Quantum Studies of Neutrinos (ID-204)

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Overview

1. Bipartite entanglement of two flavor neutrino oscillations in the uniform matter background

2. Quantum computer circuit to simulate bipartite entanglement in the two flavor neutrino oscillation on the IBMQ platform

3. Summary

Bipartite entanglement of two flavor neutrino oscillations in the uniform matter background

- The evolution equation for two-flavor states (for e.g., ν_e and ν_μ), in matter is [1,2]

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H_{eff}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{1}$$

- Assuming ultra-relativistic neutrinos, the effective Hamiltonian in symmetric form is

$$H_{eff} = [\frac{\Delta m_0^2}{4E}\begin{pmatrix}-cos2\theta_0 & sin2\theta_0\\ sin2\theta_0 & cos2\theta_0\end{pmatrix} + \begin{pmatrix}A/2 & 0\\ 0 & -A/2\end{pmatrix}] \tag{2}$$

where $\Delta m_0^2 = m_2^2 - m_1^2$ and θ_0 is the mixing angle in the vacuum, and $A = 2\sqrt{2}EG_F N_e$ is the effective matter potential induced by ordinary charge-current (contribution from W Boson exchange) weak interactions with electrons.

[1] Mattias Blennow and Alexei Yu.Smirnov, http://dx.doi.org/10.1155/2013/972485
[2] Claudio Giganti, et. al. 10.1016/j.ppnp.2017.10.001

- This H_{eff} matrix can be diagonalized by the unitary transformation,

$$U^T H_{eff} U_= H, \tag{3}$$

where, $H = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 & 0 \\ 0 & \Delta m^2 \end{pmatrix}$ is the effective matrix in the mass basis in matter.

- The unitary matrix

$$U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \equiv \begin{pmatrix} \tilde{U}_{ee} & \tilde{U}_{e\mu} \\ \tilde{U}_{\mu e} & \tilde{U}_{\mu\mu} \end{pmatrix} \tag{4}$$

is the effective mixing matrix in matter.

- Thus, in matter the time evolved flavor neutrino state in flavor basis are

$$|\nu_e(t)\rangle = \tilde{U}_{ee}(t)\,|10\rangle_e + \tilde{U}_{e\mu}(t)\,|01\rangle_\mu, \tag{5}$$

$$\text{and} \quad |\nu_\mu(t)\rangle = \tilde{U}_{\mu e}(t)\,|10\rangle_e + \tilde{U}_{\mu\mu}(t)\,|01\rangle_\mu\,, \tag{6}$$

where $|\tilde{U}_{ee}(t)|^2 + |\tilde{U}_{e\mu}(t)|^2 = 1$ and $|\tilde{U}_{\mu e}(t)|^2 + |\tilde{U}_{\mu\mu}(t)|^2 = 1$,and $|10\rangle$ and $|01\rangle$ are two-qubit states:

$$\begin{aligned} |\nu_e\rangle &= |1\rangle_e \otimes |0\rangle_\mu \equiv |10\rangle_e\,, \\ |\nu_\mu\rangle &= |0\rangle_e \otimes |1\rangle_\mu \equiv |01\rangle_\mu\,. \end{aligned}$$

- The appearance (P_a) and disappearance (P_d) probabilities for $|\nu_\mu(t)\rangle$ are:

$$P_a = |\tilde{U}_{\mu e}(t)|^2 = \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta\cos\left(\frac{\Delta m^2 t}{2E}\right)$$

$$\text{and} \quad P_d = 1 - |\tilde{U}_{\mu e}(t)|^2 = 4\sin^2\theta\cos^2\theta\sin^2\left(\frac{\Delta m^2 t}{4E}\right). \tag{7}$$

where θ and Δm^2 are the effective neutrino oscillation parameters in matter.

