

On Fuzzy gp^* - neighborhood, interior And Closure In Fuzzy Topological Spaces

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Abstract

Based on fuzzy gp^* -closed sets and fuzzy gp^* -open sets, in this paper we have introduced fuzzy gp^* -neighborhoods, fuzzy gp^* -Interior and fuzzy gp^* -Closure. Also we investigated some of their elementary properties and discuss some important theorems of fuzzy gp^* - neighborhoods, fuzzy gp^* -Interior and fuzzy gp^* -Closure.

Keywords: Fuzzy topological spaces; fuzzy gp^* -closed sets; fuzzy gp^* -open sets; fuzzy gp^* -neighborhoods; fuzzy gp^* -Interior and fuzzy gp^* -Closure.

1. INTRODUCTION

Introduced by Lotfi A. Zadeh in 1965[1] fuzzy set theory is the generalization of the classical set theory and thus fuzzy set extended the basic mathematical concept of a crisp set. So fuzzy mathematics is just a kind of mathematics developed in this framework and fuzzy topology is the generalization of topology in classical mathematics introduced by C.L Chang in 1968 [2]. Since then work started taking place in fuzzy topology at a rapid rate and various types of fuzzy sets were introduced and studied by various researchers, Like S.S Benchalli and G.P.Siddapur introduced fuzzy g^* pre continuous maps[4], Hamid Reza Moradi and Anahid Kamali introduced fuzzy strongly g^* -closed sets and g^{**} -closed sets in 2015 [5], In 2020 Firdose Habib and Khaja Moinuddin introduced fuzzy gp^* -closed sets and fuzzy gp^* -open sets [7]. And almost all the mathematical, engineering, medicinal etc. concepts have been redefined using fuzzy theory and it has further deepened the understanding of basic set theory.

Based on these newly introduced fuzzy gp^* -closed and fuzzy gp^* -open sets, in this paper we have introduced and studied fuzzy gp^* - neighborhoods, fuzzy gp^* -Interior and fuzzy gp^* -Closure. Also we investigated some of their elementary properties and discuss some important theorems.

2. PRELIMINARIES

Definition 2.1[1] Let X be a space of objects, with a generic element of X denoted by x . Then a *fuzzy set* A in X is a set of ordered pairs $\{(x, f_A(x))\}$ where $f_A(x)$ is called the membership function which associates each point in X a real number in the interval $[0,1]$.

Definition 2.2 [2] A family τ of fuzzy sets of X is called *fuzzy topology* on X if 0 and 1 belong to τ and τ is closed with respect to arbitrary union and finite intersection. The elements of τ are called *fuzzy open sets* and their complements are called *fuzzy closed sets*. The space X with topology τ is called fuzzy topological space denoted by (X, τ)

Definition 2.3 [2] For a fuzzy set α of X , the closure $Cl \alpha$ and the interior $Int \alpha$ of α are defined respectively, as

$$Cl \alpha = \bigwedge \{ \mu : \mu \geq \alpha, 1 - \mu \in \tau \} \text{ and}$$

$$Int \alpha = \bigvee \{ \mu : \mu \leq \alpha, \mu \in \tau \}$$

Definition 2.4 [2] A function f from a fts (X, τ) to a fts (Y, δ) is *fuzzy-continuous* iff the inverse of each δ -open fuzzy set in Y is τ -open fuzzy set in X .

Definition 2.5 [2] let (X, τ) be a fts. A fuzzy set h in X is a *neighborhood* of fuzzy set α in X iff there is $g \in \tau$ such that $\alpha \leq g \leq h$.

Definition 2.6 [3] A fuzzy set n in a fts (X, τ) is a neighborhood of a point $x \in X$ iff there is $g \in \tau$ such that $g \leq n$ and $n(x) = g(x) > 0$. A neighborhood of a point x is frequently denoted by n_x . A neighborhood n_x is called an open neighborhood of x iff $n_x \in \tau$.

Definition 2.7 [2] Let A and B be fuzzy sets in fts (X, τ) , and let $A \geq B$. Then B is called an interior fuzzy set of A iff A is nbhd of B . The union of all interior fuzzy sets of A is called the interior of A and is denoted by A^o .

