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Polarized radiative transfer: two Monte Carlo codes versus integral equations

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1. Introduction

While modelling theoretically the observable polarized radiation coming from geometrically irregular celestial objects and/or anisotropic effective medium (magnetized plasma, oriented nonspherical dust particles), numerical modelling of multiple scattering of radiation is needed usually. Universal means for this is Monte Carlo method – the direct statistical modelling of propagation of photon packages. In particular, this is necessary for interpretation of resolved polarimetric observations of circumstellar envelopes.

Such Monte Carlo software was created, called Ventspils RTMC [Radiative Transfer Monte Carlo]). In order to check its correctness and plausibility of results with independent means, an alternative code was developed, solving integral equations for source functions in homogeneous sphere with Rayleigh scattering. The irradiating spherical star is in the centre of the sphere. The final results of both computer codes were compared, namely, the images of the scattering sphere with star in its centre onto virtual CCD matrix.

Finally, the results of Ventspils RTMC and integral equation solving codes were compared with the results of the powerful software RADMC3D v2.0 [1].

2. The basic physical assumptions

1) Radiation propagates in Minkowski spacetime (the effects of general relativity are not essential).
2) The cloud of scattering particles (dust, electrons, molecules) is sufficiently rarified, and radiative transfer equation (an approximate corollary of Maxwell electrodynamics) is valid.
3) The effective extinction matrix of the medium is scalar. The effective refraction coefficient is constant in the medium, and the refraction is unobservable.
4) The frequency of radiation does not change during scattering process (we do not consider Compton effect and the redistribution of radiation over frequencies within spectral line).

The scattering in the medium is characterized by single scattering albedo $\lambda = C_{sca} / C_{ext}$, where C_{sca} and C_{ext} are statistically averaged scattering and extinction cross sections, and the scattering matrix [2]

$$\begin{Vmatrix} I_{sca}(\vartheta_{sca},\varphi_{sca}) \\ Q_{sca}(\vartheta_{sca},\varphi_{sca}) \\ U_{sca}(\vartheta_{sca},\varphi_{sca}) \\ V_{sca}(\vartheta_{sca},\varphi_{sca}) \end{Vmatrix} \propto C_{sca} \begin{Vmatrix} F_{11}(\boldsymbol{\theta}) & F_{12}(\boldsymbol{\theta}) & F_{13}(\boldsymbol{\theta}) & F_{14}(\boldsymbol{\theta}) \\ F_{21}(\boldsymbol{\theta}) & F_{22}(\boldsymbol{\theta}) & F_{23}(\boldsymbol{\theta}) & F_{24}(\boldsymbol{\theta}) \\ F_{31}(\boldsymbol{\theta}) & F_{32}(\boldsymbol{\theta}) & F_{33}(\boldsymbol{\theta}) & F_{34}(\boldsymbol{\theta}) \\ F_{41}(\boldsymbol{\theta}) & F_{42}(\boldsymbol{\theta}) & F_{43}(\boldsymbol{\theta}) & F_{44}(\boldsymbol{\theta}) \end{Vmatrix} \begin{Vmatrix} I_{inc}(\vartheta_{inc},\varphi_{inc}) \\ Q_{inc}(\vartheta_{inc},\varphi_{inc}) \\ U_{inc}(\vartheta_{inc},\varphi_{inc}) \\ V_{inc}(\vartheta_{inc},\varphi_{inc}) \end{Vmatrix}, \qquad (1)$$

where (ϑ, φ) are the spherical angles characterizing the direction of propagation of radiation, θ is the scattering angle, and $\boldsymbol{\theta} = (\vartheta_{inc}, \varphi_{inc}, \vartheta_{sca}, \varphi_{sca})^{\mathrm{T}}$ is the vector characterizing the directions of both incidence and scattering. Here the plane of scattering is the polarization reference plane. In the isotropic medium the scattering matrix is dependent only on the scattering angle θ. Optical cross sections and the scattering matrix are statistically averaged over types, chemical composition, shapes, sizes, orientation of the scattering particles etc. In macroscopically isotropic and mirror symmetric medium the extinction matrix is scalar, the effective cross sections C_{ext} and C_{sca} are isotropic and independent of polarization of the incident radiation, and the scattering matrix is [2]

$$\mathbf{F}(\theta) = \begin{Vmatrix} F_{11}(\theta) & F_{12}(\theta) & 0 & 0 \\ F_{12}(\theta) & F_{22}(\theta) & 0 & 0 \\ 0 & 0 & F_{33}(\theta) & F_{34}(\theta) \\ 0 & 0 & -F_{34}(\theta) & F_{44}(\theta) \end{Vmatrix}. \qquad (2)$$

3. Ventspils RTMC code

At this stage we assume that the medium (dust cloud) is macroscopically isotropic and mirror symmetric.

