Convective turbulent viscosity acting on equilibrium tidal flows:

new frequency scaling of the effective viscosity

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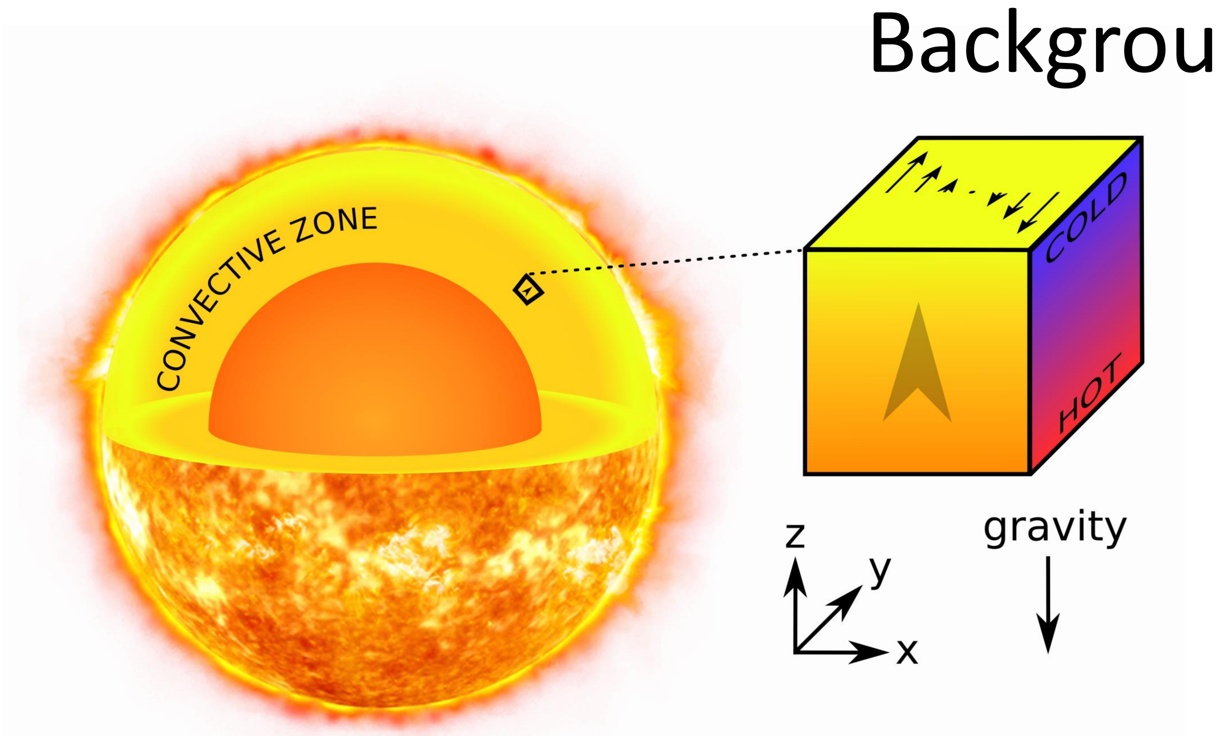
Background



Turbulent convection is thought to act as an effective viscosity (ν_E) in damping tidal flows in stars and giant planets. However, the efficiency of this mechanism has long been debated, particularly in the regime of fast tides, when the tidal frequency (ω) exceeds the turnover frequency of the dominant convective eddies (ω_c).

How does the effective viscosity (ν_E) scale with ω?

- Zahn [1] adapted mixing length theory by considering the mixing length as how far an eddy can travel in one tidal period.
- Goldreich and Nicholson [2] considered a similar argument but assumed a Kolmogorov cascade, resonant eddies provide the dominant contribution to the effective viscosity.

Zahn (constant Q')

$\nu_E \propto \omega^{-1}$

Goldreich (quadratic reduction)

$\nu_E \propto \omega^{-2}$

It is essential to determine which (if any) of these prescriptions is correct. This is because for Hot Jupiters, $\omega \sim {}^1/_2$ (days^{-1}) and $\omega_c \sim {}^1/_{20}$ (days^{-1}) $\rightarrow \omega \gg \omega_c$, and both scalings differ significantly in their predictions for the orbital decay of the planet.

model	ν_E prescription	$Q'_\star$	τ_a (years)
constant time lag	1	$\sim 10^5$	$\sim 10^6$
constant $Q'_\star$	ω^{-1}	$\sim 10^6$	$\sim 10^9$
Quadratic reduction	ω^{-2}	$\sim 10^8$	$\sim 10^{11}$

To highlight this we can crudely estimate the inspiral time (τ_a) (and stellar $Q'_\star$) using each of the three prescriptions for a Jupiter mass planet on a one day aligned circular orbit around a slowly rotating sun-like star. This is analogous to WASP-12b [4].