Definition 2.8 [2] Let A and B be two fuzzy sets in a space $X = \{x\}$ with the grades of membership of x in A and B denoted by $\mu_A(x)$ and $\mu_B(x)$, respectively. Then

$$A = B \iff \mu_A(x) = \mu_B(x) \text{ for all } x \in X$$

$$A \subset B \iff \mu_A(x) \leq \mu_B(x) \text{ for all } x \in X$$

$$C = A \cup B \iff \mu_C(x) = \max [\mu_A(x), \mu_B(x)] \text{ for all } x \in X$$

$$D = A \cap B \iff \mu_D(x) = \min [\mu_A(x), \mu_B(x)] \text{ for all } x \in X$$

Definition 2.9 [6] A fuzzy set A of a fuzzy topological space (X, τ) is called *fuzzy generalized pre-closed* or *gp-closed* set if $pcl(A) \leq U$ whenever $A \leq U$ and U is a fuzzy open set in (X, τ) . And complement of a Fuzzy gp-closed set is called *fuzzy generalized pre-open* or *gp-open* set.

Definition 2.10 [7] A fuzzy set λ of a fuzzy topological space (Y, τ) is called fuzzy generalized pre star closed (briefly fuzzy gp*-closed) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy generalized pre-open in Y .

Definition 2.11 [7] suppose a fuzzy set λ is fuzzy generalized pre star closed set in fuzzy topological space (Y, τ) , Then its complement I.e. $1 - \lambda$ is called fuzzy generalized pre star open (briefly fuzzy gp*-open) in (Y, τ) .

Remark 2.12 [7] All fuzzy closed sets are fuzzy gp* closed sets.

Remark 2.13 [7] All fuzzy open sets are fuzzy gp*-open.

3. Fuzzy generalized pre star (fuzzy gp*) neighborhood

Definition 4.1 Suppose (Y, τ) is a fuzzy topological space & suppose that $y \in Y$. Then a subset N of Y is called *fuzzy gp*-neighborhood* of y if and only if there exists a fuzzy gp*-open subset M of Y such that $y \in M \leq N$.

Definition 4.2 Suppose (Y, τ) is a fuzzy topological space then a subset N of Y is said to be *fuzzy gp*-neighborhood* of a fuzzy set L in Y if and only if there exists a fuzzy gp*-open set M in Y such that $L \leq M \leq N$.

Theorem 4.3 Every fuzzy neighborhood α of $y \in Y$ is fuzzy gp*-neighborhood of y .

Proof: Suppose α is a fuzzy neighborhood of point $y \in Y$. So by definition of fuzzy neighborhood, we have a fuzzy open set μ in Y such that $y \in \mu \leq \alpha$. Now as every fuzzy open set is fuzzy gp*-open in Y , implies μ is fuzzy gp*-open set. So $y \in \mu \leq \alpha$, where μ is fuzzy gp*-open set implies α is fuzzy gp*-neighborhood of y .

Theorem 4.4 α is fuzzy gp*-neighborhood of each of its points in fuzzy topological space (Y, τ) , if α is fuzzy gp*-open in Y .

Proof: Let α is a fuzzy gp*-open set in Y and let $y \in \alpha$. Now, the proof that α is fuzzy gp*-neighborhood of y is clear as $y \in \alpha \leq \alpha$, and α is fuzzy gp*-open set in Y . So α follows the definition of fuzzy gp*-neighborhood. Now as y is any arbitrary point in α implies that α is fuzzy gp*-neighborhood of each of its points.

Theorem 4.5 Suppose (Y, τ) is a fuzzy topological space & A is a fuzzy gp*-closed subset of Y , and let $y \in 1 - A$. Then \exists fuzzy gp*-neighborhood B of y such that $A \wedge B = \phi$.

Proof: Given that A is a fuzzy gp*-closed subset of Y , implies $1 - A$ is a fuzzy gp*-open subset in Y , such that $y \in 1 - A$. So by Theorem 4.4, $(1 - A)$ is a fuzzy gp*-neighborhood of y . Now by Definition 4.2 \exists a fuzzy gp*-open set B , which is also fuzzy gp*-neighborhood of y (by Theorem 4.4) in Y such that $y \in B \leq 1 - A$ I.e. $A \wedge B = \phi$.

4. Fuzzy generalized pre star interior (fuzzy gp*-int).

Definition 5.1 Suppose (Y, τ) is a fuzzy topological space and α is a fuzzy subset in Y . Then a point $y \in \alpha$ is called *fuzzy gp*-interior point* of α , if α is fuzzy gp*-nbhd of y . The collection of all fuzzy gp*-interior points of α is said to be *fuzzy gp*-interior* of α , denoted by *fuzzy gp*-int* (α).