- These effective neutrino oscillation parameters Δm^2 and θ are related to the vacuum neutrino oscillation parameters Δm_0^2 and θ_0 are given by

$$\Delta m^2 = \sqrt{[\Delta m_0^2 Cos(2\theta_0) - A]^2 + [\Delta m_0^2 Sin(2\theta_0)]^2} \tag{8}$$

$$Sin^2 2\theta = \frac{(\Delta m_0^2 Sin2\theta_0)^2}{(\Delta m_0^2 Cos2\theta_0 - A)^2 + (\Delta m_0^2 Sin2\theta_0)^2} \tag{9}$$

- The density matrix for $|\nu_\mu(t)\rangle$ can be expressed as,

$$\rho^{\mu e}(t) = |\nu_\mu(t)\rangle \langle \nu_\mu(t)| = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |\tilde{U}_{\mu e}(t)|^2 & \tilde{U}_{\mu e}(t)\tilde{U}^*_{\mu\mu}(t) & 0 \\ 0 & \tilde{U}_{\mu\mu}(t)\tilde{U}^*_{\mu e}(t) & |\tilde{U}_{\mu\mu}(t)|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

- The concurrence is the study of non-locality of a bipartite quantum system and it is defined as: [3]:

$$C(\rho^{\mu e}(t)) = [max(\mu_1 - \mu_2 - \mu_3 - \mu_4, 0)], \tag{11}$$

where $\mu_1, ..., \mu_4$ are the square roots of the eigenvalues of non-Hermitian matrix $\rho^{\mu e}(t)\tilde{\rho}^{\mu e}(t)$ in decreasing order.

- The $\tilde{\rho}^{\mu e}(t)$ is the spin-flipped density matrix

$$\tilde{\rho}^{\mu e}(t) = (\sigma_y \otimes \sigma_y)\rho^{*\mu e}(t)(\sigma_y \otimes \sigma_y), \tag{12}$$

where the asterisk denotes the complex conjugation in the standard basis $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)$, and σ_x, σ_y are Pauli matrices.

[3] W.K.Wooters, doi:10.1103/PhysRevLett.80.2245

- We find only one square root of eigenvalue of matrix $\rho^{e\mu}(t)\tilde{\rho}^{e\mu}(t)$ is non zero i.e., $\mu_4 = 2\sqrt{|\tilde{U}_{\mu e}(t)|^2(1 - |\tilde{U}_{\mu e}(t)|^2)}$, thus concurrence is quantified as:
$$C(\rho^{e\mu}(t)) = 2\sqrt{P_a P_d}. \tag{13}$$
- Note that $P_d < 1$, immediately implies $P_a > 0$. Hence, entanglement is non-zero if the transition probabilities are non-zero.
- This result shows that the two flavor neutrino oscillation is a bipartite entangled system of two qubit pure states [4,5].

[4] A.K.Jha et al., arXiv:2004.14853v2 (2020) [hep-ph], To be published in Modern Physics letters A, doi: 10.1142/S0217732321500565.

[5] A.K.Jha et al. arXiv:2010.06458 [hep-ph]

Concurrence Plot in vaccum

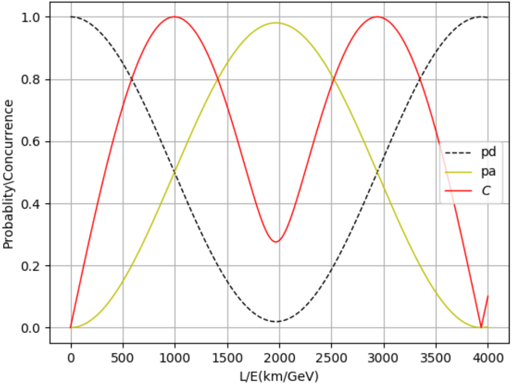

Figure: The time evolution of the concurrence (Red line) compared to the appearance P_a (Green line) and disappearance P_d (Black Dashed line) probabilities of $|\nu_\mu(t)\rangle$ in the vaccum A=0 [6].