Model

We consider a small patch of the convective region of the host star, where the large-scale tidal flow is represented by an oscillatory background shear flow. The convection is modelled using the Rayleigh-Bénard setup, adopting the Boussinesq approximation. The nondimensional equations are:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{u}_0\cdot\nabla\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}_0 = -\nabla P + \mathrm{RaPr}\,\theta\mathbf{e_z} + \mathrm{Pr}\nabla^2\mathbf{u}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta + \mathbf{u}_0\cdot\nabla\theta = u_z + \nabla^2\theta$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathrm{Pr} = \frac{\nu}{\kappa} = 1 \qquad \mathrm{Ra} = \frac{-N^2 d^4}{\nu\kappa} \qquad R = \frac{\mathrm{Ra}}{\mathrm{Ra}_c}$$

Tidal shear flow: $\boldsymbol{u}_0 = a_0\omega x\cos(\omega t)\boldsymbol{e}_y$

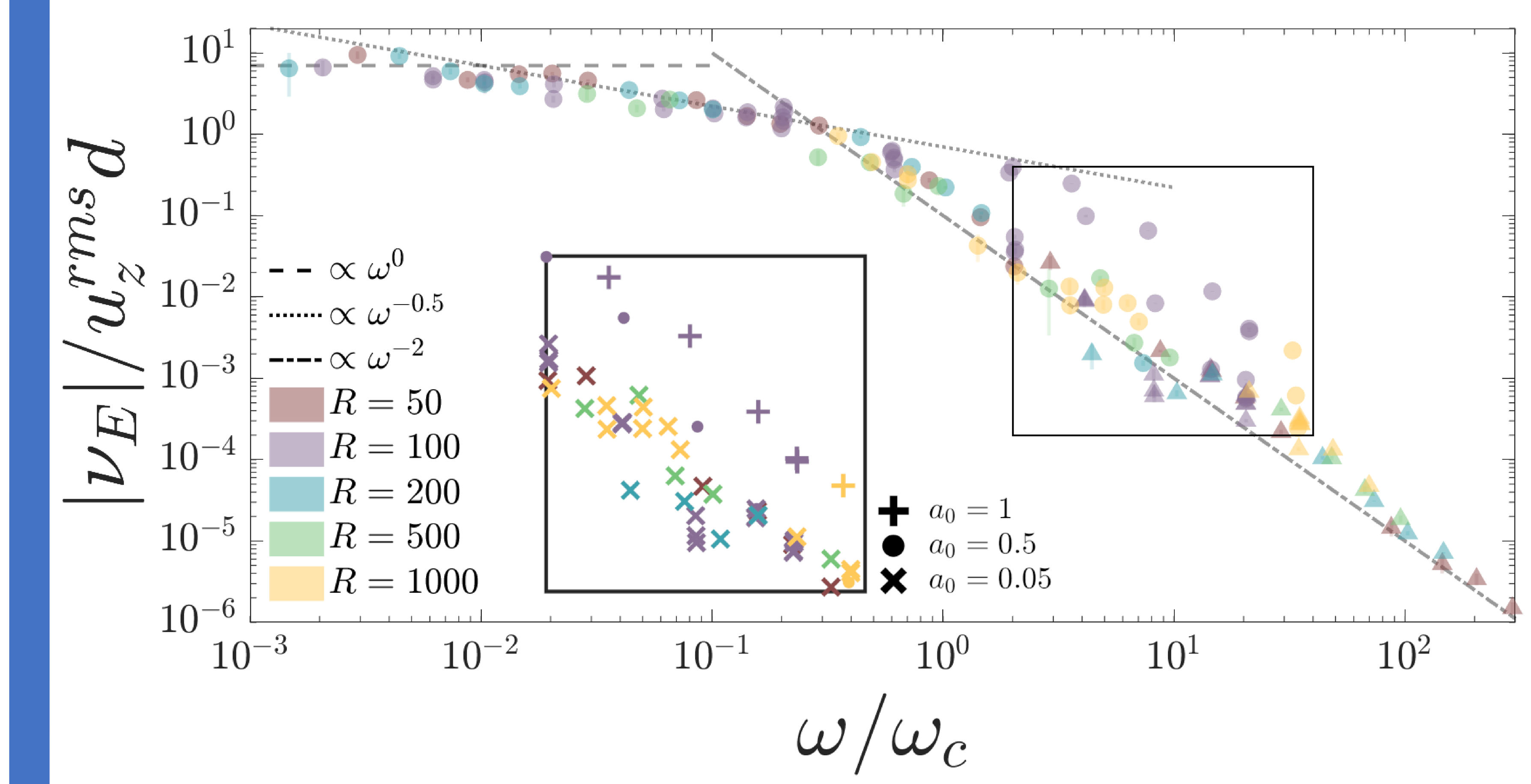
Boundary conditions: $u_z = \theta = \partial_z u_x = \partial_z u_y = 0 \quad \text{on} \quad z=0 \;\;\&\;\; z=1$

