i = F_e = F \\ \alpha_i = \alpha_e = \alpha \end{cases} \qquad (1)$$
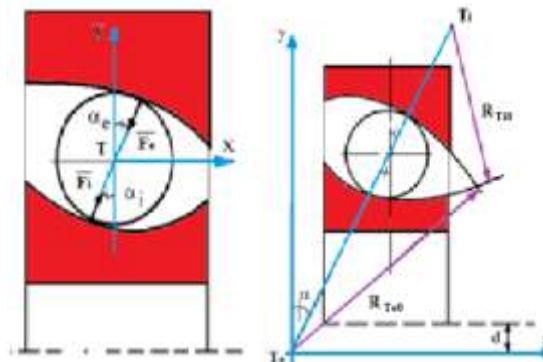

Figure 1. Technological elements of construction for the idle bearing;

It is considered a benchmark attached to the exterior ring (fixed) with the origin in the center of the processing arch of the exterior ring, T_e, and longitudinal axe T_{ex} and radial axis T_{ey} (figure 1). [5,6]
We have:

$R_{T_{e_0}}$ = the processing range of the exterior ring;
$R_{T_{i_0}}$ = the processing range of the interior ring;
The T_i position of the center of the interior ring from the stated benchmark will have the following coordinates:

$$\begin{cases} X_{T_i} = (R_{T_{e_0}} + R_{T_{i_0}} - 2r)sin\alpha \\ X_{T_i} = (R_{T_{e_0}} + R_{T_{i_0}} - 2r)cos\alpha \end{cases} \qquad (2)$$

ANALYSIS OF THE AXIAL RIGIDITY. ESTABLISHING A MATHEMATICAL MODEL FOR THE SIMULATION.

In dynamic mode, the bearing`s vibrations will produce relative movements between the interior and the exterior rings. It is considered a movement of the interior ring equal to u_1.
This is equivalent to a displacement of the interior ring center along $T_x$axis with a distance equal to u_1. The T_i point will become T_i` point, which has the following coordinates:

$$\begin{cases} X_{T_i`} = X_{T_i} + u_1 = \left(R_{T_{e_0}`} \cdot R_{T_{i_0}} - 2r\right)sin\alpha + u_1 \\ \quad X_{T_i`} = Y_{T_i} = (R_{T_{e_0}} + R_{T_{i_0}} - 2r)cos\alpha \end{cases} \qquad (3)$$

This will produce a displacement of the bead (of its center), its deformation and the runways deformation. We can write:

$$\begin{cases} X_{T_i`} = (R_e - r)sin\alpha_e + (R_i - r)sin\alpha_i \\ Y_{T_i`} = (R_e - r)cos\alpha_e + (R_i - r)cos\alpha_i \end{cases} \qquad (4)$$

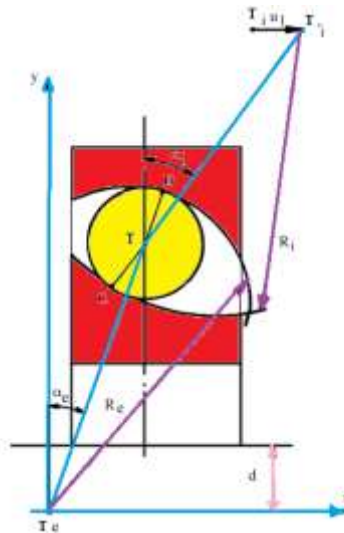By making simplifications, we have:

Figure 2. Technological elements of construction for the bearing in case of dynamic stress;
The angle at contact with the rings will become:

$$\begin{cases} \alpha_i = \alpha + \varphi_i \\ \alpha_e = \alpha + \varphi_e \end{cases} \qquad (5)$$

The rings deformation will be produced under F_e and F_i forces action; these forces are calculated using the following relations, based on the Hertziancontact theory:

$$\begin{cases} F_i = K(R_i - R_i^0) = K \cdot \Delta_i \\ F_e = K(R_e - R_e^0) = K \cdot \Delta_e \end{cases} \qquad (6)$$

Where K is a constant which depends on the nature of the materials being in contact, while Δ_i and Δ_e are the interior, respectively the exterior ring deformations.