[6] I. Esteban et al.JHEP **09** (2020), 178

Concurrence Plot in the uniform matter background

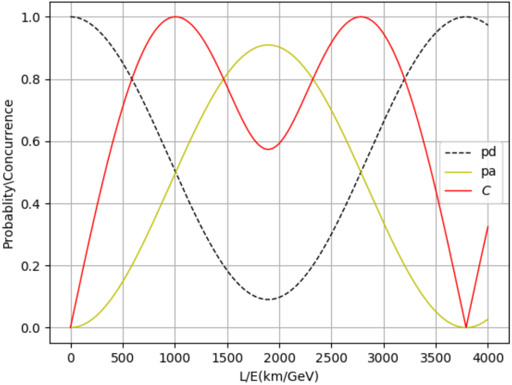

Figure: The time evolution of the concurrence (Red line) compared to the appearance P_a (Green line) and disappearance P_d (Black Dashed line) probabilities of $|\nu_\mu(t)\rangle$ in the constant effective matter potential A=0.000435ev^2.[6].

[6] I. Esteban et al.JHEP **09** (2020), 178

Quantum computer circuit to simulate bipartite entanglement in the two flavor neutrino oscillation on the IBMQ platform

- We identified that the SU(2) rotation matrix $R(\theta)$ can be encoded in the IBM quantum computer by using the universal quantum gate $U3$ [7]:

$$U3(\Phi,\phi,\lambda) = \begin{pmatrix} cos\frac{\Phi}{2} & -sin\frac{\Phi}{2}e^{i\lambda} \\ sin\frac{\Phi}{2}e^{i\psi} & cos\frac{\Phi}{2}e^{i(\lambda+\psi)} \end{pmatrix} \tag{14}$$

- For the two-neutrino system, oscillation probabilities are depend on one of the parameters of U3 gate. Thus,

$$R(\theta) = U3(-2\theta,0,0) = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \equiv \begin{pmatrix} \tilde{U_{ee}} & \tilde{U_{e\mu}} \\ \tilde{U_{\mu e}} & \tilde{U_{\mu\mu}} \end{pmatrix} \tag{15}$$

- The time-evolution operation can be identified as S-gate, where $\psi = \frac{\Delta m^2 t}{2E}$

$$S(\psi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{pmatrix} = U1(t). \tag{16}$$

[7] C.A. Argüelles et al.Phys.Rev.Research. 1 (2019) 033176

- Using the Controlled-NOT (CNOT) gate, we find the gate arrangement of the time evolved muon flavor neutrino state in the two qubit mode (flavor) system:

$$
\begin{aligned}
|\nu_\mu(t)\rangle = CNOT_{12}[U3(2\theta,0,0)U1(t)U3(-2\theta,0,0) \\
X|0\rangle_1 \otimes X|0\rangle_2] \\
\rightarrow \tilde{U}_{\mu e}(t)|10\rangle_e + \tilde{U}_{\mu\mu}(t)|01\rangle_\mu .
\end{aligned}
\tag{17}
$$

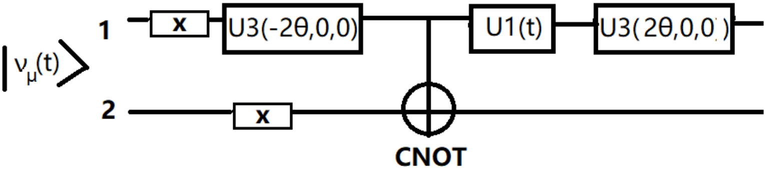

Figure: The circuit represent the time evolved muon flavor neutrino state in a linear superposition of flavor basis ($\nu_\mu \rightarrow \nu_e$), in the two qubit system: $|\nu_\mu(t)\rangle = \tilde{U}_{\mu e}(t)|10\rangle_e + \tilde{U}_{\mu\mu}(t)|01\rangle_\mu$. Here, 1 and 2 represent the input qubits first and second, respectively

- In order to measure concurrence, first we prepare two decouple copies of bi-partite neutrino state $|\nu_\alpha(t)\rangle \otimes |\nu_\alpha(t)\rangle$ in the two flavor system (where $\alpha = e, \mu$), and apply a "spin-flipped" operation $\sigma_y \otimes \sigma_y$ on one of the two copies to prepare an arbitrary global state of neutrino in the four qubit system followed by CNOT and Hadamard gate (H) .
- We find that the concurrence value of the time evolved flavor neutrino oscillation can be extracted from the global state [8].