Theorem 5.2 Suppose (Y, τ) is a fuzzy topological space and α, μ be two fuzzy subsets of Y . Then we have

- (a) *Fuzzy gp*-int*(1_Y) = 1_Y & *fuzzy gp*-int*(0_Y) = 0_Y .
- (b) *Fuzzy gp*-int*(α) $\leq \alpha$.
- (c) Suppose α is any fuzzy gp*-open set contained in μ , Then $\alpha \leq \text{fuzzy gp*-int}(\mu)$.
- (d) Suppose $\alpha \leq \mu$, Then *fuzzy gp*-int*(α) $\leq \text{fuzzy gp*-int}(\mu)$.

Proof: (a) We know that 1_Y and 0_Y are fuzzy gp*-open sets in Y , as every fuzzy open set is fuzzy gp*-open. So *fuzzy gp*-int* (1_Y) = $\vee \{H: H \text{ is fuzzy gp*-open } H \leq 1_Y\}$

$$\Rightarrow \text{Fuzzy gp*-int} (1_Y) = 1_Y \vee \{ \text{all fuzzy gp*-open sets} \}$$

$$\Rightarrow \text{Fuzzy gp*-int} (1_Y) = 1_Y.$$

Now, as 0_Y is the only fuzzy gp*-open set contained in 0_Y . So *fuzzy gp*-int* (0_Y) = 0_Y

(b) Suppose $y \in \text{fuzzy gp*-int}(\alpha)$

$$\Rightarrow y \text{ is fuzzy interior point of } \alpha.$$

$$\Rightarrow \alpha \text{ is fuzzy neighborhood of } y.$$

$$\Rightarrow y \in \alpha.$$

So, $y \in \text{fuzzy gp*-int} (\alpha) \Rightarrow y \in \alpha$, I.e. *fuzzy gp*-int*(α) $\leq \alpha$.

(c) Suppose that α is any fuzzy gp*-open set contained in μ . Now, let $y \in \alpha$. So as α is fuzzy gp*-open set contained in μ , implies y is fuzzy gp*-interior point of μ . I.e. $y \in \text{gp*-int}(\mu)$. Hence $\alpha \leq \text{gp*-int}(\mu)$.

(d) Suppose α & μ are two fuzzy subsets of fuzzy topological space (Y, τ) , such that $\alpha \leq \mu$. Now we have to show that

$$\text{fuzzy gp*-int}(\alpha) \leq \text{fuzzy gp*-int}(\mu).$$

Let $y \in \text{fuzzy gp*-int}(\alpha)$, which implies y is fuzzy gp*-interior point of α I.e. α is fuzzy gp*-nbhd of y . Now as $\alpha \leq \mu$, implies μ is also fuzzy gp*-nbhd of y I.e. $y \in \text{fuzzy gp*-int}(\mu)$.

So $y \in \text{fuzzy gp*-int}(\alpha) \Rightarrow y \in \text{fuzzy gp*-int}(\mu)$ I.e.

$$\text{fuzzy gp*-int}(\alpha) \leq \text{fuzzy gp*-int}(\mu).$$

Theorem 5.3 Suppose (Y, τ) is a fuzzy topological space & α is any fuzzy gp*-open subset in Y , then *fuzzy gp*-int* (α) = α .

Proof: Suppose α is a fuzzy gp^* -open subset in Y . Now from Theorem 5.2(b) we have $\text{fuzzy } gp^*\text{-int}(\alpha) \leq \alpha$. Also α is fuzzy gp^* -open subset contained in α . So by Theorem 5.2 (c), we have $\alpha \leq \text{fuzzy } gp^*\text{-int}(\alpha)$. Hence $\text{fuzzy } gp^*\text{-int}(\alpha) = \alpha$.

Theorem 5.4 Suppose (Y, τ) is a fuzzy topological space & α, μ are fuzzy subsets of Y . Then, $\text{fuzzy } gp^*\text{-int}(\alpha) \vee \text{fuzzy } gp^*\text{-int}(\mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha \vee \mu)$.