In order to analyze the k_{11} rigidity variation, first of all, we will study the effect of the bearing rotation movement. For this, we must solve the system (5) for an $u_1 = 0$ movement. We will have the solutions:

$$\varphi_i = \frac{-2\left(R_{T_{i0}} - r\right) tg\alpha}{R_{T_{i0}} - R_{T_{e0}}} \qquad \varphi_e = \frac{2\left(R_{T_{e0}} - r\right) tg\alpha}{R_{T_{i0}} - R_{T_{e0}}}$$

$$\Delta_i = -\frac{D\Omega^2 cos\alpha}{2k} \qquad \Delta_e = \frac{D\Omega^2 cos\alpha}{2k} \qquad (7)$$

Where: $D = m_b \left| R_{T_{e0}} \cdot r cos\alpha \right| B^2 \qquad (8)$

The interaction between the bead and the interior ring (respectively the exterior ring) will be:

$$F_i = -\frac{D\Omega^2 cos\alpha}{2}$$

$$F_e = \frac{D\Omega^2 cos\alpha}{2} \qquad (9)$$

By introducing a longitudinal displacement u_1, the bead will continuously deform the rings, with the shifts $\Delta_i \rightarrow \Delta_i + \Delta_t t$ and $\Delta_e \rightarrow \Delta_e + \Delta_e I$, while the rotations are $\varphi_i \rightarrow \varphi_i + \varphi_i I$ and $\varphi_e \rightarrow \varphi_e + \varphi_e I$.
The system becomes:

$$\begin{cases} sin\alpha \Delta_i^I + sin\alpha \Delta_e^I + \left(R_{T_{i0}} - \Delta_i - r\right)cos\alpha \varphi_i^I + \left(R_{T_{e0}} + \Delta_e - r\right)cos\alpha \varphi_e^I = u_1 \\ cos\alpha \Delta_i^I + cos\alpha \Delta_e^I - \left(R_{T_{i0}} - \Delta_i - r\right)sin\alpha \varphi_i^I - \left(R_{T_{e0}} + \Delta_e - r\right)sin\alpha \varphi_e^I = 0 \\ k(sin\alpha + \varphi_i cos\alpha)\Delta_i^I - k(sin\alpha + \varphi_e cos\alpha)\Delta_e^I + Fcos\alpha \varphi_i^I + Fcos\alpha \varphi_e^I = 0 \\ k(cos\alpha - \varphi_i sin\alpha)\Delta_i^I - k(cos\alpha + \varphi_e sin\alpha)\Delta_e^I - Fsin\alpha \varphi_i^I - Fsin\alpha \varphi_e^I = 0 \end{cases} \qquad (10)$$

This is a linear system with the unknown Δ_i, Δ_e, φ_i, φ_e, where:

$$\Delta_i^I = \frac{m_1}{m}; \quad \Delta_e^I = \frac{m_2}{m}; \quad \varphi_i^I = \frac{m_3}{m}; \quad \varphi_e^I = \frac{m_4}{m}; \qquad (11)$$

Also, the axial force, which determines the u_1 displacement, is calculated using eq. (12)

$$F_{ax} = n \cdot k \cdot \Delta_i^I (sin\alpha + (\varphi_i + \varphi_i^I)cos\alpha)$$
$$F_{ax} = n \cdot k \cdot \frac{d_1}{m} u_1 (sin\alpha + (\varphi_i + \frac{d_3}{m}u_1)cos\alpha) \qquad (12)$$