With the physical assumptions mentioned above, **software complex in C++ language was created,** modelling **polarized radiative transfer** in the scattering medium **of arbitrary shape** using **Monte Carlo method**. We call this software code **Ventspils RTMC** (*Ventspils Radiative Transfer Monte Carlo*). Up to this moment it has been used for the case of uniform dust properties in the whole cloud, allowing for spatially variable dust concentration. It is possible to include spatially variable properties of dust as well. Radiation outcoming from the scattering medium and hitting the virtual telescope and the pixels of virtual CCD matrix is calculated using the “peeling off” method (see, e.g., [3]).

Parallellized calculations were done on VIRAC’s computing cluster (24 cores, 2.8 GHz base frequency); up to 10^9 photon packages were followed up.

4. Integral equations for source functions in homogeneous sphere. Rayleigh scattering

The aim was to create polarized radiative transfer code, fully independent of Monte Carlo method, for at least one physically simple case. By comparison of the results, to check the correctness of results of Monte Carlo code and to determine the limits of applicability of its current version.

Let us consider homogeneous, isotropic, optically inactive sphere of geometrical radius r_0 and optical radius $\tau_0 = \alpha r_0$, where α is the scalar extinction coefficient. There are absorption and Rayleigh scattering in the sphere, and single scattering albedo is λ. The sphere emits nothing by itself, but there is a concentric spherical, opaque and nonreflecting star of radius $r_* < r_0$ inside the sphere, homogeneously emitting isotropic unpolarized radiation. Luminosity of the star is L_*, and its geometrical radius can be replaced with conventional optical radius $\tau_* = \alpha r_*$. The real optical thickness of medium in radial direction is $\alpha(r_0 - r_*) = \tau_0 - \tau_*$.

If the linear polarization reference plane goes through the direction of propagation of radiation and the radius of the sphere, then only two Stokes parameters (I, Q) are nonzero. System of Fredholm integral equations of the second kind for the source functions was written, and Python computer code for their solution was created, with discretization by the optical radius τ and solution of the corresponding system of linear algebraic equations using standard software library **numpy.linalg**. After that, the radiation outcoming from the sphere is found by numerical integration along the ray.

This physically very simple model was used in order to compare the results of three radiative transfer codes – Ventspils RTMC, the solution of integral equations for source functions, and RADMC3D.

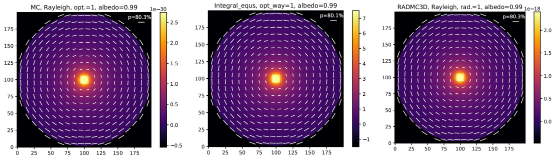
5. RADMC3D

This is a Monte Carlo radiative transfer software complex created by C.Dullemond et al. [1]. It provides many possibilities, including calculation of cosmic dust temperature, radiative transfer in both continuum and spectral lines, adaptive refinement of the spatial grid. The current version of the code is 2.0, and it can be freely downloaded from the Internet. It exists for at least 10 years and has been used for radiative transfer calculations by many authors, but in most cases neglecting polarization. Some of the very rare exceptions of astrophysical cases were polarized radiative transfer was calculated indeed are papers [4] and [5]. It is remarkable that even in [6], while interpreting the polarimetric imaging of AGB star L_2 Pup, RADMC3D is used only in “unpolarized” mode for dust temperature calculations, producing images with the assumption of isotropic scattering.

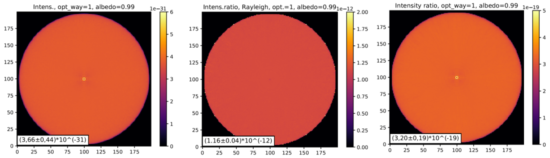
All Monte Carlo codes experience serious problems for large optical depth of the scattering medium. In such cases RADMC3D accelerates the calculations using diffusion approximation.