Proof: We have $\alpha \leq \alpha \vee \mu$ & $\mu \leq \alpha \vee \mu$. Now by Theorem 5.2(d) we have,

$$\text{Fuzzy } gp^*\text{-int}(\alpha) \leq \text{fuzzy } gp^*\text{-int}(\alpha \vee \mu)$$

$$\& \quad \text{Fuzzy } gp^*\text{-int}(\mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha \vee \mu)$$

The above two inequalities implies that,

$$\text{Fuzzy } gp^*\text{-int}(\alpha) \vee \text{fuzzy } gp^*\text{-int}(\mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha \vee \mu).$$

Theorem 5.5 Suppose (Y, τ) is a fuzzy topological space & α, μ are fuzzy subsets of Y . Then, $\text{fuzzy } gp^*\text{-int}(\alpha \wedge \mu) = \text{fuzzy } gp^*\text{-int}(\alpha) \wedge \text{fuzzy } gp^*\text{-int}(\mu)$.

Proof: Since we know that $\alpha \wedge \mu \leq \alpha$ and $\alpha \wedge \mu \leq \mu$, so by Theorem 5.2(d) we have

$$\text{Fuzzy } gp^*\text{-int}(\alpha \wedge \mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha) \quad \&$$

$$\text{Fuzzy } gp^*\text{-int}(\alpha \wedge \mu) \leq \text{fuzzy } gp^*\text{-int}(\mu)$$

The above two inequalities imply that

$$\text{Fuzzy } gp^*\text{-int}(\alpha \wedge \mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha) \wedge \text{fuzzy } gp^*\text{-int}(\mu) \rightarrow (1)$$

Now, Let $y \in \text{fuzzy } gp^*\text{-int}(\alpha) \wedge \text{fuzzy } gp^*\text{-int}(\mu)$.

$$\Rightarrow y \in \text{fuzzy } gp^*\text{-int}(\alpha) \& y \in \text{fuzzy } gp^*\text{-int}(\mu).$$

So it follows that y is fuzzy gp^* -interior point of both the sets α & μ . Which implies that both α & μ are fuzzy gp^* -nbhds of y . Implying that $\alpha \wedge \mu$ is also fuzzy gp^* -neighborhood of y .

$$\Rightarrow y \in \text{fuzzy } gp^*\text{-int}(\alpha \wedge \mu). \text{ Therefore,}$$

$$\text{Fuzzy } gp^*\text{-int}(\alpha) \wedge \text{fuzzy } gp^*\text{-int}(\mu) \leq \text{fuzzy } gp^*\text{-int}(\alpha \wedge \mu) \rightarrow (2)$$

From (1) and (2), we have

$$\text{Fuzzy } gp^*\text{-int}(\alpha \wedge \mu) = \text{fuzzy } gp^*\text{-int}(\alpha) \wedge \text{fuzzy } gp^*\text{-int}(\mu).$$

Theorem 5.6 Let (Y, τ) be a fuzzy topological space and α is a fuzzy subset of Y , Then $\text{fuzzy } gp^*\text{-int}(\alpha) = \bigvee \{\mu : \mu \text{ is a fuzzy } gp^*\text{-open set in } Y \text{ and } \mu \leq \alpha\}$.

Proof: Suppose that α is a fuzzy subset of Y . Now we know that,

$$\begin{aligned}
 y \in \text{fuzzy gp}^*\text{-int}(\alpha) &\Leftrightarrow y \text{ is a fuzzy gp}^*\text{-interior point of } \alpha. \\
 &\Leftrightarrow \alpha \text{ is fuzzy gp}^*\text{-nbhd of } y. \\
 &\Leftrightarrow \exists \text{ a fuzzy gp}^*\text{-open set } \mu \text{ such that } y \in \mu \leq \alpha \text{ (by Definition 4.1)} \\
 &\Leftrightarrow y \in \bigvee \{ \mu : \mu \text{ is a fuzzy gp}^*\text{-open set in } Y \text{ such that } y \in \mu \leq \alpha \}. \\
 &\Rightarrow \text{fuzzy gp}^*\text{-int}(\alpha) = \bigvee \{ \mu : \mu \text{ is a fuzzy gp}^*\text{-open set in } Y \text{ and } \mu \leq \alpha \}.
 \end{aligned}$$

Theorem 5.7 Suppose (Y, τ) is a fuzzy topological space & α is any fuzzy subset in Y . Then, $\text{fuzzy int}(\alpha) \leq \text{fuzzy gp}^*\text{-int}(\alpha)$.