And the rigidity

$$k_{11} = \frac{\delta F_{ax}}{\delta u_1} \qquad (13)$$

$$m_1 = \begin{bmatrix} u_1 & sin\alpha & (R_{T_{i0}} + \Delta_i - r)cos\alpha & (R_{T_{e0}} + \Delta_e - r)cos\alpha \\ 0 & cos\alpha & -(R_{T_{i0}} + \Delta_i - r)sin\alpha & -(R_{T_{e0}} + \Delta_e - r)sin\alpha \\ 0 & -k(sin\alpha + \varphi_e cos\alpha) & Fcos\alpha & Fcos\alpha \\ 0 & -k(cos\alpha - \varphi_e sin\alpha) & -Fsin\alpha & -Fsin\alpha \end{bmatrix}$$

$$m_2 = \begin{bmatrix} sin\alpha & u_1 & (R_{T_{io}} + \Delta_i - r)cos\alpha & (R_{T_{e0}} + \Delta_e - r)cos\alpha \\ cos\alpha & 0 & -(R_{T_{io}} + \Delta_i - r)sin\alpha & -(R_{T_{e0}} + \Delta_e - r)sin\alpha \\ k(sin\alpha + \varphi_e \cos\alpha) & 0 & Fcos\alpha & Fcos\alpha \\ k(cos\alpha - \varphi_e \sin\alpha) & 0 & -Fsin\alpha & -Fsin\alpha \end{bmatrix}$$

$$m_3 = \begin{bmatrix} sin\alpha & sin\alpha & u_1 & (R_{T_{e0}} + \Delta_e - r)cos\alpha \\ cos\alpha & cos\alpha & 0 & -(R_{T_{e0}} + \Delta_e - r)sin\alpha \\ k(sin\alpha + \varphi_e \cos\alpha) & -k(sin\alpha + \varphi_e \cos\alpha) & 0 & Fcos\alpha \\ k(cos\alpha - \varphi_e \sin\alpha) & -k(cos\alpha - \varphi_e \sin\alpha) & 0 & -Fsin\alpha \end{bmatrix}$$

$$m_2 = \begin{bmatrix} sin\alpha & sin\alpha & (R_{T_{io}} + \Delta_i - r)cos\alpha & u_1 \\ cos\alpha & cos\alpha & -(R_{T_{io}} + \Delta_i - r)sin\alpha & 0 \\ k(sin\alpha + \varphi_e \cos\alpha) & -k(sin\alpha + \varphi_e \cos\alpha) & Fcos\alpha & 0 \\ k(cos\alpha - \varphi_e \sin\alpha) & -k(cos\alpha - \varphi_e \sin\alpha) & -Fsin\alpha & 0 \end{bmatrix}$$

(14)

We have:

$$k_{11} = M + 2 \cdot N \cdot u_1$$

$$M = n \cdot k \cdot \frac{d_1}{m}(sin\alpha + \varphi_i \cos\alpha)$$

$$N = n \cdot k \cdot \frac{d_1 \cdot d_3}{m^2} \cdot cos\alpha$$

$$k_{11} = \sqrt{M^2 + 4NF_{ax}}$$

$$m = 2kF\left[R_{T_{io}} - R_{T_{e0}} - \frac{\Omega^2 cos\alpha}{k}m_b(R_{T_{e0}} - rcos\alpha)\frac{(R_e - 2rcos\alpha - d)^2}{4 \cdot (R_e - rcos\alpha - d)^2}\right]$$

$$d_1 = kFsin\alpha\left[R_{T_{io}} - R_{T_{e0}} - \frac{\Omega^2 cos\alpha}{k}m_b(R_{T_{e0}} - rcos\alpha)\frac{(R_e - 2rcos\alpha - d)^2}{4 \cdot (R_e - rcos\alpha - d)^2}\right]$$

$$d_3 = 2kFcos\alpha + 2k^2 sin\alpha \cdot tg\alpha \cdot \left[R_{T_{io}} - R_{T_{e0}} - \frac{\Omega^2 cos\alpha}{k}m_b(R_{T_{e0}} - rcos\alpha)\frac{(R_e - 2rcos\alpha - d)^2}{4 \cdot (R_e - rcos\alpha - d)^2}\right]$$