Box size: $(L_x, L_y, d) \quad \text{where} \quad d=1$

The effective viscosity (ν_E) can be evaluated by equating the rate at which the shear does work on the flow with a viscous dissipation rate for the tidal flow:

$$\nu_E(\omega) = \frac{-2}{a_0\omega(T-T_0)}\int_{T_0}^{T} R_{xy}(t)\cos(\omega t)\,\mathrm{d}t$$

Effective viscosity dependence and spatial structure

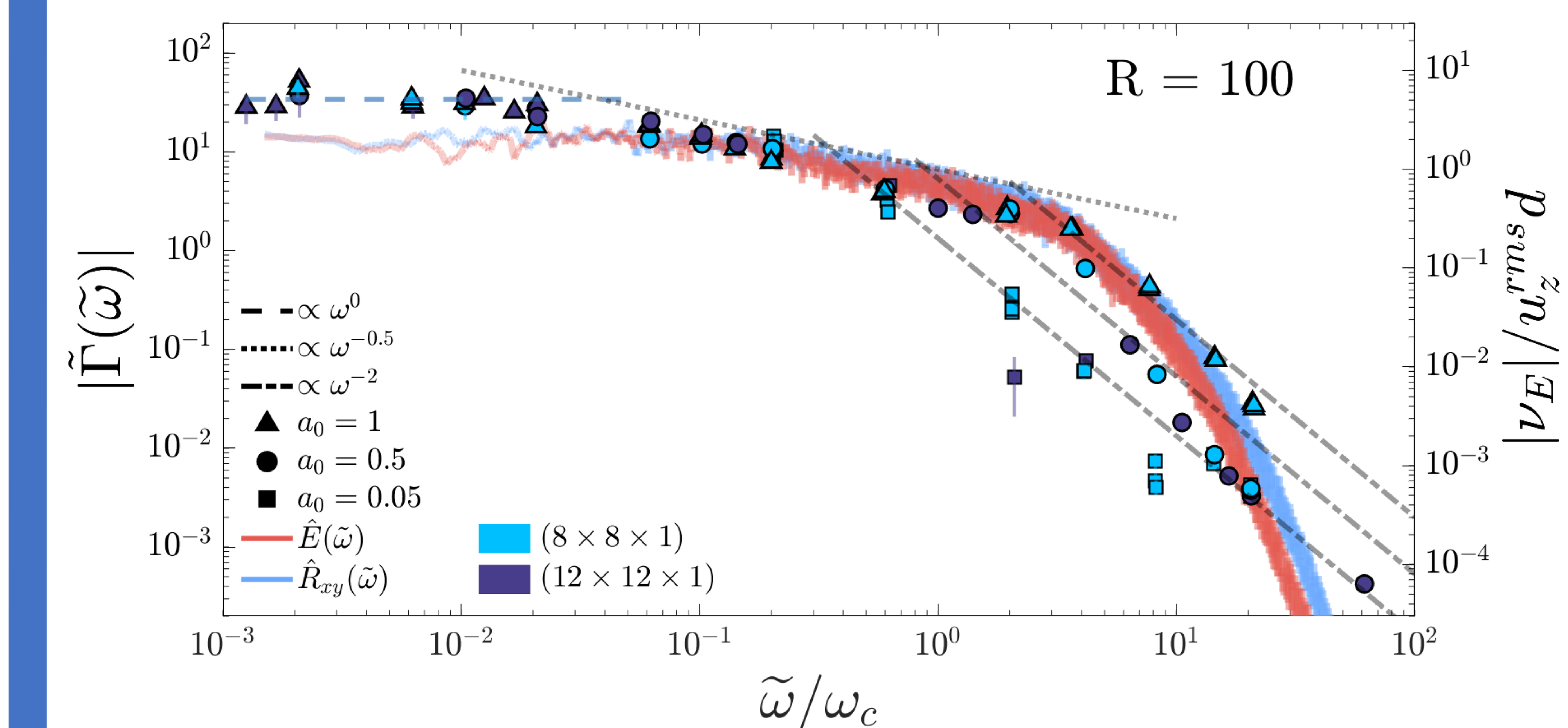


Left top - we show the scaled frequency dependence of the scaled effective viscosity for various strengths of convection in domains of size (8,8,1). Note – triangles represent negative values (antidissipation).

In the high frequency regime we observe agreement with Goldreich and Nicholson in the scaling law.

There is a significant frequency range with a new $\omega^{-0.5}$ power law (not previously predicted or observed).