Proof: Suppose that α is any fuzzy subset of Y . And let $y \in \text{fuzzy int}(\alpha)$.

$$\begin{aligned}
 &\Rightarrow y \in \bigvee \{ \mu : \mu \text{ is fuzzy open in } Y, \mu \leq \alpha \} \\
 &\Rightarrow \exists \text{ A fuzzy open set } \mu \text{ such that } y \in \mu \leq \alpha
 \end{aligned}$$

Now as every fuzzy open set is fuzzy gp*-open, Implies there exists a fuzzy gp*-open set μ such that $y \in \mu \leq \alpha$.

$$\begin{aligned}
 &\Rightarrow y \in \bigvee \{ \mu : \mu \text{ is fuzzy gp}^*\text{-open in } Y, \mu \leq \alpha \} \\
 &\Rightarrow y \in \text{fuzzy gp}^*\text{-int}(\alpha).
 \end{aligned}$$

So $y \in \text{fuzzy int}(\alpha) \Rightarrow y \in \text{fuzzy gp}^*\text{-int}(\alpha)$ I.e. $\text{fuzzy int}(\alpha) \leq \text{fuzzy gp}^*\text{-int}(\alpha)$.

5. Fuzzy generalized pre star closure (fuzzy gp*-Closure).

Definition 6.1 Suppose (Y, τ) is a fuzzy topological space and $\alpha \leq Y$. Then *fuzzy gp*-closure* of α is defined as

$$\text{fuzzy gp}^*\text{-cl}(\alpha) = \bigwedge \{ \mu : \alpha \leq \mu, \mu \text{ is fuzzy gp}^*\text{-closed set in } Y \}$$

Theorem 6.2 Suppose (Y, τ) is a fuzzy topological space and α, μ are fuzzy subsets of Y . Then

- (a) $\text{fuzzy gp}^*\text{-cl}(1_Y) = 1_Y$ and $\text{fuzzy gp}^*\text{-cl}(0_Y) = 0_Y$.
- (b) $\alpha \leq \text{fuzzy gp}^*\text{-cl}(\alpha)$.
- (c) Suppose $\mu \leq \alpha$, where α is fuzzy gp*-closed set. Then $\text{fuzzy gp}^*\text{-cl}(\mu) \leq \alpha$.
- (d) If $\alpha \leq \mu$, then $\text{fuzzy gp}^*\text{-cl}(\alpha) \leq \text{fuzzy gp}^*\text{-cl}(\mu)$.

Proof: (a) We know that $\text{fuzzy gp}^*\text{-cl}(1_Y)$ is the intersection I.e. minimum of all fuzzy $\text{gp}^*\text{-closed}$ sets in Y containing 1_Y and since 1_Y is the minimum fuzzy $\text{gp}^*\text{-closed}$ set containing 1_Y . So $\text{fuzzy gp}^*\text{-cl}(1_Y) = 1_Y$. Now $\text{fuzzy gp}^*\text{-cl}(0_Y)$ is the intersection I.e. minimum of all fuzzy $\text{gp}^*\text{-closed}$ sets in Y containing 0_Y and we know that 0_Y is the minimum fuzzy $\text{gp}^*\text{-closed}$ set containing 0_Y . So $\text{fuzzy gp}^*\text{-cl}(0_Y) = 0_Y$.

Proof: (b) Since we know that $\text{fuzzy gp}^*\text{-cl}(\alpha)$ is the intersection of all fuzzy $\text{gp}^*\text{-closed}$ sets containing α . So $\alpha \leq \text{fuzzy gp}^*\text{-cl}(\alpha)$ is obvious

Proof: (c) Suppose $\mu \leq \alpha$, where α is fuzzy $\text{gp}^*\text{-closed}$ set. Now,

$$\text{fuzzy gp}^*\text{-cl}(\mu) = \bigwedge \{ \pi : \mu \leq \pi, \pi \text{ is fuzzy gp}^*\text{-closed set in } Y \}$$

I.e. $\text{fuzzy gp}^*\text{-cl}(\mu)$ is contained in all fuzzy $\text{gp}^*\text{-closed}$ sets, so in particular $\text{fuzzy gp}^*\text{-cl}(\mu) \leq \alpha$

Proof: (d) Suppose $\alpha \leq \mu$, and we know that

$$\text{fuzzy gp}^*\text{-cl}(\mu) = \bigwedge \{ \pi : \mu \leq \pi, \pi \text{ is fuzzy gp}^*\text{-closed set in } Y \} \rightarrow (d.1)$$

Now if $\mu \leq \pi$, where π is fuzzy $\text{gp}^*\text{-closed}$ in Y , then by (c) of this theorem we have $\text{fuzzy gp}^*\text{-cl}(\mu) \leq \pi$.

