Field linkage and magnetic helicity density

M. Jardine¹, K. Lund¹, A. J. B. Russell², J.-F. Donati³, R. Fares⁴, C. P. Folsom³, S. V. Jeffers⁵, S. C. Marsden⁶, J. Morin⁷, P. Petit⁴ and V. See⁸



- Introduction

The helicity (H) of a magnetic field is a fundamental property that is conserved in ideal MHD. It measures the linkage of poloidal and toroidal fields:

$$H = \int \boldsymbol{A} \cdot \boldsymbol{B} \mathrm{dV}, \quad \boldsymbol{B} = \boldsymbol{\nabla} \boldsymbol{\times} \boldsymbol{A}$$

Through the Sun's magnetic cycle, its helicity varies as the poloidal and toroidal fields wax and wane.

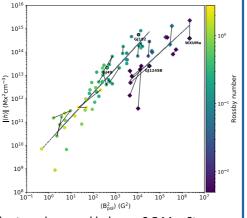
Helicity is lost from the Sun during Coronal Mass Ejections.

How does helicity density vary with magnetic energy?

Magnetic helicity density follows a single power law

 $|\langle h \rangle| \propto \langle \mathrm{B_{tor}}^2 \rangle^{0.86 \pm 0.04}$

when plotted against the *toroidal* field energy, but splits into two branches when plotted against the *poloidal* field energy (Lund+2020).



These two branches divide stars above and below $\sim 0.5~M_{\odot}$. Stars on the right-hand lower-mass branch (such as GJ1245B) may have a higher magnetic energy than stars (such as GJ49) on the left-hand higher-mass branch but they have the same helicity density.

Results: Low-mass stars are inefficient producers of helicity density

We find that stars on the lower-mass branch tend to have toroidal fields that are non-axisymmetric and so link through regions of positive and negative poloidal field. A lower-mass star may have the same helicity density as a higher-mass star, despite having a stronger poloidal field.

The "helicity energy fraction"

$$\tilde{h} \equiv |\langle h \rangle| / \langle R_{\star} B^2 \rangle$$

is lowest in the lowest mass stars.

How do we map helicity density?

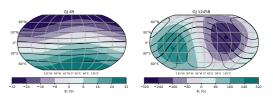
We have mapped the helicity density

$$h = A \cdot B$$

across the surface of 51 stars in the mass range 0.1-1.34 M_{\odot} using Zeeman-Doppler imaging to determine the poloidal and toroidal fields at the surface. Toroidal field loops lie on spherical surfaces, while the radial component of the poloidal field passes through these loops.

How can we increase the magnetic energy but not the helicity density?

We present here a novel method of visualising the helicity density in terms of the linkage of the surface toroidal and poloidal fields. This allows us to classify the field linkages that provide the helicity density for stars of different masses and rotation rates. Colours show the strength of the radial poloidal field, while contours show toroidal magnetic field lines.



GJ49: most of the poloidal field GJ1245B: toroidal field loops are linked by links through the toroidal field both positive and negative poloidal field

Result: both have similar helicity density