(15)

By calculating M and N, we have:

$$C_1 = \frac{nksin2\alpha}{F} \cdot \left[Fcos\alpha + ksin\alpha \cdot tg\alpha \cdot (R_{T_{e0}} - r)\right]$$

$$C_2 = \frac{nksin2\alpha}{F} \cdot \frac{sin^2\alpha}{8} \cdot \frac{(R_{T_{e0}} - 2rcos\alpha)^2}{R_{T_{e0}} - rcos\alpha}$$

$$C_3 = \frac{m_b cos\alpha}{4k} \cdot \frac{(R_{T_{e0}} - 2rcos\alpha)^2}{R_{T_{e0}} - rcos\alpha}$$

$$k_{11} = \sqrt{M^2 + 4 \cdot \frac{C_1 + C_2 \cdot \Omega^2}{-C_3 \cdot \Omega^2} \cdot F_{ax}}$$

$$A_1 = \frac{4C_1}{C_3} \; ; A_2 = \frac{4C_2}{C_3}$$

$$k_{11} = \sqrt{M^2 - \left(\frac{A_1}{\Omega^2} + A_2\right) \cdot F_{ax}}$$

$$(16)$$

For the purpose of establishing a mathematical model for the simulation, it is considered the expression of the dynamic accommodation base load at the rolling contact, which is given by the relation:

$$Q_C = A \cdot \left[\frac{D_R \cdot D_{C_{i,e}}}{D_w \cdot \left(D_{C_{i,e}} - D_R\right)}\right]^{0,41} \cdot \frac{(1\pm\gamma)^{1,39}}{(1\pm\gamma)^{1/3}} \cdot \left(\frac{\gamma}{cos\alpha}\right)^{0,3} \cdot D_w^{1,8} \cdot Z^{-1/3} \qquad (17)$$

Where:
Q_C = the basis dynamic load at the rolling contact, for bearings with contact   point [N];
A = material constant = 100;
D_R = the double of the curvature radius of the rolling body in the area of contact with the rolling track;
$D_{Ci,e}$ = the diameter of the rolling track of the interior/ exterior ring;
D_w = the diameter of the rolling body;
[4, 6, 7, 8]
The simulation was done using MATLAB MathWorks for two typo dimensions of angular contact bearings with beads. The constructive dimensions are given in Table 1 [6].

Table 1. The constructive dimensions used in the mathematical simulation of the models;

d (mm)	A	D_R (mm)	D_{Ci} (mm)	D_w (mm)	γ	d_m (mm)	α^0	Y	Q_C (N)	n
40	100	10,16	46	7,94	0,142	53,94	15	1	3298,68	16
25	100	8,62	31	6,75	0,159	41	15	1	2650,97	14

The resulting mathematical models for each case are given by the following expressions:

$$k_{11} = \sqrt{18438 - \left(\frac{7265104,8}{\Omega^2} + 1403,39\right) F_{ax}}$$

$$k_{11} = \sqrt{7377,766 - \left(\frac{3450741}{\Omega^2} + 1046,956\right) F_{ax}}$$