Now by (b) of this theorem $\mu \leq \text{fuzzy gp}^*\text{-cl}(\mu)$ implies $\alpha \leq \mu \leq \pi$, where π is fuzzy $\text{gp}^*\text{-closed}$. So we have $\text{fuzzy gp}^*\text{-cl}(\alpha) \leq \pi$ (by (c) of this theorem). Therefore

$$\text{fuzzy gp}^*\text{-cl}(\alpha) \leq \bigwedge \{ \pi : \mu \leq \pi, \pi \text{ is fuzzy gp}^*\text{-closed set in } Y \}$$

$$\Rightarrow \text{fuzzy gp}^*\text{-cl}(\alpha) \leq \text{fuzzy gp}^*\text{-cl}(\mu). \quad (\text{using (d.1)})$$

Theorem 6.3 Suppose (Y, τ) is a fuzzy topological space and α is a fuzzy $\text{gp}^*\text{-closed}$ set in Y , then $\text{fuzzy gp}^*\text{-cl}(\alpha) = \alpha$.

Proof: Suppose α is fuzzy $\text{gp}^*\text{-closed}$ subset in Y . Now by Theorem 6.2 (b) $\alpha \leq \text{fuzzy gp}^*\text{-cl}(\alpha)$. Also $\alpha \leq \alpha$ & given that α is fuzzy $\text{gp}^*\text{-closed}$ set in Y , so by Theorem 6.2(c) $\text{fuzzy gp}^*\text{-cl}(\alpha) \leq \alpha$. Therefore we have $\text{fuzzy gp}^*\text{-cl}(\alpha) = \alpha$.

Theorem 6.4 Suppose α & μ are fuzzy subsets in fuzzy topological space (Y, τ) . Then we have $\text{fuzzy gp}^*\text{-cl}(\alpha \wedge \mu) \leq \text{fuzzy gp}^*\text{-cl}(\alpha) \wedge \text{fuzzy gp}^*\text{-cl}(\mu)$.

Proof: Suppose α & μ are fuzzy subsets in Y . Then clearly $\alpha \wedge \mu \leq \alpha$ and $\alpha \wedge \mu \leq \mu$. Now by Theorem 6.2 (d) $\text{fuzzy gp}^*\text{-cl}(\alpha \wedge \mu) \leq \text{fuzzy gp}^*\text{-cl}(\alpha)$ and $\text{fuzzy gp}^*\text{-cl}(\alpha \wedge \mu) \leq \text{fuzzy gp}^*\text{-cl}(\mu)$. Implying that

$$\text{fuzzy gp}^*\text{-cl}(\alpha \wedge \mu) \leq \text{fuzzy gp}^*\text{-cl}(\alpha) \wedge \text{fuzzy gp}^*\text{-cl}(\mu).$$

Theorem 6.5 Suppose (Y, τ) is a fuzzy topological space and α is a fuzzy set in Y , then $\text{fuzzy gp}^*\text{-cl}(\alpha) \leq \text{cl}(\alpha)$.

Proof: Suppose α is a fuzzy subset in Y . Now, we know that $cl(\alpha) = \bigwedge \{\pi: \alpha \leq \pi, \pi \text{ is fuzzy closed}\}$. So if $\{\alpha \leq \pi: \pi \text{ is fuzzy closed set in } Y\} \Rightarrow \{\alpha \leq \pi: \pi \text{ is fuzzy } gp^*\text{-closed set in } Y\}$, as every fuzzy closed set is fuzzy gp^* -closed. So by Theorem 6.2(c) $fuzzy\ gp^*\text{-}cl(\alpha) \leq \pi$. Therefore $fuzzy\ gp^*\text{-}cl(\alpha) \leq \bigwedge \{\pi: \alpha \leq \pi, \pi \text{ is fuzzy closed}\} = cl(\alpha)$ I.e. $fuzzy\ gp^*\text{-}cl(\alpha) \leq cl(\alpha)$

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