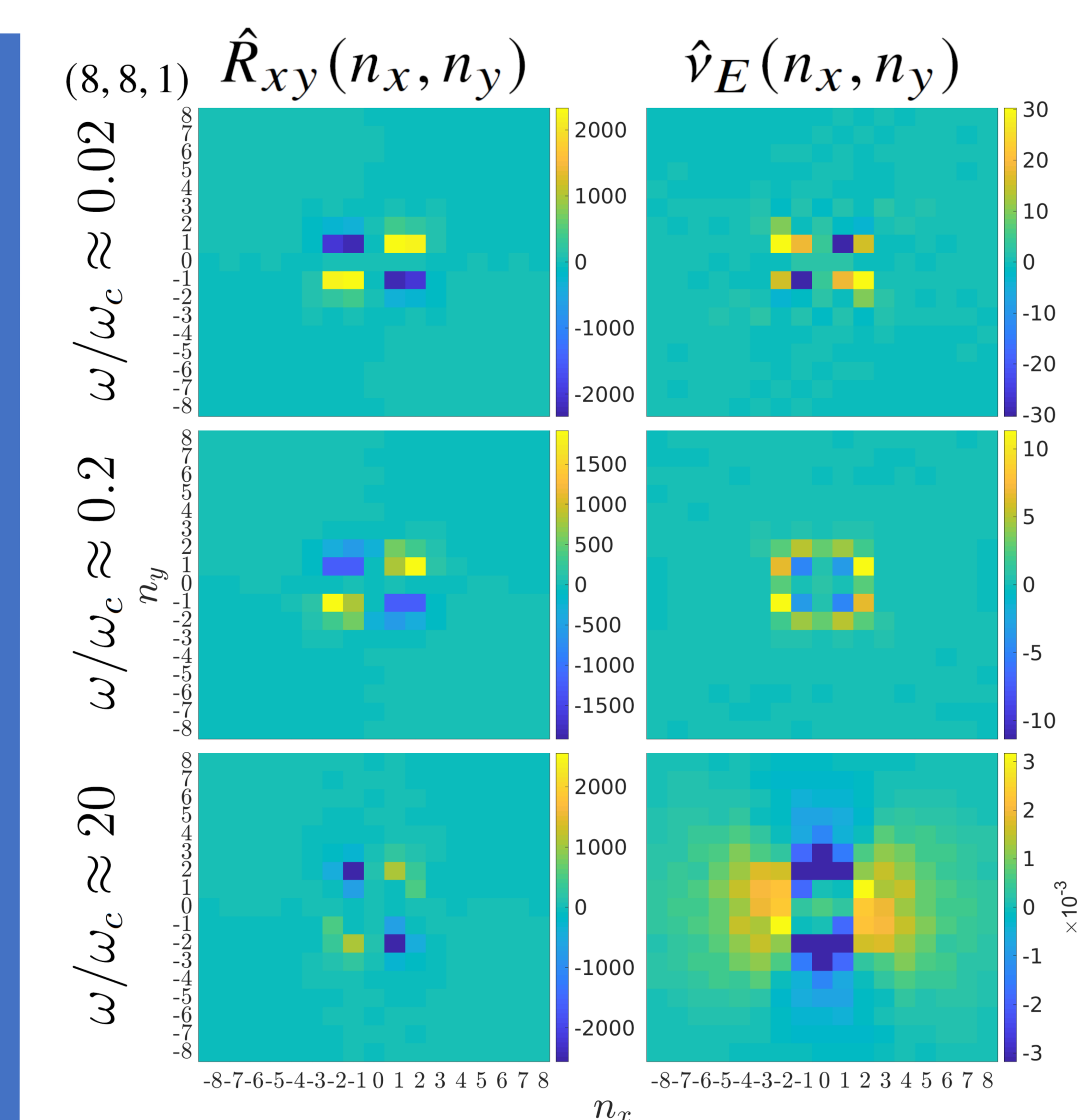
The scaled effective viscosity (α_{prop}) is larger than 1/3 (from kinetic theory) which is typically used.



Left bottom - comparisons of the effective viscosity trend with the frequency (temporal) spectrum (Γ) of the kinetic energy and Reynolds stress.

Note significant amplitude dependence in the high frequency regime.

Below – Plot showing which spatial wavenumbers provide contributions to the Reynolds stress (left) and effective viscosity (right). It is clear that the large scales (smaller wavenumber) dominate the contributions.



Key takeaways

1. High frequency ω^{-2}

- Can be "explained" through asymptotic analysis
- $-\nu_E$ is possible (antidissipation)
- Amplitude dependence

2. Low frequencies independent of ω

- Dissipation is more efficient than the naïve $\alpha_{prop} = 1/3$ from kinetic theory

3. New scaling law with -0.5 exponent

- Scaling exponent in agreement with the power law in the frequency spectrum

4. The energetically dominant modes contribute the most to the effective viscosity

- Resonance in the frequency spectrum may be important

See our papers [5] and [6] for further details…. Or ask me!

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