[8] G.Romero et al.Phys.Rev.A.75.032303 (2007)

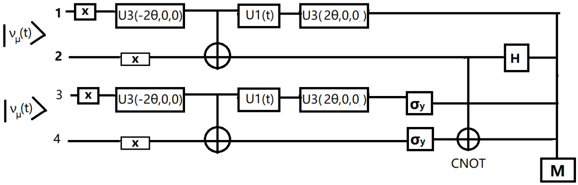

Figure: The circuit represent the concurrence measurement of ν_μ disappearance in two-flavor neutrino oscillations.

- The first two channels (1 and 2) stand for the entangled state $|\nu_\mu(t)\rangle$ that we want to measure. The third and fourth channel (3 and 4) denote the copy of $|\nu_\mu(t)\rangle$

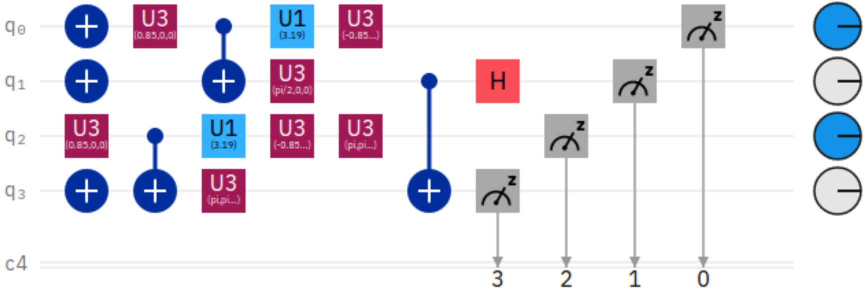

Figure: Implementation of concurrence circuit for $|\nu_\mu(t)\rangle$ on IBMQ processor. [6].

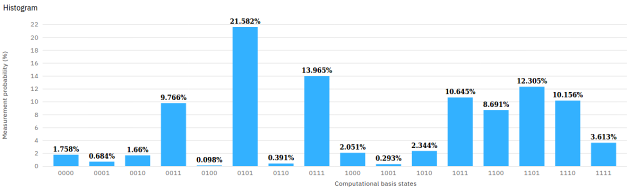

Figure: The concurrence varies with time at the IBMQ computer for an initial muon neutrino flavor state in vaccum (A=0) . The concurrence information encoded in the coefficients of four qubit global state basis are shown through Histogram (probabilities in percentage) plot [6].

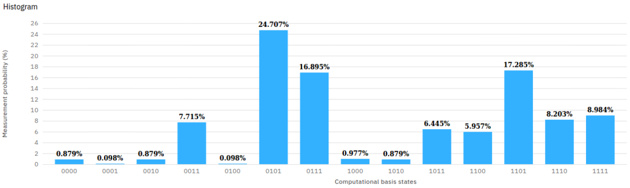

Figure: The concurrence varies with time at the IBMQ computer for an initial muon neutrino flavor state in the uniform matter background (A=0.000435)[6].

Summary

- We mapped the flavor state to two mode states which are like qubits and shows the quantification of bipartite measures like concurrence in two flavor neutrino oscillations in matter and vaccum.
- We constructed quantum computer circuit using Universal $U(3)$ gate, S-gate, Controlled-NOT and Pauli (X) gate to outline the simulation of two flavor neutrino oscillations on a quantum computer.
- We directly measure the concurrence using spin-flipped $\sigma_y \otimes \sigma_y$ gate, and Hadamard gate (H), and proposed the implications of the implementation of entanglement in the two neutrino system on the IBM quantum processor in vaccum and in the uniform matter background.
- Quantum simulation of bipartite entanglement of neutrino oscillations in matter with Non-Standard Interactions (NSIs), and incorporation of Lorentz violation quantum circuits on IBM quantum computer is in progress.

Thank You