6. The results

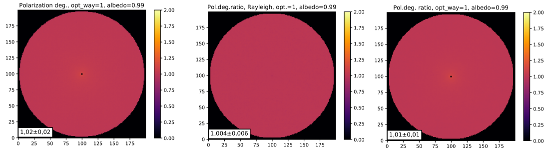
Considering the Rayleigh scattering sphere as described above, we constructed pictures of the observable polarized radiation on the virtual CCD matrix having 200x200 pixels, assuming that the spatially resolved images cover all the CCD matrix. Single scattering albedo was assumed to be 0.4664; 0.9; 0.99 and 1, while the optical thickness of the scattering sphere in the radial direction was assumed to be 0.1; 1; 5 and 10. The direction of the observable polarization turned out to be always perpendicular to the radial direction of the image. Further, we constructed the maps of intensity ratios and ratios of polarization degree. The intensities are not uniformly normalized, and this is the reason why their ratios have very big orders. We calculated the statistical mean values and standard deviations of both the intensity ratios and polarization degree ratios.



Intensity ratios:


Ventspils RTMC / integral equations — Ventspils RTMC / RADMC3D — RADMC3D / integral equations

Polarization degree ratios:


Ventspils RTMC / integral equations — Ventspils RTMC / RADMC3D — RADMC3D / integral equations

Intensity ratio, Ventspils RTMC / integral equations

Intens. ratio	Albedo=0.4664	Albedo=0.9	Albedo=0.99	Albedo=1
opt_way =0.1	(3,62±0,5)*10^(-31)	(3,62±0,5)*10^(-31)	(3,62±0,5)*10^(-31)	(3,62±0,5)*10^(-31)
opt_way =1	(3,65±0,5)*10^(-31)	(3,66±0,4)*10^(-31)	(3,66±0,4)*10^(-31)	(3,66±0,4)*10^(-31)
opt_way =5	(3,77±0,4)*10^(-31)	(3,75±0,4)*10^(-31)	(3,74±0,4)*10^(-31)	(3,74±0,4)*10^(-31)
opt_way =10	(4,07±0,6)*10^(-31)	(3,89±0,4)*10^(-31)	(3,91±0,4)*10^(-31)	(3,92±0,4)*10^(-31)

Polarization degree ratio, Ventspils RTMC/ integral equations

Polariz. deg.	Albedo=0.4664	Albedo=0.9	Albedo=0.99	Albedo=1
opt_way =0.1	1,02±0,01	1,02±0,01	1,02±0,01	1,02±0,01
opt_way =1	1,02±0,01	1,02±0,02	1,02±0,02	1,02±0,02
opt_way =5	1,02±0,02	1,02±0,02	1,02±0,03	1,03±0,03
opt_way =10	1,03±0,09	1,03±0,07	1,03±0,12	1,04±0,15

Intensity ratio, RADMC3D / integral equations

Intens. ratio	Albedo=0.4664	Albedo=0.9	Albedo=0.99	Albedo=1
opt_way =0.1	(3,18±0,2)*10^(-19)	(3,18±0,2)*10^(-19)	(3,18±0,2)*10^(-19)	(3,18±0,2)*10^(-19)
opt_way =1	(3,16±0,2)*10^(-19)	(3,19±0,2)*10^(-19)	(3,20±0,2)*10^(-19)	(3,20±0,2)*10^(-19)
opt_way =5	(3,00±0,1)*10^(-19)	(3,15±0,1)*10^(-19)	(3,22±0,2)*10^(-19)	(3,23±0,2)*10^(-19)
opt_way =10	(2,81±0,1)*10^(-19)	(3,11±0,1)*10^(-19)	(3,33±0,1)*10^(-19)	(3,40±0,1)*10^(-19)

Polarization degree ratio, RADMC3D / integral equations

Polar. deg.	Albedo=0.4664	Albedo=0.9	Albedo=0.99	Albedo=1
opt_way =0.1	1,02±0,01	1,02±0,01	1,02±0,01	1,02±0,01
opt_way =1	1,02±0,01	1,02±0,01	1,01±0,01	1,01±0,01
opt_way =5	1,01±0,01	1,01±0,02	1,00±0,03	1,00±0,03
opt_way =10	1,01±0,01	1,01±0,09	1,02±0,20	1,03±0,25

7. Conclusions and acknowledgements

All three codes tested give essentially compatible results until optical thickness of 10 and for single scattering albedos from 0.4664 until 1.0. There are some small systematic trends in intensity ratios, and some differences in the statistical noise.

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