$$(18)$$

The simulations were done by varying the definition domain of the two parameters $n_\Omega = 15100 \div 19000$ r/m and $F_{ax} = 1\div10$ (N), corresponding to the results obtained by direct measurements. The results are shown in Figure 3.
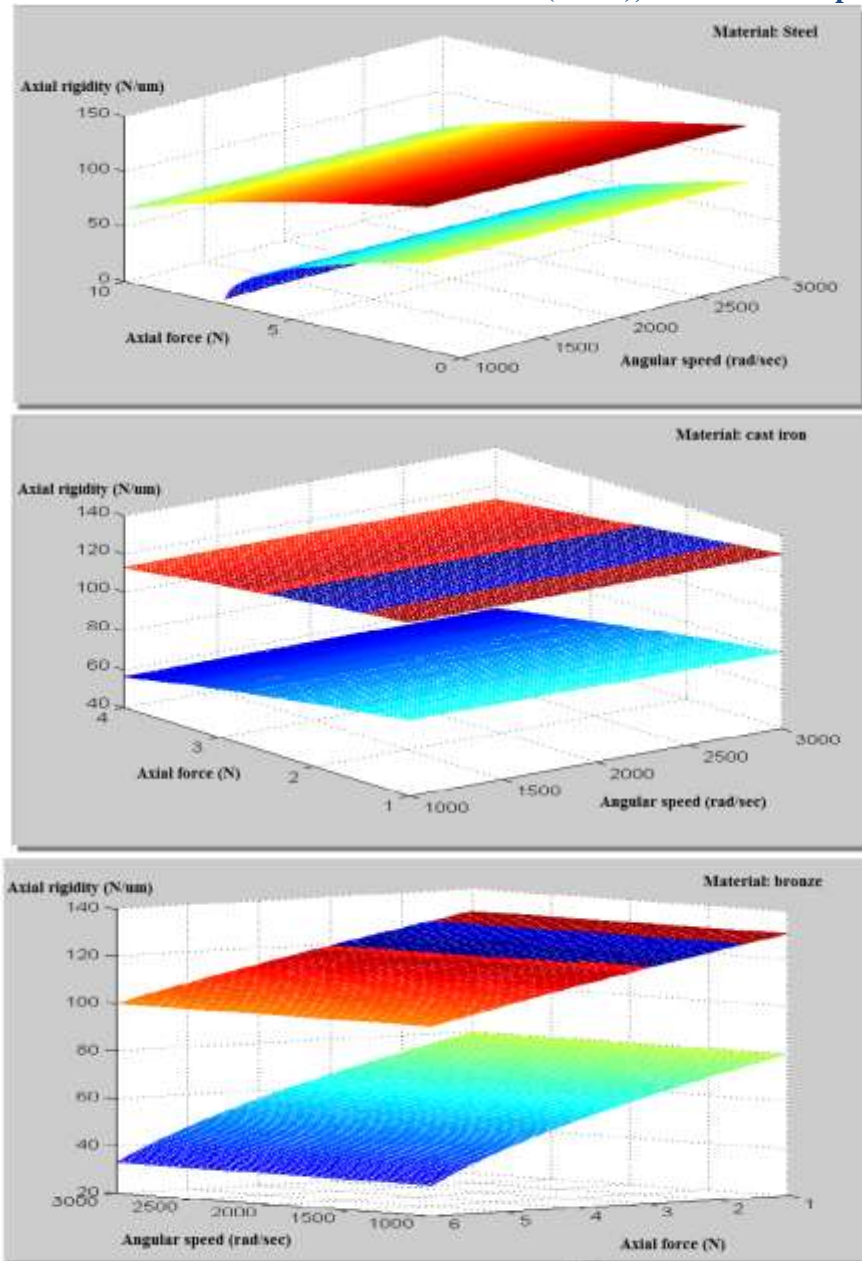
Figure 3- Analysis of bearing axial rigidity in case of the main axis from high rotation speed technological systems;

CONCLUSION
1) The simulations were made for materials for processing like steel, cast iron and bronze;
2) The range of the interior grinding spindleangular speedcorresponds to the direct experimental measurement ranges:

$$n_{w_1} = 15100 \frac{r}{m}; \; n_{w_2} = 17500 \frac{r}{m}; \; n_{w_3} = 19000 \frac{r}{m};$$

3) The spindle`s final load range corresponds to the direct experimental measurement ranges:

$$F_{ax\,steel} = 0 \div 10 \; (N); \; F_{ax\,cast\,iron} = 1 \div 4 \; (N); \; F_{ax\,bronze} = 1 \div 6 \; (N);$$

4) In both cases one can observe a decrease of the axial rigidity with the increase of the final load, with a higher gradient for the real model than the one obtained after the simulation;

5) The axial rigidity remains constant or varies very little with the variation of the angular speed of the interior grinding spindle axis.

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